Strong Ground Motion

Seismographs

- Strong ground motions are measured by accelerographs and expressed in the form of accelerograms.

Location and Intensity of Earthquake

- Seismographic stations around the World work together to record earthquake location and determine earthquake strength.
"Displacement of the pendulum is proportional to ground motion \( V_g \).

- If \( T \) of pendulum > \( T \) of ground motion and if appropriate damping for the pendulum is chosen, this type of seismogram is called Displacement Seismograph or long period seismograph.

- If \( T \) of pendulum < \( T \) of ground motion and if appropriate damping for the pendulum is chosen, this type of seismograph is called Acceleration Seismograph or short period seismograph.

- If \( T \) of pendulum = \( T \) of ground motion and if appropriate damping for the pendulum is chosen, this type of seismograph is called velocity Seismograph.

\[
\omega = \sqrt{\frac{k}{m}}
\]

\[
f = \frac{\omega}{2\pi}
\]

\[
T = \frac{1}{f} = \frac{2\pi}{\omega}
\]
\[ m\ddot{u} + c\dot{u} + ku = -m\ddot{u}_g \]

\[ k_d = \frac{\beta^2}{\sqrt{(1 - \beta^2)^2 + (2\beta\xi)^2}} \]

\[ k_p = \frac{1}{\omega_0^2 \sqrt{(1 - \beta^2)^2 + (2\beta\xi)^2}} \]

\[ \beta = \frac{\omega}{\omega_0} \]

**Seismographs**

Typical Seismograms

- The Bottom part of curves is the recording made at La Paz in Bolivia of the vertical component of the initial P wave from the same earthquake that was recorded by a short-period seismograph and a long-period seismograph in the World wide Network.

**Earthquake Ground Motion**

Newmark and Rosenblueth (1971) classified earthquake ground motions into four types:

- **Single-Shock Type.** The focus is at a shallow depth and the bedrock is hard.
- **A moderately long, extremely irregular motion.** The depth of the focus is intermediate and the bedrock is hard as in the El Centro Earthquake of 1940.
- **A long ground motion exhibiting pronounced prevailing period.** The wave is filtered by many soft layers, and the successive reflections occur at the boundaries, as in the Mexican earthquake of 1964.
- **A ground motion involving large-scale permanent deformation of the ground.** This occurred at Anchorage in Alaska earthquake of 1964.

**Ground Motion Time History**

North-South ground acceleration recorded at Canyon during M6.4 San Francisco (Feb 9, 1971)

- The instrument was located at 20 miles from the causative fault, and at this distance the duration of strong ground shaking was approximately 8 second, this being the same as the duration of the slipping process on the fault.
**Strong Ground Motion**

- Significant characteristics of earthquake motion:
  - Amplitude
  - Frequency Content
  - Duration of the Motion

**Strong Motion Processing**

- The raw data from a strong-motion instrument may include errors from:
  - Traffic
  - Construction activity
  - Wind
  - Ocean waves
  - The triggering of analog seismograph

- An acceleration error as small as 0.001g at the beginning of a 30-sec long acceleration would predict a permanent displacement of 441 cm.

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**Amplitude Parameters**

**Numerical Differentiation**

First order Forward
\[
 f_i^{(1)} = \frac{1}{\Delta t} \left( f_{i+1} - f_i \right)
\]

2nd order forward
\[
 f_i^{(2)} = \frac{1}{2\Delta t} \left( -3f_i + 4f_{i+1} - f_{i+2} \right)
\]

First Backward Interpolation
\[
 f_i^{(b)} = \frac{1}{\Delta t} \left( f_i - f_{i-1} \right)
\]

2nd Backward Interpolation
\[
 f_i^{(2b)} = \frac{1}{2\Delta t} \left( f_i - 4f_{i-1} + 3f_{i-2} \right)
\]

Central Interpolation
\[
 f_i^{(c)} = \frac{1}{2\Delta t} \left( f_{i-1} + f_{i+1} \right)
\]
**Numerical Integration**

Trapezoidal Rule:

\[ I = \frac{h}{2} \left( f(x_0) + 2 \sum_{i=1}^{n-1} f(x_i) + f(x_n) \right) \]

Simpson’s Rule:

\[ I = \frac{h}{3} \left( f(x_0) + 4 f(x_1) + f(x_2) \right) \]

**Peak Acceleration**

- Peak horizontal acceleration (PHA) is the largest (absolute) value of horizontal acceleration obtained from the accelerogram of that component.

- Peak vertical acceleration (PVA): a PVA of 1.74g in Imperial Valley 1979.

**Amplitude Parameters**

- The acceleration time history shows a significant proportion of relatively high frequencies.
- Integration produces a smoothing or filtering effect in frequency domain.
- The velocity time history shows substantially less high-frequency motion than the acceleration time history.
- The displacement time history shows substantially less high-frequency motion than the velocity time history.

**Relationship between PHA and MMI**
Peak Velocity

- The peak horizontal velocity (PHV) can be used to characterize ground motion amplitude accurately at intermediate frequencies.
- For structures within the intermediate frequency range (flexible buildings, bridge, etc.), the PHV may provide a much more accurate indication of the potential damage than PHA.

Peak Displacement

- Peak displacements are generally associated with the lower-frequency components of an earthquake motion.
- However, it is often difficult to determine accurately, due to signal processing errors in filtering and integration of accelerograms and due to long-period noise.

Sustained Maximum Acceleration

- Nuttli (1979) used lower peaks of acceleration for three (or five) cycles as the third (or fifth) highest (absolute) value of acceleration in the time history.

Effective Design Acceleration

- The Effective Design Acceleration (EDA) is the peak acceleration that remains after filtering out accelerations above 8 to 9 Hz.
- Kennedy (1980) proposed that the EDA be 25% greater than the third (absolute) peak acceleration obtained from a filtered time history.
The frequency content describes how the amplitude of ground motion is distributed among different frequencies.

Since the frequency content of an earthquake motion will strongly influence the effect of that motion, characterization of the motion cannot be complete without consideration for frequency content.

A plot of Fourier amplitude versus frequency is known as a Fourier Amplitude Spectrum.

A plot of Fourier phase angle versus frequency is known as a Fourier Phase Spectrum.

Fourier amplitude spectrum may be narrow or broad.

A narrow spectrum implies that the motion has a dominant frequency, which can produce a smooth, almost sinusoidal time history.

A broad spectrum corresponds to motion that contains a variety of frequencies that produces a more jagged, irregular time history.
Fourier Spectra

- Corner frequency $f_c$
- Cutoff frequency $f_{\text{max}}$

$$f_c = 4.9 \times 10^6 \beta \left( \frac{\Delta \sigma}{M_g} \right)^{1/3}$$
Brune (1970)

Power Spectra

- Using Parseval’s theorem, the total intensity can be expressed in the frequency domain as

$$I_0 = \frac{1}{\pi} \int_0^{\omega_n} c_n^2 d\omega$$

$\omega_n = \pi / \Delta t$

- Is the Nyquist Frequency (the highest frequency in the Fourier series).

Power Spectra

- The frequency content of a ground motion can be described by a power spectrum or power spectrum density function.

- The power density function can be used to estimate the statistical properties of a ground motion and to compute stochastic response using random vibration techniques.

- The total density of a ground motion of duration of $T_o$ is given in the time domain by the area of the time history squared acceleration:

$$I_o = \int_0^T [a(t)]^2 dt$$

Power Spectra

- The average intensity, $\lambda_o$ can be obtained by dividing the previous equations by the duration

$$\lambda_o = \frac{1}{T_o} \int_0^T [a(t)]^2 dt = \frac{1}{\pi T_o} \int_0^{\omega_n} c_n^2 d\omega$$

- The average intensity is equal to the mean-squared acceleration.
The power spectral density, $G(\omega)$ is defined such that:

$$\lambda_0 = \int_0^\infty G(\omega)d\omega$$

From which

$$G(\omega) = \frac{1}{\pi T_d} c_n^2$$

Response Spectra are divided into
- Acceleration controlled (high frequency)
- Velocity controlled (intermediate frequency)
- Displacement controlled (low frequency)
Response Spectra

Equations for Response Spectrum

\[ u(t) = \text{displacement of mass} \]
\[ z(t) = \text{displacement of ground} \]
\[ y(t) = \text{relative displacement} \]

\[ y(t) = u(t) - z(t) \]
\[ m\ddot{u} + c(u - \dot{z}) + k(u - z) = 0 \]
\[ m\ddot{y} + c(\dot{y}) + k(y) = 0 \]
\[ m\ddot{y} + c\ddot{y} + ky = -m\ddot{z} \]

\[ Y(\Omega) = \frac{m\ddot{z}(\Omega)}{k - m\Omega^2 + ic\Omega} \times \frac{1}{m\omega_n^3} \]

\[ \frac{1}{\omega_n^2} \frac{\ddot{z}(\Omega)}{\omega_n^2 + i\frac{c\Omega}{m\omega_n^2}} \]

\[ Y(\Omega) = \frac{1}{k} \frac{\ddot{z}(\Omega)}{\omega_n^2 - \omega_n^2 + i\frac{c\Omega}{m\omega_n^2}} \]

\[ c = 2\xi\omega_n \quad \text{and} \quad \omega_n = \frac{k}{m} \]

\[ Y(\Omega) = \frac{1}{\omega_n^2 + 2\xi\Omega} \frac{\ddot{z}(\Omega)}{\omega_n^2} \]

\[ y(t) = \text{IFFT}(Y(\Omega)) \]
### Spectral Parameters

- Predominant Period
- Bandwidth
- Central Frequency
- Shape Factor
- Kanai-Tajimi Parameters

### Predominant Period

- The predominant period is the period of vibration corresponding to the maximum value of the Fourier amplitude spectrum.

### Bandwidth

- The predominant period can be used to locate the peak of the Fourier amplitude spectrum; however, it provides no information on the dispersion of the spectral amplitude about the predominant period.
- The bandwidth of the Fourier amplitude spectrum is the range of frequency over which some level of Fourier amplitude is exceeded.
- Bandwidth is measured at the level where the power of the spectrum is half its maximum value - this corresponds to a level of $1/\sqrt{2}$ time the maximum Fourier amplitude.

### Central Frequency

Define the nth spectral moment of a ground motion as:

$$\lambda_n = \int_0^{\infty} \omega^n G(\omega) d\omega$$

The central frequency is given by:

$$\Omega = \sqrt{\frac{\lambda_2}{\lambda_0}}$$

The central frequency is a measure of the frequency where the power density is centered.
The ratio (for a 5% damping spectrum) of the spectral response at the appropriate period to EPA or the EPV is set at a standard value of 2.5 in both cases.

According to NEHRP 94, the effective peak acceleration ($A_e$) may be determined by dividing map values of the maximum 0.3 second (3 Hz) spectral response acceleration by 2.5.

For earthquake motions that include many frequencies, the quantity $2 \cdot V_{max}/A_{max}$ can be interpreted as the period of vibration of an equivalent harmonic wave indicating which period of the ground motion are most significant.

Seed and Idriss (1982) suggested the following representative average values for different site conditions less than 50 km from the source:

<table>
<thead>
<tr>
<th>Site Condition</th>
<th>Vmax/Amax</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rock</td>
<td>55 cm/sec/g, $\geq 0.056$ sec</td>
</tr>
<tr>
<td>Stiff Soils (&gt;200ft)</td>
<td>110 cm/sec/g, $\geq 0.112$ sec</td>
</tr>
<tr>
<td>Deep Stiff Soils (&gt;200ft)</td>
<td>135 cm/sec/g, $\geq 0.138$ sec</td>
</tr>
</tbody>
</table>

Because peak velocity and peak acceleration are usually associated with motions of different frequency, the ratio of $V_{max}/A_{max}$ should be related to the frequency content of the motion.

For simple harmonic motion:

$$V_{max}/A_{max} = \frac{T}{2\pi}$$
The duration of strong ground motion can have a strong influence on earthquake damage. A motion of short duration may not produce enough load reversals for damaging response to build up in a structure, even if the amplitude of the motion is high. On the other hand, a motion with moderate amplitude but long duration can produce enough load reversals to cause substantial damage.

Bracketed duration (Bolt, 1969) is defined as the time between the first and last exceedance of a threshold acceleration (usually 0.05g).

Tifunac and Brady (1975) is based on the time interval between the point at which 5% and 95% of the total energy has been recorded.

Boore (1983) duration to be equal to the corner period (i.e., inverse of the corner frequency).
Typical Earthquake Durations at Epicentral Distances Less than 10km

<table>
<thead>
<tr>
<th>Magnitude</th>
<th>Rock Site</th>
<th>Soil Sites</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.0</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>5.5</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>6.0</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>6.5</td>
<td>11</td>
<td>23</td>
</tr>
<tr>
<td>7.0</td>
<td>16</td>
<td>32</td>
</tr>
<tr>
<td>7.5</td>
<td>22</td>
<td>45</td>
</tr>
<tr>
<td>8.0</td>
<td>31</td>
<td>62</td>
</tr>
<tr>
<td>8.5</td>
<td>43</td>
<td>86</td>
</tr>
</tbody>
</table>

Other Ground Motion Parameters

- A single parameter that includes the effects of the amplitude and frequency content of a strong motion record is the rms acceleration

\[ a_{rms} = \sqrt{\frac{1}{T_d} \int_0^{T_d} [a(t)]^2 dt} = \sqrt{\lambda_0} \]

Duration

- Hermann (1985)
  - \( T = T_s + bR \)
  - \( b = \) constant to consider path = 0.05
  - \( T_s = 1/fc \)

Arias Intensity

\[ I_a = \frac{\pi}{2g} \int_0^{\infty} [a(t)]^2 dt \]
The CAV correlates well with structural damage potential. For example, a CAV of 0.3g-sec corresponds to lower limit for MMI VII Shaking.

- **Cumulative Absolute Velocity**

  \[ CAV = \int_0^T |a(t)| \, dt \]

- **Response Spectrum Intensity**

  \[ SI(\xi) = \int_{\xi_1}^{\xi_2} PSV(\xi, T) \, dT \]

  - It captures important aspects of the amplitude and frequency content.

- **Acceleration Spectrum Intensity**

  \[ ASI = \int_{0.1}^{0.5} S_a(\xi = 0.05, T) \, dT \]