Example of Bar Cutoff

A floor system consists of single span T-beams 8 ft on centers, supported by 12 in masonry walls spaced at 25 ft between inside faces. The general arrangement is shown in below. A 5-inch monolithic slab to be used in heavy storage warehouse. Determine the reinforcement configuration and the cutoff points.

Check the provisions of ACI 318 for bar cutoff.

\[ f'c = 4,000 \text{ psi (normal weight)} \]

\[ f_y = 60,000 \text{ psi} \]

\[ f'c = \frac{4,000}{12} = 333.3 \text{ psi} \]

Dead Load

Weight of slab \( = \frac{5}{12} \text{ ft}(7 \text{ ft})(150 \text{ lb/ft}^3) = 440 \text{ lb/ft} \)

Weight of beam \( = \frac{12}{12} \text{ ft}(\frac{22}{12} \text{ ft})(150 \text{ lb/ft}^3) = 275 \text{ lb/ft} \)

\[ w_D = 440 + 275 = 715 \text{ lb/ft} \]

\[ 1.2w_D = 860 \text{ lb/ft} \]

Live Load

Referring to Table of 1.1 in your notes, for Storage Warehouse – Heavy, \( w_L = 250 \text{ psf} \)

\[ w_L = (250 \text{ lb/ft}^2)(8 \text{ ft}) = 2,000 \text{ lb/ft} \]

\[ 1.6w_L = 3,200 \text{ lb/ft} \]
Find Flange Width

\[ \frac{L}{4} = \frac{26 \times 12}{4} = 78 \text{ inches} \quad \leftrightarrow \text{Controls } b = 48 \text{ inches} \]

16\(h_j + b_w = 80 + 12 = 92 \text{ inches} \)

Centerline spacing = 8\(\times 12 = 96 \text{ inches} \)

Determine Factored Load

\[ w_u = 1.2w_D + 1.6w_L = 860 + 3,200 = 4060 \text{ lb/ft} \]

4.06 \text{ kips/ft}

Determine Factored Moment

\[ M_u = \frac{1}{8} w_u l^2 \]

\[ M_u = \frac{1}{8} (4.06)(26)^2 = 343 \text{ ft-kips} \]

Design the T-beam

Use a trial and error procedure. First, assume for the first trial that the stress block depth will be equal to the slab thickness (\(a = 5 \text{ inches}\)):

\[ A_s = \frac{M_u}{\phi f_y (d - a/2)} = \frac{343 \times 12}{0.9 \times 60(18 - 5/2)} = \frac{76.2}{18 - 5/2} = 4.92 \text{ in}^2 \]

\[ a = \frac{A_s f_y}{0.85 f_c b} = \frac{4.92 \times 60}{0.85 \times 4 \times 78} = 4.92 \times 0.226 = 1.11 < h_j = 5 \text{ inches } \rightarrow \text{ok.} \]

The stress block depth is less than the slab thickness; therefore, the beam will act as a rectangular beam and the rectangular beam equations are valid.

Adjust trial

\[ A_s = \frac{M_u}{\phi f_y (d - a/2)} = \frac{76.2}{18 - 1.11/2} = 4.37 \text{ in}^2 \]

\[ a = \frac{A_s f_y}{0.85 f_c b} = 4.73 \times 0.226 = 0.99 \]

Next trial

\[ A_s = \frac{M_u}{\phi f_y (d - a/2)} = \frac{76.2}{18 - 0.99/2} = 4.35 \text{ in}^2 \]

Close enough to previous iteration of 4.37 \text{ in}^2. Stop here.
Use 6-#8 bars $A_s = 4.71$ in$^2$

Check ACI for Maximum Steel:

Using similar triangles:

$$\frac{\varepsilon_u}{0.004} = \frac{c}{d-c} \quad \Rightarrow \quad \frac{0.007}{0.004} = \frac{c}{18-c} \quad \rightarrow \quad c = 7.71 \text{ inches}$$

$$a = \beta_c c = 0.85 \times 7.71 = 6.65 \text{ inches}$$

$$A_{s_{\text{max}}} f_y = 0.85 f'_c [78 \times 5 + 12 \times 1.56] \rightarrow A_{s_{\text{max}}} = 23.16 \text{ in}^2$$

Since $A_s = 4.71 \text{ in}^2 \leq 23.16 \text{ in}^2$, we satisfy the ACI code and we will have tension failure.
CASE 1
6 bars
As = 4.71 in²
d = 18 in

CASE 2
4 bars
As = 3.14 in²
d = 19 in

CASE 3
2 bars
As = 1.57 in²
d = 19 in

Distance From Support (ft)
0 1 2 3 4 5 6 7 8 9 10 11 12 13

V_u (kips)
0 10 20 30 40 50 60

Distance From Support (ft)
0 1 2 3 4 5 6 7 8 9 10 11 12 13

M_u (ft-kips)
0 50 100 150 200 250 300 350 400
Note: Code allows discontinuing 2/3 of longitudinal bars for simple spans. Therefore, let’s cut 4 bars.

Capacity of section after 4 bars are discontinued:
\[
a = \frac{A_f}{f_y} b = \frac{1.57 \times 60}{0.85 \times 4 \times 78} = 0.355 \text{ inches}
\]
\[
M_u(2 \text{ bars}) = \phi M_n = \phi A_f f_y (d - \frac{a}{2})
\]
\[
M_u(2 \text{ bars}) = 0.9 \times 1.57 \times 60 (19 - \frac{0.355}{2}) \times \frac{1}{12} = 133 \text{ ft-kips}
\]

Capacity of section after 2 bars are discontinued:
\[
a = \frac{A_f}{f_y} b = \frac{3.14 \times 60}{0.85 \times 4 \times 78} = 0.71 \text{ inches}
\]
\[
M_u(4 \text{ bars}) = \phi M_n = \phi A_f f_y (d - \frac{a}{2})
\]
\[
M_u(4 \text{ bars}) = 0.9 \times 3.14 \times 60 (18 - \frac{0.71}{2}) \times \frac{1}{12} = 250 \text{ ft-kips}
\]

Find the location where the moment is equal to \( M_u(2 \text{ bars}) \):
\[
M = 52.7x - \frac{1}{2}(4.06)x^2
\]
\[
M = 52.7x - 2.03x^2
\]
\[
M_u(2\text{ bars}) = 52.7x - 2.03x^2 = 133
\]
\[
2.03x^2 - 52.78x + 133 = 0 \rightarrow x = \frac{52.78 \pm \sqrt{52.78^2 - 4 \times 133 \times 2.03}}{2 \times 2.03} = 2.8 \text{ ft}
\]

Find the location where the moment is equal to \( M_u(4 \text{ bars}) \):
\[
M_u(4 \text{ bars}) = 52.7x - 2.03x^2 = 250
\]
\[
2.03x^2 - 52.78x + 250 = 0 \rightarrow x = \frac{52.78 \pm \sqrt{52.78^2 - 4 \times 250 \times 2.03}}{2 \times 2.03} = 6.3 \text{ ft}
\]
CASE 1
6 bars
As = 4.71 in²
d = 18 in
clear spacing = 2.63 in
center to center spacing = 3.63 in

CASE 2
4 bars
As = 3.14 in²
d = 18 in
clear spacing = 6.25 in
center to center spacing = 7.25 in

CASE 3
2 bars
As = 1.57 in²
d = 19 in
clear spacing = 6.25 in
center to center spacing = 6.25 in

Note: Clear bar spacing is equal to:
\[
\frac{1}{\text{no. of bars in one row} - 1} = \left[ 12 - 2 \left( \frac{3}{8} \right) \right] - \text{no. of bars} \times \left( \frac{8}{8} \right) - 2(1.5)
\]

Determine the development length
\[
\psi_t = 1.0 \quad \psi_s = 1.0
\]
\[
\psi_t \lambda = 1.0
\]
\[
A_{tr} = 0.22 in^2
\]
\[
n = 3
\]
\[
s = 9 in
\]
\[
k_{tr} = \frac{A_{tr} f_{yt}}{1500 s n} = \frac{0.22 \times 60,000}{1500 \times 9 \times 3} = 0.33
\]
\[
c = \left[ \frac{1}{2} (3.63) = 1.8 in \right. \quad \left. \text{control} \right] \quad c + k_{tr} = \frac{1.8 + 0.33}{1.00} = 2.13 < 2.5 \quad \text{ok}
\]
\[
l_{d} = \frac{3 f_{c} \psi_t \psi_s \psi_t \lambda}{40 \sqrt{f_{c} \left( c + k_{tr} \right)}}
\]
\[
l_{d} = \left( \frac{3 \times 60,000 \times 1 \times 1 \times 1}{40 \times \sqrt{4,000 \times 2.13}} \right) \times 1 = 33 \quad in
\]
\[
l_{d} = 33 \quad in = 2.75 \quad ft
\]
\[
l_{d} = 2.75 \times \frac{A_{s\text{required}}}{A_{s\text{provided}}} = 2.75 \times \frac{4.35}{4.71} = 2.54 \quad ft
\]
Extend bars:
\[
\begin{align*}
12d_s &= 12 \times 1.00 = 12 \text{ inches } = 1 \text{ ft} \\
\therefore d &= 18 \text{ inches } = 1.5 \text{ ft} & \leftarrow \text{ controls}
\end{align*}
\]
Check Zero Moment:

\[ l_d \leq 1.3 \frac{M_n}{V_u} + l_a \]

\[ M_n = \frac{M_u}{\phi} = \frac{343}{0.9} = 381 \text{ ft.kips} \]

\[ l_d \leq 1.3 \frac{381 \times 12}{52.78} + 3.00 = 116 \text{ inches} \]

\[ l_d = 2.54 \text{ ft} = 2.54 \times 12 = 31 \text{ inches} \leq 116 \text{ inches} \rightarrow \text{ok} \]

This is to ensure that the continued steel is of sufficiently small diameter and the required anchorage requirement of the ACI code is satisfied.

Check for shear Complication (ACI 12.10.5)

\[ V_c = 2 \sqrt{f_c b_w d} = 2 \sqrt{4,000 \times 12 \times 18} = 27.3 \text{ kips} \]

\[ V_s = \frac{A_v f_y d}{s} = \frac{(0.22) \times 60 \times 18}{9} = 26.4 \text{ kips} \]

\[ V_u = \phi (V_c + V_s) = 0.75 (27.3 + 26.4) = 40.3 \text{ kips} \]

\[ Vu(x = 1.3) = 52.78 - 4.06 \times 1.3 = 47.5 \text{ kips} \]

\[ Vu(x = 1.3) = 47.5 \text{ kips} > (2/3) \times 40.3 = 26.9 \]

\[ Vu(x = 4.8) = 52.78 - 4.06 \times 4.8 = 33.3 \text{ kips} \]

\[ Vu(x = 4.8) = 33.3 \text{ kips} > (2/3) \times 40.3 = 26.9 \]

Need additional reinforcements at both cutoff points.
Check for Shear Complications (ACI12.10.5), Continued

\[
s = \frac{A_v f_y}{60b_n} = \frac{(0.22)b_n 60,000}{60 \times 12} = 18.33 \text{ in}
\]

\[
s = \frac{d}{8\beta_d} = \frac{18}{8 \times \left(\frac{2}{6}\right)} = 6.7 \text{ in} \leftrightarrow \text{controls use 6 inches}
\]

Provide additional reinforcement for a length of \((3/4)d/\)

\[
\frac{3}{4}d = \frac{3}{4} \times 18 = 13.5 \text{ inches}
\]