11.13. Example of Bar Cutoff

A floor system consists of single span T-beams 8 ft on centers, supported by 12 in masonry walls spaced at 25 ft between inside faces. The general arrangement is shown below. A 5 inch monolithic slab to be used in heavy storage warehouse. Assume a superimpose dead load of 118 k/ft and additional live load of 15 k/ft. Determine the reinforcement configuration and the cutoff points. Check the provisions of ACI 318 for bar cutoff.

\[ f'_c = 4,000 \text{ psi (normal weight)} \text{ and } f_y = 60,000 \text{ psi} \]

---

Dead Load

Weight of slab = \[ 5 \text{ (in)} \times \left( \frac{1}{12} \text{ (ft/in)} \right) \times 150 \left( \frac{\text{lb/ft}^3}{\text{ft}^3} \right) \times 7 \text{ (ft)} = 440 \text{ lb/ft} \]

Weight of beam = \[ \frac{12}{12} \text{ (ft)} \times \frac{22}{12} \text{ (ft)} \times 150 \left( \frac{\text{lb/ft}^3}{\text{ft}^3} \right) = 275 \text{ lb/ft} \]

Total \( w_d = 715 + 118 = 833 \text{ lb/ft} \)

Live Load

1.2\( w_d = 1000 \text{ lb/ft} \)

Referring to Table 1.1 in your text book, for Storage Warehouse – Heavy, \( w_L = 250 + 15 \text{ psf} \)

\[ w_L = 265 \left( \frac{\text{lb/ft}^2}{\text{ft}^2} \right) \times 8 \text{ (ft)} = 2,125 \text{ lb/ft} \]

1.6\( w_L = 3,400 \text{ lb/ft} \)
Find Flange Width (ACI 8.10):

\[ \frac{L}{4} = 26 \times 12 / 4 = 78" \quad \text{Controls} \]

\[ 16 h_f + b_w = 80 + 12 = 92" \]

Centerline spacing = 8’ x 12 = 96 inches

\[ w_u = 1.2w_D + 1.6w_L \]
\[ (1000 + 3,400)/1000 = 4.4 \text{ kips/ft} \]

Maximum moment is:

\[ M_u = \frac{1}{8} w_u l^2 \]

\[ M_u = \frac{1}{8} \times 4.4 \times 26^2 = 371.8 \text{ ft} \times \text{kips} \]

Design the T-beam:

Assuming for trial that the stress block depth will be equal to the slab thickness:

\[ A_s = \frac{M_u}{\phi f_y (d - \frac{a}{2})} = \frac{[371.8 \text{ (ft} \times \text{kips}) \times 12]}{0.9 \times 60 \times (18 - 5/2)} = \frac{82.622}{18 - 5/2} = 5.33 \text{ in}^2 \]

\[ a = \frac{A_s f_y}{0.85 \times f_c b} = \frac{5.33 \times 60}{0.85 \times 4 \times 78} = 5.33 \times 0.226 = 1.2 < h_f = 5 \text{ inches} \rightarrow o.k. \]

The stress block depth is less than the slab thickness, therefore, rectangular beam equations are valid.

Adjustment trial:

\[ A_s = \frac{M_u}{\phi f_y (d - \frac{a}{2})} = \frac{82.622}{18 - 1.2/2} = 4.75 \text{ in}^2 \]

\[ a = 4.75 \times 0.226 = 1.08 \text{ inches} < h_f = 5 \text{ inches} \]

Next trail

\[ A_s = \frac{M_u}{\phi f_y (d - \frac{a}{2})} = \frac{82.622}{18 - 1.08/2} = 4.73 \text{ in}^2 \]

Close enough to previous iteration of 4.75. Stop here
Use 6-#8 bars $A_s = 4.71 \text{ in}^2$

Check ACI for maximum steel:

$$A_{sf} = 0.85 \frac{f'_c}{f_y} (b - b_w) h_f = 0.85 \times \frac{4}{60} \times (78 - 12) \times 5 = 18.7 \text{ in}^2$$

$$\rho_f = \frac{A_{sf}}{b_w d} = \frac{18.7}{12 \times 18} = 0.08657$$

$$\bar{\rho}_b = 0.85 \times \beta_1 \times \frac{f'_c}{f_y} \times \frac{87,000}{87,000 + f_y} = 0.0285$$

$$\rho_{bal} = \frac{b_w}{b} (\rho_f + \bar{\rho}_b) = \frac{12}{78} (0.08657 + 0.0285) = 0.0177$$

$$\rho_{max} = 0.75 \times \rho_{bal} = 0.013828$$

$$A_{s max} = \rho_{max} bd = 0.013828 \times 78 \times 12 = 19.4 \text{ in}^2$$

$$A_s = 4.71 \text{ in}^2 < A_{s max} = 19.4 \text{ in}^2 \quad \text{O.K.}$$
1.5” clear
6 - #8
$A_s = 4.71 \text{ in}^2$
$d = 18 \text{ in}$

4 - #8
$A_s = 3.14 \text{ in}^2$
$d = 19 \text{ in}$

2 - #8
$A_s = 1.57 \text{ in}^2$
$d = 19 \text{ in}$

6 bars
$A_s = 4.71$
$d = 18 \text{ in}$

4 bars
$A_s = 3.14$
$d = 18 \text{ in}$

2 bars
$A_s = 1.57$
$d = 19 \text{ in}$

---

![Graphs showing $V_u$ and $M_u$ as functions of distance from support, with phi notation for load and moment.
**Input information**

- $A_i = 4.71 \text{ in}^2$
- $A_v = 0.22 \text{ in}^2$
- $f_t = 4000 \text{ psi}$
- $f_y = 60000 \text{ psi}$
- Bar length = 26 ft
- $s_s = (1), \text{ Cant. (2)}$
- $W_{L} = 2.123 \text{ kft}$
- $W_{d} = 0.833 \text{ kft}$
- $b_w = 1 \text{ in}$
- $d = 18 \text{ in}$
- $W_{u} = 4.40 \text{ kft}$
- Reaction at Support = 57.19
- $\rho_o = 0.0218$
- $V_{L\text{max}} = 35.42 \text{ kips}$
- $4\sqrt{f_c}^2 = 54.64 \text{ kips}$

**$S_{max}$**

- $d_2$ or $d_4 = 9 \text{ inches}$
- $24^\circ$ or $12^\circ = 12 \text{ inches}$
- $A^{4\sqrt{f_c}(50\%)} = 22.0 \text{ inches}$

---

<table>
<thead>
<tr>
<th>Dist from</th>
<th>Mu</th>
<th>$V_i$</th>
<th>$\phi_i/V_c$</th>
<th>$V_o-\phi_o/V_c$</th>
<th>req'd</th>
<th>$V_o/d/Mu$</th>
<th>$\phi_o/V_c$</th>
<th>$V_o-\phi_o/V_c$</th>
<th>req'd</th>
</tr>
</thead>
<tbody>
<tr>
<td>support (ft)</td>
<td>(kips)</td>
<td>(kips)</td>
<td>(kips)</td>
<td>spacing</td>
<td>(kips)</td>
<td>(kips)</td>
<td>spacing</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w \cdot a^2(L^2-x^2)^2/2$</td>
<td>$w \cdot a^2L^2/w \cdot a^2$</td>
<td>Eq 1.1-3</td>
<td>Eq 1.1-3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.00</td>
<td>0.01</td>
<td>50.60</td>
<td>20.49</td>
<td>30.10</td>
<td>5.92</td>
<td>1.00</td>
<td>28.30</td>
<td>22.30</td>
<td>7.99</td>
</tr>
<tr>
<td>1.00</td>
<td>55.00</td>
<td>50.60</td>
<td>20.49</td>
<td>30.10</td>
<td>5.92</td>
<td>1.00</td>
<td>28.30</td>
<td>22.30</td>
<td>7.99</td>
</tr>
<tr>
<td>1.50</td>
<td>80.84</td>
<td>50.60</td>
<td>20.49</td>
<td>30.10</td>
<td>5.92</td>
<td>0.94</td>
<td>27.76</td>
<td>22.84</td>
<td>7.80</td>
</tr>
<tr>
<td>2.00</td>
<td>105.59</td>
<td>48.40</td>
<td>20.49</td>
<td>27.90</td>
<td>6.39</td>
<td>0.69</td>
<td>25.54</td>
<td>22.86</td>
<td>7.80</td>
</tr>
<tr>
<td>3.00</td>
<td>151.79</td>
<td>44.00</td>
<td>20.49</td>
<td>23.50</td>
<td>7.58</td>
<td>0.43</td>
<td>22.31</td>
<td>20.69</td>
<td>8.61</td>
</tr>
<tr>
<td>4.00</td>
<td>153.58</td>
<td>39.60</td>
<td>20.49</td>
<td>15.10</td>
<td>9.00</td>
<td>0.51</td>
<td>22.18</td>
<td>17.42</td>
<td>9.00</td>
</tr>
<tr>
<td>5.00</td>
<td>230.98</td>
<td>35.20</td>
<td>20.49</td>
<td>14.71</td>
<td>9.00</td>
<td>0.23</td>
<td>21.49</td>
<td>13.71</td>
<td>9.00</td>
</tr>
<tr>
<td>6.00</td>
<td>263.98</td>
<td>30.80</td>
<td>20.49</td>
<td>10.31</td>
<td>9.00</td>
<td>0.18</td>
<td>21.01</td>
<td>9.78</td>
<td>9.00</td>
</tr>
<tr>
<td>7.00</td>
<td>322.57</td>
<td>26.40</td>
<td>20.49</td>
<td>5.91</td>
<td>9.00</td>
<td>0.14</td>
<td>20.66</td>
<td>5.74</td>
<td>9.00</td>
</tr>
<tr>
<td>8.00</td>
<td>316.77</td>
<td>22.00</td>
<td>20.49</td>
<td>1.51</td>
<td>9.00</td>
<td>0.10</td>
<td>20.39</td>
<td>1.61</td>
<td>9.00</td>
</tr>
<tr>
<td>9.00</td>
<td>336.57</td>
<td>17.60</td>
<td>20.49</td>
<td>0.00</td>
<td>9.00</td>
<td>0.08</td>
<td>20.16</td>
<td>-2.56</td>
<td>9.00</td>
</tr>
<tr>
<td>10.00</td>
<td>351.97</td>
<td>13.20</td>
<td>20.49</td>
<td>0.00</td>
<td>9.00</td>
<td>0.06</td>
<td>19.96</td>
<td>0.00</td>
<td>9.00</td>
</tr>
<tr>
<td>11.00</td>
<td>382.97</td>
<td>8.80</td>
<td>20.49</td>
<td>0.00</td>
<td>9.00</td>
<td>0.04</td>
<td>19.79</td>
<td>0.00</td>
<td>9.00</td>
</tr>
<tr>
<td>12.00</td>
<td>389.57</td>
<td>4.40</td>
<td>20.49</td>
<td>0.00</td>
<td>9.00</td>
<td>0.02</td>
<td>19.62</td>
<td>0.00</td>
<td>9.00</td>
</tr>
<tr>
<td>12.75</td>
<td>371.63</td>
<td>1.10</td>
<td>20.49</td>
<td>0.00</td>
<td>9.00</td>
<td>0.00</td>
<td>19.51</td>
<td>0.00</td>
<td>9.00</td>
</tr>
</tbody>
</table>

**Shear**

![Shear diagram]

**Strain-Stress**

![Strain-Stress diagram]
Capacity of section after 4 bars are discontinued:

\[
a = \frac{A_s f_y}{0.85 f_y' c b} = \frac{1.57 \times 60}{0.85 \times 4 \times 78} = 0.355 \text{ inches}
\]

\[
M_u(2 \text{ bars}) = \phi \times M_n = \phi A_s f_y (d - \frac{a}{2})
\]

\[
M_u(2 \text{ bars}) = 0.9 \times 1.57 \times 60 \times (19 - \frac{0.355}{2}) \times \frac{1}{12} = 133 \text{ ft} - \text{kips}
\]

Capacity of section after 2 bars are discontinued:

\[
a = \frac{A_s f_y}{0.85 f_y' c b} = \frac{3.14 \times 60}{0.85 \times 4 \times 78} = 0.71 \text{ inches}
\]

\[
M_u(4 \text{ bars}) = \phi \times M_n = \phi A_s f_y (d - \frac{a}{2})
\]

\[
M_u(4 \text{ bars}) = 0.9 \times 3.14 \times 60 \times (18 - \frac{0.71}{2}) \times \frac{1}{12} = 250 \text{ ft} - \text{kips}
\]

Find the location where the moment is equal to \(M_u(2 \text{ bars})\)

\[
M = 57.2x - \frac{1}{2}(4.4)(x)^2
\]

\[
M = 57.2x - 2.2x^2
\]

\[
M_u(2\text{bars}) = 133 = 57.2x - 2.2x^2
\]

\[
2.2x^2 - 57.2x + 133 = 0
\]

\[
x = \frac{57.2 \pm \sqrt{57.2^2 - 4 \times 133 \times 2.2}}{2 \times 2.2} = 2.6 \text{ ft}
\]

Find the location where the moment is equal to \(M_u(4 \text{ bars})\)

\[
M_u(4 \text{ bars}) = 250 = 57.2x - 2.2x^2
\]

\[
2.2x^2 - 57.2x + 250 = 0
\]

\[
x = \frac{57.2 \pm \sqrt{57.2^2 - 4 \times 250 \times 2.2}}{2 \times 2.2} = 5.6 \text{ ft}
\]

Note:

Code allows discontinuities of 2/3 of longitudinal bars for simple spans. Therefore, let’s cut 4 bars.
Note: Clear bar spacing is equal to:

\[
\frac{1}{(\text{no. of bars in one row} - 1)} \sqrt{12 - 2\left(\frac{3}{8}\right) - \text{No. of bars} \times \left(\frac{8}{12}\right) - 2(1.5)}
\]

stirrups bars cover

CASE 1

6 bars
\( A_s = 4.71 \) in²
\( d = 18 \) in

clear spacing = 2.63 in
center to center spacing = 3.63 in

CASE 2

4 bars
\( A_s = 3.14 \) in²
\( d = 18 \) in

clear spacing = 6.25 in
center to center spacing = 7.25 in

CASE 3

2 bars
\( A_s = 1.57 \) in²
\( d = 19 \) in

clear spacing = 6.25 in
center to center spacing = 6.25 in

**Determine the Development Length**

\[ \alpha = 1.0 \quad \gamma = 1.0 \]
\[ \beta = 1.0 \quad \lambda = 1.0 \]

\[ A_{tr} = 0.22 \]
\[ n = 3 \]
\[ s = 9 \text{ in} \]

\[ K_{tr} = \frac{A_{tr} f_{ys}}{1500 s n} \]

\[ K_{tr} = \frac{0.22 \times 60,000}{1500 \times 3 \times 9} = 0.33 \]

\[ c = \left\{ \begin{array}{l}
\frac{1}{2} \times 3.63 = 1.8 \\
1.5 + \frac{3}{8} + 0.5 = 2.375
\end{array} \right. \]

\[ c + K_{tr} = 1.8 + \frac{0.33}{1} = 2.13 < 2.5 \text{ ok} \]

\[ l_d = \frac{3 f_y}{f_c' \sqrt{c + K_{tr}}} d_b \]

\[ l_d = \frac{3 \times 60,000}{\sqrt{4,000 \times 1 \times 1 \times 1 \times 1 \times 1}} \times 1 \]

\[ l_d = 33 \text{ in} = 2.75 \text{ ft} \]

\[ l_d = 2.75 \times \frac{A_{req's}}{A_s \text{ provided}} = 2.75 \times \frac{4.71}{4.71} = 2.75 \text{ ft} \]
Extend Bars:

\[
\begin{align*}
12d_b &= 12\times (1.0) = 12\text{ inches} = 1.0\text{ ft} \\
\text{or} \\
d &= \frac{18}{12} = 1.5\text{ ft} & \quad \text{Governs}
\end{align*}
\]
Check Zero Moment:

\[
l_d \leq 1.3 \frac{M_n}{V_u} + l_a
\]

\[
M_n = \frac{M_u}{\phi} = \frac{371.8}{0.9} = 413 \text{ ft-kips}
\]

\[
l_d \leq 1.3 \times \frac{413 \times 12}{57.2} + 3 = 116 \text{ inches}
\]

\[
l_d = 2.75 \times 12 = 33 \text{ in} \leq 116 \text{ in} \rightarrow \text{o.k.}
\]

This is to ensure that the continued steel is of sufficiently small diameter and the required anchorage requirement of the ACI code is satisfied.

Check for shear Complications (ACI 12.10.5)

\[
V_c = 2\sqrt{f_c' b_w d} = 2\sqrt{4000 \times 12 \times 18} = 27.3 \text{ kips}
\]

\[
V_s = \frac{A_v f_y d}{s} = \frac{(2 \times 0.11) \times 60 \times 18}{9} = 26.4 \text{ kips}
\]

\[
V_u = \phi V_n = \phi (V_c + V_s) = 0.85 \times (27.3 + 26.4) = 45.6 \text{ kips}
\]

\[
V_u(x = 1.1) = 57.2 - 4.4 \times 1.1 = 52.4 \text{ kips}
\]

\[
V_u(x = 1.1) = 52.4 \text{ kips} > (2/3) \times 45.6 = 30.4
\]

\[
V_u(x = 4.1) = 57.2 - 4.4 \times 4.1 = 39.2 \text{ kips}
\]

\[
V_u(x = 4.1) = 39.2 \text{ kips} > (2/3) \times 48.5 = 32.3
\]

Need additional reinforcements at both cutoff points.
Check for shear Complications (ACI 12.10.5), Continued

\[
s = \frac{A_{sfv}}{60b_w} = \frac{0.22 \times 60,000}{60 \times 12} = 18.33 \text{ in}
\]

\[
s = \frac{d}{8\beta_d} = \frac{18}{8 \times \frac{2}{6}} = 6.7\text{in} \quad \text{Governs use 6 inches}
\]

provide additional shear reinforcement for a length of \((3/4)d\)

\[
\frac{3}{4} \times d = \frac{3}{4} \times 18 = 13.5 \text{ in}
\]