



# Basic Regression Analysis: Time Series Data

#### CIVL 7012/8012

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# Nature of time series data

- Temporal ordering of observations; may not be arbitrarily reordered
- Time series data has a separate observation for each time period
  - e.g. annual traffic volume on a corridor,
  - census observations over multiple decades
  - Population of a city over multiple years

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# Data features

- Time periods to consider
  - Daily, Weekly, Monthly, Quarterly, Annually,
     Quinquennially (every five years), Decennially (every 10 years)
- Since not a random sample, different problems to consider
  - Trends and seasonality will be important

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# Data issues

- Stationary issue
  - Loosely speaking a time series is stationary if its mean and standard deviation does not vary systematically over time
- How should we think about the randomness in time series data?
- The outcome of economic variables (e.g. GNP, Dow Jones) is uncertain; they should therefore be modeled as random variables
- Time series are sequences of r.v. (= stochastic processes)
- Randomness does not come from sampling from a population

# Example data

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 US inflation and unemployment rates 1948-2003

<b>TABLE 10.1</b> Partial Listing of Data on U.S. Inflation and Unemployment Rates,1948–2003		
Year	Inflation	Unemployment
1948	8.1	3.8
1949	-1.2	5.9
1950	1.3	5.3
1951	7.9	3.3
•		
1998	1.6	4.5
1999	2.2	4.2
2000	3.4	4.0
2001	2.8	4.7
2002	1.6	5.8
2003	2.3	6.0

Here, there are only two time series. There may be many more variables whose paths over time are observed simultaneously.

Time series analysis focuses on modeling the dependency of a variable on its own past, and on the present and past values of other variables.



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#### Example of time series regression model

#### • Static models

- In static time series models, the current value of one variable is modeled as the result of the current values of explanatory variables
- Examples for static models

There is a contemporaneous relationship between unemployment and inflation (= Phillips-Curve).

 $inf_{0} = \beta_{0} + \beta_{1}unem_{0} + u_{t}$ 

 $mrdrte_{f} = \beta_0 + \beta_1 convrte_{f} + \beta_2 unem_{f} + \beta_3 yngmle_{f} + u_t$ 

The <u>current</u> murderrate is determined by the <u>current</u> conviction rate, unemployment rate, and fraction of young males in the population.



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# Finite distributed lag models

- Finite distributed lag models
  - In finite distributed lag models, the explanatory variables are allowed to influence the dependent variable with a time lag
- Example for a finite distributed lag model
  - The fertility rate may depend on the tax value of a child, but for biological and behavioral reasons, the effect may have a lag



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# Interpretation of coefficients: finite distributed lag models

Interpretation of the effects in finite distributed lag models

 $y_t = \alpha_0 + \delta_0 z_t + \delta_1 z_{t-1} + \dots + \delta_q z_{t-q} + u_t$ 

 Effect of a past shock on the current value of the dep. variable

 $\frac{\partial y_t}{\partial z_{t-s}} = \delta_s$ 

<u>Effect of a transitory shock:</u> If there is a one time shock in a past period, the dep. variable will change temporarily by the amount indicated by the coefficient of the corresponding

$$\frac{\partial y_t}{\partial z_{t-q}} + \dots + \frac{\partial y_t}{\partial z_t} = \delta_1 + \dots + \delta_q$$

#### Effect of permanent shock:

If there is a permanent shock in a past period, i.e. the explanatory variable permanently increases by one unit, the effect on the dep. variable will be the cumulated effect of all relevant lags. This is a long-run effect on the dependent variable. THE UNIVERSITY OF **MEMPHIS**.

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## Lagged effects





For example, the effect is biggest after a lag of one period. After that, the effect vanishes (if the initial shock was transitory).

The long run effect of a permanent shock is the cumulated effect of all relevant lagged effects. It does not vanish (if the initial shock is a permanent one).

# Example-1

• Example: Static Phillips curve

$$\widehat{inf}_t = 1.42 + .468 unem_t$$
(1.72) (.289)

$$n = 49, R^2 = .053, \bar{R}^2 = .033$$

Discussion of CLM assumptions

<u>Assumption.1:</u>  $inf_t = \beta_0 + \beta_1 unem_t + u_t$ 

The error term contains factors such as monetary shocks, income/demand shocks, oil price shocks, supply shocks, or exchange rate shocks

# Assumption.2: A linear relationship might be restrictive, but it should be a good approximation. Perfect collinearity is not a problem as long as unemployment varies over time.

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Contrary to theory, the estimated Phillips Curve does not suggest a tradeoff between inflation and unemployment

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# Example-2

Example: Effects of inflation and deficits on interest rates



<u>Assumption.2</u>: A linear relationship might be restrictive, but it should be a good approximation. Perfect collinearity will seldomly be a problem in practice.



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# Example-3

• Using dummy explanatory variables in time series



$$n = 72, R^2 = .473, \bar{R}^2 = .450$$

- Interpretation
  - During World War II, the fertility rate was temporarily lower
  - It has been permanently lower since the introduction of the pill in 1963





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# Modeling a linear time trend

$$y_t = \alpha_0 + \alpha_1 t + e_t \quad \Leftrightarrow \quad E(\Delta y_t) = E(y_t - y_{t-1}) = \alpha_1$$

Abstracting from random deviations, the dependent variable increases by a constant amount per time unit

 $E(y_t) = \alpha_0 + \alpha_1 t \longleftarrow$ 

 $\partial y_t / \partial t = \alpha_1 \bullet$ 

Alternatively, the expected value of the dependent variable is a linear function of time



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### Modeling an exponential time trend

 $\log(y_t) = \alpha_0 + \alpha_1 t + e_t \quad \Leftrightarrow \quad E(\Delta \log(y_t)) = \alpha_1$ 

 $(\partial y_t/y_t)/\partial t = \alpha_1$ 

Abstracting from random deviations, the dependent variable increases by <u>a constant percentage</u> per time unit



# Example of a time series with exponential trend

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# Using trending variables

- Using trending variables in regression analysis
  - If trending variables are regressed on each other, a spurious re- lationship may arise if the variables are driven by a common trend
  - In this case, it is important to include a trend in the regression
- Example: Housing investment and prices





# Using trending variables (2)

• Example: Housing investment and prices (cont.)

$$\widehat{\log}(invpc) = -.913 + .381 \log(price) + .0098 t$$

$$(.136) (.679)$$
There is no significant relationship
between price and investment
anymore

• When should a trend be included?

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- If the dependent variable displays an obvious trending behaviour
- If both the dependent and some independent variables have trends
- If only some of the independent variables have trends; their effect on the dep.
   var. may only be visible after a trend has been substracted



## Modeling seasonality in time series

- Modelling seasonality in time series
- A simple method is to include a set of seasonal dummies:

$$y_t = \beta_0 + \delta_1 feb_t + \delta_2 mar_t + \delta_3 apr_t + \dots + \delta_{11} dec_t$$
  
+  $\beta_1 x_{t1} + \beta_2 x_{t2} + \dots + \beta_k x_{tk} + u_t$  =1 if obs. from december  
=0 otherwise



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#### Further exploration (not covered in class)

- Problem of stationarity
- Auto regressive models
- Moving average models
- Autoregressive and moving average models
- Other advanced time series models