1. Complete homework assignments 16-25; compile your work in a separate notebook (20 points).

2. Complete your individual project assigned in class and submit a final project report. The report should include a complete description of the problem, a rationale for the problem discretization, plots of the solution, and a summary of the results. Please write this as if you were submitting this for publication in a technical journal (20 points).

3. Use your program T6QUAD (T6 element stiffness matrix) to compute the stiffness matrix for the following two elements (10 points).

4. Set up and solve a variational finite element formulation for the following heat transfer problem using: (a) two six-node triangular elements of equal area and (b) one eight-node quadrilateral element. When computing the terms associated with the internal heat generation use MathCAD (or some other symbolic mathematical software) to evaluate the integrations. Justify any assumptions you make in evaluating the forcing function. Plot the resulting temperature distribution in terms of isothermal lines. Use the shape functions of the element to interpolate value of $T$ at points that do not correspond to node locations (10 points).

5. Use program POIS36 to find the torsion function in a circular shaft weakened by a radial crack. Assume the material properties $G$, $J$, $\theta$, and the radius of the shaft to be equal to unity. Present a justification for the problem discretization. Solve the problem for $\alpha = 1$ and $\frac{1}{2}$. What are the values of the torque for the two problems? Present your results in terms of a contour plot of the torque function. Hint: Adjust POIS36 to calculate the torque - see your class notes (10 points).
6. Consider the quadratic element shown below. What are the range of values for $a$ and $b$ such that the transformation from global to element coordinates is a valid one-to-one mapping (10 points).

\[
x^T_e = [1 \ 3 \ 1 \ 2 \ 2 \ a]
\]
\[
y^T_e = [1 \ 1 \ 3 \ b \ 2 \ 2]
\]

7. Set up the Helmholtz problem for the $L$-shaped region and mesh shown below. Using 4-node quadrilateral elements, determine the first four eigenvalue-eigenvector pairs. Assume $\psi = 0$ everywhere on the boundary. Hint: Choose your node number scheme in such a way as to minimize your effort in assembling the global matrices (10 points).

\[
\nabla^2 \psi - \lambda \psi = 0 \quad \text{in } \Omega
\]
\[
\psi = 0 \quad \text{on } \Gamma_1
\]
\[
\frac{\partial \psi}{\partial n} + \alpha \psi = 0 \quad \text{on } \Gamma_2
\]

8. For the following heat conduction problem:

\[
\frac{\partial}{\partial x} \left( kA \frac{\partial u}{\partial x} \right) = \rho c_p A \frac{\partial u}{\partial t} \quad 0 \leq x \leq L, \quad t \geq 0
\]
\[
u(0,t) = u_0 \quad \text{and} \quad u(L,t) = 0 \quad t \geq 0
\]
\[
u(x,0) = 0 \quad 0 \geq x \geq L
\]

Show that consistent mass matrices using for two equally-spaced quadratic elements results in:

\[
\begin{bmatrix}
16 & -8 & 0 \\
-8 & 14 & -8 \\
0 & -8 & 16
\end{bmatrix}
+ \begin{bmatrix}
16 & 2 & 0 \\
2 & 8 & 2 \\
0 & 2 & 16
\end{bmatrix}
\begin{bmatrix}
8\psi \nu_0 \\
-\psi \nu_0 \\
0
\end{bmatrix}
\]

\[
\psi = \frac{40\alpha}{L^2}
\]

Determine the analytical solution and compare it with results obtained using four linear elements and with the exact solution.