

1 **Locating the Most Connected Transit Stop, Route and Transfer Center: A** 2 **Tool for Users and Decision Makers**

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1 **ABSTRACT**

2 Agencies at federal, state and local level are aiming to augment the public transportation system (PTS) as
3 an alternative to alleviate congestion and to cater the needs of captive riders. One of the ways to
4 determine the efficiency of the PTS is connectivity. In a multimodal transportation system, transit is a
5 component and unlike highway connectivity, transit connectivity is relatively complex to determine as
6 one has to consider, fare, schedule, capacity, frequency and other features of the system at large. Thus,
7 assessing transit connectivity requires a systematic approach to consider all parameters involved in the
8 real world. The purpose of this paper is two-fold: (1) to propose a methodology for evaluating transit
9 connectivity at various levels such as nodes, lines, and transfer centers in multimodal transportation
10 system; and (2) to provide a platform for extending the methodology for use in large scale applications,
11 including a medium to visualize results to assist public transit decision making. A graph theory approach
12 is developed to incorporate transit specific variables and detailed formulation is discussed in the paper.
13 Two-example problems are discussed to demonstrate the methodology. Following, the proposed
14 framework is applied to the comprehensive transit network in the Washington-Baltimore region. Then a
15 novel web based interface designed with HTML5 is demonstrated to visualize the transit connectivity in
16 various platforms such as mobile phones, tablets and personal computers. The proposed methodology can
17 be a useful tool for both users and decision makers in assessing transit connectivity in a multimodal
18 transit network in a number of ways such as the identification of under-served transit areas, prioritizing
19 and allocating funds to locations for improving transit service.

20 **INTRODUCTION**

21
22 Transit service is widely used as means of transport for captive riders in urban and suburban areas. The
23 connectivity of transit is crucial in providing adequate levels of service to these riders. However,
24 measuring connectivity can be a complex and elusive task. Analyzing a transit network is significantly
25 different from analyzing a highway or road network. In a transit network, nodes are called stops and the
26 lines are called links or route segments. Links in a multimodal transit network have different
27 characteristics from those in a road network. While a link in a road network is a physical segment that
28 connects one node to another, a link in a multi-modal transit network is part of a transit line that serves a
29 sequence of transit stops (nodes). Since different transit lines can serve a single stop, multiple transit links
30 may exist between nodes in a multi-modal transit network. But in the case of a highway network, only
31 one link exists between two nodes. Similarly, transit nodes are composed of a different set of
32 characteristics than highway nodes. While some defining characteristics of transit links are common to
33 both types of networks such as speed and capacity, their meaning in the context of connectivity are
34 completely different. Other terms like headway and frequency are completely foreign to road networks,
35 but are critical characteristics of transit networks.

36 Among riders, transit choice among modes of travel depends on two principal components. First,
37 the number of factors related to service quality, such as walking distance, in-vehicle travel time, waiting
38 time, number of destinations served and number of transfers needed to reach destinations makes transit
39 connectivity a multidimensional problem. Second, a transit system consists of many different routes.
40 Determining the extent to which routes are integrated and coordinated so that the transit system is
41 connected is another task (1). The structure of a public transit network is critical in determining its
42 performance, coverage, and service of the network. Network connectivity can be used as a measure to
43 study the performance of the transit system which will assist decision makers in prioritizing transit
44 investments and deciding which stops/lines need immediate attention in regard to operation and/or
45 maintenance (2). In this context, connectivity is one of the index measures that can be used to quantify
46 and evaluate transit performance (3).

47 Measures of transit connectivity can be used for a number of purposes. First, in a public or quasi-
48 public agency, connectivity can be used to quantify transit stop and route performance and to evaluate

1 overall system performance so as to direct public spending in the most efficient manner. Second, in a
2 rural or suburban area where exact information on transit ridership, boardings, and alightings are not
3 available (such data are generally obtained from a sophisticated travel demand model or from an
4 advanced transit system where smart cards are used to keep track of revenues) connectivity measurement
5 can be used to obtain a measure of performance for developing service delivery strategies. Third,
6 connectivity can serve as a performance measure in a large scale urban multi-modal transit network
7 containing local buses, express buses, metro, local light rail, regional light rail, bus rapid transit, and other
8 transit services, where such services are provided by multiple public and private agencies with little
9 coordination. For these reasons it is clear that a system should be developed for small and medium scale
10 transit agencies that (1) provides a methodological framework for transit connectivity, and (2) delivers a
11 visualization tool for users and decision makers to best utilize transit.

12 The research presented in this paper comprises a unique approach to measuring transit
13 connectivity, particularly for applications where transit assignment models or ridership tracking tools are
14 not available. This method incorporates a graph theory approach to determine the performance of large-
15 scale multimodal transit networks by quantifying measures of connectivity at multiple levels, including
16 node, line and transfer center. This is achieved through an assessment of connectivity that incorporates
17 the unique qualities of each transit line and stop, as well as measures of accessibility. By combining these
18 criteria in a single connectivity index, a quantitative measure of transit performance is developed that
19 goes beyond the traditional measure of centrality. The new connectivity index significantly extends the
20 set of performance analysis tools decision makers can use to assess the quality of a transit system.

21 The next section presents a literature review highlighting the evolution of connectivity measures
22 in past research, followed by the identification of a gap in current understanding and technique to which
23 this research could make a significant contribution. The methodology section describes a step-by-step
24 process of obtaining the transit connectivity. A case study shows how the concept can be applied in real
25 world applications. The next section shows how the results of the study can be used as a powerful
26 network visualization tool. Finally, findings of the study are discussed in the conclusion section.

27 LITERATURE REVIEW

28 Centrality measures have been studied extensively in past research, however, their application to public
29 transit is rare. One of the more common measures of connectivity is known as the degree of centrality.
30 This measure is calculated by summing the total number of direct connections from a particular node to
31 other nodes in a system, then dividing by the total number of system nodes minus 1. Degree centrality is the
32 most widely used measure of connectivity in the literature. Its use spans multiple disciplines including
33 network and graph theory (3), (4), (5), (6), (7); computer and information science (7), (8), (9) (10), (11);
34 gene-disease research (12), (13); epidemiology (14), (15); shortest path (3), (16), (17); and transportation
35 (18) (15) (19).

36 The degree centrality $D_c(n)$ simply counts the number of direct connections a node has to other
37 nodes in the network, but does not account for the quality of the connection or indirect accessibility to
38 other nodes. A more advanced measure of connectivity is known as eigenvector centrality. This measure
39 acknowledges that not all connections are equal. It assigns relative 'scores' to all nodes in the network
40 based on the principle that connections to high-scoring nodes contribute more to the score of the node in
41 question than equal connections to low-scoring nodes. The eigenvector centrality succeeded the
42 development of degree centrality and is used for a number of studies including research on network and
43 graph theory (20), (21), (22), (21) and in the social science (17), (23), (24), (25), (26), (27).

44 Another formulation of connectivity is called closeness centrality. In this measure, nodes with
45 low closeness scores are short distances from others and will tend to be more accessible. Nodes with
46 higher closeness scores, meanwhile, represent longer distances from other nodes and are not easily

1 accessible. In topology and related fields in mathematics, closeness is one of the basic concepts in a
2 topological space.

3 Betweenness Centrality is defined as the share of times that a specific node relies on another
4 specific node (whose centrality is being measured) in order to reach a third node via the shortest path. In
5 other words, betweenness centrality essentially counts the number of geodesic paths that pass through a
6 node. Betweenness centrality has been adopted by researchers in network and graph theory (28), (29),
7 (30), (31); computer and information science (8), (9), (32), (33); and to find the shortest path (17), (34).

8 Previous node indexes did not take into account transit characteristics. Park and Kang (2011)
9 introduced transit characteristics into the node centrality measures and proposed a connectivity index as a
10 true measure of a transit node (35). The Connectivity index of a node can be defined as the sum of
11 connecting powers of all lines crossing through a node n . The total connecting power of a node is the
12 multiple of connecting power of a line at node n . The characteristics of a link contain the performance of
13 a series of nodes in that link. A link is a part of the transit route, which in turn is a function of the speed,
14 distance, frequency, headway, capacity, acceleration, deceleration, and other factors. Since a route will
15 contain both in-bound and out-bound, the line performance will in part depend upon the directionality of
16 the transit route, that is, whether the line is circular or bidirectional. The total connecting power of line l at
17 node n is the average of outbound and inbound connecting power. This paper develops a more advanced
18 connectivity measure that incorporates the quantification of underlying socio-economic data and the cost
19 of transfers in a multimodal system.

Notation	Explanation
D_l^i	: Inbound distance of link l
D_l^o	: Outbound distance of link l from node n to destination
L_{n,n_1}	: Shortest distance between node n_1 to n
$P_{l,n}^i$: Inbound connecting power of link l
$P_{l,n}^o$: Outbound connecting power of link l
$P_{l,n}^t$: Total connecting power of line l at node n
S_R	: Set of stops in region R
S_l	: Set of stops in line l
S_σ	: Set of stops in region center σ
V_l	: Average Speed of link l
n_0	: Initial stop
$t_{n_1,n}$: Transfer time from n_1 to n
δ_{n_1,n_2}	: Total number of paths between n_1 and n_2
$\delta_{n_1,n_2}(n)$: Number of paths exist between n_1 and n_2 those pass through n
δ_{np}	: A binary indicator variable for determining the degree centrality, which takes the value of 1 when
θ_R	: Connectivity index for region R
θ_l	: Connectivity index for line l
θ_n	: Connectivity index for node n
$\rho_{n_1,n}$: Passenger acceptance rate from node n_1 to n
ρ_R	: Density measure for region R
a	: Parameter for passenger acceptance rate
b	: Parameter for passenger acceptance which is sensitive to travel time
L	: Link
N	: Node

N	:	Network system
P	:	Node dependent on n
α	:	Scaling factor coefficient for Capacity of line l
β	:	Scaling factor coefficient for Speed of line l
γ	:	Scaling factor coefficient for distance of line l
$A_{l,n}$:	Activity density of line l , at node n
ϑ	:	Scaling factor for activity density
$H_{l,n}^z$:	Number of households in zone z containing line l and node n
$E_{l,n}^z$:	Employment for zone z containing line l and node n
$\Theta_{l,n}^z$:	Area of z containing line l and node
Θ_l^n	:	Number of lines l at node n

1

2 **MOTIVATION**

3 Many measures of transit service and accessibility have been put forth in the literature, but few offer a
4 metric to measure the quality of service and performance of a large multi-modal regional transit system.
5 The literature does purport to offer such insight requires significant amounts of data not only about the
6 transit system, but also of the complete demographics of the service area. Other methods require full
7 transportation demand and transit assignment models, tools that are prohibitively expensive for many
8 localities.

9 Measuring transit system performance and the level of service at many different levels is vital to
10 funding decisions. Agencies with the objective to improve the transit system using external funds must
11 make the case that the project will make worthwhile improves to the system. At the same time, agencies
12 in the quest for investigating the potential effect of removing a stop, group of stops or transit line from
13 service must know the potential effect it will have on the performance of the system. In the absence of
14 complex transportation demand models, this information is nearly impossible to obtain. A methodology
15 that reduces the need for heavy data inputs yet provides important information on system performance is
16 critical to the decision making process. Transit planning agencies may also be interested in applying such
17 an index to determine the best use of land surrounding well-connected transit nodes. Beyond Transit
18 Oriented Development (TOD) style plans, the connectivity index provides a way for planners to measure
19 passenger acceptance rates and accessibility for a single node based on its access within an entire multi-
20 modal regional transportation network.

21 The objectives of this paper are several-fold, with the overall goal of providing a strong measure
22 of system performance with the lowest possible data requirements. This paper will first seek to construct a
23 list of node and link based commonly encountered flow processes and define them in terms of a few
24 underlying characteristics; second, to determine and propose the best suited measures in terms of transit
25 connectivity; third, to examine these measures by running simulations of flow processes and comparing
26 the results in a real world case study; and fourth, to suggest the best practices which can be adopted for
27 decision making. All the aforementioned problems require development of a tool to quantify connectivity
28 of a public transportation system. The proposed methodology is presented in the next section.

29 **METHODOLOGY**

30 The methodology presented in this paper is for transit systems at different levels. The very nature of
31 nodes, lines and transfer centers each necessitate a unique formulation. The description below explains
32 the mathematical construct of these transit levels in a step-by-step manner.

1 Network Connectivity

2 The connectivity index is shown in equation (1.1). The total connecting power of a node is the multiple of
 3 connecting power of a line at node n ($P_{l,n}^t$). The conditional value of presence of a line is represented a by
 4 a binary indicator variable ($\mu_{l,n}$), which takes the value 1 if line l contributes to the connectivity at node n ,
 5 and 0 otherwise. The characteristics of a link contain the performance of a series of nodes in that link. A
 6 link is a part of the transit route, which in turn is a function of the speed, distance, frequency, headway,
 7 capacity, acceleration, deceleration, and other factors. Since a route will contain both in-bound and out-
 8 bound, the line performance will in part depend upon the directionality of the transit route, that is,
 9 whether the line is circular or bidirectional. The total connecting power of line l at node n is the average of
 10 outbound and inbound connecting power and can be defined as

$$P_{l,n}^t = \frac{P_{l,n}^o + P_{l,n}^i}{2} \quad (1.1)$$

11

12 The outbound connecting power of a line l , at node n can be defined as (35)

$$P_{l,n}^o = \alpha C_l \times \beta V_l \times \gamma D_{l,n}^o \quad (1.2)$$

13

14 where, C_l is the capacity of line l , V_l is the speed of line l , and $D_{l,n}^o$ is the distance of line l from
 15 node n to the destination. The parameter α is the scaling factor coefficient for capacity, β is the scaling
 16 factor coefficient for speed, and γ is the scaling factor coefficient for distance. Similarly, the inbound
 17 connecting power of line l can be defined as

$$P_{l,n}^i = \alpha C_l \times \beta V_l \times \gamma D_{l,n}^i \quad (1.3)$$

18 where, $P_{l,n}^i$ is the inbound connecting power of line l at node n . While the outbound connecting
 19 power of a transit line at a certain transit stop represents connectivity from the stop to the downstream
 20 stops of the transit line, the inbound connecting power measures connectivity from the upstream stops of
 21 the transit line to the stop under consideration.

22 Node Connectivity

23 The proposed methodology consists of better representations of transit node index measures. In the
 24 proposed formulation we consider the congestion effects achieved because of lane sharing of transit lines
 25 of buses, light rail, bus rapid transit, and other similar transit facilities. We have redefined the connecting
 26 power of a transit line as the other measures have not incorporated the transit attractiveness as per the land
 27 use and transportation characteristics of the area the transit line is passing through. As discussed
 28 previously, the connecting power of a transit line is a function of the inbound and outbound powers, as
 29 the connecting power may vary depending on the direction of travel. The inbound and outbound
 30 connecting power of a transit line can be redefined as follows.

$$P_{l,n}^o = \alpha \left(C_l \times \frac{60}{F_l} \times H_l \right) \times \beta V_l \times \gamma D_{l,n}^o \times \vartheta A_{l,n} \quad (2)$$

$$P_{l,n}^i = \alpha \left(C_l \times \frac{60}{F_l} \times H_l \right) \times \beta V_l \times \gamma D_{l,n}^i \times \vartheta A_{l,n} \quad (3)$$

1 In equation 3 ($P_{l,n}^i$) is a term for activity density of transit line "l" at node "n", and ϑ is the scaling factor
 2 for the variable. The density measurement represents the development pattern based on both land use and
 3 transportation characteristics. The literature defines the level of development a number of ways, but for
 4 simplification purposes we have considered it to be the ratio of households and employment in a zone to
 5 the unit area. Mathematically, activity density (equation (4)) is defined as:

$$A_{l,n} = \frac{H_{l,n}^z + E_{l,n}^z}{\Theta_{l,n}^z} \quad (4)$$

6 The connectivity index measures aggregate connecting power of all lines that are accessible to a
 7 given node. However not all lines are equal; nodes with access to many low quality routes may attain a
 8 connectivity index score equal to a node with only a couple very high quality transit lines. This means
 9 that while both nodes are able to provide good access, the node with the fewest lines provides the most
 10 access with the lowest need to transfer. To scale the index scores based on the quality of individual lines,
 11 that is, scaling for the least number of transfers needed to reach the highest number and quality of
 12 destinations, the node scores are adjusted by the number of transit lines incident upon the node. The
 13 inbound and outbound connecting power of a transit line can be further refined as:

$$P_{l,n}^o = \alpha(C_l \times \frac{60}{F_l} \times H_l) \times \beta V_l \times \gamma D_{l,n}^i \times \vartheta A_{l,n} \times \varphi T_{l,n} \quad (5)$$

$$P_{l,n}^i = \alpha(C_l \times \frac{60}{F_l} \times H_l) \times \beta V_l \times \gamma D_{l,n}^i \times \vartheta A_{l,n} \times \varphi T_{l,n} \quad (6)$$

14 This equation adds the number to transit lines "l" at node "n", and φ is the scaling factor for the
 15 number of transit lines. The transfer scale is simply the sum of the connectivity index scores for each of
 16 the transit lines that cross a node divided by the count of the number of lines that incident upon the node.
 17 The transfer scaled index (equation (7)) is defined as:

$$T_{l,n} = \frac{\sum P_{l,n}^t}{\Theta_l^n} \quad (7)$$

18 **Line Connectivity**

19 The total connecting power of a line is the sum of the averages of inbound and outbound connecting
 20 powers for all transit nodes on the line as presented in equation (1.1) scaled by the number of stops on
 21 each line. The scaling measure is used to reduce the connecting score of lines with many stops like bus
 22 lines to properly compare to lines with only a few stops like rail. The line connectivity can be defined as
 23 following:

$$\theta_l = (|S_l| - 1)^{-1} \sum P_{l,n}^t \quad (8)$$

24 **Transfer Center Connectivity**

25 The concept of a connectivity index of a transfer center is different from the connectivity measure of a
 26 conventional node. Transfer centers are groups of nodes that are defined by the ease of transfer between
 27 transit lines and modes based on a coordinated schedule of connections at a single node or the availability
 28 of connections at a group of nodes within a given distance or walk time. In this paper, we define a transfer

1 center as the group of nodes within half mile of any rail station in the transit network. The sum of the
 2 connecting power of each node in the transfer center is scaled by the number of nodes on the transfer
 3 center. Thus, a transfer center in a heavily dense area is made comparable to a transfer center in a less
 4 dense area. This scaling procedure is particularly important when comparing transfer centers in a
 5 multimodal network where one transfer center may be primarily served by a well-connected commuter
 6 rail line and other may have many bus lines and rail lines connecting to the center. The following
 7 equation shows the connectivity index of a transfer center.

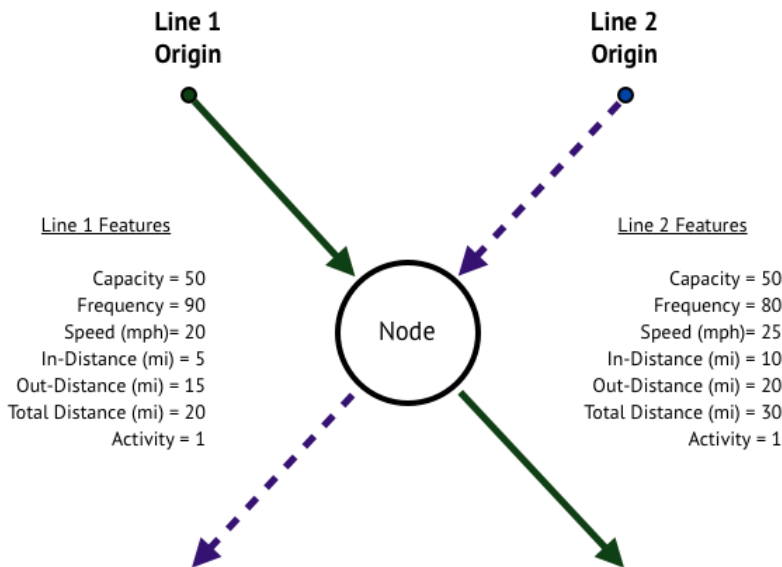
$$\theta_{tc} = (|S_{\omega}| - 1)^{-1} \sum P_{l,n}^t (\rho_{n_1,n}) \quad (9)$$

8 **Example Problem**

9 To demonstrate the methodology two example problems are illustrated. In the example problems it is
 10 shown that how to estimate the parameters used for connectivity estimation.

11 **Example-1: One-Node Problem**

12 The one node problem is shown in Figure 1. There are two bus lines passing through the node. The
 13 capacity, frequency (or number of operations), speed of the bus, distance from the origin, and distance to
 14 the destination is given as the input data. The first task is to estimate the parameters to obtain
 15 connectivity.



16

17 **FIGURE 1 One Node Example Problem**

18 α = the sum-product of capacity and frequency is estimated as $[(50*90) + (50*80)]/(90+80) =$

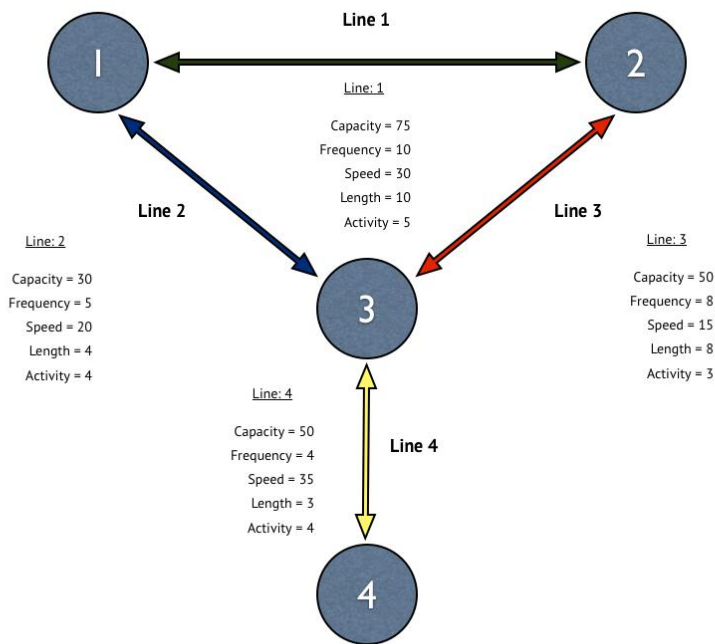
19 β = Average of speeds = $(20+25)/2 = 22.5$

20 γ = Average of distances = $(20+30)/2 = 25$

- 1 $\phi = \text{Average of activities} = (1+1)/2 = 1$
- 2 Connectivity of Line 1 = $[(50*90)/4250] * [20/22.5] * [20/25] * [1] = 0.3951$
- 3 Connectivity of Line 2 = $[(50*80)/4250] * [25/22.5] * [30/25] * [1] = 0.5926$
- 4 The result shows that connectivity of line 2 is higher than that of line 1.
- 5 Point connectivity is the sum of connectivity of all lines passing through the node.
- 6 Point connectivity = $0.3951 + 0.5926 = 0.9877$.

7 **Example-2: Four-Node Problem**

- 8 A four-node example problem is presented in Figure 2. Four transit lines serve the four nodes in the
- 9 second example problem. Each line is bi-directional. The input data for each line is also shown in Figure
- 10 2. The first task is to estimate the parameters. For example looking at the first row of Table 1, α is the
- 11 product of average capacity and frequency.



12

13 **FIGURE 2 Four-Node Example Problem**

1 **TABLE 1 Step by Step Estimation of Three Node Problem**

Line	Distance	Node	Origin Distance	Destination Distance	Speed	Operations	Capacity	Activity	α	β	γ	φ	$P_{l,n}^o$	$P_{l,n}^i$	$P_{l,n}^t$
1	10	1	10	0	30	10	50	4	332.22	28.33	7.33	4.5000	1.9316	0.0000	0.9658
1	10	2	0	10	30	10	50	4	332.22	28.33	7.33	4.5000	0.0000	1.9316	0.9658
2	4	1	4	0	25	5	30	4	332.22	28.33	7.33	4.5000	0.1932	0.0000	0.0966
2	4	3	0	4	25	5	30	5	332.22	28.33	7.33	4.5000	0.0000	0.2414	0.1207
3	8	2	8	0	30	8	50	5	332.22	28.33	7.33	4.5000	1.5453	0.0000	0.7726
3	8	3	0	8	30	8	50	5	332.22	28.33	7.33	4.5000	0.0000	1.5453	0.7726
4	3	4	3	0	35	4	50	4	332.22	28.33	7.33	4.5000	0.2704	0.0000	0.1352
4	3	3	0	3	35	4	50	4	332.22	28.33	7.33	4.5000	0.0000	0.2704	0.1352
1	10	1	0	10	30	10	50	4	332.22	28.33	7.33	4.5000	0.0000	1.9316	0.9658
1	10	2	10	0	30	10	50	4	332.22	28.33	7.33	4.5000	1.9316	0.0000	0.9658
2	4	1	0	4	25	5	30	4	332.22	28.33	7.33	4.5000	0.0000	0.1932	0.0966
2	4	3	4	0	25	5	30	5	332.22	28.33	7.33	4.5000	0.2414	0.0000	0.1207
3	8	2	0	8	30	8	50	5	332.22	28.33	7.33	4.5000	0.0000	1.5453	0.7726
3	8	3	8	0	30	8	50	5	332.22	28.33	7.33	4.5000	1.5453	0.0000	0.7726
4	3	4	0	3	35	4	50	4	332.22	28.33	7.33	4.5000	0.0000	0.2704	0.1352
4	3	3	3	0	35	4	50	4	332.22	28.33	7.33	4.5000	0.2704	0.0000	0.1352

2

3 **TABLE 2 Summary of Network Connectivity for Example-2**

Network	Number	Connectivity
Line	1	3.7344
	2	2.7879
	3	3.6893
	4	1.5517
Node	1	0.5312
	2	0.8692
	3	0.3429
	4	0.1352
Transfer Center	3	2.3284

4

1 Similarly β is the average of all speeds and γ is the average of all distances. ϕ is the average of all area
2 types to include urbanization of transit nodes. Using equation (6) and (7) the outbound and inbound
3 connecting power of lines are determined. The last column shows the total connecting power, which is the
4 sum of inbound and outbound connecting powers. The detailed parameter estimates and estimation of
5 connectivity is shown in Table 1. The total connectivity of all lines and nodes are summarized in Table 2.

6 **5. Network Visualization and Interaction**

7 While the development and implementation of a true multimodal connectivity index is a major hurdle, the
8 result produces significant amounts of data that may be even more difficult to use conceptualize in a
9 meaningful way. To aid in the use of such data, we developed a visualization tool for planners and users
10 to examine the performance of a given node, line and/or transfer center. The geovisualization component,
11 which allows connectivity measures to be embedded in a software platform and displayed, is discussed in
12 the next section.

13 **5.1 Geovisualization Structure**

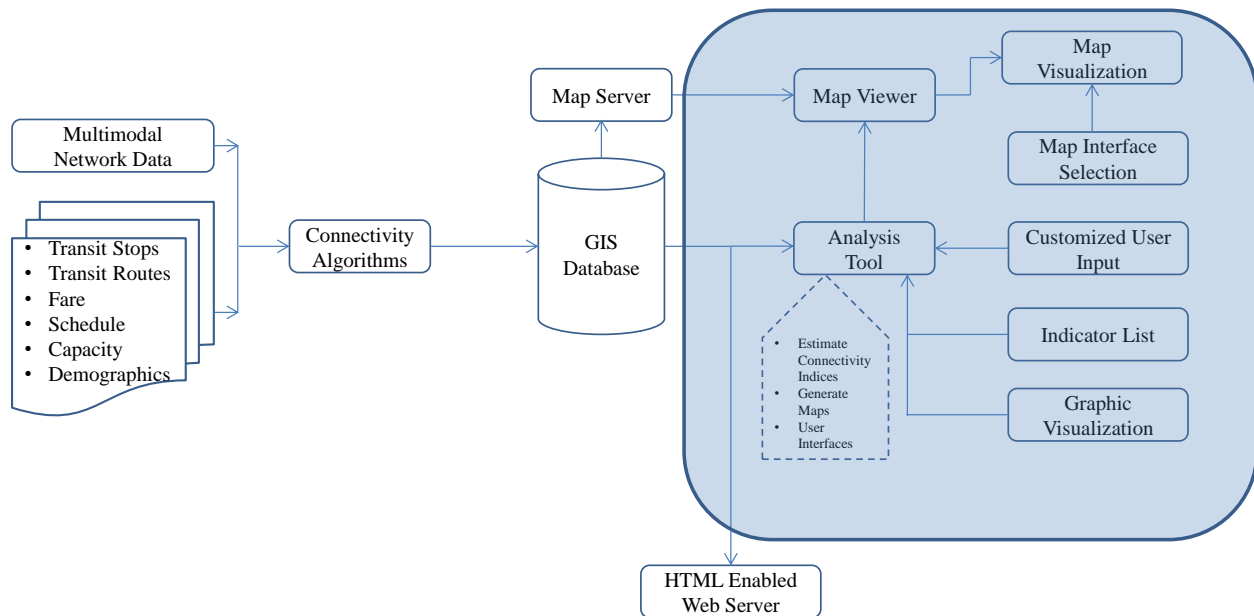
14 In addition to standalone access to these metrics and standard geovisualization using traditional desktop
15 or server-side Geographic Information Systems, we built an on-demand access and visualization
16 infrastructure using HTML5 for to visualize transit connectivity measures. In doing so, our design goals
17 were to provide (1) a rich user interactive experience; (2) in an on-demand capacity that could shift
18 agnostically to mobile as well as tablet and desktop media; (3) using self-service modes of access that
19 require little prior knowledge of GIS; (4) free from licensing costs; (5) scalable to large numbers of users
20 and query-loads; and (6) provided in an extensible framework that supports future modification and
21 alteration.

22 We used a three-layer approach to the on-demand infrastructure. First, we developed a spatial
23 database management and spatial data access layer, with standard spatial data models, metadata schemes,
24 access procedures, query abilities, and so on. This “back-end” infrastructure can actually be reused, using
25 desktop GIS or client/server access schemes. Second, we developed a middle layer using MapDotNet
26 (although other layers could easily be substituted in its place). The middle layer provides a set of *Web*
27 *Services* that can broker data requests and exchange between the interface and the “back-end” data
28 infrastructure. The outer layer was developed in HTML5 and JavaScript, and provides “slippy map”
29 geovisualizations as an interface to the other layers of the scheme. The key here, is *tiling* of data layers
30 and cartography, which is rendered in the browser and is therefore free from the need of any client-side
31 software. Standard queries and returns for information or shifting map views are easily handled using
32 JavaScript calls and queries to the other layers. This permits access from a wide variety of devices and
33 platforms—iPhone OS, Android, desktop, tablet, Kindle, cell-phone, etc., while maintaining the same
34 user interaction experience.

35 **5.2 On-Demand GIS Implementation**

36 The on-demand GIS infrastructure has several advantages. First, it is relatively inexpensive. No
37 proprietary software licenses are required. Moreover, underlying base maps can be pulled from a variety
38 of sources such as an organization’s own database, commercial providers, or free “citizen-volunteered”
39 data-sets such as OpenStreetMap. Second, the user experience is relatively straightforward and requires
40 no prior knowledge of GIS or data-query. Users can interact with the data by tapping, and using multi-
41 touch gestures on their devices in schemes that are not common to most “apps.” Third, the infrastructure
42 is secure. Data that are sent to devices are tiled as rasters; the underlying information remains on the
43 “back-end” database and no shapefiles are delivered to the user. Fourth, the scheme is extensible. Because
44 it is based on HTML5 and Web Markup conventions, it can be coupled with a wide variety of other Web

1 services, such as syndication services, geolocation services that can automatically pull a user's location
 2 (and deliver the necessary map services for that location), animation schemes using WebGL (the Web-
 3 based version of the Open Graphics Library, for 2D and 3D visualization and animation), and any number
 4 of other "mash-ups". It can also be integrated with other markup schemes, such as the Geographic
 5 Markup Language (GML; <http://www.opengeospatial.org/standards/gml/>), or other emerging markup
 6 schemes in urban and transportation planning and management, such as CityGML
 7 (<http://www.citygml.org/>) or the NCHRP's TransXML for transportation data (<http://www.transxml.org/>).
 8 A schematic of the web interface development is shown in Figure 3.



9

10 **FIGURE 3 Methodology of Web Interface Development**

11

12 CASE STUDY

13 The proposed framework is applied to a comprehensive transit network in the Washington-Baltimore
 14 region. The complete transit network is adapted from Maryland State Highway Administration data. The
 15 transit database consists of the two largest transit systems namely in the region, Washington Metropolitan
 16 Area Transit Authority (WMATA), and Maryland Transit Administration (MTA). WMATA is a tri-
 17 jurisdictional government agency that operates transit service in the Washington, D.C. metropolitan area,
 18 including the Metrorail (rapid transit), Metrobus (fixed bus route) and Metro Access (demand response),
 19 and is jointly funded by the District of Columbia, together with jurisdictions in suburban Maryland and
 20 northern Virginia. There is approximately \$300 million spent in the WMATA capital, operating and
 21 maintenance cost of which \$150 million per year of Federal funds available that are required to be
 22 matched by \$50 million in annual contributions from DC, Northern Virginia and suburban Maryland,
 23 each for ten years.

24 WMATA has the second highest rail ridership in the US with over 950,000 passengers per day.
 25 This is second only to New York. The WMATA Metro provides an extensive heavy rail system with
 26 106.3 route miles. The WMATA bus system also serves an extensive ridership of over 418,000 unlinked
 27 daily trips. Figure 4(a) shows the WMATA network at Union Station.

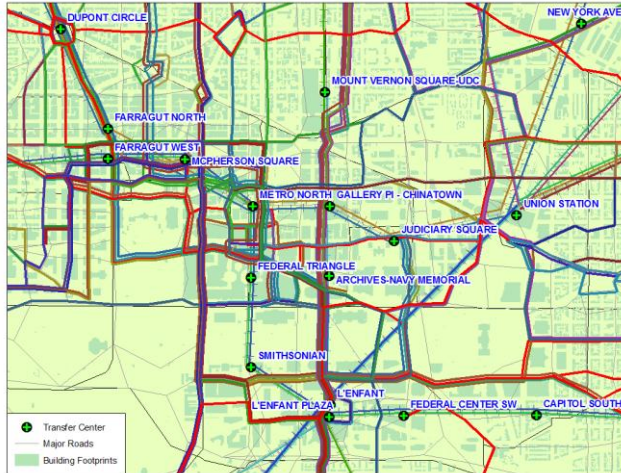


FIGURE 4(a) Thematic of the transit lines in Washington DC

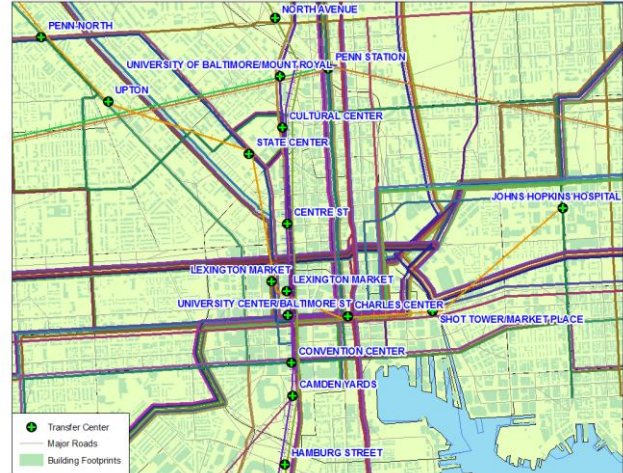


FIGURE 4(a). Thematic of the transit lines in Baltimore

1 On the other hand, MTA is a state-operated mass transit administration in Maryland. MTA
 2 operates a comprehensive transit system throughout the Baltimore-Washington Metropolitan Area. There
 3 are 77 bus lines serving Baltimore's public transportation needs. The system has a daily ridership of
 4 nearly 300,000 passengers along with other services that include the Light Rail, Metro Subway, and
 5 MARC Train. The Baltimore Metro subway is the 11th most heavily used system in the US with nearly
 6 56,000 daily riders. Nearly half the population of Baltimore lack access to a car, thus the MTA is an
 7 important part of the regional transit picture. The system has many connections to other transit agencies
 8 of Central Maryland: WMATA, Charm City Circulator, Howard Transit, Connect-A-Ride, Annapolis
 9 Transit, Rabbit Transit, Ride-On, and TRANSIT. Figure 4(b) shows MTA network around Camden in
 10 station downtown Baltimore. Both the WMATA Metro rail system and the Baltimore transit system are
 11 connected by the MARC commuter rail system. This system has a daily ridership of over 31,000. In the
 12 next section, results of the proposed methodology are discussed (APTA 2011). The complete
 13 methodology is integrated in a Geographic Information System (GIS) user interface using ArcInfo (ESRI
 14 2010).

15 RESULTS

16 The results reported in the following sections are based on the application of methods developed in this
 17 paper on a large-scale multi-modal network of the Washington DC and Baltimore region. The system
 18 represents one of the largest and most heavily patronized transit systems in the county. The application of
 19 the methodology to this complex network provides a demonstration of the scalability of the connectivity
 20 index.

21 Region-wide View

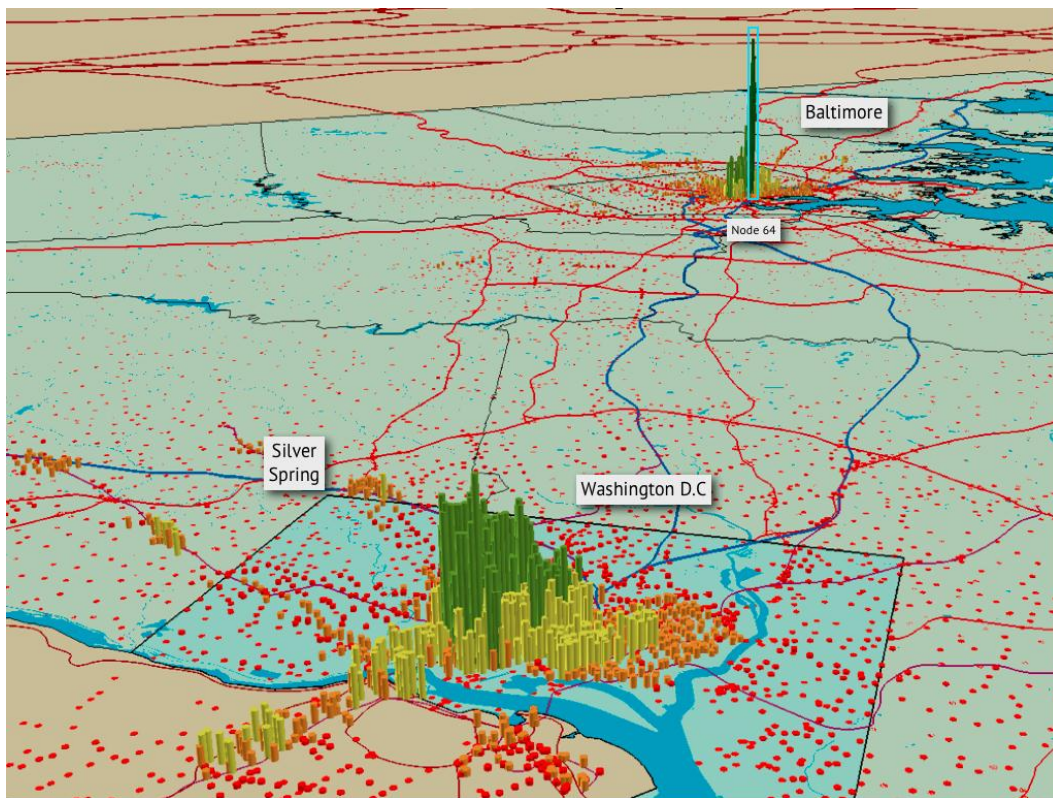
22 The Washington/Baltimore region has a significant number of transit nodes, each of which provide a
 23 varying degree of connectivity to the network. Determining network connectivity and funding
 24 prioritization is a highly complex task in a multi-modal network. Funding prioritization is additionally
 25 aided by the connectivity index by providing decision makers with a tool to measure network resilience.
 26 As with any network, transit systems are designed to interact with many different nodes, while remaining
 27 functional in the event that a particular node becomes inaccessible. Additionally, resiliency tests based on
 28 connectivity can reveal if there is an over concentration of connections which rely on a given node,
 29 or region. Figure 5 provides a three-dimensional view of connectivity for transit network in the study
 30 area. The image illustrates how useful visualization can be in understanding the topography of network
 31 connectivity. However, this type of visualization requires significant amounts of computing power,

1 knowledge of GIS software and lacks interactivity both for planners and end-users. In the following
2 sections we describe a computationally feasible, interactive tool we have developed to make the type of
3 visualization seen in Figure 5, useful for a broad audience. Baltimore, Washington DC, and Silver Spring
4 are three areas with extensive transit connectivity. The highlighted node 64 is the most connected station
5 in the Baltimore-Washington DC region.

6 **The Graphical User Interface**

7 Several innovations are provided in the Graphical User Interface (GUI). First, the graphical experience of
8 the interface remains the same, regardless of the device from which the map is accessed, and regardless of
9 the browser that is used to view the map. This is significant as it allows the tool to be used on different
10 screen-sizes, different platforms, and different operating systems, without any required intervention from
11 the user, while constantly preserving a similar experience.

12 Second, a variety of data-layers can be added to the interface. Above, we show four dimensions
13 of transit connectivity (transfer stops, transit nodes, rail lines, and bus lines), overlaid and georeferenced
14 to a base map that illustrates major landmarks, political boundaries, street-names, routes, and features
15 along the D.C./Northern Virginia border. This canvas “backdrop” could show anything: historical maps,
16 dynamic weather patterns, population density, aerial photography, and so on.



17

18

19 **FIGURE 5 Regional Node Connectivity in a GIS Map**

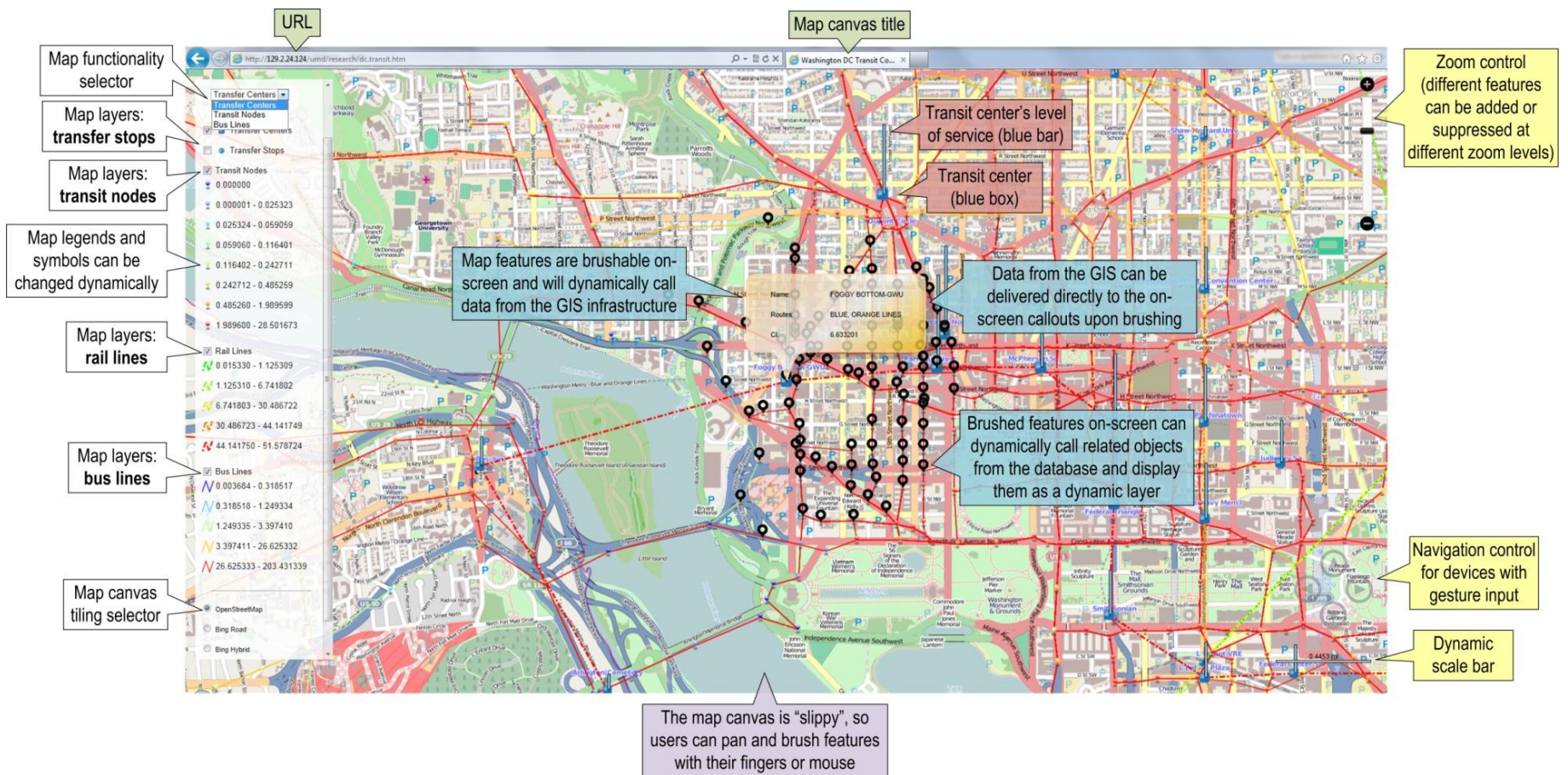


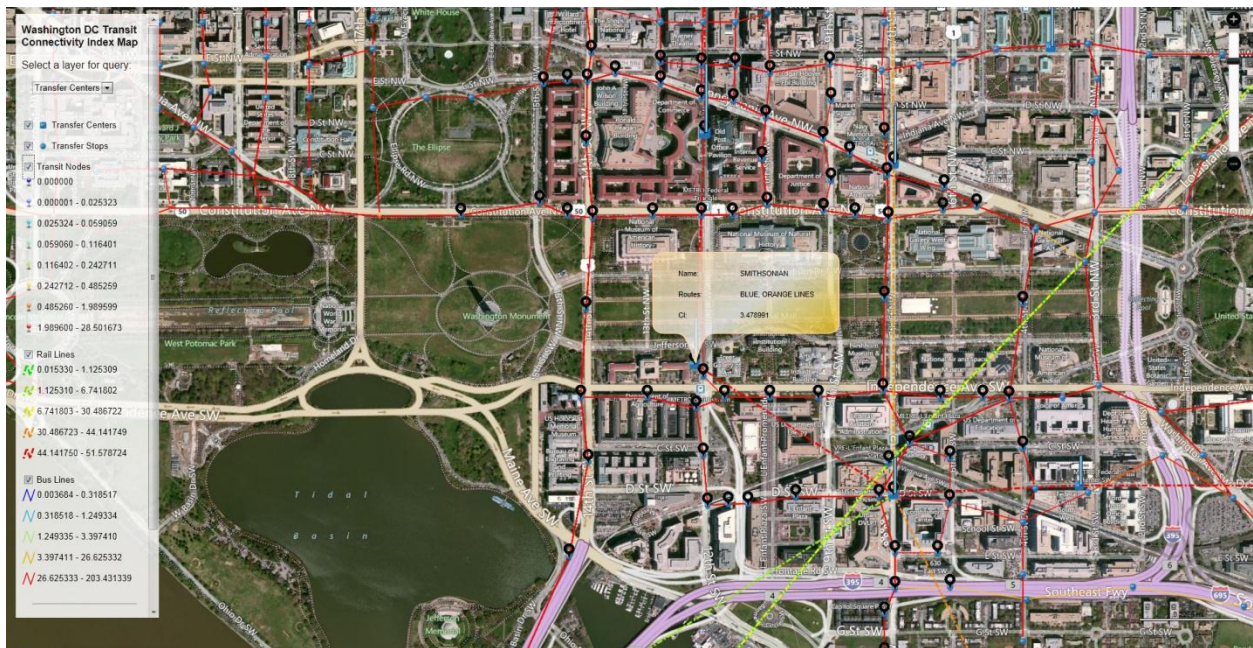
FIGURE 6 GUI Interface of the Transit Connectivity Tool

1 Third, the symbology on the map can be swapped on-the-fly using Cascading Style Sheets (CSS).
 2 In essence, CSS allows for the specification of a set of themes that can be substituted at will, for example,
 3 when a particular scaling factor is invoked, when a particular functionality is called, or when a particular
 4 action is initiated. The CSS schema can be ported from other applications and they are easily loaded
 5 without further input from the user.

6 Fourth, all of the features on the GUI can be rendered dynamically, with minimal update
 7 requirements. For example, as new entries are populated in the database, they can be updated on-screen.
 8 Similarly, as variables change, the data can refresh on the GUI. Users could, for example, edit
 9 information through callouts on the GUI, and these changes could be pushed to the database. They would,
 10 very quickly, be available to any other users simultaneously accessing the data.

11 Fifth, data elements on the canvas can be dynamic (or not). The entire canvas can be panned and
 12 zoomed using standard gestures on mobile devices (or using overlaid zoom and pan controls that we have
 13 provided for devices that do not support gesture control or mouse-based input). Similarly, users can use
 14 their fingers to tap or brush objects on-screen, which will return data for that feature from the database.
 15 As shown above, these tap-queries can also be used to perform spatial (or temporal) queries, returning
 16 nearby features of relevance.

17 Figure 6 shows the graphical user interface (GUI) to the transit connectivity tool. Above, a
 18 transfer center (blue square) at Foggy Bottom has been selected by brushing the icon on-screen. This
 19 action, in turn, sends a query to the database to return the transfer stops (highlighted a black circles) that
 20 have been used in calculating the center's level of service (which is illustrated by a blue bar above the
 21 center's icon). The symbology in the map legend can be changed dynamically, or allied to Cascading
 22 Style Sheets. Different map features can be added and suppressed at different zoom levels. Different
 23 canvases can be loaded as a backdrop to the data layer. Above, an OpenStreetMap canvas is shown. As
 24 data are altered in the spatial database, the results will be immediately available in the mapping interface.
 25 An increased zoom level of connectivity is shown in Figure 7.



26
 27 **FIGURE 7** The same data, shown with increased zoom and aerial photography as the canvas
 28 **backdrop.**

1 7.3 Multi-level Network Visualization

2 The network visualization interface allows users to view connectivity at three distinct levels. At the
 3 highest level, line connectivity provides an overview of the entire transportation system, its interaction
 4 with each mode and a quick way to determine the best and least connected parts of the network. Figure
 5 8(a) provides a screenshot of the line-level overview. In the image, the red lines represent the highest
 6 levels of connectivity while the blue lines show the lowest connectivity for buses and green show the
 7 lowest connectivity for rail. In this case, the lines that service the Baltimore and Washington cores and the
 8 major links between the two cities are very well connected.

9 At a less macro level, the transfer center analysis provides users with connectivity scores of major
 10 intermodal transit stations. For each transfer center connectivity score, the index value is derived from a
 11 combination of the walking time from smaller facilities and their respective score, for stops that are
 12 within a half-mile of the major facility. Figure 8(b) shows the transfer center results in the user interface
 13 window. Transfer centers are given a vertical bar that represents the level of connectivity. When the bar is
 14 clicked, all of the associated transit stops (the stops that contribute to the transfer center score) are
 15 highlighted.

16 At the lowest level, node connectivity shows how well connected each transit stop is in the entire
 17 system, relative to the rest of the network nodes. Figure 8(c) shows the Baltimore transit network at the
 18 node level. Like the line level index, the red node denote the best connected transit stops while yellow is
 19 less connected, and blue shows the nodes with the least amount of connectivity. The visualization at this
 20 level provides a good tool for planners to quickly assess locations that may be important but not fully
 21 connected to the rest of the transit network. Typically several adjacent nodes have similar levels of
 22 connectivity and the ability to easily display them provides one import way to prioritize the need for
 23 future transit investment.

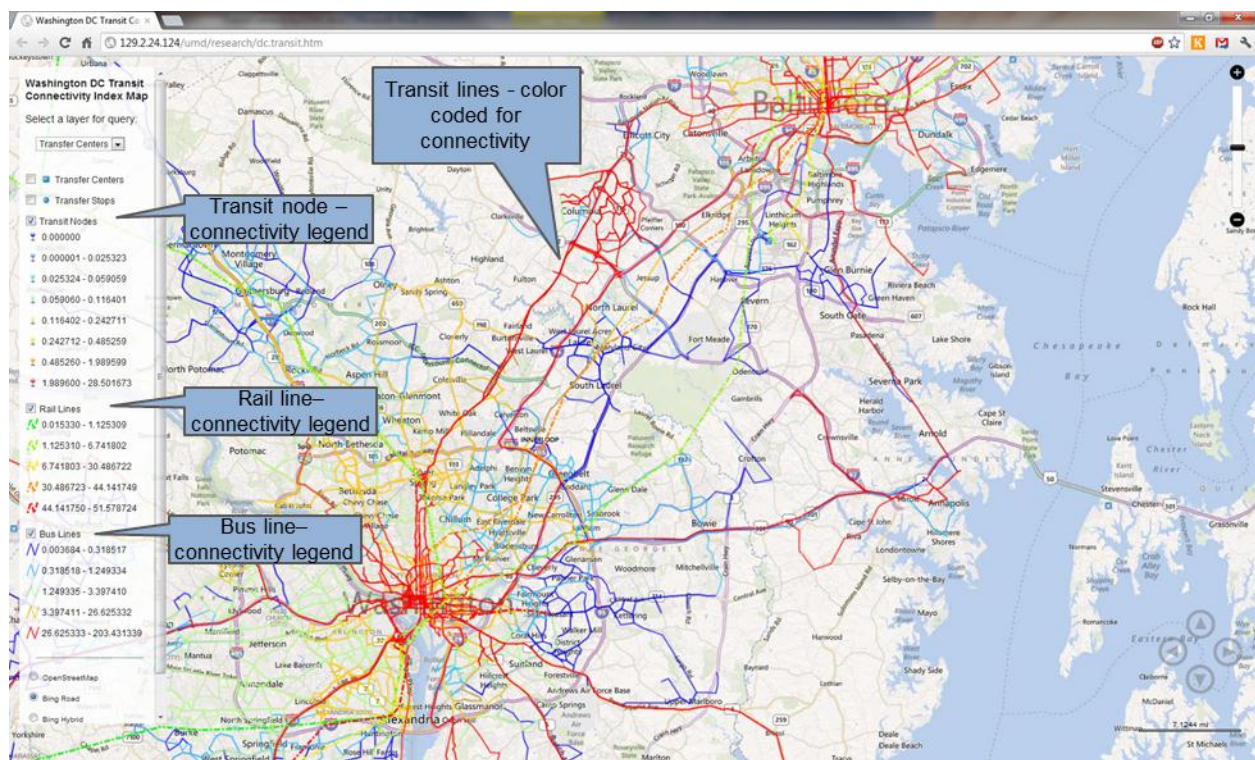


FIGURE 8(a) Line level connectivity

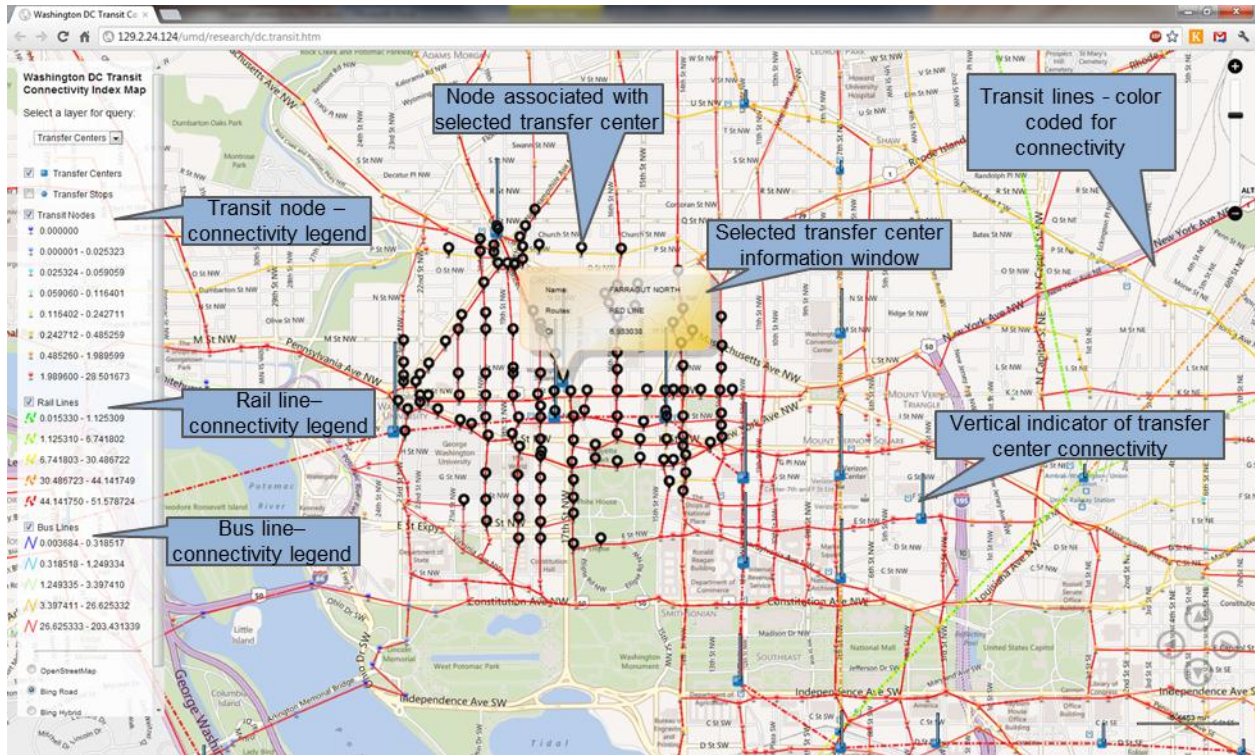


FIGURE 8(b) Transfer center level connectivity

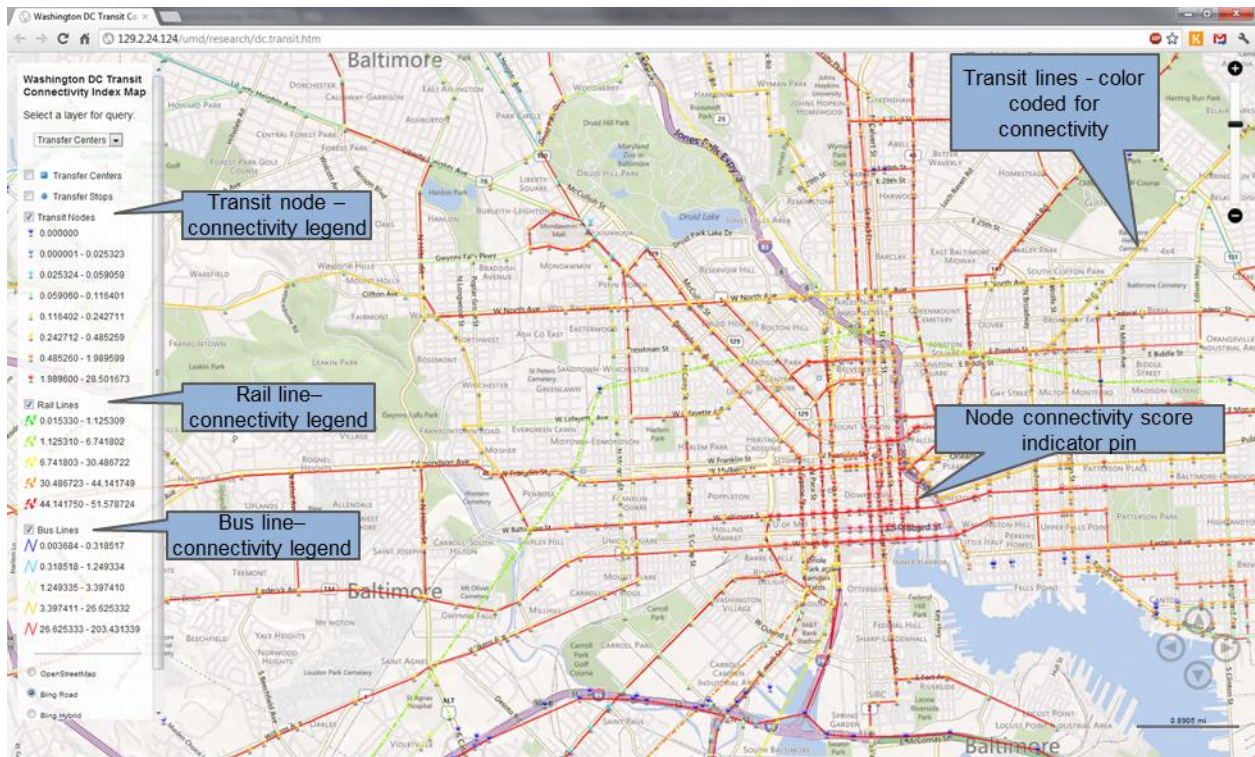


FIGURE 8(c) Node level connectivity

1 **Cross Platform Compatibility**

2 The connectivity GUI shown in the previous section is compatible with a broad range of software and
 3 hardware platforms. This cross-compatibility makes the index useful for a variety of purposes and end
 4 users. In figure 9(a) and 9(b), the interface is shown on a PC. The first figure (a) shows what users will
 5 see when the first load the interface, providing an overview of the entire network. The second figure (b)
 6 provides an example of the zoomed view of then network with connectivity scores. Figures 9(c) and 9(d)
 7 show the mobile capabilities of the index and user interface, displayed here on an iPad. The network
 8 levels are the same as with the PC view. With a mobile device, users can pan and scroll using the built-in
 9 navigation controls, or by using the finger gestures supported by their mobile device.

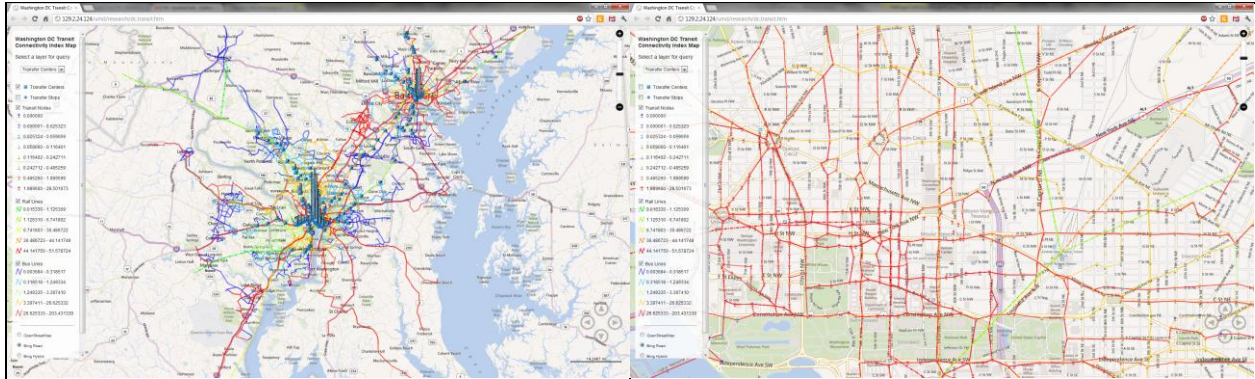


FIGURE 9(a) GUI Interface on PC – Full Network

FIGURE 9(b) GUI Interface on PC – Node and Links

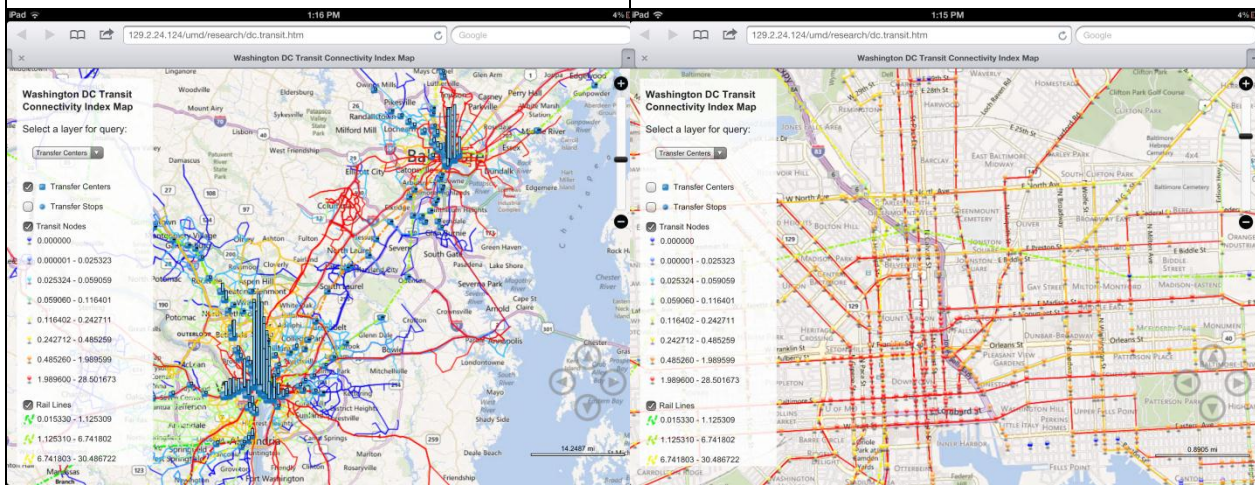


FIGURE 9(c) GUI Interface on iPad – Full Network

FIGURE 9(d) GUI Interface on iPad – Node and Links

10

11 **CONCLUSION**

12 In this paper, we present connectivity indicators to represent the potential ability of a transit system
 13 encompassing comprehensive clustered development in a multimodal transportation network. The paper
 14 is constructed around two broad themes. The first theme discusses the concept of connectivity depicted in
 15 terms of a graph theoretic approach. The second theme emphasizes a visualization aspect using an
 16 HTML5 platform with state of the art GIS technology.

1 Connectivity defines the level of coordination of the transit routes, coverage, schedule, speed,
2 operational capacity, urban form characteristics, and is an influential element of the image of any transit
3 network. The difficulty for development of connectivity indicators lies in the complex interacting factors
4 embedded in a multimodal transit network that encompasses various public transportation modes with
5 different characteristics, such as buses, express buses, subways, light rail, metro rail, commuter and
6 regional rail. In addition, multimodal transit networks, like road networks, consist of nodes and links.
7 However, links in a multimodal transit network have different characteristics from those in a road
8 network as links in a multimodal transit network are part of a transit line that serve a sequence of transit
9 stops (nodes) and a stop can be served by different transit lines; multiple links may exist between nodes in
10 a multimodal transit network. The indicator development process is further complicated as connectivity
11 varies by urban form with differences among geographical, land use, highway and trip pattern
12 characteristics between regions. A good performance indicator should, therefore include all the
13 aforementioned complexities and should be quantified to portray connectivity of the multimodal
14 transportation network.

15 We developed a set of connectivity indexes for (1) nodes, (2) links, and (3) transfer centers. The
16 node connectivity index includes the transit lines passing through it, their characteristics such as speed,
17 capacity, frequency, distance to destination, activity density of the location, and degree centrality. The
18 link connectivity index is the sum of connectivity indexes of all stops it passes through and normalized to
19 the number of stops. The concept of a connectivity index for a transfer center is different from the
20 connectivity measure of a conventional node. Transfer centers are groups of nodes that are defined by the
21 ease of transfer between transit lines and modes based on a coordinated schedule of connections at a
22 single node or the availability of connections at a group of nodes within a given distance or walk time.
23 The sum of the connecting power of each node in the transfer center is scaled by the number of nodes in
24 the transfer center. Thus, a node in a heavily dense area is made comparable to a transfer center located in
25 a less dense area.

26 Network connectivity is a complex concept, especially in the context of a multi-model
27 transportation system. In this paper we develop theoretically robust connectivity index for such a system.
28 Though the theory is complex, the application of this index is simple and its results provide important
29 insights on system performance, yielding useful tools for transportation planners and policy-makers.
30 Though such an index alone can be a powerful tool, we also develop an interface for user interaction with
31 the connectivity index. With this interface, planners and other end-users can easily locate highly
32 connected or poorly connected nodes, links, routes and transfer centers. The interface can reveal much
33 more about system performance than pages of tables or numbers. Planners can easily use this tool to
34 develop, prioritize and justify transit investments. Other users can interact with the index and map to
35 determine the level of transit service in their city or neighborhood.

36 The paper has significant importance to research and practice. Major contributions of the paper
37 include (1) extending the graph theory approach to determine the performance of the multimodal transit
38 network; (2) quantifying the measures of connectivity at the node, line, and transfer center; (3) developing
39 a web GIS tool using an HTML 5 interface which can be used in cross platform applications with online
40 query, browsing and exploring all features of the connectivity performances in a multimodal transit
41 network; (4) providing a comprehensive framework for analyzing connectivity, and efficiency of transit
42 networks for agencies that do not have access to well-developed travel demand and transit assignment
43 models, and (5) demonstrating the applicability of the proposed framework in a heavily used multimodal
44 transit network in the Baltimore-Washington region. In future research, transit network resiliency can be
45 incorporated in a web GIS platform with features such as updating connectivity measures in real time; and
46 adding connectivity features of which are catered to pedestrians.

47

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