EARLY FINITE ELEMENT RESEARCH AT BERKELEY[1]

by
Ray W. Clough
Nishkian Professor of Structural Engineering, Emeritus
University of California, Berkeley
and
Edward L. Wilson
T. Y. Lin Professor of Structural Engineering, Emeritus
University of California, Berkeley

ABSTRACT

Significant finite element research was conducted at the University of California at Berkeley during the period 1957 to 1970. The initial research was a direct extension of classical methods of structural analysis which previously had been restricted to one-dimensional elements. The majority of the research conducted was motivated by the need to solve practical problems in Aerospace, Mechanical and Civil Engineering. During this short period the finite element method was extended to the solution of linear and nonlinear problems associated with creep, incremental construction or excavation, crack closing, heat transfer, flow of water in porous media, soil consolidation, dynamic response analysis and computer assisted learning of structural analysis. During the last six years of this period the fields of structural analysis and continuum mechanics were unified.

The computer programs developed during this early period at Berkeley were freely distributed worldwide allowing practicing engineers to solve many new problems in structural mechanics. Hence, the research was rapidly transferred to the engineering profession. In many cases the research was used professionally prior to the publication of a formal paper.

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INTRODUCTION

Prior to 1952 structural analysis was restricted to elements connected to only two points in space. Structural engineers used the ***lattice analogy,*** as developed by Hrennikoff [1] and McHenry[2], to model membrane and plate bending parts of the structure. However, this analogy could not be applied to nonrectangular areas. Ray Clough first faced this problem in the summers of 1952 and 1953 after joining the Boeing Summer Faculty Program. During this period he worked with Jon Turner, head of the Structural Dynamics Unit, and was asked to calculate the bending and torsional flexibility influence coefficients on low aspect wings. Static experimental results had been obtained for the swept-back box wing structure shown in Figure 1 and they did not agree with the results produced by a structural analysis model using one-dimensional elements only. This significant historical work has been documented in detail by Clough [3] where Turner is given principal credit for conceiving the procedure for the development of the constant strain triangle.

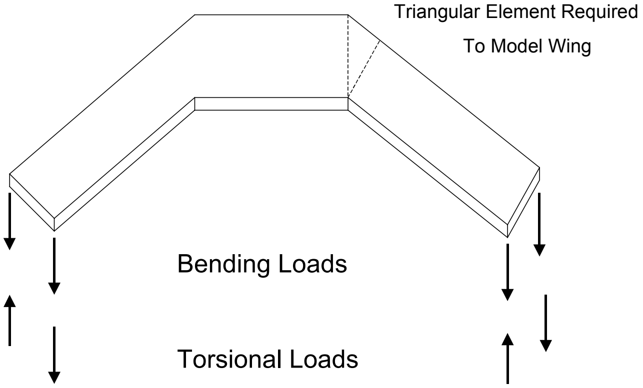


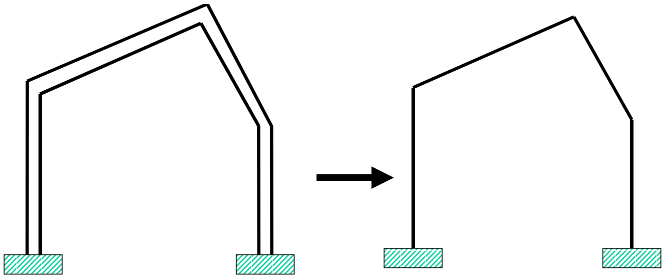
Figure 1. Swept-back Box Wing Test Structure

Turner presented the Boeing pioneering work at the January 1954 meeting of the Institute of Aeronautical Sciences in New York. However, the paper was not published until September 1956 [7]. In addition to the constant strain triangular membrane element, a rectangular membrane element, based on equilibrium stress patterns, was presented which avoided shear locking. The node equilibrium equations were formed by the direct stiffness method. The purpose of the two-dimensional element development at Boeing was to accurately model the dynamic stiffness properties and displacements of the structure, but was not proposed as a general solution method for stress analysis of continuous structures.

In 1956 and 1957 Clough was on sabbatical leave in Trondheim, Norway. During this period he had time to reflect on his work at Boeing and to study the new developments in the field. The comprehensive series of papers by Argyris and Kelsey, published in Aircraft Engineering between October 1954 and May 1955, unified many different approximate methods for the solution of both continuous and one-dimensional frame structures [6]. By using matrix transformation methods it was clearly shown that most structural analysis methods could be categorized as either a force or a displacement method.

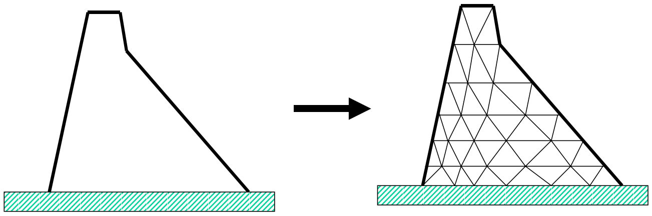
It was in Norway where Clough concluded that two-dimensional elements, connected to more than two nodes, could be used to solve problems in continuum mechanics. For the Turner triangular element the ***stress strain relationship*** within the element, ***displacement compatibility*** between adjacent elements, and ***force equilibrium*** on an integral basis at a finite number of node points within the structure were satisfied. It was apparent that the satisfaction of these three fundamental equations proved convergence to the exact elasticity solution as the mesh was refined.

This discrete element idealization was a different approach to the solution of continuum mechanics problems; hence, Clough coined the terminology ***finite element method.*** Therefore, analysis models for both continuous structures and frame structures were modeled as a system of elements interconnected at joints or nodes as indicated in Figure 2. Other researchers in structural analysis may have realized the potential of solving problems in continuum mechanics by using discrete elements; however, they were all using the direct stiffness terminology at that time.



Actual Frame

Finite Element Model



Actual Dam

Finite Element Model

Figure 2. The Finite Element Idealization

It should be pointed out that during the nineteen sixties there were many different research activities being pursued at Berkeley. First, it was the height of the Cold War and the Defense Department was studying the cost and ability to reinforce buildings and underground structures to withstand nuclear blasts. Second, a very significant program on Earthquake Engineering Research, including the construction of the world's largest shaking table, was initiated by Professors Bouwkamp, Clough, Penzien and Seed. Third, the Federal Government and the California Department of Transportation were rapidly expanding the freeway system in the state and were sponsoring research at Berkeley, led by Professors Scordelis and Monismith, concerning the behavior of bridges and overpass structures. Fourth, the manned space program was a national priority and Professors Pister, Penzien, Popov, Sackman, Taylor and Wilson were very active conducting research related to these activities. Fifth, the offshore drilling for oil in deep water and the construction of the Alaska pipeline required new technology for steel structures, which was developed by Professors Popov, Bouwkamp and Powell. Finally, the construction of nuclear reactors and cooling towers required the development of new methods of analysis and new materials. Also, Professors Popov, Scordelis and Lin were consultants on the design and construction of many significant long-span shell structures. To support this research and development a new Structural Engineering Building, Davis Hall, was built on the Berkeley Campus and a new shaking table, to simulate earthquake motions, was constructed at the Richmond Field Station. ***The Finite Element Method was an analysis tool that complemented all of these analytical and experimental research activities.***

THE YEARS 1957 TO 1960

After Clough returned from sabbatical leave in Norway in 1957 he initiated a new structural analysis research program at Berkeley. He applied for, and received, a small NSF grant to support research on computer analysis of structures. In addition, he initiated a new graduate course entitled Matrix Analysis of Structures. During the Fall semester of 1957 he listed several possible graduate student research areas. This list contained research topics on the Finite Element Analysis of Plane Stress Structures, Finite Element Analysis of Plates, and Finite Element Analysis of Shells.

An IBM 701 digital computer, with 4k of 16 bit memory, had been installed in the College of Engineering the previous year. The maximum number of equations that could be solved by this computer was approximately 40. Clough worked with the computer group on campus to develop a matrix algebra program in order that students would not be required to immediately learn programming in order to solve finite element problems. Therefore, by using submatrix techniques and tape storage it was possible to solve larger systems.

Under the direction of Clough, graduate student Ari Adini used the matrix algebra program to solve several plane stress problems using triangular elements. Since all matrices were calculated by hand the analysis of even a simple structure required a significant amount of time. Hence, only coarse mesh solutions were possible as shown in Figure 3. However, this approach was used to produce all examples in the paper ***The Finite Element Method In Plane Stress Analysis*** by Clough, presented at the 2nd ASCE Conference on Electronic Computations in September of 1960 [8]. This was the first use of the Finite Element terminology in a published paper outside the Berkeley Campus and first demonstrated that the structural analysis method could be used to solve for the stresses and displacements in continuous structures.

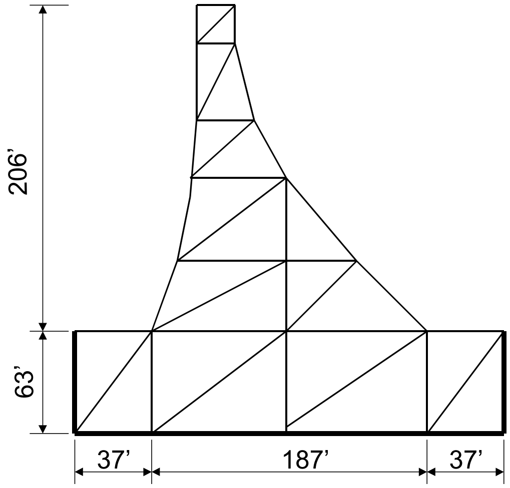


Figure 3. First Finite Element Mesh Used for the Analysis of Gravity Dam

Ed Wilson, a graduate student who shared an office with Adini, was not satisfied with the large amount of work required to solve finite element problems by using the matrix algebra program. In 1958 Wilson, under the direction of Clough, initiated the development of an automated finite element program based on the rectangular plane stress finite element developed at Boeing. After several months of learning to program the IBM 701, Wilson produced a limited capacity, semi-automated program which was based on the force method. A MS research report was produced, which has long since been misplaced, with the approximate title of Computer Analysis of Plane Stress Structures.

In 1959 the IBM 704 computer was installed on the Berkeley Campus. It had 32K of 32 bit memory and a floating point arithmetic unit which was approximately 100 times faster than the IBM 701. This made it possible to solve practical structures using fine meshes. While working on the NSF project Wilson, under the direction of Clough, wrote a two-dimensional frame analysis program with a nonlinear, moment-curvature relationship defined by the classical Ramburg-Osgood equation. The loads were applied incrementally and produced a pushover type of analysis.

The resulting research paper was also presented at the 1960 ASCE Conference [9]. The incremental load approach was general and could be used for all types of finite element systems.

Adini continued his finite element research by using the matrix algebra program to solve plate bending problems using rectangular finite elements and demonstrated that this class of structures could be modeled accurately by the method. The resulting research paper [10] demonstrated that plate bending problems could also be solved by the finite element method; however, it was not accepted for presentation at the 1960 ASCE Conference since two other papers from Berkeley had been accepted.

Adini solved several simple shell structures using the matrix algebra approach and additional commands to form membrane and bending stiffness matrices for rectangular elements. In 1962 he completed his Ph.D. thesis on the Finite Element Analysis of Shell Structures [11].

In 1960 Clough and Wilson developed a fully automated finite element program in which the basic input was the location of the nodes and the node numbers where the triangular plane stress elements were attached. The node equilibrium equations were stored in compact form and solved using Gauss-Seidel iteration with an over-relaxation factor. Hence, it was then possible for structural engineers, without a strong mathematical background in continuum mechanics, to solve practical plane stress structures of arbitrary geometry built by using several different materials. The work required to prepare the computer input data was simple and could be completed in a few hours for most structures. Wilson later added incremental loading and nonlinear material capability to this program and wrote his thesis on this topic [16]

THE NORFORK DAM PROJECT

Prior to the development of the finite element method the University of California at Berkeley had a long tradition of research on concrete, earth and rockfill dams and their material testing. In the nineteen twenties Professor R. E. Davis conducted material studies for Hoover Dam. In the late nineteen fifties model and material studies for the Oroville Dam project were conducted by Professors J. Raphael, H. Eberhart and D. Pirtz. At that time, the majority of the faculty in the Civil Engineering Department had conducted significant research on the design and construction of dam structures. Therefore, it was not surprising that the first real application of the newly developed plane finite element program was to a dam structure.

On the recommendation of Dr. Roy Carlson, a consultant to the Little Rock District of the Corps of Engineers, Clough submitted a proposal to perform a finite element analysis of Norfork Dam, a gravity dam that had a temperature induced vertical crack near the center of the section. The proposal contained a coarse mesh solution of a section of the dam that was produced by the new program and clearly indicated the ability of the method to model structures of arbitrary geometry with different orthotropic properties within the dam and foundation. The Clough finite element analysis proposal was accepted by the Corps of Engineers over an analog computer proposal submitted by Professor Richard MacNeal of Cal-Tech, which at that time was considered as the state-of-the-art method for solving such problems.

The Norfork Dam project provided an opportunity to improve the numerical methods used within the program and to extend the finite element method to the nonlinear solution of the crack closing due to hydrostatic loading. Wilson and a new graduate student, Ian King, conducted the detailed analyses that were required by the study. The significant engineering results of the project indicated that the cracked dam was safe with the existence of the vertical crack, as indicated in Figure 4.

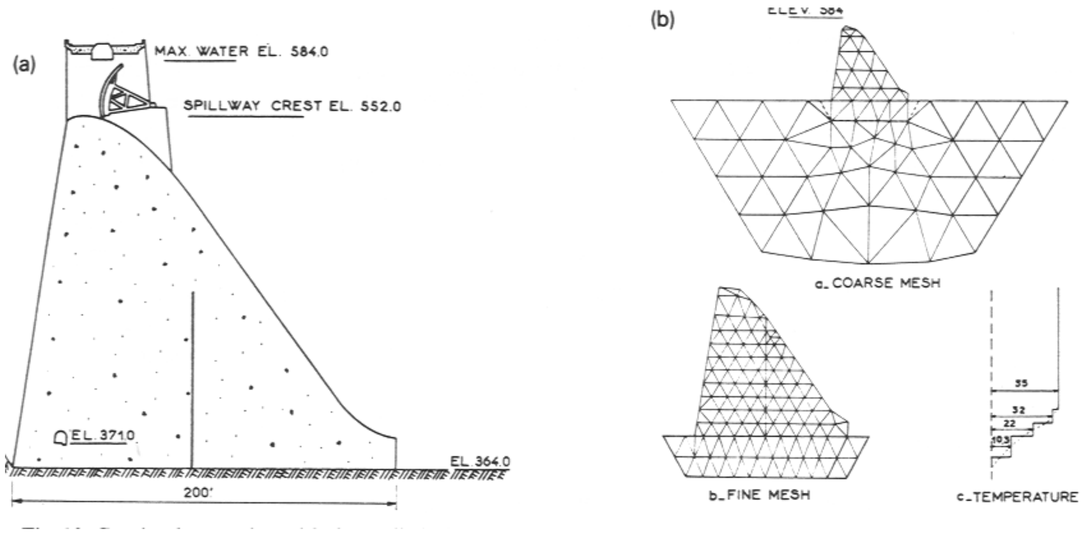


Figure 4. The Finite Element Analysis of Norfork Dam

The crack closing behavior, as the reservoir is increased in height, is summarized in Figure 5. Looking back on the Norfork dam study one is impressed by the sophistication of the analysis considering that such nonlinear behavior is rarely taken into account in dam analysis today.

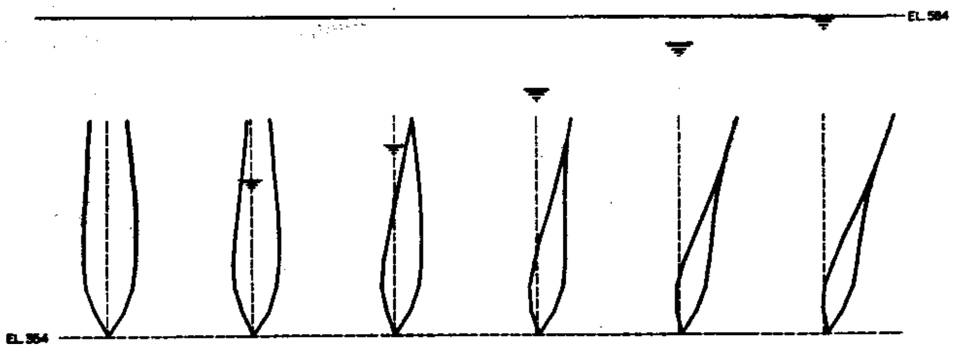


Figure 5. Crack Closing as Reservoir is Filled

In addition to the report to the Corps of Engineers on the analysis of Norfork Dam [13], a paper was prepared and presented at the Symposium on the Use of Computers in Civil Engineering that was held in Lisbon, Portugal, in 1962 [14]. This was only the second time that the finite element name appeared in the title of a paper published externally to the Berkeley Campus. Wilson and Clough presented another important paper at the Lisbon Symposium on the step-by-step dynamic response analysis of finite element systems [15]. This paper formulated Newmark's method of dynamic analysis in matrix form and eliminated the need for iteration at each time step.

REACTION FROM THE CONTINUUM MECHANICS COMMUNITY

During the Norfork Dam project Berkeley colleague Karl Pister was skeptical of the validity of the finite element approach and challenged Clough to solve some of the classical problems of plane stress analysis by the finite element method [22]. The problem selected was a plate with an elliptical hole subjected to the loading shown in Figure 6a. An inexperienced student was asked to establish the finite element idealization and to prepare the computer input. The only guidance given the student was that small elements be used in regions of high stress gradient (around the opening) and larger elements be used elsewhere. The resulting mesh is shown in Figure 6b.

Results of this study are presented in Figure 6c in the form of principal stress contours and plots of the normal stresses on the horizontal and vertical axes. The major error was at the stress concentration where the finite element stress is 564 compared to the theoretical value of 700.

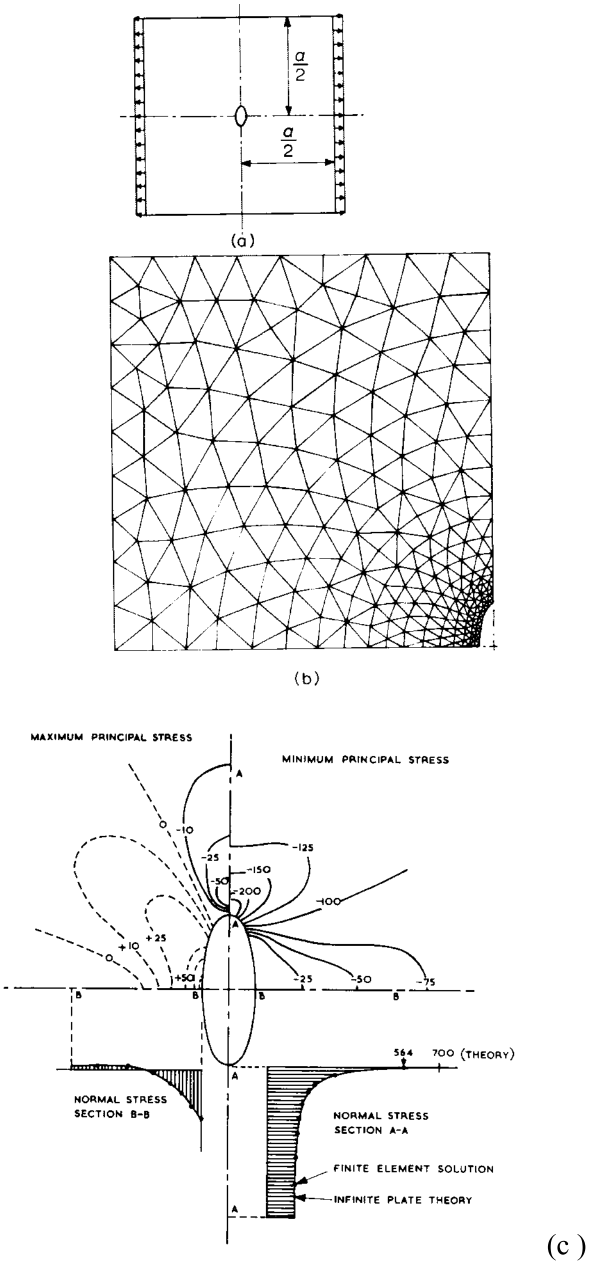


Figure 6. Finite Element Approximation of Infinite Plate with Elliptical Hole

Pister then recognized that the finite element method was a special formulation of the Ritz Method, *where the trial functions were independent within each element and were compatible at the element edges,* and he was actively conducting research in the area within a short period of time. In general, however, the continuum mechanics researchers outside the Structural Engineering and Structural Mechanics group at Berkeley were very reluctant to accept the method.

It is interesting to note that the assumption of constant strain fields within the elements is essentially equivalent to the concept of regional descretization that had been proposed many years earlier by Courant [4] and by Prager and Synge [5]. However, these ideas were not pursued at that time due to the lack of high-speed computers. It is apparent that Clough and the Boeing group were unaware of these mathematical references when they conducted their development work. It was not until 1964, after researchers in continuum mechanics discovered these early mathematical papers, that the finite element approach was accepted as a method of solution for problems in continuum mechanics [23]. Also in 1965, the finite element method was used for the solutions of heat transfer problems; therefore, the direct stiffness terminology was no longer the appropriate name [19,20,21].

THE TRIANGULAR PLATE-BENDING ELEMENT

In 1960 Jim Tocher, a Ph.D. student working under the direction of Clough, started a search for a practical triangular plate-bending element. After two years of tedious work Tocher produced a dissertation that indicated real plate structures could be modeled by triangular elements where the normal displacements were approximated by a ten term polynomial [12]. However, the element was too flexible and he could not prove that the results converged to the exact solution as the mesh was refined. It did indicate, however, that slope compatibility was required for plate bending elements.

In 1962-63 Tocher spent a year as a post doctoral fellow in Norway and continued to work on the triangular plate element while in communication with Clough. A former student of Clough, T. K. Hsieh, suggested a complicated method of creating a triangular element which satisfied the displacement compatibility conditions.

This procedure involved the use of three ten-term polynomials within each triangular region of a triangular element as shown in Figure 7. The three normal rotations at sides 4, 5 and 6 were constrained to be linear functions. The normal displacement and two rotations at node 9 are eliminated by requiring the three tangential rotations at the internal sides 6, 7 and 8 of the triangle to be linear functions. The resulting plate bending element was implemented and tested by Tocher while working at Boeing. The element produced excellent results and was named the HCT element, after Hsieh, Clough and Tocher. The results were published in 1965 at the Wright-Patterson Conference on Matrix Methods [25]. This element was used by the profession for over twenty years to solve thin plate bending problems. It was later replaced by the DKT element.

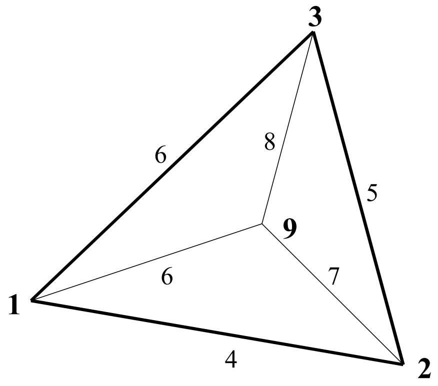


Figure 7. The HCT Compatible Plate Bending Element

APPLICATION TO CREEP AND INCREMENTAL CONSTRUCTION

During the Norfork Dam project it was necessary to assume the state of stress within the concrete dam prior to the application of the hydrostatic loading. It was apparent that this initial state of stress within the dam was a function of the construction sequence where the geometry, the temperature changes due to the heat of hydration, and the modulus and creep properties of the concrete were all a function of time. In 1962 Ian King selected this topic for his Ph.D. thesis [29].

King, under the direction of Professors Clough and Raphael, made extensive modifications to Wilson's program in order to solve this problem. King's research was sponsored by the Walla Walla District of the Corps of Engineers and was used to approximate the construction stresses in the 687 foot high Dworshak Dam shown in Figure 8. The time required to construct a typical monolith was approximately two years and the maximum thermal stresses occurred prior to the completion of the monolith.

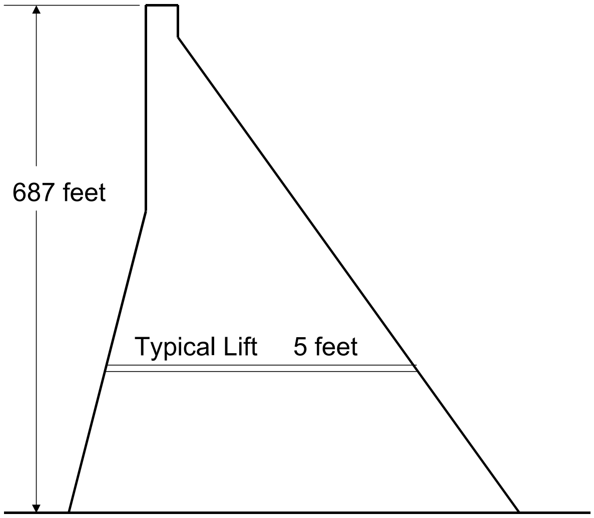


Figure 8. Evaluation of Construction and Creep Stresses in Dworshak Dam

It was necessary for King to devise a complicated re-meshing scheme during the solution procedure in order for the problem to be solved on the mainframe computers of 1964. The project clearly demonstrated the importance of the construction sequence and creep during construction [24]. However, to our knowledge, this type of analysis has not been conducted on any other dam.

Later in the nineteen seventies Professor Scordelis and his graduate students conducted significant research on the incremental construction and creep of bridges, shells and other concrete structures.

ANALYSIS OF UNDERGROUND STRUCTURES

One phase of the Oroville Dam project initiated in 1957 was the photoelastic analysis of the underground power plants under the direction of Professor Eberhart. Several possible designs were investigated using photoelasticity; however, the experimental approach was expensive and time consuming. Hence, after the development of the automated finite element program used for the Norfork dam project it was possible to rapidly investigate the stress concentration within underground concrete and rock structures. In 1962 Clough and Raphael received a research grant from the California State Department of Resources to use the finite element approach for the solution of this class of problems [17]. A new graduate student, Joe Rashid, was hired to work on this project.

In addition to working on the project, Rashid completed his Ph.D. work in 1964 under the direction of Pister on the Finite Element Analysis of Axisymmetric solids [18]. In addition, he modified Wilson's program to solve axisymmetric structures subjected to axisymmetric loads [27].

INTERACTION WITH PROFESSOR O. C. ZIENKIEWICZ

When Clough presented the first paper using the finite element terminology in 1960 it attracted the attention of his friend, Professor O. C. Zienkiewicz, who was then on the faculty at Northwestern University. A few weeks after the presentation of the paper Zienkiewicz invited Clough to present a seminar on the finite element method to his students. Zienkiewicz was considered one of the world's experts on the application of the finite difference method to the solution of continuum mechanics problems in Civil Engineering; therefore, Clough was prepared to debate the relative merits of the two methods. However, after a few penetrating questions about the finite element method, Zienkiewicz was almost an instant convert to the method.

During the academic year 1964-65 Clough was a Visiting Professor at Cambridge University. He continued to supervise a number of students by mail, met other international experts in the field and wrote several research papers on earthquake engineering and the finite element method. During this period Dr. Zienkiewicz, then installed as a Professor at the University of Wales in Swansea, asked Clough and many other leading specialists on the development of new methods of analysis to take part in a conference on Stress Analysis at the University. These lectures were compiled in a book entitled ***Stress Analysis*** [26], where each chapter was written by a different expert. Clough, Zienkiewicz and B. Fraeijs de Veubeke wrote chapters on the finite element method. De Veubeke referenced the 1947 work of Prager and Synge [5] and introduced the six node plane triangular element; however, he did not present any numerical examples.

The long friendship between the research group at Swansea and the Berkeley finite element group has been of significant mutual benefit and has continued for the past thirty five years.

THE AEROJET EXPERIENCE

Wilson accepted a position, in August of 1963, as a senior research engineer in the Solid Rocket Plant at Aerojet General in Sacramento, California, approximately 80 miles from Berkeley, where he continued computer program development and research on the automation of the finite element method. There he worked with Stan Dong (now Professor Emeritus of UCLA) and Len Herrmann (now Professor Emeritus of UC Davis) former Ph.D. students of Professor Pister. Therefore, the Aerojet group was a direct extension of the research programs of Clough and Pister at Berkeley. During the next two years the classical fields of structural analysis and continuum mechanics would be unified at Aerojet, Berkeley and Swansea. This new field has since been given the name ***computational mechanics***.

One of the first problems at Aerojet for which Wilson used his two-dimensional finite element program from Berkeley to solve, was the stress analysis of a cross-section of a solid rocket propellant, called a grain, subjected to internal pressure, as shown in Figure 9. The stresses were compared to a photoelastic analysis which had previously been conducted. To his amazement, at the point of maximum stress, the finite element stress was approximately five percent larger than the stress obtained from the photoelastic analysis. Since one would expect the stresses obtained from a displacement-based finite element analysis to be less than the exact results, a new photoelastic analysis was conducted. The new results were closer to the computer results; however, due to three-dimensional effects near the stress concentration the photoelastic results were still a few percent less than the finite element stresses. Wilson then created a special purpose version of his program with mesh generation to automatically conduct stress analyses of solid rocket motors of arbitrary geometry [19]. Within the next few years at Aerojet the photoelastic group was reduced in size and most of the photoelasticity engineers were writing or using finite element programs.

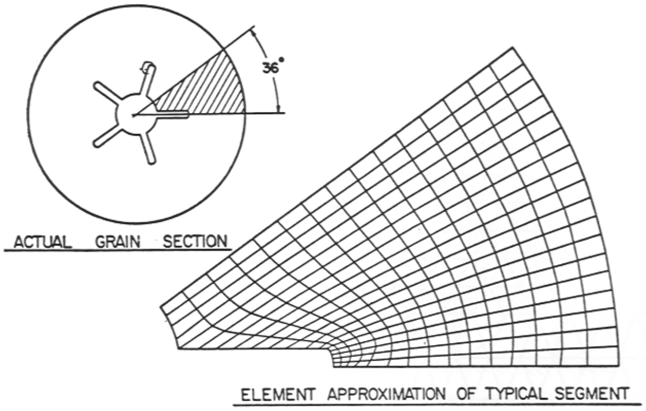


Figure 9. Finite Element Analysis of Solid Rocket Motor

Perhaps the most important class of problems at Aerojet was the analysis of axisymmetric solids and shells. The addition of the hoop stress and temperature-dependent orthotropic material properties allowed the finite element program at Berkeley to solve these problems. The axisymmetric rocket nozzle shown in Figure 10 was the first such problem solved. It clearly illustrated the power of the finite element method. Since each element could have different material properties, special material interface equations were not required as in the case of finite differences which was the existing solution method used at Aerojet at that time. In addition, the finite element model ***looked like*** the real structure; therefore, the method had immediate appeal to design engineers. Within a few months several different departments, within both the solid and liquid rocket plants at Aerojet, were using the program.

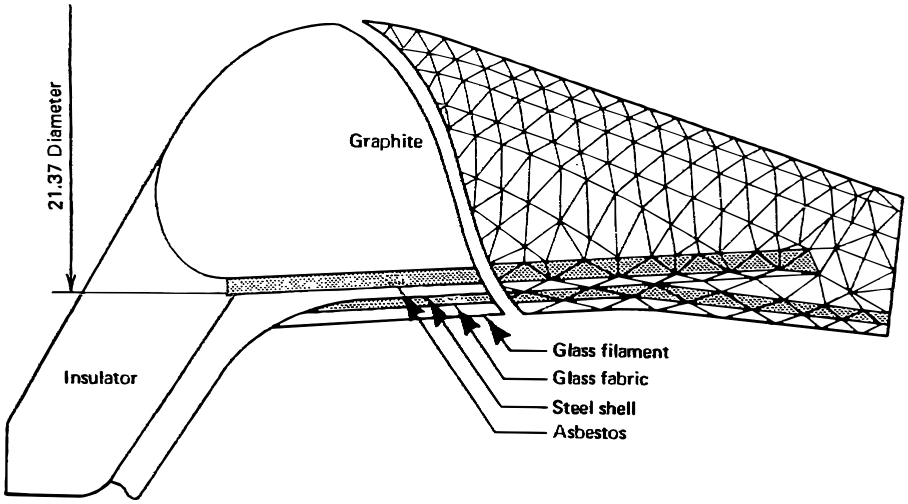


Figure 10. Finite Element Analysis of Rocket Nozzle

The Berkeley program had several deficiencies, specifically: limited capacity, no mesh generation and an unpredictable iterative solution algorithm. Next, Wilson wrote a completely new program based on a direct, blocked, out-of-core memory, equation solver, introduced the quadrilateral element composed of four triangles with static condensation, and added a powerful mesh generation option. This new program was more accurate, had large capacity, was several times faster than the previous iterative program, and was easy to use.

Both Dong and Herrmann realized the power of the finite element approach and had recognized that it was a form of the Ritz method. It become apparent to them that it was necessary to learn a computer programming language in order to ***quickly*** conduct research and development with the finite element method. Dong directed his research to the finite element solution of shells and Herrmann worked on the finite element solution of plates and large deformation analysis of solid rocket motors.

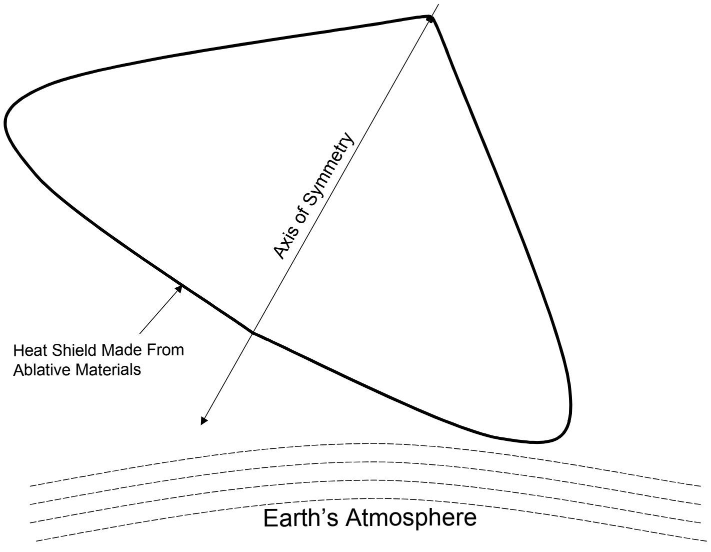
The group at Aerojet obtained a contract with NASA for the thermal stress analysis of the Apollo Spacecraft during re-entry to the Earth's atmosphere. The problem could be approximated with an axisymmetric structure subjected to non-axisymmetric loading. At the suggestion of Herrmann, Wilson used a semi-analytical solution approach in which the structure was modeled with axisymmetric quadrilateral ring elements and the three displacements and loads were expanded in the hoop direction by a series of the following ***separation of variable*** form:

$$u_r(r,z,\theta) = \sum_{n=0}^{N} u_r(r,z)\cos(n\theta),$$

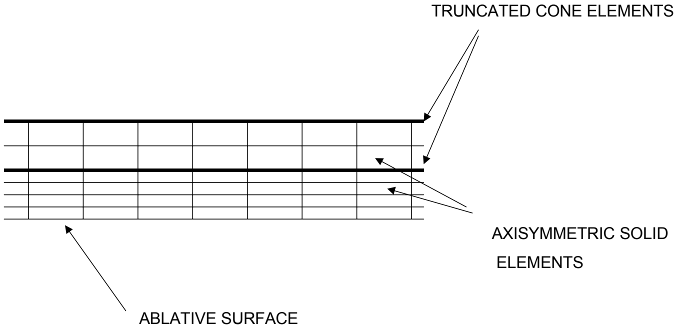
$$u_z(r,z,\theta) = \sum_{n=0}^{N} u_z(r,z)\cos(n\theta)$$

$$u_\theta(r,z,\theta) = \sum_{n=1}^{N} u_\theta(r,z)\sin(n\theta)$$

Therefore, due to the orthogonality properties of the functions, it was possible to obtain the solution of the three-dimensional thermal stress analysis by the sum of solutions of two-dimensional problems. Hence, it was possible to conduct the three-dimensional thermal stress analysis of the Apollo Spacecraft heat shield, as shown in Figure 11, by using the relatively slow computers that existed in 1964.



a. The Apollo Spacecraft



b. *Finite Element Model of Heat Shield*

Figure 11. Finite Element Analysis of the Apollo Spacecraft Heat Shield

Most of the research work conducted at Aerojet was presented at the "AIAA 2nd Aerospace Science Meeting" in New York during January 1965 [20]. At the same session Wilson met Professor Pian of MIT who presented a paper using the direct stiffness method for the analysis of axisymmetric shell structures [21].

In August 1965, Dong accepted a faculty position at UCLA, Wilson returned to Berkeley as a member of the faculty and Herrmann joined the faculty at the University of California at Davis and continued to consult for Aerojet for the next several years. It is of interest to note that a few days prior to their departure, word was received from NASA that Aerojet had been selected as one of the top three firms to be interviewed for the development of the NASTRAN computer program. Due to the loss of key personnel, the Aerojet proposal was withdrawn. One other finalist, Lockheed Aerospace, also withdrew their proposal. Hence, Richard MacNeal of MacNeal-Schwendler Corporation devoted the next thirty years to the innovative development and distribution of the NASTRAN structural analysis program.

THE WRIGHT-PATTERSON CONFERENCE IN 1965

In October 1965 a conference on ***Matrix Methods in Structural Analysis*** was held at the Wright-Patterson Air Force Base [32]. It brought together the major structural analysis research groups from many areas of the world. Most papers used two and three-dimensional elements to solve problems in continuum mechanics. However, only one session, containing six papers on Finite Element Properties and chaired by Professor Richard Gallagher, was devoted to the Finite Element Method.

A most impressive 180 page paper ***Continua And Discontinua*** was presented by John Argyris and contained a large number of applications on the analysis of solids, plates and shells. In addition, he presented the six node triangular plane element formulated in a natural area coordinate system and a ten node solid tetrahedral element formulated in a natural volume coordinate system. In the paper he did not use the finite element terminology to describe his work. However, in a seminar presented a short time later at Berkeley he referred to the finite element method extensively. Therefore, it was not until the end of 1965 that the name “finite element method” was accepted as replacement terminology for the direct stiffness method.

FINITE ELEMENT RESEARCH AT BERKELEY 1965 TO 1970

Finite element research at Berkeley moved rapidly in many different directions during the years 1965 to 1970. Many Professors within the Department of Civil Engineering were involved and the number of graduate students in the department increased significantly during this period. We believe many excellent students from all areas of the world were attracted to Berkeley because it was known as the ***home of the finite element method***. Therefore, we will only present a general summary of the research conducted during this period.

Working under the direction of Professors Penzien and Popov, Z. A. Lu completed a dissertation in 1965 on the "Finite Element Analysis of Axisymmetric Thin Shells" [28]. He used the exact solution for a conical shell element to model shells of revolution. Since the element could be very small, axisymmetric thin shells of arbitrary geometry could be modeled. Professor Popov had several other excellent students working on shell structures during the next several years.

Anil Chopra worked with Penzien and Clough in 1964-65 to include dam/reservoir interaction, during earthquakes, for both earth and concrete dams [30,31]. He used the constant strain triangle and a frequency domain time solution method. This work attracted the attention of Professors Brekke, Duncan, Lysmer and Seed in the Berkeley geotechnical group in Civil Engineering. Within a few years their students were using the finite element method for the solution of problems in soil and rock mechanics.

After Clough returned from Cambridge in 1964, Carlos Felippa, a new Ph.D. student, started working on the application of the higher order six node triangular plane element to problems where nonlinear behavior and buckling were important. He was a very creative individual with an excellent background in mechanics, numerical methods and computer programming. He clearly illustrated, for the same number of equations to solve, that the use of six node triangles produced more accurate results for both displacements and stresses as compared to use of three node triangles. He introduced nonlinear material properties and solved several examples. He formed the consistent mass and geometric stiffness and solved elementary buckling problems. All work was formulated in the natural area coordinate system and several new numerical integration formulas were presented [35].

Wilson had developed a plane and axisymmetric transient heat transfer program while working at Aerojet [22]; however, he had not completed a formal paper on the work. In 1966 Bob Nickell, who later completed his Ph.D. with Sackman [71], prepared additional examples and formulated the method in variational form. Prior to acceptance by the Journal of Nuclear Engineering and Design [33] the paper was submitted to a Mechanical Engineering Heat Transfer Journal and was rejected because ***it was the same as finite differences.*** It was not until approximately seven years later that researchers in classical heat transfer appreciated the full potential of the use of the finite element method.

In 1965-66 Professors Robert Taylor and Colin Brown used the finite element method to develop a numerical procedure for solving two-dimensional problems of fluid-flow in porous media [34]. They introduced an iterative method to solve for the free surface in partially saturated solids.

Professors Pister, Herrmann, Taylor and Sackman had a large group of students working on the application of finite elements to many different problems in mechanics, including nearly incompressible materials, large displacements and nonlinear materials. Some of these activities are summarized in references [36,39,41,49,56,71]

In 1967 Clough had three graduate students complete their Doctor's Degrees. Phil Johnson combined the HCT bending element with a triangular membrane element to solve shell structures of arbitrary geometry [42]. Athol Carr combined beam and thin shell elements to solve shells subjected to both static and dynamic loads [38]. Ojars Greste used the thin shell element to study the behavior of the steel joints in off-shore structures [37]. Also, he developed a practical automated mesh generation for the intersection of large steel pipes of different sizes.

In 1967 Dave Murray, under the direction of Wilson and Clough, used the HCT bending element and the constant strain triangle to solve out-of-plane buckling of plates [45]. In addition, he applied incremental loads to solve for the post buckling behavior of plates. By satisfying equilibrium in the deformed position for each load increment he showed that the same results are obtained with or without the use of the geometric stiffness. The use of the geometric stiffness for each load increment increased the rate of convergence.

Wilson continued to conduct research on construction, thermal and creep stresses in mass concrete structures. R. Sandhu worked in this area as summarized in references [40,44,47,52]. Also, Wilson and I Farhoomand conducted research on the linear and nonlinear dynamic (blast) analysis of underground structures [48,59,69]. In addition, Peter Smith completed his dissertation on the determination of membrane shapes for shell structures [68].

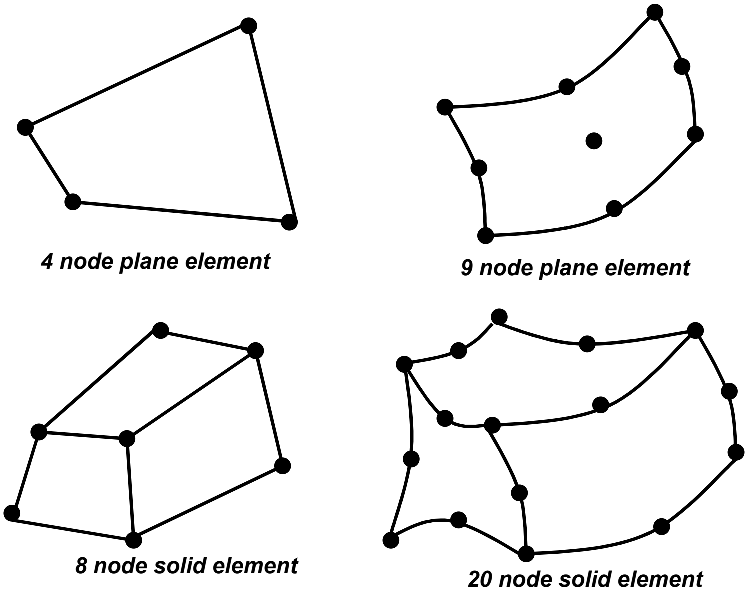
Professor Popov supervised several students who worked on the static and dynamic analysis of shell structures. Z. A. Lu worked on axisymmetric shells [28], as was mentioned above. John Abel's Ph.D. thesis was on the static and dynamic analysis of sandwich shells with viscoelastic damping [50]. S. Yaghmai's dissertation was on the incremental analysis of large deformation of axisymmetric shells [53].

Professor Scordelis conducted both experimental and analytic research on the behavior of long span structures, including reinforced concrete shells and bridges, during the 1963-70 time period. Kam Lo completed a dissertation on the finite element analysis of box girder bridges [72]. De Ngo completed work on the finite element analysis of reinforced concrete beams [73]. Arthur Nilson completed a Ph.D. dissertation on the nonlinear interaction between reinforcing steel and concrete [74]. The dissertation by Andy Franklin was on the nonlinear behavior of reinforced concrete beams and panels [75]. Kasper Willam completed work on the analysis of folded plates and cellular structures [62,64] and Christian Meyer was actively working on his Ph.D research [63] concerning structures combining beam elements with plate elements.

In addition, Scordelis and Bouwkamp conducted experimental and finite element analysis of tubular joints [51]. Under the direction of Bouwkamp, Mike Mehrain completed a Ph.D. dissertation on the finite element analysis of skew bridges [43].

ISOPARAMETRIC ELEMENTS

In 1968 Irons and Zienkiewicz presented the isoparametric formulation of finite element stiffness matrices and this work had an immediate and significant impact on the finite element research being conducted at Berkeley. Professor Taylor was the first to program this new formulation at Berkeley and to demonstrate the power of this new type of element formulation. In a very short period of time many other faculty members and students were using this new type of element as shown in Figure 12.


4 node plane element

9 node plane element

8 node solid element

20 node solid element

Figure 12. Examples of Isoparametric Elements

In 1969, William Doherty, working under the direction of Taylor, developed the first program for analysis of three-dimensional, steady state flow of fluids in porous media using the eight node isoparametric element. Today, several firms are still using a version of this program to predict the hydrostatic pressures under dams. At approximately the same time, Kenneth Kavanagh, working under the direction of Clough, used the eight node solid element for the structural analysis of three-dimensional solids [57].

One of the significant problems with the use of the four node plane element and the eight node solid element was ***shear locking*** when the elements were subjected to pure bending. Taylor and Wilson experimented with reduced integration [56] and incompatible displacement modes [70] to eliminate the problem.

THE SMIS and SAP PROGRAMS

In 1963 Wilson and Clough developed a **S**ymbolic **M**atrix **I**nterpretive **S**ystem, SMIS, for the purpose of teaching the static and dynamic analysis of structures. The purpose of this program was to bridge the gap between traditional hand calculation methods and matrix methods of structural analysis. This FORTRAN program was freely distributed and was modified many different times at different universities. The latest release of this program is CAL 91 and it is still used for teaching modern structural analysis [76].

During the early years of finite element research at Berkeley each student developed his own computer program or modified another student's program to solve a specific type of structure. Often these programs were not documented and could not be used by anyone else but the developer. In many cases, members of the engineering profession could not use the research without extensive development costs. For these reasons Wilson, in 1969, initiated the development of the general purpose static and dynamic **S**tructural **A**nalysis **P**rogram, SAP.

The SAP program used the existing technology of that time. Each node or joint could have zero to six displacement degrees-of-freedom. Within SAP an integer pointer array was created that was six times the number of nodes. This array allowed each node to have a different number of displacements. Therefore, during the assembly of the element stiffness only the equilibrium equations for the unknown displacements were formed. Hence, the program was just as efficient as special purpose programs that had a fixed number of displacements per node.

In less than one year Wilson and three students developed the first SAP program [66]. It was freely distributed to all students and members of the profession and became the basic starting program for many different finite element projects. In

1973, Dr. Jurgen Bathe updated the dynamic response options and developed additional documentation to produce SAP IV [77]. At the time of completion SAP IV was one of the fastest and largest capacity structural analysis programs in the world. The free distribution of this program served as an effective way of transferring the finite element research developed at Berkeley to the profession and to other universities.

FINAL REMARKS

In this paper we have attempted to summarize the early research at the University of California at Berkeley from 1957 to 1970. Most of this research was conducted over thirty years ago. Therefore, it has been difficult to recreate all the research accurately and to present the material in the proper sequence. We apologize to those individuals whose research activities we may have omitted or not accurately reported.

Professors DeVogelaere and Parlette of the U. C. Department of Mathematics taught numerical methods, served on many C. E. dissertation committees and were an important part of the early finite element research effort. It was not until approximately 1970 that a formal course on the finite element method, within the Civil Engineering Department, was introduced; instead, a free and open interchange of research activities existed between all faculty and students. In addition, there was no formal research institute on computational mechanics at Berkeley where the research was conducted. We believe that this informal atmosphere was the major reason for the high research productivity during this initial period.

Approximately 50 percent of the students, after receiving their doctor's degrees during this period, obtained teaching appointments at major universities and continued to make important contributions in the general area of computational mechanics. This fact has been of great personal satisfaction to the authors and other faculty members at the University of California at Berkeley.

Many of the references in this paper are from the Structural Engineering and Structural Mechanics report series, UCB/SESM. Copies of these reports may be obtained from NISEE at the Earthquake Engineering Research Center of the University of California, Berkeley. Some of the reports are summarized online at www.eerc.berkeley.edu.

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