A Simulation Approach for Estimating Value at Risk in Transportation Infrastructure Investment Decisions

by

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Abstract

Traditional economic analysis techniques used in the assessment of Public Private Partnership (PPP) projects are based upon the assumption that future cash flows are fully deterministic in nature and are not designed to account for risks involved in the assessment of future returns. In reality, many of these infrastructure projects are associated with significant risks stemming from the lack of knowledge about future cost and benefit streams. The fundamental premise of the PPP concept is to efficiently allocate risks between the public and the private partner. The return based on deterministic analysis may not depict a true picture of future economic outcomes of a PPP project for the multiple agencies involved. This deficiency underscores the importance of risk-based economic analysis for such projects. In this paper, the authors present the concept of Value-at-Risk (VaR) as a measure of effectiveness (MOE) to assess the risk share for the public and private entity in a PPP project. Bootstrap simulation is used to generate the risk profile savings in vehicle operating cost, and in travel time resulting from demand-responsive traffic. The VaR for Internal Rate of Return (IRR) is determined for public and private entity. The methodology is applied to a case study involving such a joint venture in India, the Mumbai Pune

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Expressway/National Highway 4 (MPEW/NH4), and fiscal implications from the perspective of the public and the private entities are examined. A comparison between deterministic and risk based economic analysis for MPEW/NH4 is presented. Risk analysis provides insightful results on the economic and financial implications from each participant’s viewpoint.

Key words: value-at-risk, bootstrap simulation, economic and financial analysis, internal rate of return

1. Introduction

Transportation infrastructure investments typically undertaken by the public sector have recently attracted private entities, thereby forming a joint participation commonly referred to as Public Private Partnership (PPP). There are a number of reasons for the growing trend of private participation in public projects. These include among other things: scarcity of fiscal resources at the public sector level, the perception that the private sector is more efficient in managing large projects, and the ability to share risks, thereby reducing exposure levels to financial losses for both entities.

Most investment decisions share three important characteristics in varying degrees. First, the investment is generally irreversible in that the funds invested are completely “sunk” in the project. Thus the agencies responsible for managing the project must be fully committed to the project, once the investment is made. Second, there are uncertainties over the future outcome from the investment. One way to address this is to assess the probabilities of alternative outcomes representing varying degrees of profits or losses. The third characteristic is related to timing of the investment. With proper planning, investment decisions maybe postponed until credible information about future outcomes may be available. These three characteristics interact
to determine the optimal decision of investors (Weston and Brigham, 1976). Experts suggest that purely public and purely private delivery mechanisms are unreliable, unstable, averse to innovation, and hence undesirable (Miller and Evje, 1999). A disparity between infrastructure needs and limited public resources has given rise to an increasing use of PPP.

India (the subject case for this paper) has, in the past, used the traditional approach of road financing, where roads are treated as publicly owned/operated facilities, and are funded from a myriad of sources including general revenue, and road user taxes. Private sector financing is being sought increasingly to fund infrastructure programs; and tolls are being applied to generate revenues. However, private sector financing “cannot replace the role of the public sector, nor can it reduce the importance of rational, fair and transparent public financing system” (World Bank 2004, p. ii). Thus, joint public-private ventures appear to be the key to the financial success of such projects.

A recent World Bank Report shows that India currently has 3.5 million kilometers (km) of roads, of which approximately 170,000 km are under the national and state highway category (mostly two-lane facilities), representing modest design standards. The National Highway System in India totaling 58,000 km of two-lane facilities, carries 45 percent of total traffic (World Bank, 2004). In spite of significant public investment on roads by the Government of India, there is a great need today for high quality, high capacity highways to accommodate the ever-increasing traffic in India’s metropolitan areas.
2. Problem Statement

Traditional economic analysis techniques used in the assessment of PPP projects are based upon the assumption that future cash flows are fully deterministic in nature. Thus, these are not designed to account for risks involved in the assessment of future returns. In reality, many of these infrastructure projects are associated with significant risks stemming from a lack of knowledge about future cost and benefit streams. Such projects typically involve huge initial costs, take longer to complete and are reliant on future cash flows to meet financial obligations. Most of the economic analysis techniques that are used to compute future returns, fall into two categories, i.e. predictive (ex-ante) or evaluative (ex-post) (Boardman et al., 2005). Predictive analysis is used to forecast the likely economic impacts of a proposed investment, whereas evaluative techniques are used to gauge the effect of the investment after it has been implemented (Cambridge Systematics, 1998). The topic of this paper is ex-ante analysis.

The fundamental premise of the PPP concept is to efficiently allocate risks between the public and the private partner and to deliver the project at a lower total cost to the public. The economic and financial measures derived through deterministic analysis do not reflect possible risks. The purpose of the paper is twofold: first, to demonstrate the use of risk as a measure of effectiveness (MOE) in the assessment of infrastructure project feasibility; second to demonstrate the use of this MOE, using a real world case study.

3. Modeling Risk in PPP

Risk may be looked upon as the probability of occurrence of an undesirable outcome, and is defined in literature in many ways (Al-Bahar, 1988; Newman, 1983; Hammer, 1972; Lowrance, 1976; Petak and Atkisson, 1982; Chapman, 1991; Kerzner, 2005; Sanchez, 1998). While there are a
variety of methods to measure risk, the choice of one depends mostly on the objectives of the analysis to be performed. For an infrastructure project, the risk measure can be quantified by determining the combined effect of risks in traffic, economic factors, cash flow needs, construction and maintenance costs, etc. Examples of risk measurement techniques include risk probability of occurrence, volatility, risk on return of capital, and value at risk. Other forms of analysis such as sensitivity and stochastic analysis, measure the tradeoff on the economic outcome (in terms of Net Present Value (NPV) and Internal Rate of Return (IRR), etc.) by altering the effects of risk factors (traffic, toll, cost etc.).

Fig.1. Volatile Factors and Value-at-Risk
Risk can be quantified in different ways (Mun, 2006). The term “Value-at-Risk” (VaR) is one of such methods, and has been used as a decision tool for risk analysis in this paper. VaR can be defined as the maximum expected loss over a target horizon, with a given level of confidence (Jorion, 1997). VaR describes the quantile of the projected distributions of gains/losses over the target horizon. If \( \alpha \) is the selected confidence level, VaR corresponds to the \((1 - \alpha)\) lower tail level. For example, for 95 percent confidence level, VaR should be such that it exceeds 5 percent of the total number of observations in the distribution. VaR can be determined as the maximum expected loss over a target horizon, with a 95 percent level of confidence as the IRR at \( \alpha \) of 0.05 (Figure 1).

3.1 VaR Estimation Techniques: Bootstrap Simulation

With the ability to predict the consequences under different circumstances, the technique of simulation is being used increasingly to unveil the effects of risks on the MOE. The early VaR models are also referred as parametric because of the strong theoretical assumptions they impose on the underlying properties of the data\(^4\). One such assumption is that the density function of risk factors influencing asset returns must conform to the multivariate normal distribution\(^5\). Empirical evidence indicates that speculative asset price changes, especially the daily ones, are not necessarily normal.

The problems of earlier models spurred the search for better estimates of VaR. A number of recent VaR techniques are based on nonparametric statistical methods. Re-sampling techniques are often used to re-construct the distribution of a population starting from limited samples. The

\(^4\) Parametric VaR models are based on strong theoretical assumptions and rules. They impose that the distribution of the data (daily price changes) conforms to a known theoretical distribution.

\(^5\) The normality assumption is frequently used because the normal distribution is well described; it can be defined using only the first two moments (mean and standard deviation) and it can be understood easily. Other distributions can be used, but at a higher computational cost.
principle of bootstrap distribution uses re-sampling with replacement from the original sample (selected from a studied population) that results in a new distribution. An advantage of the bootstrap approach is that it can include unusual traits, such as fat tails, jumps, or other departures from normal distribution.

VaR can be computed once the price path is simulated using bootstrap, and the resulting MOE (say NPV or IRR) can be developed at the end of the selected horizon. The simulation can be carried out in the following steps.

- **Step-1**: Choose a stochastic process and parameters. Traffic volume is considered as the stochastic parameter as both cost (operation and maintenance cost) and benefit (revenue) elements are associated with it.

- **Step-2**: Generate random numbers of variables from which the MOEs are computed as \( S_{t+1}, S_{t+2}, \ldots, S_{t+n} \). In this case random numbers are generated for traffic volume variation.

- **Step-3**: Calculate the value of the infrastructure under this particular sequence of MOEs at the target horizon.

- **Step-4**: Repeat steps 2 and 3 for higher number of iterations

### 4. Methodology

The algorithm and step by step approach for VaR estimation is presented in Fig.2. The first step is to set up the risk model for desired MOE, say IRR in this case. For IRR estimation, the cost and benefit elements need to be estimated. As these are expected to vary over time, the next step is to determine the PDF of each variable. Next, the simulation cycle is started, say at \( t=0 \), with generating random numbers by bootstrap simulation. The idea is to draw samples from the past
historical data with repetition using random numbers. At $t=0$, the present worth of costs (PWOC) (discounted values of $C_t$) and benefits (PWOB) (Discounted values of $B_t$) are determined by assuming the Minimum Attractive Rate of Return (MARR) as the interest rate. IRR is the interest rate at which POWC is equal to PWOB. The IRR is recorded at $t=0$, and the process is repeated.
to estimate the IRR for \( t = t+1 \). The simulation cycle is run till the desired number of iterations is reached. Finally, the VaR for the simulation profile of IRR is estimated.

### 4.1 IRR as MOE

IRR provides an estimate of the return or yield of the investment, given a set of expenditure and revenue data along with their expected dates over the life of the project. It is the annualized compounded return rate derived from an investment comprising payments and earnings at different points/periods in time during the tenure of the project. IRR is defined as the interest rate at which the Net Present Worth (or Net Annual Worth or Net Future Worth) of the investment is equal to zero. The generic form of the model is:

\[
C_c + P_{om} + R_{om} = B_t + B_{voc} + B_{tt} + B_{sa}
\]  

(1)

where,

- \( C_c \): present worth (PW) of construction cost incurred prior to the implementation of the project
- \( P_{om} \): PW of periodic operation and maintenance (POM) incurred in different points in time
- \( R_{om} \): PW of regular operation and maintenance (ROM) incurred in every year
- \( B_t \): PW of benefit accrued from toll receipts in different years
- \( B_{voc} \): PW of benefit accrued from vehicle operating cost savings in different years
- \( B_{tt} \): PW of benefit accrued from travel time savings in different years
- \( B_{sa} \): PW of the amount received by “leasing the facility” to operate during the concession period

Equation (1) can be re-written as the following
\[
\sum_{n=1}^{p} \left[ (C^n_p) \left( \frac{F}{P} \right)_{n,i} + (P_{in}^n) \left( \frac{P}{F} \right)_{n,i} + (R_{im}^n) \left( \frac{P}{F} \right)_{n,i} \right] = \\
\sum_{n=1}^{p} \left[ (B^n_i + B^n_{coc} + B^n_{ni} + B^n_{mi}) \left( \frac{P}{F} \right)_{n,i} \right]
\]

(1)

where,

\( p \): project life in years

\( n \): year under consideration

\( i \): minimum attractive rate of return (MARR)

\( x \): periodic interval investment in POM

\( \left( \frac{F}{P} \right) \): compound amount factor for year \( n \) and a particular MARR

\( \left( \frac{P}{F} \right) \): present worth factor for year \( n \) and a particular MARR

Since the cash flow elements described above are incurred at different points / periods in time, appropriate interest factors are to be applied to convert them to their PW equivalent. The specific interest factors would depend upon the exact nature and time frame of the cost/benefit items and are not shown in the generic model in Equation 1. A heuristic procedure is used to estimate \( i \) at which the NPW approaches a zero value. This is accomplished by systematically changing the value of \( i \) in Equation 1 until convergence is reached.

Further, the IRR derived using Equation 1 must be at least equal to the MARR for the project to be viable, where MARR is the rate of return below which the investment proposal is to be deemed unacceptable (Blank and Tarquin, 2005). The MARR value is based upon
predominant lending rates and economic conditions, and is often treated as an exogenous variable, a detailed discussion of which is beyond the scope of this paper. It is however conceivable, that the public and private entities may identify different MARR’s as a benchmark of accepting investment proposals.

4.1.1 Economic Internal Rate of Return (EIRR) vs. Financial Internal Rate of Return (FIRR)

The IRR technique has been used in the literature to evaluate project viability. Examples include decision economic models for parking facility planning in urban locations (Merino, 1989), ranking of the transportation projects by their financial rate of return (Bonnafoius and Jensen, 2005), sensitivity analysis of various transportation investment projects (Borgonovo and Peccati, 2005), and testing the feasibility of multiple agency development projects (Khasnabis et al., 1982; Khasnabis et al., 2010).

For joint public private projects, questions are often raised about the inclusion of externalities, or social costs/benefits such as: environment damages, pollution, savings in travel time, and in travel cost, etc., that are not reflected in the markets (Johnson and Kasarda, 2007). It is customary to ignore these externalities in any analysis dealing with the private sector, as these do not affect its decision. The term used for this return is Financial Internal Rate of Return (FIRR), where only direct expenditures and revenues are included. For the public sector, two sets of returns are generally estimated. First, the FIRR is used to benchmark public sector performance with that of the private sector. An additional analysis that includes the externalities is conducted to estimate the Economic Internal Rate of Return (EIRR). Further discussion on this topic is available in the literature (ADB, 2000).
4.2 Procedure for Estimation of Costs and Benefits

Vehicle operating cost (VOC) and savings in travel time (TT) are the critical elements in estimating costs and benefits. For Indian highways, the formulation for VOC and TT savings are adapted from the Central Road Research Institute (CRRI), India, and reported by the authors earlier (Khasnabis et al., 2010). The deterministic part of the analysis is presented in this paper for continuity, and is adapted from the authors’ earlier work. The risk-based part of the analysis presented represents the authors’ continued research efforts on this topic. The details of VOC and TT estimation are presented in Appendix-A.

Based on the historical traffic volume data over five years, the future data was extrapolated by vehicle class (Car, LCV, truck, bus, 3-axle, and multi-axle). In the extrapolation procedure, the data for a given year depends upon that for the past year. For the risk model, the variation in traffic volume is drawn from a range with an upper bound (positive 20 percent) and a lower bound (negative 20 percent). In the stochastic process, the random traffic volume drawn from the range is not deterministic, the volume drawn at time $t=0$, is different from volume that drawn from time $t= t+1$.

5. Case study of MPEW and NH4

Mumbai is the commercial and financial capital of India with a population of more than 15 million. Pune, with a population of over 5 million is a major urban center in the state of Maharashtra, and is growing into a major industrial and commercial center, being the automotive capital of India. Hence, the importance of Mumbai-Pune travel corridor increased tremendously in last decade. The travel demand is currently served by a multimodal system comprising rail, air
and highways. The road traffic demand warrants a ten-lane system between the two cities (MSRDC, 2007).

5.1 Background Information

For a long time, the National Highway 4 (NH4) was the only available roadway connecting the cities of Mumbai and Pune. This section of NH4, a two-lane roadway built to modest design standards, is a part of the National Highway System and is one of the most congested facilities in the country. The poor operating condition on NH4 made it necessary for the Government of Maharashtra to build an independent toll facility the Mumbai Pune expressway (MPEW), virtually parallel to NH4 to meet the increasing travel demands between the two cities. MPEW is a six-lane facility with high design standards built in the year 2000 and was authorized to collect tolls from road users (MSRDC, 2007). Mahararastra State Road Development Corporation (MSRDC) invested $525 million, over a four year period to construct the 95 km long six-lane carriageway using high design standards and modern machineries. A foreign exchange conversion rate of 40 Indian rupees per US dollar was used in this paper. MPEW was opened to traffic as a toll facility in the year 2001 (Fig. 3).

In the year 2004, MSRDC entered into an agreement with a private entity to operate, maintain and collect tolls on MPEW and to rebuild, maintain, operate and collect tolls on NH4, both for a period of 15 years (2005-2019). Further, the private operator paid MSRDC $225 million (900 crores of Indian rupees) and agreed to invest necessary capital to upgrade NH4 to a four-lane facility.

During the lease period, the private entity was required to widen NH4 from two lanes to four lanes and to open the facility to traffic in 2007 (Fig. 3). MPEW and NH4 are to be delivered back to MSRDC and the Government of Maharashtra respectively in 2020 in fully operational
condition. MSRDC is expected to resume its role of operating and maintaining MPEW for the remainder of its service life up to the year 2030. Since NH4 is an older facility, it was assumed for the purpose of this case study, that NH4 will not have any further life left after 2019. Realistically however, to meet the traffic demand, the Government may be required to make a significant investment to keep the facility operational. No decision on the disposition of NH4 beyond 2019 has been made at this point.

**Fig. 3.** MPEW and NH4 between Mumbai and Pune (MSRDC 2007)

Table 1 shows the traffic and other key features of the two facilities obtained mostly from MSRDC. Six categories of vehicles with different toll rates and operating characteristics were considered: Car, Light Commercial Vehicles (LCV), Small Truck, Bus, 3 Axle (3AX) and Multi
Axle (MA) trucks. LCV’s, small trucks, 3AX and MA trucks are generally used for long haul freight transportation. The equivalent $525 million (2100 crores of Indian rupees) was invested in the MPEW facility during the period 1997-2000. Per MSRDC estimates, approximately 20 percent of the capital cost of $525 million was incurred, each at the end of the first and fourth year and 30 percent was incurred, each at the end of the second and third year.

The cost of improving NH4 to a four lane facility was estimated as the equivalent of $100 million, mostly invested in 2004.

### Table 1. Typical Traffic Characteristics of the Mumbai - Pune Expressway and NH-4 Section*

<table>
<thead>
<tr>
<th>Facility</th>
<th>AADT*</th>
<th>Traffic Growth (%)</th>
<th>Speed Limit (km/hr)</th>
<th>Toll Rate ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car</td>
<td>MPEW (2001)</td>
<td>NH4 (2005)</td>
<td>MPEW</td>
<td>NH4</td>
</tr>
<tr>
<td></td>
<td>8,416</td>
<td>7,740</td>
<td>5.7</td>
<td>2.73</td>
</tr>
<tr>
<td>LCV</td>
<td>924</td>
<td>3,434</td>
<td>9.47</td>
<td>2.65</td>
</tr>
<tr>
<td>Truck</td>
<td>3,348</td>
<td>3,394</td>
<td>-8.08</td>
<td>1.94</td>
</tr>
<tr>
<td>Bus</td>
<td>1,209</td>
<td>1,721</td>
<td>0.07</td>
<td>2.78</td>
</tr>
<tr>
<td>3AX</td>
<td>321</td>
<td>560</td>
<td>1.45</td>
<td>7.47</td>
</tr>
<tr>
<td>MA</td>
<td>122</td>
<td>463</td>
<td>3.38</td>
<td>20.89</td>
</tr>
</tbody>
</table>

Note: * Data presented in Table 1 is from the year 2005; **Annual Average Daily Traffic.

### 5.2 Private Entity Perspective

The objective of the private entity is to maximize its revenue by way of the toll collected, and thereby derive a healthy profit. The financial aspects of the private entity for two cases analyzed are presented in Table 2.

- First, the private entity was involved in MPEW operation in the year 2005 by making a capital investment of $225 million, with license to operate and maintain the facility till 2019. The operation and maintenance costs, regular, and periodic and toll value for the first year (2005) for MPEW are shown in the first row of Table 2. The investment in operation and maintenance cost consists of two components: (1) regular operation and
Table 2. Financial Aspects of the Private Entity (Base Condition)

<table>
<thead>
<tr>
<th>Project</th>
<th>Initial Cost (Million $)</th>
<th>Periodic Investment at every 5th year (Million $)</th>
<th>Regular Operation and Maintenance Cost (Million $)</th>
<th>Project Life</th>
<th>Toll for first year (Million $)</th>
</tr>
</thead>
</table>

- maintenance cost, (2) periodic operation and maintenance cost. Regular operation and maintenance cost is demand responsive, hence sensitive to the traffic volume, and estimated from the equations shown in Appendix-1. Periodic operation and maintenance cost is assumed as 10 percent of the initial cost and take effect in every five years.

- Second, the private entity spent $100 million in rebuilding of NH4, and was authorized to collect toll till 2019. The rebuilding of NH4 was necessary to make it an attractive facility as MPEW.

5.3 Public Entity Perspective

The objective of the public entity is to maximize social welfare, which include benefits received from travel time savings and road user costs, in addition to the tangible benefits received from toll. Cost and benefit elements of four different cases analyzed are presented in Table 3.

- Case (a) represents MPEW current operation with an initial investment of $525 million during the four year (1997 to 2001) of construction period, an operating and maintenance (O & M) cost of $7.43 million per year, and $52.5 million of periodic O & M cost (for
every fifth year). The project life of MPEW is 30 years. In the year 2005, MPEW was “sold” to a private entity for $225 million to operate and maintain the facility from 2005 to 2019. MPEW is expected to generate savings in VOC and VOT along with toll charges, shown of Table 3 for the first year.

- Case (b) represents rebuilding of NH4 to make it a competitive facility as MPEW. The public entity was not responsible for any investment, and nor does it derive any benefits from toll. But, the public entity derives the savings in VOC and in TT without any investment.

- Case (c) is a hypothetical scenario of the public entity continuing to operate MPEW without any private investment whatsoever from 2001 to 2030. In this case, the public entity is responsible for all expenditures and system operation.

- Case (d) is also a hypothetical scenario for the public entity that would allow the private entity to continue the operation of MPEW beyond the year 2019, up to the year 2030. The selling amount needs to be determined, which is presented later in section 5.4.2.1.

- Case (e) and (f) are two hypothetical scenarios considered to experiment on few “what-if” type analysis in the investment decision making. Case (e) is a combination of Case (a) and (b); and Case (f) is a combination of Case (b) and (c).
<table>
<thead>
<tr>
<th>Case</th>
<th>Project</th>
<th>Initial Cost (Million $)</th>
<th>Periodic Investment at every 5th year (Million $)</th>
<th>Regular Operation and Maintenance Cost (1st Year) (Million $)</th>
<th>Project Life</th>
<th>Selling Amount (Million $)</th>
<th>Savings in VOC (1st Year) (Million $)</th>
<th>Savings in VOT (1st Year) (Million $)</th>
<th>Toll (1st Year) (Million $)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b)</td>
<td>NH4 (Current operation)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>2005-2019</td>
<td>-</td>
<td>32.42 (2005)</td>
<td>15.32 (2005)</td>
<td>-</td>
</tr>
</tbody>
</table>

Note:

a, b: Represents current operation

c: Represents a hypothetical scenario, where the private sector is not involved at all, and the public entity is responsible for all expenditures and system operation

d: Represents a hypothetical scenario, where the private sector is responsible for all expenditures and system operation, but it provides onetime payment at two stages 2005 and 2019
5.4 Financial and Economic Evaluation

The financial and economic evaluation for the private and public entity is presented in below. For IRR determination, a MARR of 12 percent is assumed.

5.4.1 Private Entity Perspective

The FIRR for the private entity is presented in two cases: (1) Deterministic Financial Evaluation, and (2) Risk Based Financial Evaluation.

5.4.1.1 Deterministic Financial Evaluation

The deterministic FIRR is based upon the assumption of the fixed future costs, returns, and traffic volumes (Table 4). The FIRR’s for MPEW and NH4 are 12.77 percent and 30.35 percent respectively. The NH4 project generates much higher return, because it requires much smaller investment. Case (d) representing a combination of the two projects produces an FIRR of 20.21 percent.

Table 4: (VaR) IRR for Private Entity

<table>
<thead>
<tr>
<th>Case</th>
<th>Project</th>
<th>Deterministic FIRR (%)</th>
<th>(VaR)0.05 FIRR (%)</th>
<th>(VaR)0.10 FIRR (%)</th>
<th>Difference in IRR for (VaR)0.05 (%)</th>
<th>Difference in IRR for (VaR)0.10 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>MPEW</td>
<td>12.77</td>
<td>11.03</td>
<td>11.2</td>
<td>1.74</td>
<td>1.57</td>
</tr>
<tr>
<td>(b)</td>
<td>NH4</td>
<td>33.05</td>
<td>30.08</td>
<td>30.97</td>
<td>2.97</td>
<td>2.08</td>
</tr>
<tr>
<td>(c)</td>
<td>Combination (a + b)</td>
<td>20.21</td>
<td>17.81</td>
<td>17.95</td>
<td>2.4</td>
<td>2.26</td>
</tr>
</tbody>
</table>
5.4.1.2. Risk Based Economic Evaluation

A risk-based financial evaluation represents situations where the future costs, returns, and traffic volume are not fully defined, and may vary in the future. In the case study, the traffic volume being a risk variable is assumed to vary by 20 percent (in both directions; higher and lower) from the expected value.

A bootstrap simulation approach is adapted to draw random seeds with repetition from the previously known traffic volume. As discussed earlier, bootstrap simulation does not assume any predefined probability distribution of the risk variable, and may thus reflect a realistic behavior of the MOE. A total of 1,000 simulation runs were performed and the FIRR value is recorded. Fig.4 shows simulation results for MPEW. The expected IRR is shown on the X-axis, the frequency on the primary Y-axis, and the cumulative probabilities on secondary Y-axis. To determine the 95th percent confidence level IRR, an imaginary horizontal line can be drawn from the 5 percent of the secondary Y-axis to the cumulative probability distribution profile. Further, a vertical line can be drawn to the X-axis, to determine the 95th percent confidence level IRR to be 11.03 percent. The VaR for MPEW extension is 11.03 percent. In other words, the maximum expected loss (or the lower level) in IRR can be 11.03 percent. The 95th percent confidence level value is also shown in Table 4. The deterministic IRR for MPEW was 12.77 percent which made the facility financially viable considering MARR of 12 percent. The introduction of risk into the IRR estimation made the IRR lower than MARR. The 95th percent confidence level VaR for NH4is 30.08 percent and the combined case is 17.95 percent. Similarly, the 90th percent confidence level VaR for all three cases is presented in Table 4. As the confidence level decreases the VaR become larger.
The last two columns of Table 4 show the differences between the two sets of IRR’s representing deterministic and risk-based analysis. For Case (a), the 95\textsuperscript{th} percent confidence level difference in IRR from its deterministic counterpart is 1.74\% percent, which suggests that the maximum loss in IRR at 95\textsuperscript{th} percent confidence level level of confidence can not exceed 1.74\%. Similarly difference in IRR for VaR at 90\textsuperscript{th} percent confidence level for Case (a) is 1.57\%. It is observed that the difference in deterministic and risk based return is higher when the absolute FIRR is higher, representing higher risk in high FIRR.

**Fig. 4.** Value-at-Risk for MPEW Private Case Only
5.4.2 Public Entity Perspective

Results of economic analysis for the public entity are presented in two cases: (1) Deterministic Economic Evaluation, and (2) Risk Based Economic Evaluation.

5.4.2.1 Deterministic Economic Evaluation

The deterministic FIRR-values for cases (a), (b), (c), (d), (e), and (f) vary from a low of 5.07 percent to 7.05 percent, and are lower than the MARR of 12 percent. The EIRR has a range of 13.95 percent to 17.89 percent, all exceeding the MARR of 12 percent. Since the public entity did not invest any money on NH4 rebuilding, and does not receive any toll from NH4, the FIRR calculation is not applicable (NA). The public entity, on the other hand, derives the savings in VOC and in TT without any investment so that corresponding EIRR is very high or infinity.

For Case (c), the expected FIRR and EIRR for the public entity (7.05 percent and 14.75 percent respectively), are both considerably higher than those for case (a). This is expected, as the public entity must be ready to share the project returns with its partners to attract private capital. The differential between the two sets of EIRR’s/FIRR’s can be looked upon as the
Table 5: (VaR) IRR for Public Entity

<table>
<thead>
<tr>
<th>Case</th>
<th>Project</th>
<th>Deterministic</th>
<th>(VaR)$_{0.05}$</th>
<th>(VaR)$_{0.10}$</th>
<th>Difference in IRR for (VaR)$_{0.05}$</th>
<th>Difference in IRR for (VaR)$_{0.10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>MPEW (Current operation)</td>
<td>5.07</td>
<td>4.68</td>
<td>4.77</td>
<td>0.39</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td>13.95</td>
<td>13.68</td>
<td>13.74</td>
<td>0.27</td>
<td>0.11</td>
</tr>
<tr>
<td>(b)</td>
<td>NH4 (Current operation)</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>(c)</td>
<td>MPEW (No private involvement)</td>
<td>7.05</td>
<td>6.69</td>
<td>6.75</td>
<td>0.36</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td></td>
<td>14.75</td>
<td>13.62</td>
<td>13.69</td>
<td>1.13</td>
<td>1.05</td>
</tr>
<tr>
<td>(d)</td>
<td>MPEW* (Continued private operation)</td>
<td>5.07</td>
<td>5.07</td>
<td>5.07</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>13.95</td>
<td>13.79</td>
<td>13.84</td>
<td>0.16</td>
<td>0.11</td>
</tr>
<tr>
<td>(e)</td>
<td>Combination (a + b)</td>
<td>5.07</td>
<td>4.65</td>
<td>4.74</td>
<td>0.42</td>
<td>0.31</td>
</tr>
<tr>
<td></td>
<td></td>
<td>16.32</td>
<td>14.84</td>
<td>14.90</td>
<td>1.48</td>
<td>1.42</td>
</tr>
<tr>
<td>(f)</td>
<td>Combination (b + c)</td>
<td>7.05</td>
<td>6.63</td>
<td>6.74</td>
<td>0.42</td>
<td>0.31</td>
</tr>
<tr>
<td></td>
<td></td>
<td>17.89</td>
<td>15.37</td>
<td>15.84</td>
<td>2.52</td>
<td>2.05</td>
</tr>
</tbody>
</table>

*: the minimum “asking price” by the public entity will be $524.31 millions in the year 2019 for it to earn the same FIRR of 5.07 percent
“price” paid by the public entity to secure private participation. The FIRR and EIRR values for different cases shown in Table 5 follow logical trends (Khasnabis et.al 2010).

5.4.2.2 Risk Based Economic Evaluation

In the risk based economic evaluation, traffic volume is considered as the risk variable. Both the benefit and cost elements are dependent on traffic volume. The savings in VOC, and TT will vary with changes in traffic volume. The 95th percent confidence level VaR values for FIRR and EIRR for Case (a) are 4.68 percent and 13.68 percent respectively, compared to 5.07 percent and 13.95 percent for deterministic scenarios (Table 5). The reduction in FIRR and EIRR can be attributed to future risks associated with traffic volumes. Similar numbers for the 90th percent confidence level VaR for FIRR and EIRR for Case (a) are presented in Table 5.

A VaR for FIRR for Case (b) is not applicable for reasons cited earlier. The public entity, on the other hand, derives the savings in VOC and in TT without any investment, so that corresponding EIRR is very high or infinity. The 95th percent confidence level VaR values for FIRR and EIRR for Case (c) are 6.69 percent and 13.62 percent respectively. Even with the future risks involved, it is observed that both EIRR would be viable (greater than MARR) for the public entity. Similar figures for the 90th percent confidence level VaR are shown in Table 5.

A VaR for FIRR for Case (b) is not applicable for reasons cited earlier. On the other hand, the public entity derives the savings in VOC and in TT without any investment, so that corresponding EIRR is very high or infinity. The 95th percent confidence level VaR for FIRR for Case (d) is 5.07 percent which is the same as the deterministic FIRR. This is because the public entity was not involved in any risk in the FIRR estimation. The public entity in this case received
a premium of $225 million, but does not receive any share on toll during the concession period. In contrast, the 95th percent confidence level VaR for EIRR for Case (d) is 13.79 percent, which is lower than the deterministic EIRR, as expected. For Case (e), the 95th percent confidence level VaR values for EIRR and FIRR are 6.63 percent and 15.37 percent respectively (Table 5). Similar results for the 90th percent confidence level VaR are shown in Table 5. For Case (f), the VaR values of FIRR and EIRR are the highest. The combination of current NH4 operation and the hypothetical scenario of MPEW without any private involvement would have resulted in 6.63 percent and 15.37 percent of FIRR and EIRR respectively for the public entity. Similar numbers for the 90th percent confidence level values are shown in Table 5. The differences between the deterministic and risk-based IRR’s for various combinations are also shown in Table 5.

5.5. Effect of Risk on Pay-Back Period

Fig. 5 shows a typical diagram of the relationship between NPV, IRR, break-even period, and pay-back period. Fig. 5 represents the cash flow diagram for the MPEW operation for the private entity. The X-axis represents the time period of MPEW operation. The NPV is shown in primary Y-axis, where the bars represent the net present value for the particular year in question. The IRR is shown in secondary Y-axis, and the series represents growth of IRR over time. The break-even period is achieved in the year 2018, and by end of 2019 (pay-back period) the IRR for private entity is 11.2 percent.
In the risk based analysis for the MPEW, the private entity can only obtain an IRR of 11.2 percent, which is lower than the MARR of 12 percent, during the concession period of 15 years (end of 2019). To obtain an IRR of 12 percent, the pre-determined concession period of 15 years should be extended. For demonstration purposes, the risk based assessment of pay-back period further analyzed. A series of IRR’s is used as the interest rate to compute the return on investment as a function of number of years. Fig.6 represents the pay-back period for a series of IRR values.

**Fig.5.** IRR, break-even period, and pay-back period

In the risk based analysis for the MPEW, the private entity can only obtain an IRR of 11.2 percent, which is lower than the MARR of 12 percent, during the concession period of 15 years (end of 2019). To obtain an IRR of 12 percent, the pre-determined concession period of 15 years should be extended. For demonstration purposes, the risk based assessment of pay-back period further analyzed. A series of IRR’s is used as the interest rate to compute the return on investment as a function of number of years. Fig.6 represents the pay-back period for a series of IRR values.
First, the 90\textsuperscript{th} percent confidence level FIRR for the MPEW private entity is used as the interest rate to compute the payback period. The simulation model is set up to produce a payback period with varying traffic volume on MPEW, with the assumed interest rate. In Fig. 6, each series represents the probability of obtaining the payback period with a given IRR. For example, Fig. 6 shows that there is a 90 percent probability that MPEW will produce 11.2 percent IRR at the end of 2019. In other words, the private entity can only obtain a 11.2 percent IRR in the year 2019 if the traffic volume 20 percent from the expected value (which is also observed in Table 4).

Fig. 6 can be also used to compute the probability of obtaining a desired IRR and the corresponding pay-back period. For example, under the prescribed risk scenario, the probability
Table 6: Cumulative Probability of IRR Realization and Pay-Back Period of MPEW Private Entity

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>IRR-12</td>
<td>0</td>
<td>0</td>
<td>0.028</td>
<td>0.186</td>
<td>0.547</td>
<td>0.697</td>
<td>0.820</td>
<td>0.944</td>
<td>0.995</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>IRR-13</td>
<td>0</td>
<td>0.001</td>
<td>0.014</td>
<td>0.129</td>
<td>0.218</td>
<td>0.332</td>
<td>0.625</td>
<td>0.88</td>
<td>0.977</td>
<td>0.998</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>IRR-14</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.002</td>
<td>0.011</td>
<td>0.028</td>
<td>0.101</td>
<td>0.309</td>
<td>0.609</td>
<td>0.845</td>
<td>0.961</td>
<td>0.997</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Note*: 0.5 represents six months of the year in operation
of obtaining a 12 percent IRR by the private entity at the end of 2019 is 0.82. It is evident from figure 6 that as the IRR increases, the probability of obtaining higher returns in the assigned pay-back period diminishes, and vice versa.

Table 6 represents the probability distribution for different combinations of IRR’s and pay-back periods. For example, an IRR of 12 percent can be obtained at the end of 2020 with a probability of 99.5 percent. All other entries in Table 6 can be interpreted similarly. It is observed that as the expectation on return of investment increases, the required pay-back period increases as well. The probability path of obtaining an IRR over a time horizon shown in Table 6 might be a useful tool to assess the pay-back period with risk.

6. Conclusion

In this paper, a simulation model for determining VaR, a risk-based MOE, is proposed. The model can incorporate the complex impact of different risk variables that could affect the economic and financial outcome a PPP project. A case study involving the MPEW and NH4 in India is presented to examine the consequence of PPP in a major infrastructure project. The case study is presented in from the public and private entity viewpoints, by using deterministic and risk based economic and financial analysis. For MPEW project, the deterministic analysis shows private entity would have obtained 12.77 percent FIRR, during the concession period. While the risk analysis shows that the FIRR would have been 11.3 percent. However, an increase in the concession period would result in an increase in FIRR. With the help of risk analysis, the private entity can request either an increase in the concession period, or a cost subsidy from the public entity.
In contrast, the public entity, by transferring the operational rights of the two facilities, loses the opportunity for earning a sustained level of revenue. However even with the income foregone through future toll charges, the public sector earns a 5.07 percent FIRR and 16.32 percent EIRR based upon a deterministic analysis. Considering that a primary mission of the public sector is to derive social benefits from public investment, a 16.32 percent EIRR appears to be an excellent investment. The risk analysis shows that the public entity would have obtained 4.74 percent of FIRR and 14.9 percent of EIRR. Various combinations of VaR’s, both deterministic and risk-based, are presented in the paper for the private and public entity. The VaR analysis shows the risk elements of the cost and benefit elements may change the IRR of the investment in the PPP project.

Finally, a model demonstrating the relationship between pay-back period and IRR is presented for the MPEW private entity case. The risk-based analysis demonstrate that the private entity will not be able to obtain the expected IRR in the pre-defined timeline of 15 years (end of 2019); rather it will additional 1.5 years, to make the pay-back period at the end of 2020 and six months. Similarly the simulation model is used to determine the pay-back period for higher expected IRR of 13 percent and 14 percent. Similar pay-back period analysis for public entity is not presented for brevity.

The proposed simulation model could provide entities involved in PPP projects with useful information for determining risk based IRR and establishing an optimal pay-back period. Further improvement may be needed to make the simulation process more practical by including risk factors such as variation in interest rate, equity shares, inflation rates, and other relevant variables. Further research shall be carried out using the proposed simulation model to examine the decision making process from each participant viewpoint in the PPP project.
Appendix-A

Vehicle Operating Cost (VOC)

The VOC values for each mode are determined by the pavement roughness, rise and fall, and the carriageway width available for movement of traffic for various road types and can be estimated from the following equation:

\[ \log_{e}^{\text{VOC}} = c_1 + c_2 R_t + c_3 RF + c_4 W \]  \hspace{1cm} (2)

Where

- \( R_t \): roughness at time period \( t \)
- \( RF \): rise and fall in meters / km
- \( W \): width of carriageway in meters
- \( c_1, c_2, c_3, c_4 \): calibrated parameters to estimate vehicle operating cost

Further, roughness contains other deterioration characteristics such as cracking, rutting, patching, raveling and potholing. The roughness model adapted from CRRI is described below (CRRI 2001).

\[ R_t = R_0 + \phi N_t \]  \hspace{1cm} (3)

\[ \phi = \frac{1250}{\text{antilog}_{10} \left( a^{1/3} - b^{1/3} + 1.3841 \right)} \]  \hspace{1cm} (4)

\[ a = 0.20209 + 23.1318 C^2 - 4.809 C \]  \hspace{1cm} (5)

\[ b = 0.20209 + 23.1318 C^2 + 4.809 C \]  \hspace{1cm} (6)

\[ C = 2.1989 - MSN \]  \hspace{1cm} (7)

\[ MSN = \sum_{a=1}^{N} a_d D_a + 3.51 \times \log_{10}^{\text{CBR}} - 0.85 \left( \log_{10}^{\text{CBR}} \right)^2 - 1.43 \]  \hspace{1cm} (8)

\[ N_t = 10^{-6} \sum_{i=1}^{M} y_i^m \gamma_i^m l_d \]  \hspace{1cm} (9)
Where,

- $R_0$: initial roughness
- $N_t$: number of millions of standard axles at time period $t$
- $a, b,$ and $C$: parameters to calculate $\phi$
- MSN: modified structural number
- $\phi$: estimated coefficient for calculation of $R_t$
- $a_\omega$: the strength coefficient for layer $\omega$
- $D_\omega$: the thickness of layer $\omega$ in inches
- $m_i$: vehicular mode $i$, where $i = 1, 2, \ldots, M$
- $N$: total number of layers in the pavement
- $CBR$: California Bearing Ratio of the subgrade
- $y_{it}^{m_i}$: annual traffic volume for mode $m_i$ at year $t$
- $\gamma_{m_i}^f$: truck factor for mode $m_i$
- $l_d$: lane distribution factor

Equation 3 is used to obtain the roughness of the pavement surface. The values of initial roughness, number of millions of standard axles and its coefficients are required to calculate the parameter $\phi$ represented in Equation 4. Equations 5-7 are applied to estimate $\phi$. Equation 7 requires Modified Structural Number (MSN) to calculate $C$. The MSN (Equation 8) is a pavement characteristic which is a function of depth of pavement layers, layer coefficients and California Bearing Ratio (CBR). Equation 9 determines the number of Millions of Standard Axles (MSA), to be used in Equation 3. Traffic consists of various modes; so truck factors are used to convert all modes to their equivalent number of standard axles. The annual traffic volume
is multiplied by corresponding truck factors (for Indian conditions obtained from CRRI) and lane
distribution factor to obtain MSA.

*Travel Time (TT) Savings*

Monetary value of travel time incurred for each mode to traverse a facility can be estimated as:

\[ T_t = \sum_{i=1}^{M} v_{i}^{m} o_{i}^{m} t_{i}^{m} \]  

(10)

Where:

\( T_t \): total value of travel time for all modes at the end of year \( t \)

\( v_{i}^{m} \): value of travel time per hour for mode \( m_i \)

\( o_{i}^{m} \): occupancy of mode \( m_i \)

\( t_{i}^{m} \): travel time needed to traverse the facility for mode \( m_i \)

\( i \): type of mode; 1, 2, ..., M

TT saved for a proposed facility can be calculated as the difference between the \( T_t \) –
values between the existing facility and the proposed facility.
References

        <http://www.adb.org/Documents/Handbooks/PIA_Eco_Analysis/glossary_ref.pdf>
        (Dec. 2, 2006).

        Approach for Contractors, University of California, Berkeley.


        Concepts and Practice. Upper Saddle River, NJ.

Bonnafois, A., and Jensen, P., 2005. Ranking transport projects by their socioeconomic value or
        financial internal rate of return. Transportation Policy, 12(1), 131-136.


        SPON, London.

CRRI., 2001. Updation of Road User Cost Data. Traffic and Transportation Division, Central
        Road Research Institute (CRRI), New Delhi, India.


        (February 12, 2011).


Kerzner, H., 2005. Project Management: A Systems Approach to Planning, Scheduling, and
        Controlling, John Wiley & Sons, Inc. New York, NY, USA.


