A Model Framework for Analyzing Public Transit Connectivity and Its Application in a Large-scale Multi-modal Transit Network

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Word Count: 4,804 (Excluding References)  
Total Count: = 4,804+ (10x 250) =7,304  
Date Submitted: July 31, 2014

Submitted for consideration for presentation at the 94th Transportation Research Board Annual Meeting and Publication in the Transportation Research Record
ABSTRACT

Public transportation is essential for mobility of users especially in urban areas where transit connectivity plays a crucial role in achieving acceptable level of service. Improvement of transit connectivity is also a critical component of transit-operations planning. The objective of this research is to develop a unique approach to measure transit connectivity that does not require detailed socio-economic, demographic, transit ridership data and transit assignment models. The methodology incorporates a graph theory approach to determine the performance of large-scale multimodal transit networks by quantifying measures of connectivity at multiple levels such as transit stops, links and lines. It also considers the unique qualities of each transit line and stop, as well as their accessibility when developing a single connectivity index. The methodology is applied (as a case study) in two urban areas to determine public transit connectivity of their large multimodal transit system using General Transit Feed Specification (GTFS) data. The new connectivity index significantly extends the set of performance analysis tools that decision-makers can use to assess the level of service and improve efficiency of the transit system.
INTRODUCTION

Transit connectivity is critical in ensuring an acceptable level of service and is especially crucial for captive users, as transit is their only means of transportation. Federal, state and local transportation agencies have been putting significant effort to enhance multimodal connectivity in urban and rural network systems. One of the most vital tasks of transit-operations planning is to improve transit connectivity (1). The nature of transit networks makes measuring their connectivity a complex task. Transit systems consist of different levels: stops, links, transit lines. A link in a multi-modal transit network is part of a transit line that serves a sequence of transit stops (nodes) and a stop can be served by different transit lines (2). These characteristics require a multilevel systematic approach to measure connectivity and estimate performance measures.

Predominantly, users choose a particular transit service based on two principal components (3). The first component is service quality, such as walking distance, in-vehicle travel time, waiting time, number of destinations served and number of transfers needed to reach final destinations, etc. If all of these factors are taken into account, measuring transit connectivity becomes a multidimensional problem (4). The second component accounts for the multiple routes of a transit system. To establish transit system connectivity, it is necessary to determine the extent of route integration and coordination within the network (5). In this context, connectivity can be used as one of the index measures to quantify and evaluate transit performance effectively.

Measures of transit connectivity can be useful for transportation planning agencies in several ways. First, connectivity can be used as a performance indicator for transit stops and/or routes in order to evaluate the overall system performance, allowing public agencies to rationalize public spending in transit accordingly. Second, in rural or suburban areas, where detailed information regarding transit ridership is not available, connectivity can be translated into a measure of performance for developing service delivery strategies. Third, the connectivity measure can assess effectiveness and efficiency of a transit system to prioritize the nodes/links in a transit system, particularly in terms of emergency evacuation. Finally, transit connectivity measures offer transit users the potential to assess the quality of transit service.

Many transit agencies have resources to deploy smart transit technologies that map ridership levels, and software and data to model what-if scenarios for connected transit systems, but these endeavors are usually incredibly burdensome. Also, many transit agencies do not have access to equivalent analytical capabilities, particularly across all transit operations. Moreover, systems and software developed for one particular city may not necessarily transfer as is to other locations, and in particular they often require large volumes of data to be assembled and organized for compatibility with particular data models. However, the need for these heavily tailored solutions is changing. New mapping technologies, particularly web mapping and related application program interfaces (API) for spatial analysis and mathematical software libraries, in conjunction with burgeoning community data-sets, have become the foundation for developing such tools cost effectively and for bespoke purposes.

Therefore, a central objective of this research is to develop a novel approach to measure transit connectivity, applicable even when transit assignment models or ridership tracking tools are not available to transit or planning agencies. Additionally, we have designed a tool that is flexible, such that if different data-sets or parameters of interest become available or relevant, they may easily be added to the system. To achieve this flexibly we rely on a graph theory approach that determines large-scale multimodal transit network performance by quantifying measures of connectivity at multiple levels such as nodes, links and lines. The methodology
considers the unique qualities of each transit line and stop, as well as measures of accessibility, and combines these criteria into a single connectivity index. This connectivity index can be a more effective quantitative measure of transit performance than the traditional measure of degree centrality. The proposed approach further captures sensitivity of the connectivity index to certain parameters, assessing the change in attractiveness of all transit lines or nodes. Finally, the proposed methodology reduces the need for large amounts of data to be available, and provides important information on system performance which would be critical for the decision making process. The new connectivity index significantly extends the set of performance analysis tools that decision makers can use to assess the level of service of a transit system.

The rest of the paper is structured as follows. The next section presents the literature review on the use of connectivity as well as limitations in terms of the approach adopted and application. The third section describes different necessary components and their formulation for obtaining transit connectivity followed by an example problem to demonstrate various connectivity indexes. The fifth section presents a case study that applies the proposed concept to a large-scale multimodal transit system followed by presentation and discussion of the model application results. The last section concludes the paper, summarizes findings and proposes future research directions.

LITERATURE REVIEW
In the area of social network and graph theory, centrality measures have been studied extensively (see Prell (6) for an overview). However, their application to public transit is limited. One of the more common measures of connectivity is known as the degree of centrality (7–11) calculated as the sum of total number of direct connections from a particular node to other nodes in a system, divided by the total number of system nodes minus one. Though simple, the degree of centrality is often a highly effective measure of the node importance. A more advanced version of the degree centrality is eigenvector centrality which considers that not all connections are equal (12–20). Hence, it assigns relative ‘scores’ to all nodes in the network based on the principle that connections to high-scoring nodes contribute more to the score of the node than equal connections to low-scoring nodes.

In topology and related fields of mathematics, closeness is a widely used concept and one of the basic elements of relationship on a topological space. This can be extended further to formulate closeness centrality (8, 9, 15, 21–24). In this measure, nodes with low closeness scores are located at short distances from others and tend to be more accessible (and vice versa). Another form of centrality commonly used in the literature is betweenness centrality which can be defined as the share of times that a node relies on another node (whose centrality is being measured) to reach a third node via the shortest path. In other words, betweenness centrality essentially counts the number of geodesic paths that pass through a node.

Past studies involving connectivity in fields (other than mathematics) have very limited application to multimodal transit networks because of its distinct properties. Some studies on transit connectivity did not consider transit characteristics when calculating the node index but Park and Kang (2) introduced transit characteristics when measuring the connectivity index of a node. Hadas (25) has attempted to compare different public transit system using spatial analysis based on GTFS data in which network coverage level, average speed, intersection coverage level, stop transfer potential and route overlap were used as connectivity indicators for comparison. Recently, transit connectivity has been used as a comprehensive impedance measure to determine both location-based and potential-accessibility measures relating to equity
assessment in the transit planning process (26). Transit connectivity has been previously proposed by Ceder (27) and applied only to buses (28, 29). In the study by Ceder (27), transit connectivity was calculated for a set of multiple and feasible transit paths for each origin-destination pair, including the three shortest paths and the three most popular paths (i.e. paths with the maximum demand) to account for the probabilistic nature of transit path choice. Several studies regarding transit service and accessibility were able to develop a quantifiable measure for quality of service and performance of large multi-modal regional transit systems but required significant amounts of data on the transit system and demographics of the service area to parameterize the measures (1,30). Another method (5) required transportation demand models (that included the transit component), which may not be available at many localities. Therefore, these methods may not be applicable to regions lacking such models or ridership tracking tools.

Past studies in transit connectivity were limited in capturing sensitivity in the connectivity index as well as the connecting power of a transit line when any of the model parameters change. As a result the relative attractiveness of the transit lines remained unchanged even when there was a significant improvement in one of the factors, such as speed, capacity, etc. This paper addresses the realistic case when attractiveness of one or more transit stops and lines increases, at the expense of others.

**METHODOLOGY**

The distinct characteristics of the elements in a transit network (nodes, links, and transit lines) require a unique formulation for each of them. The methodology presented in this paper accounts for the different levels of a transit system. This section explains the mathematical construct of these transit levels. The next subsections will discuss the concepts, components and formulation of transit node connectivity and line connectivity. Notations used throughout the paper are presented next.

<table>
<thead>
<tr>
<th>Notations</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_l$</td>
<td>Scaling factor coefficient of Capacity of a particular bus line $l$</td>
</tr>
<tr>
<td>$\beta_l$</td>
<td>Scaling factor coefficient of Speed of a particular bus line $l$</td>
</tr>
<tr>
<td>$\gamma_l$</td>
<td>Scaling factor coefficient of (Origin or Destination) Distance of a particular bus line $l$</td>
</tr>
<tr>
<td>$\phi_l$</td>
<td>Scaling factor coefficient of Activity of line $l$</td>
</tr>
<tr>
<td>$L$</td>
<td>Each Bus line</td>
</tr>
<tr>
<td>$N$</td>
<td>Each Node</td>
</tr>
<tr>
<td>$C_l$</td>
<td>Capacity of Line $l$, which is the product of frequency at which the bus/rail serves line $l$ and capacity of a single bus/rail that serves</td>
</tr>
<tr>
<td>$V_l$</td>
<td>Speed of Line $l$</td>
</tr>
<tr>
<td>$D_{ln}^1$</td>
<td>Distance of Line $l$ from the origin to node $n$</td>
</tr>
<tr>
<td>$D_{ln}^2$</td>
<td>Distance of line $l$ from node $n$ to the destination</td>
</tr>
<tr>
<td>$A_l$</td>
<td>Activity of Line $l$</td>
</tr>
<tr>
<td>100</td>
<td>Constant to normalize the Connecting power</td>
</tr>
<tr>
<td>$P_{ln}^o$</td>
<td>Outbound Connecting Power of line $l$ at node $n$</td>
</tr>
<tr>
<td>$P_{ln}^i$</td>
<td>Inbound Connecting Power of line $l$ at node $n$</td>
</tr>
<tr>
<td>$P_{ln}^t$</td>
<td>The Connecting Power of line $l$ at node $n$</td>
</tr>
<tr>
<td>$CI(n)$</td>
<td>Connectivity Index of node $n$</td>
</tr>
<tr>
<td>$V \sim N(\mu_V, \sigma_V^2)$</td>
<td>$V$ (speed) is normally distributed with mean $\mu_V$ and variance $\sigma_V^2$</td>
</tr>
<tr>
<td>$\mu_V$</td>
<td>Mean of given set of Speeds in the data set</td>
</tr>
<tr>
<td>$\sigma_V$</td>
<td>Standard Deviation of a given set of Speeds in the data set</td>
</tr>
</tbody>
</table>
In the proposed formulation, transit node connectivity is estimated by considering congestion due to lane sharing of transit lines of buses, light rail, bus rapid transit, and other similar transit facilities. We introduce the concept of connecting power of a node to: (1) represent how well a node (e.g. transit stop) serves in the scheme of a multimodal transit network, (2) identify the least, moderate and most connected nodes and (3) measure the performance of transit lines at a given node.

Node connectivity is defined as a function of connecting power of transit lines incident upon that node. As the connecting power may vary depending on the direction of travel, time of day, and day of week, the connecting power of a transit line \( P_{i,n} \) is defined as the average of the inbound and outbound connecting powers. Inbound (Eq. 1) and outbound (Eq. 2) connecting powers of a transit line are a function of capacity, speed, activity and the distance that it serves.

\[
P_{i,n}^{i} = \left[ \frac{(\alpha_{i} \times C_{i})}{100} \left( \frac{\beta_{i} \times V_{i}}{100} \right) (\gamma_{i} \times D_{i,n}^{i})(\varphi_{i} \times A_{i,n}) \right]
\]

\[
P_{i,n}^{o} = \left[ \frac{(\alpha_{i} \times C_{i})}{100} \left( \frac{\beta_{i} \times V_{i}}{100} \right) (\gamma_{i} \times D_{i,n}^{o})(\varphi_{i} \times A_{i,n}) \right]
\]

A transit line with higher scaling coefficient values indicates higher attractiveness. The scaling coefficients are also responsive in a very intuitive way. If one transit line becomes more attractive, for example, due to an increase in the number of operations during the day, the other transit lines become less attractive and this change is reflected by the respective coefficient (which is \( \alpha \) in this case). A simple example will be presented later in this section to demonstrate this concept.

Scaling coefficients are calculated assuming that the respective parameter follows a normal distribution (Eq. 3) in order to account for all the values of a parameter in a given data set (however, other distributions could easily be used instead. For example, if long histories of data were available, then a distribution could be tailored to fit these data). Scaling coefficients indicate the probability that the value of that particular parameter is less than or equal to the given value (Eq. 4). For example, in order to calculate \( \beta_{i} \), we need to assume \( V \sim N (\mu_{V}, \sigma_{V}^{2}) \) where normal distribution is given by:

\[
P(V) = \frac{1}{\sigma_{V} \sqrt{2\pi}} e^{-\frac{(V-\mu_{V})^{2}}{2\sigma_{V}^{2}}}
\]

And, therefore

\[
\beta_{i} = P(V < V_{i}) = \int_{0}^{V_{i}} \left( \frac{1}{\sigma_{V} \sqrt{2\pi}} e^{-\frac{(V-\mu_{V})^{2}}{2\sigma_{V}^{2}}} \right) dV
\]
The inbound and outbound connecting power considers activity density of a transit line \( l \) at node \( n \), which represents the ambient urban development pattern in which the transit line is situated, based on both land use and transportation characteristics. Development pattern reflects the land use activity in a particular region which can be captured by the number of household, employment, spatial distribution of activities and facilities in that area. The literature defines activity density in a number of ways (31–33). Without the loss of generality, in this paper the activity density is set equal to the ratio of households and employment in a zone to the unit area (Eq. 5). Operationally, this could be extended to cover particular urban structures or compositions of interest, to account for metrics of sprawl (34), walkability (35) etc. Mathematically, activity density is defined as:

\[
A_{l,n} = \frac{H_{l,n}^z + E_{l,n}^z}{\theta_{l,n}^z}
\]  

Connectivity index of the node (Eq. 6) is then calculated as the average of the connecting power of all the transit lines passing through the node \( n \).

\[
CI(n) = \frac{\Sigma_l P_{l,n}^t}{\Theta_l^n}
\]  

**Line Connectivity**

The total connecting power of a line is defined as the sum of the averages of inbound and outbound connecting powers for all transit nodes on the line, scaled by the number of stops on each line (Eq. 7). Scaling is used to reduce the connecting score of lines with a large number of stops (e.g. bus lines) so that they can be properly compared to lines with only a few stops (e.g. rail). Line connectivity can be defined as follows:

\[
\theta_l = (|S_l| - 1)^{-1} \sum P_{l,n}^t
\]  

**EXAMPLE DEMONSTRATION**

In this section, a four-node example (Figure 1) is presented to demonstrate the concept presented in the previous section. In the example, four bi-directional transit lines serve four nodes. Input data for each line are shown in Figure 1, while table 1 shows the results obtained when the proposed methodology is implemented. Line 3 has significantly higher connecting power than the others, which can be explained by its relatively high capacity, activity density, speed, and coverage of a relatively significant distance. All those attributes are combined into a single index and connecting power index captures efficiently the effect of the parameters of connectivity. Table 1(b) summarizes the results for the components of network connectivity.
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FIGURE 1 Example problem on four nodes with bi-Directional transit.

TABLE 1(a) Step-by-Step Estimation of the Example Problem

<table>
<thead>
<tr>
<th>Line</th>
<th>Node</th>
<th>Origin Dist.</th>
<th>Dest. Dist.</th>
<th>Speed</th>
<th>Freq.</th>
<th>Bus Capital</th>
<th>CAP</th>
<th>Activity</th>
<th>α</th>
<th>β</th>
<th>γ</th>
<th>φ</th>
<th>$P_{in}^e$ (Outbound)</th>
<th>$P_{in}^e$ (Inbound)</th>
<th>$P_{in}^e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>10</td>
<td>0</td>
<td>10</td>
<td>5</td>
<td>50</td>
<td>250</td>
<td>4</td>
<td>0.40</td>
<td>0.07</td>
<td>0.96</td>
<td>0.17</td>
<td>0.05</td>
<td>0.00</td>
<td>0.02</td>
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<td>10</td>
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<td>50</td>
<td>500</td>
<td>4</td>
<td>0.96</td>
<td>0.07</td>
<td>0.96</td>
<td>0.17</td>
<td>0.00</td>
<td>0.22</td>
<td>0.11</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>4</td>
<td>0</td>
<td>25</td>
<td>5</td>
<td>30</td>
<td>150</td>
<td>5</td>
<td>0.15</td>
<td>0.46</td>
<td>0.59</td>
<td>0.83</td>
<td>0.26</td>
<td>0.00</td>
<td>0.13</td>
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<td>25</td>
<td>5</td>
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<td>0.26</td>
<td>0.13</td>
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<td>30</td>
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<td>0.82</td>
<td>0.63</td>
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<td>0.00</td>
<td>9.31</td>
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<tr>
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<td>8</td>
<td>30</td>
<td>8</td>
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<td>0.82</td>
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<td>18.63</td>
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<tr>
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<td>0.17</td>
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<td>0.18</td>
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<td>0.00</td>
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<td>0.90</td>
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<td>18.63</td>
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<td>0.17</td>
<td>0.18</td>
<td>0.00</td>
<td>0.09</td>
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Stdev: 3.85 3.85 11.18 2.16 8.95 128.94 0.52

Note: $\alpha$, $\beta$, $\gamma$, and $\phi$ are the coefficient of capacity, speed, distance, and activity respectively.
1. **TABLE 1(b) Summary of Network Connectivity for the Example Problem**

<table>
<thead>
<tr>
<th>Network</th>
<th>Number</th>
<th>Connectivity</th>
</tr>
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<tr>
<td>Line</td>
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</table>

2. **Sensitivity of the scaling coefficients**

This section demonstrates, using the example from Figure 1, how change in the value of a parameter for a given transit line affects the respective coefficient values for the other transit lines. To observe the sensitivity of each scaling coefficient, only the respective parameter is varied. For example, Table 2 illustrates that when the speed of line 1 increased from 30 to 35 mph, the scaling coefficient $\beta_1$ increased from 0.41 to 0.67 (i.e., increase of attractiveness); meanwhile scaling coefficients for the other lines ($\beta_2, \beta_3, \beta_4$) decreased from 0.13 to 0.10, 0.41 to 0.33 and 0.94 to 0.90 respectively. In this example the value of speed is varied between 0 to 70 mph and the corresponding $\beta$ is recorded for all the lines. Figure 2 shows the similar effect on the scaling coefficients for all four lines as the speed of transit line 1 varies. It can be noticed that $\beta_2$ and $\beta_4$ are inversely related as lines 2 and 4 are incident upon node 3.

3. **TABLE 2 Change in Beta**

<table>
<thead>
<tr>
<th>Line</th>
<th>Speed</th>
<th>$\beta$</th>
<th>Line</th>
<th>Speed</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30</td>
<td>0.41</td>
<td>1</td>
<td>35</td>
<td>0.67</td>
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<td>30</td>
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<td>0.94</td>
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<td>40</td>
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</tbody>
</table>
**Model Sensitivity**

In order to determine how the model responds to changes in capacity, speed, activity and distance, these parameters are varied and corresponding connectivity indices for all nodes are calculated. Figure 3 shows the change of node connectivity index as the speed of line 1 is varied. As speed of line 1 increase from 0 to 70 mph, $CI(n)$ for all the nodes change. Since line 1 is connecting node 1 and node 2, increase in the speed of line 1 will increase the connecting power of line 1 resulting in an increase of the connectivity index of nodes 1 and 2. Since line 1 is becoming more attractive, lines 2, 3 and 4 are also negatively affected, which in turn reduces the connectivity index of node 3 and 4.

Sensitivity analysis can also be performed by simultaneously changing two or more parameters to observe the combined effect. Figure 4 shows the effect of connecting power of the lines when both speed and capacity of Line 1 vary simultaneously between 0 to 500 passengers/day and 0 to 70 mph respectively. As these two parameters change for line 1, connecting power of lines 2, 3 and 4 are highest when capacity and/or speed of line 1 is at its lowest. As one of these two parameters for line 1 start to increase, the connecting power of the other three lines start to decrease (as line 1 becomes relatively more attractive). Figure 4(b) demonstrates that connecting power of line 2 is the lowest when speed and capacity of Line 1 are between 35-40 mph and 350-380 passengers/day respectively. As the value of the parameters for line 1 continue to increase, a slight increase in connecting power of line 2 is observed from the combined effect of increased mean and standard deviation of the respective parameters. Similarly, Figure 4(c) and (d) exhibit that connecting power of line 3 and 4 decrease consistently with the increase of line 1 parameter values.

**FIGURE 2 Change in corresponding $\beta$ for all of the lines as line 1 speed changes.**
FIGURE 3 Change in connectivity index of nodes as line 1 speed changes.

FIGURE 4 Change in connecting power of lines as speed and capacity of line 1 change.
CASE STUDY

To demonstrate the methods in practice, the methodology was applied to a comprehensive transit network in the Washington and Baltimore region. The complete transit network was adapted from Maryland State Highway Administration. The transit database contained the two largest transit systems in the region, namely Washington Metropolitan Area Transit Authority (WMATA), and Maryland Transit Administration (MTA).

WMATA is a tri-jurisdictional government agency that operates transit service in the Washington, D.C. metropolitan area, including the Metrorail (rapid transit), Metrobus (fixed bus route) and MetroAccess (paratransit). WMATA has the second highest rail ridership in the US after New York, with over 950,000 passengers/day. The WMATA Metro provides an extensive heavy rail system with 106.3 route miles. The WMATA bus system serves an extensive ridership of over 418,000 unlinked daily trips.

MTA is a state-operated mass transit administration in Maryland. MTA operates a comprehensive transit system throughout the Baltimore-Washington metro Area. The system has a daily ridership of nearly 300,000 passengers along with other services that include the Light Rail, Metro Subway, and MARC Train. The Baltimore Metro subway is the 11th most heavily used system in the US with nearly 56,000 daily riders. Both the WMATA Metro rail system and the Baltimore transit system are connected by the MARC commuter rail system. This system has a daily ridership of over 31,000. Together, these provide a suitably complicated system of interconnected systems with varied parameters to test the utility of the methodology that we have proposed in this paper. In the next section, results of applying this methodology tangibly to this real world system are discussed.

MODEL APPLICATION RESULTS

In this section, the results obtained using the proposed methodology is presented.

Node Level

The Washington and Baltimore region have a significant number of transit nodes, each of which provide a varying degree of connectivity to the network. Figure 5 shows connectivity of Washington and Baltimore transit system at the node level, which is a function of connecting power of transit lines incident upon a given node. From Figure 5(a), it can be noticed that, on one hand, there are some well-connected transit stops far away from the city center, and on the other hand, a significant number of poorly-connected stops can be seen in the region with highest concentration of transit stops. Similar observation can be drawn from Figure 5(b), which also shows a well-connected transit system with a few transit lines connecting a series of nodes with poor connectivity. This is an example of how capacity, speed, activity and the distance that a transit line serves are weighed-in to provide a single connectivity index.

Figure 6 (a) and (b) show a portion of WMATA and MTA respectively at Central Business District (CBD) level. In both figures, the area with high concentration of well-connected nodes is very prominent. Figure 6(a) shows a few transit lines connecting a number of low-connectivity stops. Figure 6(c) and (d) exhibit transit stop connectivity at station level. From Figure 6(c) we can see that there are several stops with connectivity less than 1000 in close proximity to Union Station and only one stop with significantly high connectivity index. Given that these stops are in the heart of the city, they raise a question for the transit authority. Figure 6(d) shows the contrast in connectivity of stops on two adjacent parallel streets (W Lombard and W Pratt St.).
FIGURE 5 Transit stop connectivity in Washington and Baltimore region.
Line Level

The line connectivity index is applied to the Washington and Baltimore transit system and Figure 7 exhibits the results obtained. While concentration of highly connected lines at CBD is prominent, a significant number of low-connectivity transit lines are also serving the same region, as can be seen from Figure 7(a) and (b). The transit lines with high connecting power are mostly the metro lines in both the region. Note that there are several transit lines providing high degree of connectivity to the suburban areas in Washington and Baltimore. With the proposed approach in the paper, transit agencies can estimate the connectivity of each transit line (also multiple line sharing portion of a larger transit path). Such line level connectivity will provide a measure of adequate, over or underutilized transit lines when interfaced with land use data to assist decision makers to better coordinate transit schedule, and routes.
FIGURE 7 Transit line connectivity.
**Sensitivity to change in transit network characteristics**

To demonstrate the sensitivity of the model to the change in parameters, Metrobus Route 10A is chosen as an example. For this analysis, speed of Route 10A (serves between Huntington Point and Pentagon in Washington DC) is varied between 0-70 mph and percentage change in $\beta$ is calculated for other routes in its vicinity. Figure 8 shows that as speed of Route 10A increases the nearby routes lose their connecting power, to some extent, consistently which are reflected by the negative change in $\beta$. For demonstration purposes we show the effect for one route. However, such type of sensitivity can be performed for any line, node and resulting changes in transit network level can be observed.

**FIGURE 8** Sensitivity analysis of transit line speeds and resulting effect on connectivity.

**CONCLUSIONS**

This paper proposed an approach to measure transit connectivity for large multimodal transit systems, without requiring detailed information about transit ridership, demographic and socio-economic data. The methodology is expected to be quite beneficial for transit agencies to estimate multimodal network connectivity and to capture the sensitivity when there is any change in the network characteristics. The graph theory based methodology analyzes properties of each transit node and line, and compares with the rest of the alternatives and proposes a
connectivity score. The proposed approach provides disaggregate and aggregate level connectivity measures without requiring detailed transit demand and transit assignment models.

This paper has several contributions in the research area of transit connectivity. First, transit agencies can utilize multimodal transit connectivity of each node, line to determine most, moderate and least connected areas, and thereby decide to improve transit service to achieve higher transit ridership. Second, the proposed connectivity index can be used as an efficient quantitative measure of transit performance than the traditional measures such as degree centrality and betweenness centrality. Third, this approach efficiently captures sensitivity of the connectivity index to certain parameters (speed, capacity, frequency etc.) by assessing the change in attractiveness of transit lines or nodes. Fourth, the use of GTFS data in the proposed approach enables any planning agencies to adapt such procedure and obtain transit connectivity measures. Future research can include transit connectivity estimation in case of emergency evacuation to provide alternative transit routing information such that transit system can be fully utilized in serving transit captive riders.

REFERENCES


