I wondered why the baseball was getting bigger.
Then it hit me.

Objectives

- Understand the vector formulation for finding the component of a moment along an axis
- Understand the idea of a couple and the moment it produces
Couples

- A couple is a system of two forces
- The forces must satisfy the following conditions for the force system to be a couple
  - The magnitudes of the forces must be the same
  - The unit vectors of the forces must be opposite in direction

\[
F_1 = F_2 \quad \quad \vec{u}_1 = \left(-1\right)\vec{u}_2
\]
A couple produces pure rotation on a system.
The sum of the two forces will be equal to 0.

\[ F_1 = F_2 \quad \vec{u}_1 = (-1) \vec{u}_2 \]

Most importantly, a couple produces the same amount of rotation about any moment center.

\[ F_1 = F_2 \quad \vec{u}_1 = (-1) \vec{u}_2 \]
We will start with two forces in space who satisfy the conditions of a couple:

\[ F_1 = F_2 \]
\[ \vec{u}_1 = (-1) \vec{u}_2 \]

We will then pick some point in space as the moment center:

\[ F_1 = F_2 \]
\[ \vec{u}_1 = (-1) \vec{u}_2 \]
Couples

- We can pick a point on each force's line of action and construct moment arms from point a to a point on the life of action on each of these forces.

\[
\begin{align*}
F_1 &= F_2 \\
\mathbf{u}_1 &= (-1) \mathbf{u}_2
\end{align*}
\]

- The total moment generated by the two forces about a is equal to

\[
M_a = r_{ab} \times F_1 + r_{ac} \times F_2
\]
Substituting the definition of $F_1$ and $F_2$

$$\overrightarrow{M_a} = \overrightarrow{r_{ab}} \otimes \left( \overrightarrow{u_1 F_1} \right) + \overrightarrow{r_{ac}} \otimes \left( \overrightarrow{u_2 F_2} \right)$$

Using our definition of the magnitudes in a couple

$$\overrightarrow{M_a} = \overrightarrow{r_{ab}} \otimes \left( \overrightarrow{u_1 F_1} \right) + \overrightarrow{r_{ac}} \otimes \left( \overrightarrow{u_2 F_1} \right)$$
And the relationship of the unit vectors

\[ \mathbf{M}_a = r_{ab} \times (\mathbf{u}_1 \mathbf{F}_1) + r_{ac} \times (-\mathbf{u}_1 \mathbf{F}_1) \]

Manipulating the cross product

\[ \mathbf{M}_a = (r_{ab} - r_{ac}) \times (\mathbf{u}_1 \mathbf{F}_1) \]
Now if we generate another position vector from \( c \) to \( b \) and by vector addition we can say

\[
\vec{r}_{ab} = \vec{r}_{ac} + \vec{r}_{cb}
\]

\[
\overrightarrow{M}_a = (\vec{r}_{ab} - \vec{r}_{ac}) \otimes (u_1 F_1)
\]

Rearranging terms

\[
\vec{r}_{ab} - \vec{r}_{ac} = \vec{r}_{cb}
\]

\[
\overrightarrow{M}_a = (\vec{r}_{ab} - \vec{r}_{ac}) \otimes (u_1 F_1)
\]
And substituting in the expression for the moment at a

\[ \overrightarrow{M_a} = (\overrightarrow{r_{cb}}) \otimes (\overrightarrow{u_1 F_1}) \]

\[ \overrightarrow{r_{ab}} - \overrightarrow{r_{ac}} = \overrightarrow{r_{cb}} \]

So no matter where a is, the moment of the couple will depend on how far apart the forces are

\[ \overrightarrow{M_a} = (\overrightarrow{r_{cb}}) \otimes (\overrightarrow{u_1 F_1}) \]
The moment of a couple can be calculated by taking a moment arm from line of action of one of the forces to the line of action of the other force and forming the cross product of the moment arm and the force that we went to.

In a two-dimensional problem, if you can find the perpendicular distance between the forces, you can calculate the magnitude of the moment by multiplying the perpendicular distance times the magnitude of either one of the forces. The sign will be given by the sense of rotation.
Problem F4-19

\[ \begin{align*}
A &\quad 400 \text{ N} \\
\quad &\quad \text{3 m} \\
\quad &\quad 200 \text{ N} \\
\quad &\quad 300 \text{ N} \\
\quad &\quad 0.2 \text{ m} \\
\quad &\quad 200 \text{ N} \\
\quad &\quad 0.3 \text{ m} \\
\end{align*} \]

Problem F4-24

\[ \begin{align*}
F_A &= 450 \text{ N} \\
A &\quad 0.4 \text{ m} \\
B &\quad 0.3 \text{ m} \\
F_B &= 450 \text{ N} \\
\end{align*} \]
Homework

- Problem 4-68 (Hint, remember that the perpendicular distance between the forces is one part of determining the magnitude of the couple-moment)
- Problem 4-71
- Problem 4-90