11.1. Anchorage of Tension Bars by Hooks

In the event that the desired tensile stress in a bar cannot be developed by bond alone, it is necessary to provide special anchorage at the ends of the bar, usually by means of a 90° or a 180° hook. The dimension and bent radii for such hooks have been standardized in the ACI Code as shown below.

Standard Bar Hooks

See ACI 7.1
7.2 — Minimum bend diameters

7.2.1 — Diameter of bend measured on the inside of the bar, other than for stirrups and ties in sizes No. 3 through No. 5, shall not be less than the values in Table 7.2.

7.2.2 — Inside diameter of bend for stirrups and ties

*For closed ties and continuously wound ties defined as hoops in Chapter 21, a 135-deg bend plus an extension of at least $5d_b$ but not less than 3 in. (See definition of “hoop” in 21.1.)

R7.2 — Minimum bend diameters

Standard bends in reinforcing bars are described in terms of the inside diameter of bend since this is easier to measure than the radius of bend. The primary factors affecting the minimum bend diameter are feasibility of bending without breakage and avoidance of crushing the concrete inside the bend.

R7.2.2 — The minimum $4d_b$ bend for the bar sizes commonly used for stirrups and ties is based on accepted industry practice in the United States. Use of a stirrup bar size not greater than No. 5 for either the 90-deg or 135-deg standard
11.2. Development of Standard Hooks

The Code requirements account for the combined contribution of bond along the straight bar leading to the hook, plus the hooked anchorage. A total development length \( l_{dh} \) is defined as shown below and is measured from the critical section to the farthest point on the bar, parallel to the straight part of the bar.

**ACI 12.5.1.** Development length \( l_{dh} \) in inches, for the deformed bars in tension terminating in a standard hook shall be computed as the product of the basic development length \( l_{hb} \) and the applicable modification factor or factors given below, but \( l_{dh} \) shall not be less than \( 8d_b \) nor less than 6 in.

<table>
<thead>
<tr>
<th>ACI 12.5.2</th>
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<tbody>
<tr>
<td>ACI 12.5.3.1</td>
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**ACI 12.5.3.4**

<table>
<thead>
<tr>
<th>( A_s ), required</th>
<th>( A_s ), provided</th>
</tr>
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<tbody>
<tr>
<td>Lightweight aggregate concrete</td>
<td>1.3</td>
</tr>
<tr>
<td>Epoxy coated reinforcement</td>
<td>1.2</td>
</tr>
</tbody>
</table>

**ACI 7.2**

- **A.** Basic development length \( l_{hb} \) for hooked bars with \( f_y = 60,000 \) psi

\[
\frac{1.20(l_b)}{\sqrt{f'_c}}
\]

- **B.** Modification factors to be applied to \( l_{hb} \)

Bars with \( f_y \) other than 60,000 psi

\[
\frac{f_y}{60,000}
\]

For No. 11 bars and smaller, side cover not less than 2 1/2 in. and for 90° hook, cover on bar extension beyond hook not less than 2 in.

For No. 11 bars and smaller, hook enclosed vertically or horizontally within ties or stirrup-ties spaced along the full development length \( l_{hb} \) with spacing not greater than 3\( d_b \)

- 0.7

**Bar Cutoff**
11.3. ACI 12.5.4

According to ACI 12.5.4, for bars hooked at the discontinuous ends of members with side cover and top or bottom cover less than 2.5 inches, as shown below, hooks must be enclosed with closed stirrups or ties along the full development length. The spacing of the confinement steel must not exceed 3 times the diameter of the hooked bar. In such cases, the factor 0.8 of ACI 12.5.3.4 does not apply.
11.4. Example of Development of Hooked Bars in Tension

Referring to the beam shown below, the No. 11 negative rebars are to be extended into the column and terminated in a standard 90° hook, keeping 2 in. clear to the outside face of the column. The column width in the direction of beam width is 16 in. Find the minimum length of embedment of the hook past the column face, and specify hook details.

\[
I_{hb} = \frac{1200d_b}{\sqrt{f_{c'}}} = \frac{(1200)(1.41)}{\sqrt{4000}} = 27 \text{ in.}
\]

Side covers exceed 2.5 in. for No. 11 bars, and cover beyond the bent bar is adequate; therefore, the modification factor of 0.7 can be applied.

\[
I_{dh} = (27) \times (0.7) \times \frac{2.90}{3.12} = 18 \text{ in.} < 19 \text{ in.} \rightarrow \text{ok}
\]
11.5. Code Provisions for Development of Web Reinforcement

12.13 – Development of web reinforcement

12.13.1 – Web reinforcement shall be carried as close to compression and tension surfaces of member as cover requirements and proximity of other reinforcement will permit.

12.13.2 – Ends of single leg, simple U-, or multiple U-stirrups shall be anchored by one of the following means:

12.13.2.1 – For #5 bar and D31 wire, and smaller, and for #6, #7, and #8 bars with $f_y$ of 40,000 psi or less, a standard hook around longitudinal reinforcement.

12.13.2.2 – For #6, #7, and #8 stirrups with $f_y$ greater than 40,000 psi, a standard stirrup hook around a longitudinal bar plus an embedment between midheight of the member and the outside end of the hook equal to or greater than $0.014d_y\sqrt{f_e}$.

$$\frac{d_y}{2} + \frac{0.014d_y f_y}{\sqrt{f_e}}$$

R12.13 – Development of web reinforcement

R12.13.1 – Stirrups must be carried as close to the compression face of the member as possible because near ultimate load the flexural tension cracks penetrate deeply.

R12.13.2 – The anchorage or development requirements for stirrups composed of bars or deformed wire were changed in the 1989 code to simplify the requirements. The straight anchorage was deleted as this stirrup is difficult to hold in place during concrete placement and the lack of a hook may make the stirrup ineffective as it crosses shear cracks near the end of the stirrup.

R12.13.2.1 – For a #5 bar or smaller, anchorage is provided by a standard stirrup hook, as defined in 7.1.3, hooked around a longitudinal bar. The 1989 code eliminated the need for a calculated straight embedment length in addition to the hook for these small bars, but 12.13.1 requires a full depth stirrup. Likewise, larger stirrups with $f_y$ equal to or less than 40,000 are sufficiently anchored with a standard stirrup hook around the longitudinal reinforcement.

R12.13.2.2 – Since it is not possible to bend a #6, #7, or #8 stirrup tightly around a longitudinal bar and due to the force in a bar with a design stress greater than 40,000 psi, stirrup anchorage depends on both the value of the hook and whatever development length is provided. A longitudinal bar within a stirrup hook limits the width of any flexural cracks, even in a tensile zone. Since such a stirrup hook cannot fail by splitting parallel to the plane of the hooked bar, the hook strength as utilized in 12.5.2 has been adjusted to reflect cover and confinement around the stirrup hook.

For stirrups with $f_y$ of only 40,000 psi, a standard stirrup hook provides sufficient anchorage and these bars are covered in 12.13.2.1. For bars with higher strength, the
11.6. Development Bars in Compression

Reinforcement may be required to develop its compressive strength by embedment under various circumstances; e.g., where bars transfer their share of column load to a supporting footing or where lap splices are made of compression bars in columns. In the case of bars in compression, a part of the total force is transferred by bond along the embedded length, an a part is transferred by end bearing of the bars on the concrete. Because the surrounding concrete is relatively free of cracks and because of the beneficial effect of end bearing, shorter basic development lengths are permissible for compression bars than for tension bars.

If lateral confinement steel is present, such as spiral column reinforcement or special spiral steel around an individual bar, the required development length is further reduced.

Hooks such are not effective in transferring compression from bars to concrete, and if present for other reasons, should be disregarded in determining required embedment length.

12.3 — Development of deformed bars in compression

12.3.1 — Development length $l_d$, in inches, for deformed bars in compression shall be computed as the product of the basic development length $l_{db}$ of 12.3.2 and applicable modification factors of 12.3.3, but $l_d$ shall not be less than 8 in.

12.3.2 — Basic development length

$$l_{db} = \frac{0.02d_y f_y}{f_c}$$

...but not less than

$$0.0003d_y f_y$$

where the constant 0.0003 carries the unit of in.$^2$/lb

12.3.3 — Basic development length $l_{db}$ shall be permitted to be multiplied by applicable factors for:

12.3.3.1 — Excess reinforcement

Reinforcement in excess of that required by analysis

$$\frac{(A_s \text{ required})}{(A_s \text{ provided})}$$

12.3.3.2 — Spirals and ties

Reinforcement enclosed within spiral reinforcement not less than 1/4 in. diameter and not more than 4 in. pitch or within No. 4 ties in conformance with 7.10.5 and spaced at not more than 4 in. on center

$$0.75$$
11.7. Bar Cutoff and Bend Points in Beams

Usually, prismatic beams with constant concrete cross-section dimensions are usually used to simplify formwork and thus to reduce cost.

It is common practice either to cut off bars where they no longer needed to resist stress, or sometimes in the case of continuous beam, to bend up the bottom steel to provide tensile reinforcement at the top of the beam over the support.

Rational: Required steel area is calculated at maximum moment location; Less steel will be required at other points.

let \( z = d - a/2 \), then \( M = Tz \), or \( T = M/z \)

The value of \( z \) (lever arm) will be smallest when at maximum moment section, but will not vary widely. Thus, we are being conservative if we proportion steel according to bending moment. (\( T \) proportional to \( M \)). For economy, we would like to keep expensive steel as highly stressed as possible.

– See the figure in the next page:

For simply supported beam with uniform loading, 100% of the tensile steel required at center of the span where the moment is maximum and 0% is required at the support.
Continuous beam with moment envelope resulting from alternate loading (e.g., see below)

Graph A.2 Location of points where bars can be bent up or cut off for simply supported beams uniformly loaded.
11.8. ACI Moment Coefficients (ACI 8.3.3)

(a) Maximum in the positive zone

(b) Maximum in the negative zone

Fig. 7.5.1
Exterior span with discontinuous end unrestrained.

Table 16.1

(a) Maximum in the positive zone

(b) Maximum in the negative zone

Fig. 7.5.3
Exterior span with exterior support built integrally with column.
11.9. ACI 12.11 Development of Positive and Negative Moment Reinforcement

12.11 – Development of positive moment reinforcement

12.11.1 – At least one-third the positive moment reinforcement in simple members and one-fourth the positive moment reinforcement in continuous members shall extend along the same face of member into the support. In beams, such reinforcement shall extend into the support at least 6 in.

12.11.2 – When a flexural member is part of a primary lateral load resisting system, positive moment reinforcement required to be extended into the support by Section 12.11.1 shall be anchored to develop the specified yield strength $f_y$ in tension at the face of support.

12.12 – Development of negative moment reinforcement

12.12.1 – Negative moment reinforcement in a continuous, restrained, or cantilever member, or in any member of a rigid frame, shall be anchored in or through the supporting member by embedment length, hooks, or mechanical anchorage.

12.12.2 – Negative moment reinforcement shall have an embedment length into the span as required by Sections 12.1 and 12.10.3.

12.12.3 – At least one-third the total tension reinforcement provided for negative moment at a support shall have an embedment length beyond the point of inflection not less than effective depth of member, $12d_o$, or one-sixteenth the clear span, whichever is greater.
11.10. Standard Cutoff or Bend Points for Bars in Approximately Equal Spans with Uniformly Distributed Loading
11.11. Shear Complication where a Bar is Cut off

(d) Shear Complications Where a bar is cut off, the rather sudden transfer of tension to the continuing bars causes early flexural cracking at the end of the bar. This flexural crack is wider than usual and turns more diagonally to become what is commonly called a shear crack (Fig. 5.4). Shear strength is lowered in a zone around the last portion of the terminated bar (maybe for 0.75d from the cutoff).

Because of this complication, Code 12.10.5 forbids cutting off bars unless one of three possible conditions is satisfied:

\[ V_n \leq \frac{2}{3} \left( \text{perm. Hf} \right) \]

\[ A_s \leq \frac{60b_w s}{f_y} \]

\[ S \leq \frac{d}{8\beta_b} \]

12.10.5.1 – Shear at the cutoff point does not exceed two-thirds that permitted, including shear strength of shear reinforcement provided.

12.10.5.2 – Stirrup area in excess of that required for shear and torsion is provided along each terminated bar or wire over a distance from the termination point equal to three-fourths the effective depth of member. Excess stirrup area \( A_s \) shall be not less than \( 60b_w s/f_y \). Spacing \( s \) shall not exceed \( d/8\beta_b \) where \( \beta_b \) is the ratio of area of reinforcement cut off to total area of tension reinforcement at the section.

12.10.5.3 – For #11 bar and smaller, continuing reinforcement provides double the area required for flexure at the cutoff point and shear does not exceed three-fourths that permitted.

12.10.6 – Adequate anchorage shall be provided for tension reinforcement in flexural members where reinforcement stress is not directly proportional to moment, such as: sloped, stepped, or tapered footings; brackets; deep flexural members; or members in which tension reinforcement is not parallel to compression face.
11.12. Special Requirement Near the Point of Zero Moment

If bars in question were fully stressed at a distance “a” to the right of the point of inflection (PI), and the moments diminish linearly to the point of inflection, as suggested by the dashed line, then pullout will not occur if the development length $l_d$ did not exceed the distance “a.”

$$l_d \leq \frac{M_n}{V_u} + l_a$$  \hspace{1cm} Eq. 12–1 of ACI

$$l_d \leq 1.3 \frac{M_n}{V_u} + l_a$$  \hspace{1cm} When ends of reinforcements are confined by compressive reaction as at the end of simply supported spans