Economic Competitiveness and Equity Based Safety Improvements Allocation Model For Urban Intersections

Sabyasachee Mishra, Ph.D., P.E.
Assistant Professor
Department of Civil Engineering
University of Memphis
3815 Central Avenue, Memphis, TN 38152
Phone: (901) 678-5043
Email: smishra3@memphis.edu
(Corresponding Author)

Sushant Sharma, Ph.D.
Assistant Research Scientist
Texas A&M Transportation Institute
Texas A&M University System
1100 N.W. Loop 410, Suite 400, San Antonio, TX 78213
Phone: (210) 979-9411 Ext. 17204
E-mail: s-sharma@tamu.edu

Mihalis Golias, Ph.D.
Assistant Professor
Department of Civil Engineering
University of Memphis
3815 Central Avenue, Memphis, TN 38152
Phone: (901) 678-3048
E-mail: mgkolias@memphis.edu

Stephen D. Boyles, Ph.D.
Assistant Professor
Department of Civil, Architectural & Environmental Engineering
The University of Texas at Austin
Ernest Cockrell, Jr. Hall (ECJ) 6.2041 University Station C1761, Austin, TX 78712
Phone: (512) 471-3548
E-mail: sboyles@mail.utexas.edu

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ABSTRACT

Economic competitiveness and equity can be two competing objectives while allocating funds for implementation of safety alternatives on urban intersections. One of the critical phase of current safety management process (hazard elimination program) undertaken by most states is resource allocation among identified crash locations. Literature underlines the importance of this phase and lack of sophisticated tools available to state planning agencies for evaluating federal and state policies. The study overcomes this limitation by proposing an optimization based resource allocation model that maximizes safety benefits, subjected to budget and policy constraints. The proposed model incorporates economic competitiveness in the allocation and distributes improvements to urban intersections such that maximum economic benefits are obtained from crash savings. However, results show that while economic competitiveness leads to optimal benefits, resource allocation is inequitable. Hence equity based models are developed by adding two policy options: equity in opportunity and equity in outcome. The resource allocation model is solved using sequential quadratic programming. The model is applied to crash prone intersections in four counties of southeast Michigan. The proposed model is generic and scalable, with flexibilty in including policy options often considered by state and local agencies.

INTRODUCTION

Moving Ahead for Progress in the 21st Century (MAP-21) funding and authorization bill sanctioned continuation of legacy Highway Safety Improvement Program (HSIP) as a core federal-aid program. HSIP envisions significant reduction in traffic fatalities and serious injuries on the highway system. Under HSIP, state Departments of Transportation (DOT's), along with the US Department of Transportation (US DOT) spend billions of dollars annually for safety improvement programs at urban intersections.

Current safety management process (referred as hazard elimination program) undertaken by most states consists of three phases: (1) selection of candidate locations where safety improvements are warranted; (2) development of countermeasures/alternatives for potential crash reduction; and (3) allocation of resources among candidate locations in conformance with budgetary and other constraints. State planning agencies often consider these three steps as independent and sequential. The third step (resource allocation and project prioritization) is the most critical phase *(1)* and suffers from sub-optimal fund allocation with low safety benefits and long-term capital loss due to lack of sophisticated analysis tools *(2–7)* .

Crash ocurrence is a stochastic process, which depends on several factors such as highway geometry (e.g. gradient, number of lanes, medians, shoulders lane marking, roadway functional class, sight distance), traffic characteristics (average annual daily traffic, right and left turning traffic volume, speed etc.), and the environment (weather, lighting, visibility etc.). There is significant research on prediction of crashes by accounting for these variables and utilizing statistical methods *(8–11)*. However, past research on highway safety resource allocation have only considered fixed growth factor for crashes or employed simple forecasts in the prediction process.

Hence, two critical components of safety improvement resource allocation are crash prediction and fund/resource allocation for preventative measures. Stochastic nature of crashes, require appropriate methods that can provide robust results in a long-term planning. Ignoring randomness by assuming deterministic growth can lead to inappropriate allocation of highway safety improvements. Further, current approach of state planning agencies lacks fund allocation policies over a planning period. Resource allocation models need to consider optimization-based tools that maximize long-term safety benefits by employing the proposed alternatives under budgetary and other constraints. Both of these critical components need to be integrated for simultaneous crash prediction and resource allocation to the hazardous locations. Hence, the first key research question in this paper is:

> "*How to simultaneously predict crashes and allocate resources at pre-determined crash locations to implement preventative alternatives that maximize benefits within constraints over a planning horizon?*"

Economic competitiveness and safety are two of the five major goals of US DOT's Strategic Plan for fiscal years 2012-2016 *(12)*. The central idea that bridges both these goals is to achieve maximum economic returns on policies and investments for safety management. National Safety Council *(13)* estimates average economical cost of all motor vehicle crashes (i.e. fatal, nonfatal injury, and property damage) as $7,640,000 on a per death basis. According to NSC, the costs are a measure of the dollars spent and income not received due to accidents, injuries, and fatalities, which is another way to measure the importance of safety resource allocation and their impact on the nation's economy *(13)*. Hence, a critical objective for optimal safety resource allocation is economic benefits from preventative measures, referred as economic competitiveness in the remaining paper.

Further, at the regional level multiple counties are competing for funds to implement preventative alternatives. Fund allocation based on crash severity or economic competitiveness may cause regional in-equity. In economics, equity and economic competitiveness is known to pose conundrum and same can be seen in terms of resource allocation for safety measures. Equity in transportation has typically been considered under the umbrella of environmental justice in terms of distributing benefits and impacts among privileged and underprivileged populations (*14, 15*). However, the concept can more generally reflect the distribution of impacts by geographic region as well. Quantitative methods used to measure equity vary, and include least-squares *(16)*, ratio-based *(17)*, or accessibility measures *(18)*. Incorporation of equity in highway safety resource allocation problem is not considered in studies or practice to date. Hence, the second key research question apart from including both these objectives in the model is:

> "*Does economic competitiveness or equity as objectives in optimal resource allocation model result in different fund allocation for safety alternatives in a region? If yes, which objective serves the best interests of state and federal agencies?"*

The remainder of the paper is organized as follows: Next section presents the literature review specific to resource allocation models followed by the methodology and model formulation. The data set used for demonstration and model application is discussed next. Finally, the research is summarized and recommendations for future research is outlined.

LITERATURE REVIEW

This section captures recent developments in resource allocation methods. The review presented is not a comprehensive one but is designed to capture a representative cross-section of studies conducted on this subject in the past two decades.

The topic of resource allocation (using optimization techniques) spans diverse areas such as operations research, manufacturing, management, finance, and transportation. Optimization usually involves maximization or minimization of an objective function comprising a set of decision variables, subject to various constraints *(19, 20)*. Constraints are designed to reflect limitations imposed by practical and/or policy considerations, expressed in the form of (in)equalities. Different optimization techniques such as linear, integer, nonlinear, and dynamic programming have been used to allocate resources on various engineering and management problems *(21, 22)*.

Resource allocation on highway safety improvement methods include application of mixed integer programming techniques, based on branch and bound algorithm for highway safety projects *(23)*; linear programming techniques to maximize savings resulting from alcohol-crash reduction *(24)*; linear programming to select safety and operational improvement on highway networks *(25)*; integer programing for reduction in crashes *(26)*; integer programming to minimize total number of crashes *(27)*; linear programming for highway safety improvement alternatives *(28)*; and linear programming to incorporate uncertainty in safety resource allocation *(29)*.

The literature review presented above shows that within the general framework of optimization approach, researchers have used different model formulations and different solution techniques to address their respective issue. Objective functions include minimizing crashes and maximizing benefits measured in monetary values. Most of the papers reviewed by the authors allocated resources for one year; with only a limited few attempting multi-year allocation. Different researchers have treated constraints differently to reflect various policy and practical considerations.

Resource allocation in highway safety research *(23, 30)* is limited due to the need for integer programming to be combined with a crash prediction model. While integer programming and crash prediction have been studied extensively separately for various applications, there is no study that integrates both in highway safety resource allocation. Since optimally considering proposed alternatives is a discrete decision variable, literature recommends complex integer programming *(21)*.

In the context of literature discussed above and in the introduction section, the paper identifies following critical gaps in literature :

- Analysis of economic competitiveness and equity issues in safety resource allocation problem.
- Integration of stochastic crash prediction models within the modeling framework.
- Optimally allocation of funds for preventative alternatives within budget and policy constraints in a region.
- Robust analysis of various policy options for multiple counties in a region for a planning period.
- Flexibility of policies to incorporate a multi-year planning period, multiple-counties within a region and consideration of several preventative alternatives.

MODELS & METHODOLOGY

The conceptual methodology of the resource allocation model is shown figure 1 and comprises of three steps: (1) crash prediction, (2) resource allocation, and (3) policy analysis. The crash prediction component consists of several sub-steps. The first task is to identify hazardous crash locations based on crash frequency and severity. Then, for each location predominant crash patterns need to be derived. Based on crash patterns, appropriate countermeasures are designed. These steps leverage the information from the first two phases of hazard elimination program (i.e. identification of hazardous locations and countermeasures) in modelling. For the development of a crash prediction model, highway, traffic, and environment data are collected. Considering the random nature of crash occurrence, an appropriate model is developed. It should be noted that details of the crash prediction model are not presented in this paper for brevity and length limitations but can be found here[*].

Next, the resource allocation component involves an integer programming approach to allocate improvements (proposed alternatives or countermeasures to reduce crashes) subject to budget and other constraints. In this step, overall objective of resource allocation and policy constraints are finalised. Input data is fed into the optimization model and simulatenously run with crash prediction model. Next step is a policy analysis tools that involves a set of useful pragmatic scenarios and alternative ways to allocate resources.

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[*] Mishra, S., Sharma, S., Golias, M, Boyles, S. (2013). Crash Prediction Results for Resource Alloicaton. http://www.ce.memphis.edu/smishra/Publications/CrashPredictionResults.pdf

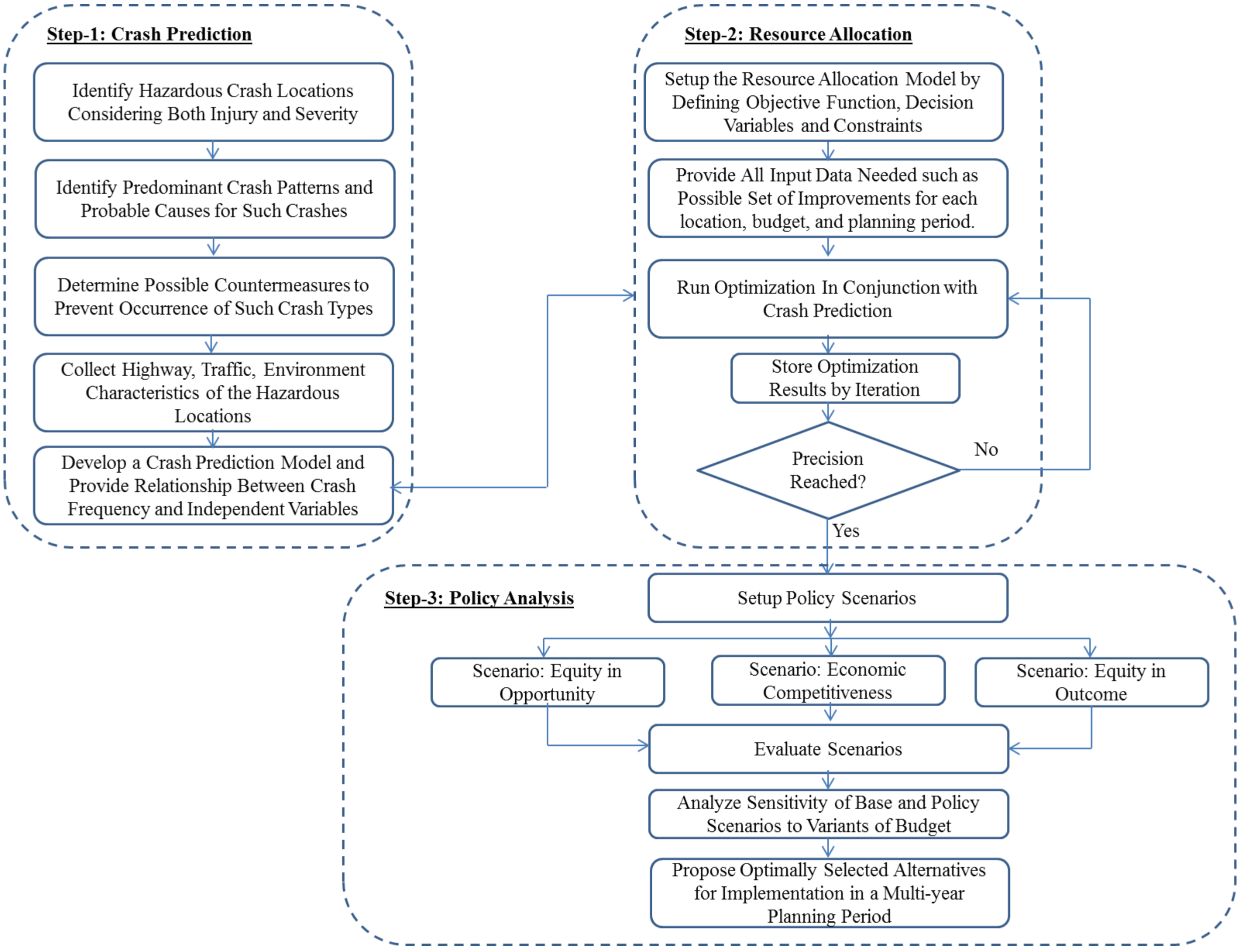

FIGURE 1 Proposed Methodology for Simultaneous Crash Prediction and Resource Allocation.

Economic Competitiveness Based Resource Allocation Model (EC-RAM)

An economic competiveness based resource allocation model is proposed to optimally allocate preventative safety alternatives at set of locations/intersections. The model maximizes total economic and safety benefits (Z) derived from prevented crashes upon safety upgrade implementation during a planning period of N years. An integer programing model is proposed based on three binary variables, indexed by the intersection i, safety improvement choice j, and year of implementation n. Each improvement j has an effective duration of l_j years. The binary variable $x_{i,j}^{n} = 1$ if alternative j is implemented at location i during year n and zero otherwise, and $y_{i,j}^{n,n'} = 1$ if alternative j is implemented at location i during year n, and is still active during year n' and zero otherwise (i.e. $y_{i,j}^{n,n'} = 1$ if $x_{i,j}^{n} = 1$ and $0 \le n' - n \le l_j$). The model has three sets of constraints: a) budget constraint, b) constraints based on the feasible alternatives for each intersection, and c) definitional constraints relating x and y. Complete list of notations can be found in the appendix section attached in the end of the paper.

Objective Function

Let f_i^n, m_i^n, and p_i^n denote the expected number of fatal crashes, injury or non-fatal crashes, and property damage only (PDO) collisions at location i during year n. Similarly, let $r_{i,j}^{f}$, $r_{i,j}^{f}$, and $r_{i,j}^{f}$

denote the crash reduction factors for these three types of crashes if treatment j is applied at intersection i, and c^f, c^m, and c^p the economical costs of each type of crash (obtained from National Safety Council (NSC, 2013)). The economic competitiveness based objective function can then be written as:

$$Maximize\ Z = \sum_{n=1}^{N}\sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{n'=1}^{N}\left[f_i^n r_{i,j}^f c^f + m_i^n r_{i,j}^m c^m + p_i^n r_{i,j}^p c^p\right] y_{i,j}^{n,n'} \qquad (1)$$

Where, $y_{i,j}^{n,n'}$ is a binary decision variable that assumes a value of 1 if alternative *j* implemented at location *i* in year *n* is still active in *n'* year and it's summation over *n* and *n'* gives total number of active alternatives.

Constraints

Equation (2) is a budget constraint, that ensures the total capital investment,operation, and maintenance (O&M) costs does not exceed the total budget of the planning period. However, there is a flexibility of expenditure between the years in the planning period. Such flexibility in expenditure can be incorporated into the procedure through a planning based budget model (Mishra et al. 2013). In these models a planning period budget is based on the assumption that the agency has the flexibility of borrowing monies from subsequent years' allocation or past year surplus. Let $\pi_{i,j}^n$ represent the capital cost of constructing improvement *j* at intersection *i* in year *n*, and $o_{i,j}^{n,n'}$ the operating costs in year *n'*. $x_{i,j}^n$ is a binary decision variable equal to 1 if alternative *j* is implemented at location *i* in year *n*. Also, let b_n be the available budget available for year *n*. Then the budget constraint is as follows:

$$\sum_{n=1}^{N}\sum_{j=1}^{J}\sum_{i=1}^{I}\left[\pi_{i,j}^n x_{i,j}^n + \sum_{n'=1}^{N} o_{i,j}^{n,n'} y_{i,j}^{n,n'}\right] \leq \sum_{n=1}^{N} b_n \qquad (2)$$

For a variety of reasons, not all alternatives can be implemented at all locations. Further, based on engineering design, the suggested alternatives tend to be location- specific. Accordingly, constraint (3) ensures that the alternatives implemented at a location, using pre-specified parameters $\hat{x}_{i,j}^n$:

$$x_{i,j}^n \leq \hat{x}_{i,j}^n, \forall i,j,n \qquad (3)$$

Equation (4) denotes that each location *i* can have a limited number of active alternatives (γ_i^n) during the analysis year *n*, pre-specified by the planning agency.

$$\sum_{j=1}^{J}\sum_{n=1}^{N} y_{i,j}^{n,n'} \leq \gamma_i^n \quad, \forall i, n' \qquad (4)$$

When the alternatives are mutually exclusive, as in the economic competitiveness case, γ_i^n is equal to one. This provides the following features:

- *Feature 1*: A location can receive only one alternative in a given year.
- *Feature 2:* A location, that has the carry-over effect from an alternative implemented in previous years, may not receive any funds during the service life of the alternative. (Note: This constraint can be modified as desired).

Furthermore, the definitions of *x* and *y* require:

$$y_{i,j}^{n,n'} \leq \begin{cases} x_{i,j}^{n} & 0 \leq n' - n \leq l_j \\ 0 & \text{otherwise} \end{cases}, \forall i, j, n, n' \tag{5}$$

$$x_{i,j}^{n} \leq y_{i,j}^{n,n'}, \forall i, j, n, n' \tag{6}$$

$$x_{i,j}^{n}, y_{i,j}^{n,n'} \in \{0,1\}, \forall\ i, j, n' \tag{7}$$

Equation (5) requires an improvement cannot be active at a given year unless it was implemented in a year within its duration of effectiveness. Equation (6) prohibits an already-active improvement from reselection during its duration of effectiveness. Finally, Equation (7) reflects the binary nature of the decision variables.

Equity based Resource Allocation Models (EQ-RAM)

Recent federal and state policies emphasize equity in transportation projects. Hence an equity based resource allocation model (EQ-RAM) will provide an additional policy option for state agencies. Literature makes a sharp distinction between “equality of outputs” and “equality of outcomes” *(31)*. “Equality of outputs” refers to an equal allocation of resources, such as funding, while “equality of outcomes” refers to an equal allocation of benefits. In this paper we refer “equality of outputs” as Equity in Opportunity.

Equity in Opportunity based Resource Allocation Model

This policy is designed to ensure that each county receives an equitable distribution of funds or number of preventative alternatives within a threshold. This condition can be achieved in the adding following constraint to EC-RAM: let I_δ denote the set of intersections in county δ, I_ε denote the set of intersections in county ε, and θ (a constant greater than or equal to 1) the upper limit for the ratio between the numbers of alternatives selected for any pair of counties. This constraint ensures number of alternatives allocated to a particular county, as compared to other county are within a bound (θ). Then the following inequality must hold:

$$\sum_{n=1}^{N} \sum_{j=1}^{J} \sum_{i \in I_\delta} x_{i,j}^{n} \leq \theta \sum_{n=1}^{N} \sum_{j=1}^{J} \sum_{i \in I_\varepsilon} x_{i,j}^{n} \text{ for all } I_\varepsilon, I_\delta \tag{8}$$

Equity in Outcome based Resource Allocation Model

This policy is based on the assumption that all counties should receive economic and safety benefits in an equitable manner. Even if counties receive an equitable number of projects (as in equity in opportunity), the amount of benefits they receive because of crash savings is inconsistent. Equity in outcome is a critical measure that guarantees equal benefits, hence justifies investments in preventative measures. Following constraint is incorporated in EC-RAM, as it ensures benefits from the alternatives allocated to a particular county, are within a reasonable limit as compared to other counties :

Assume τ be a constant greater than or equal to 1. This policy measure is framed by addition of following constraint in the EC-RAM and assuming suitable value for τ.

$$\sum_{n=1}^{N}\sum_{j=1}^{J}\sum_{i\in I_\delta}^{I}\sum_{n'=1}^{N}\left[f_i^n r_{i,j}^f c^f + m_i^n r_{i,j}^m c^m + p_i^n r_{i,j}^p c^p\right] y_{i,j}^{n,n'} \le \tau \sum_{n=1}^{N}\sum_{j=1}^{J}\sum_{i\in I_\varepsilon}^{I}\sum_{n'=1}^{N}\left[f_i^n r_{i,j}^f c^f + m_i^n r_{i,j}^m c^m + p_i^n r_{i,j}^p c^p\right] y_{i,j}^{n,n'} \tag{9}$$

MODEL APPLICATION

Study Area

The state of Michigan is used as the study area in this paper. The resource allocation model for highway safety improvements is applied to a set intersections in the Southeast Michigan region comprising of four counties (Wayne:County-1, Washtenaw:County-2, St. Clair: County-3, and Oakland: County-4). The 20 highest crash frequency locations from each of the four counties were selected (a total of 80 intersections) representing a sub-set of 25,000 intersections in the region. A practical application of the model would consider a larger subset of intersections, but a smaller subset is used in this paper for demonstration purpose. An implied assumption in limiting the study to intersections is that there is a targeted budget for the treatment of these types of locations. Annualized crash data (over a 10-year period) compiled from the website of the Southeast Michigan Council of Governments (SEMCOG) is presented in Figure 2 *(32)* for each intersection, listed in decreasing order of total crashes for each county. Figure 2 show that locations in County-4 has the highest wheras County-3 has the least number of crashes. Detailed input data for all locations is shown in Table 1. In addition to total crashes, type of crash data is also shown for each location.

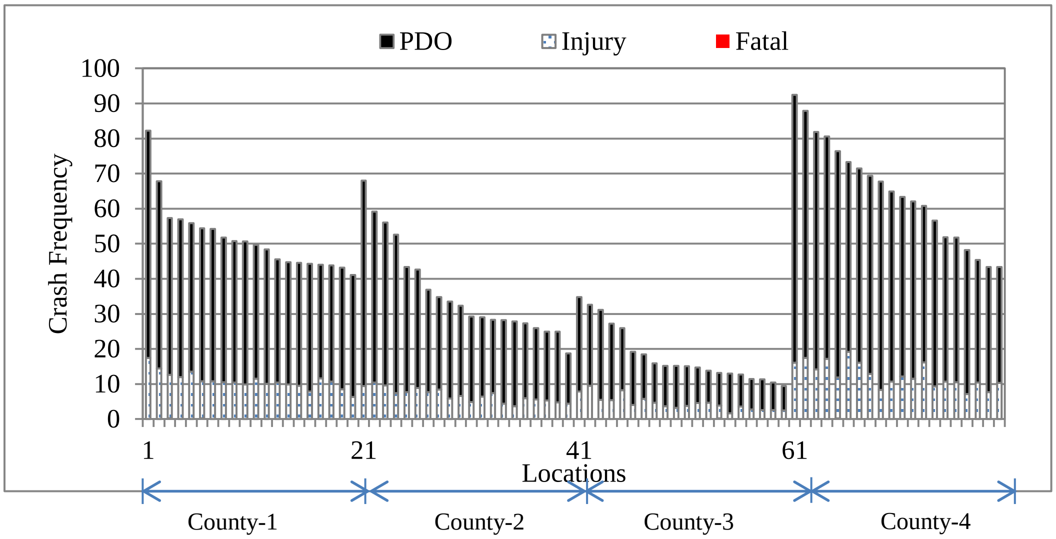

FIGURE 2 Crashes by Severity for all locations.

TABLE 1 Input Data for All Locations

County	Intersection	Crash Frequency				Crash Type					
		Total	Fatal	Injury	PDO	Rear-end	Angle	Swipe-same	Swipe-opp.	Head-on	Other
County-1	1	82.2	0.1	17.4	64.7	33.4	24.9	5.7	3.3	9.0	5.9
	2	57.0	0.0	12.0	45.0	25.1	12.8	6.1	1.6	8.9	2.5
	.	.	.	.	.	.	.	.	.	.	.
	19	44.0	0.0	11.6	32.4	20.8	8.2	4.9	0.6	7.5	2.0
	20	43.8	0.0	10.7	33.1	14.3	12.5	7.2	1.7	3.0	5.1
County-2	21	68.0	0.0	9.4	58.6	33.2	14.8	11.9	1.2	3.9	3.0
	22	59.2	0.0	10.4	48.8	25.8	21.5	4.1	0.5	4.9	2.4
	.	.	.	.	.	.	.	.	.	.	.
	39	29.2	0.1	4.8	24.3	21.1	2.6	4.0	0.4	0.4	0.7
	40	18.7	0.0	4.4	14.3	10.3	1.6	3.2	0.1	0.7	2.8
County-3	41	34.8	0.1	7.8	26.9	18.5	9.2	3.1	0.6	1.1	2.3
	42	32.6	0.0	9.5	23.1	28.0	1.7	1.0	0.5	0.5	0.9
	.	.	.	.	.	.	.	.	.	.	.
	59	12.7	0.0	3.6	9.1	2.1	7.5	1.4	0.1	0.5	1.1
	60	9.4	0.0	2.5	6.9	2.9	2.4	1.9	0.1	0.6	1.5
County-4	61	92.5	0.0	16.1	76.4	41.5	20.1	21.1	1.5	3.4	4.9
	62	76.4	0.2	11.7	64.5	29.8	26.2	14.1	2.8	0.5	3.0
	.	.	.	.	.	.	.	.	.	.	.
	79	45.4	0.0	10.4	35.0	18.8	12.3	3.9	2.2	5.0	3.2
	80	43.4	0.0	10.3	33.1	21.0	8.7	4.1	1.5	5.1	3.0

Input data

Five hypothetical safety alternatives (Table 2) are proposed as countermeasures for potential reduction in crashes. Further, it is assumed that a maximum of four alternatives can be applied to each intersection in Table 1. Each alternative is assumed to be mutually exclusive. In reality, these alternatives are developed as a second (sequential) step of the hazard elimination program and are based on engineering judgment, and an analysis of the probable causes of the crashes. Comprehensive design of alternatives is beyond the scope of this paper and hence alternatives in this study are adapted from a past study in Michigan *(32)*.

The capital cost of the proposed alternatives is presented in Table 2 (in increasing order). For simplicity, O&M costs are assumed as 10% of capital costs, and service life for the alternatives is assumed to be proportional to capital costs. Each alternative has been assumed to consist of a set of countermeasures and with crash reduction factors (CRF) for each alternative. Crash reduction factors for each countermeasure, along with their expected service life, can be derived from the literature *(33)*. An alternative may consist of a single or multiple countermeasures. In the latter case, CRF's associated with each countermeasure are combined, following a linear function, to derive a combined CRF. The CRF values listed in Table 2 can be assumed to be associated with each alternative (that may be a combination of countermeasures).

In this study an initial annual budget of $1.6 million is considered. The future year budgets are assumed to increase by six percent every alternate year over a five year planning horizon. Information on factors that need to be considered from year to year for all the proposed models: mutually exclusive feature, carry-over factor[†], and year end surplus are tracked internally within the model. The model is applied to a sub-set of locations depicting reality to the extent possible to ensure a connection between the proposed process and its application. An analysis period of five years is assumed in the example demonstration.

[†] An alternative installed for the first year remains effective for the remainder of its service life.

The annual savings measured in monetary terms from the reduction in number of crashes is termed as “benefit”, and the savings over the five year planning period is termed as “total benefit”. These two terms are used in the following sections as a measure of the monetary savings from reduction in crashes. Surplus is defined as difference between available budget and the amount committed for implementation of alternatives. The terms annual surplus and total surplus are used in the remainder of the paper for unused budget for annual and planning period respectively.

TABLE 2 Crash Reduction Factors, Cost and Service Life of Alternatives

Alternatives	Crash Reduction Factors			Capital Cost ($)	O&M Cost ($)	Service Life (Years)
	Fatal	Injury	PDO			
I	0.06	0.05	0.04	20,000	2,000	2
II	0.13	0.11	0.09	35,000	3,500	2
III	0.25	0.23	0.18	80,000	8,000	3
IV	0.30	0.29	0.25	100,000	10,000	4
V	0.46	0.45	0.42	150,000	15,000	4

Single Year Allocation

Both resource allocation models (ES-RAM & EQ-RAM) are solved with sequential quadratic programming evolutionary algorithm in a VBA based solver platform *(33)*. Table 3 presents single year allocation of projects with minimum budget considering mutually exclusive nature of alternatives. Single year allocation results illustrates working principle of model for the first year in the planning period. If minimum cost alternative ($20,000) is chosen for 80 locations then budget will be $1,600,00 and the resulting economic benefits can be calculated as $2,980,000. By employing the proposed resource allocation model (ES-RAM) for single year planning period the resulting benefits are $6,788,149 (Table 3). This shows that the proposed optimization model did not allocate alternatives to all locations, rather to location that needs improvement to result maximum benefits in terms of crash savings. Results in Table 3 (for brevity only sample number of locations is presented) shows that no locations in county 2 and 3 received any improvements. From optimization viewpoint this is logical because these locations consists of relatively low number of crashes compared to county 1 and county 4 and therefore did not warrant any improvements to maximize the total benefit. The model resulted in 11 alternatives (1 alternative IV and 10 alternative V) using the proposed budget as opposed to choosing all 80 locations with minimum budget.

Figure 3 shows yearly allocation of alternatives to all locations. For example, Figure 3(a) shows in the first year only three alternative IV, and five alternative III are chosen. Most of the locations received improvements of alternatives of I and II. No locations received alternative V. Similarly, Figure 3(b) shows location received alternatives for the second year. It should be noted that locations funded in the first year were not allowed by the optimization process to be eligible for funding for the second year because of the mutually exclusive nature of the alternatives specified in the model. Similarly, allocation of alternatives for each year in the planning period is shown in Figure 3.

TABLE 3 Results of Single Year Allocation

County	Intersection	Improvements (Optimized)					Min Project Cost ($)	Minimum Benefit ($)	Optimized Project Cost ($)	Optimized Benefit ($)
		I	II	III	IV	V				
County-1	1	0	0	0	0	1	20,000	76,272	150,000	708,677
	2	0	0	0	0	0	20,000	47,760	0	0
	.	.	.	.	.	.	.			
	19	0	0	0	1	0	20,000	42,527	100,000	361,415
	20	0	0	0	0	0	20,000	40,282	0	0
County-2	21	0	0	0	0	0	20,000	45,071	0	
	22	0	0	0	0	0	20,000	44,606	0	
	.	.	.	.	.	.	.	.	.	.
	39	0	0	0	0	0	20,000	28,370	0	0
	40	0	0	0	0	0	20,000	16,790	0	0
County-3	41	0	0	0	0	0	20,000	37,473	0	0
	42	0	0	0	0	0	20,000	33,702	0	0
	.	.	.	.	.	.	.	.	.	.
	59	0	0	0	0	0	20,000	12,885	0	0
	60	0	0	0	0	0	20,000	9,138	0	0
County-4	61	0	0	0	0	1	20,000	69,334	150,000	661,597
	62	0	0	0	0	1	20,000	67,731	150,000	622,113
	.	.	.	.	.	.	.	.	.	.
	79	0	0	0	0	0	20,000	40,080	0	0
	80	0	0	0	0	0	20,000	39,182	0	0
Total		**0**	**0**	**0**	**1**	**10**	**1,600,000**	**2,980,006**	**1,600,000**	**6,788,149**

Alternatives allocated to each county by year is shown in Figure 4. Each alternative will have its effect in reduction of crashes till the end of service life termed as carry over effects. New projects allocated and the projects carried over to each county by year are shown in Figure 4(a) and 4(b) respectively. Figure 4(a) shows in the first year County-4 received highest number of projects and County-1 and County-3 received least number of projects. On the contrary, in year 3, County-1 received highest number of projects. Similarly, new project allocation for all the years by county is shown in Figure 4(a). Number of projects carried over is shown in Figure 4(b). It is observed County-1 and County-4 has highest number of project carried over at the end of year five.

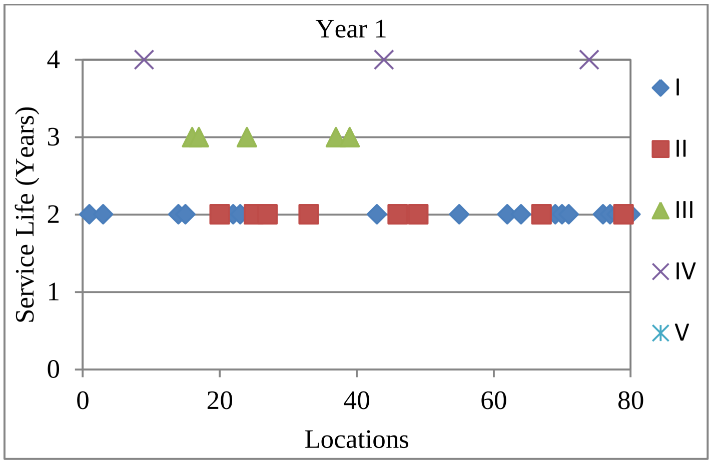

Figure 3(a): Project allocation for year-1

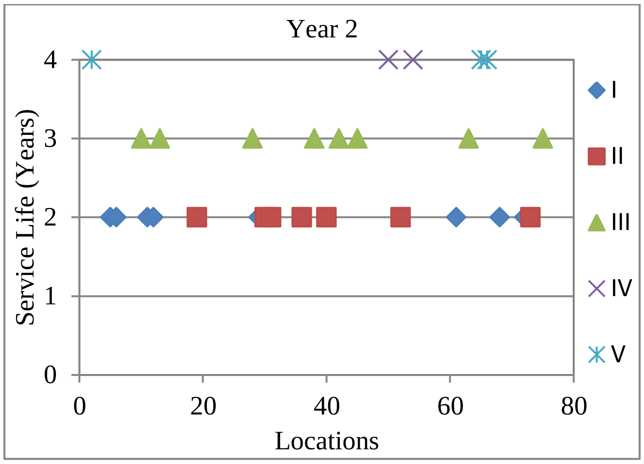

Figure 3(b): Project allocation for year-2

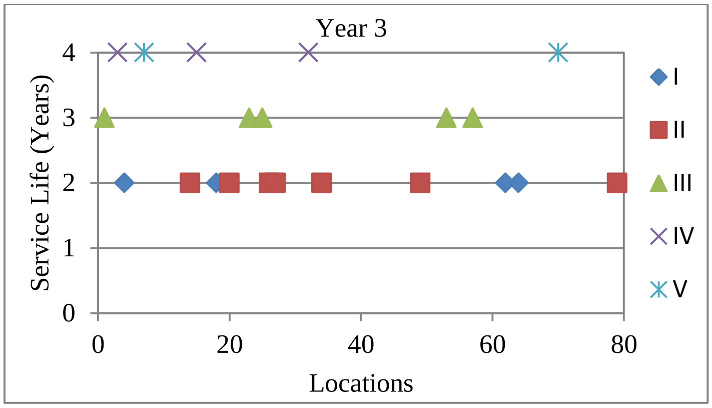

Figure 3(c): Project allocation for year-3

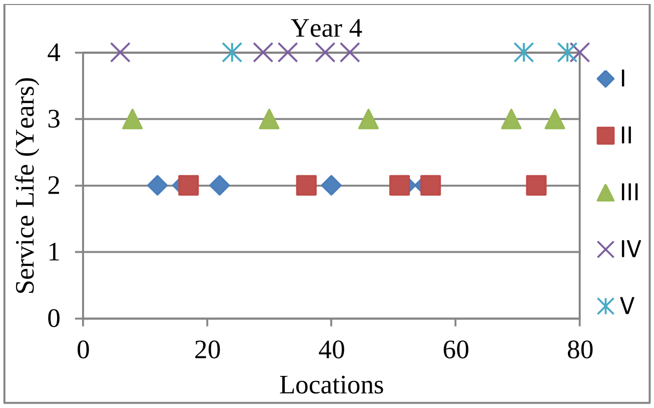

Figure 3(d): Project allocation for year-4

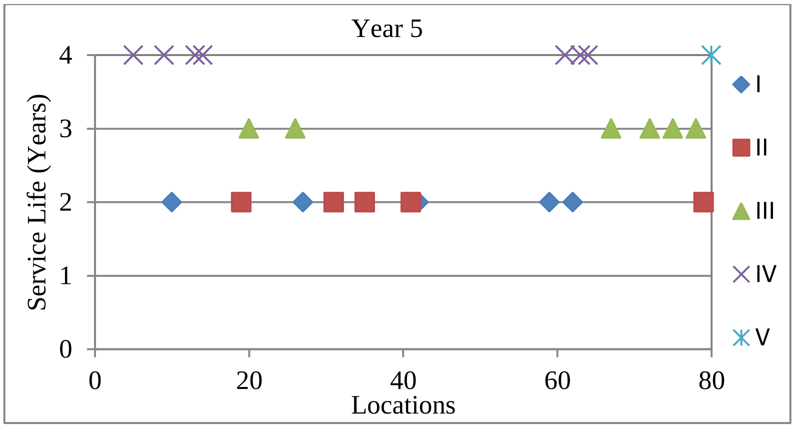

Figure 3(e): Project allocation for year-5

FIGURE 3 Yearly Project Allocation.

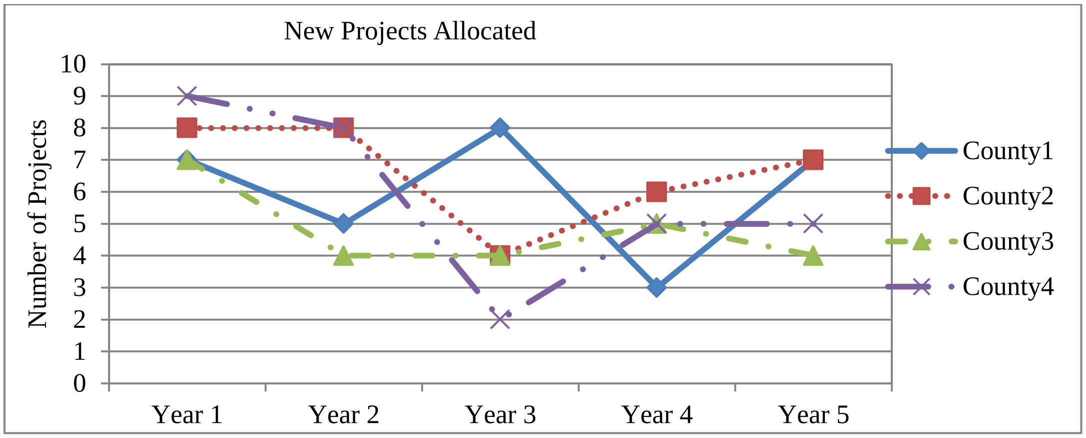

Figure 4(a): New Projects Allocated

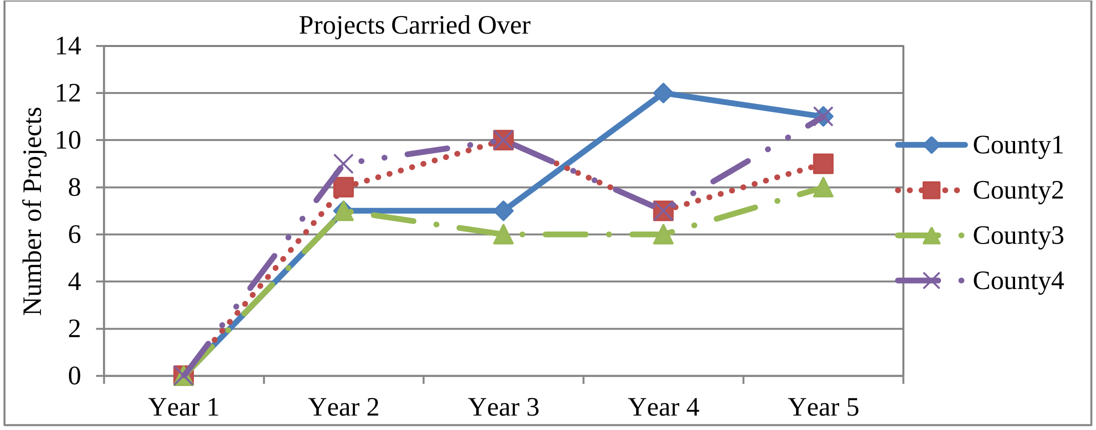

Figure 4(b): Projects Carried Over

FIGURE 4 County-by-County Allocation.

MODEL RESULTS

Economic Competitiveness based Model (EC-RAM) Results

Results of Economic competitiveness based resource allocation model (EC-RAM) for a planning period of five years are shown in Table 4. The model resulted in 35 new alternatives in the first year with capital cost for implementing these alternatives as $1.36 million leaving surplus of $240,000. The operationa and maintenance (O&M) cost is zero, as these costs are incurred one year after the alternative is implemented. The optimum benefit for the first year is computed as $2.86 million. In the second year, optimization resulted in selection of 28 new alternatives with a capital cost of $1.69 million and benefit of $7.05 million. The effect of carry-over alternatives from the previous year is also included in the estimation of the benefits derived. Similar allocations are made for next five years. On comparison the benefit for the first year resulting from a single year analysis was $2.98 million (Table 3), while the first year benefit from a multi-year analysis is $2.86 million (Table 4). The difference in the benefit is simply a reflection of the fact that the model allocates resources over the five-year period optimally resulting in a greater a flexibility of investment from year to year. An analysis of one year at a time, on the other hand, is blind to availability of future funds, and may not necessarily result in maximization of total benefit over the five-year period. Also, for the first year planning period model resulted in a surplus of $240,000. Table 4 also shows that a total of 133 new alternatives are selected in the five year planning period for economic competitiveness. The total benefit achieved is worth $30.63 million at an expense of $7.73 million of capital cost and $583,500 of O&M cost, leaving a surplus of $10,500.

TABLE 4 Summary of Allocation for a Five Year Planning Period

Model	Year	Number of Alternatives Allocated						Benefit ($)	Allocated ($)	O&M Cost ($)	Budget ($)	Surplus ($)	Cumulative ($)
		I	II	III	IV	V	Total						
Economic Competitiveness	1	19	8	5	3	0	35	2,864,987	1,360,000	0	1,600,000	240,000	240,000
	2	8	7	8	2	3	28	7,052,356	1,695,000	136,000	1,600,000	-231,000	9,000
	3	4	7	5	3	2	21	7,441,855	1,325,000	123,500	1,680,000	231,500	240,500
	4	6	5	5	6	3	25	6,325,140	1,745,000	159,000	1,680,000	-224,000	16,500
	5	5	5	6	7	1	24	6,947,227	1,605,000	165,000	1,764,000	-6,000	10,500
	Total	**42**	**32**	**29**	**21**	**9**	**133**	**30,631,565**	**7,730,000**	**583,500**	**8,324,000**	**10,500**	
Equity in Opportunity	1	1	1	2	3	0	7	1,186,829	515,000	0	1,600,000	1,085,000	1,085,000
	2	7	2	3	6	5	23	5,538,559	1,800,000	51,500	1,600,000	-251,500	833,500
	3	5	3	6	2	3	19	7,664,491	1,335,000	91,000	1,680,000	254,000	1,087,500
	4	2	4	6	6	3	21	6,367,482	1,710,000	189,000	1,680,000	-219,000	868,500
	5	4	1	6	1	4	16	6,995,142	1,295,000	248,000	1,764,000	221,000	1,089,500
	Total	**19**	**11**	**23**	**18**	**15**	**86**	**27,752,503**	**6,655,000**	**579,500**	**8,324,000**	**1,089,500**	
Equity in Outcome	1	17	3	1	2	0	23	1,044,359	725,000	0	1,600,000	875,000	875,000
	2	9	4	6	8	1	28	4,781,175	1,750,000	72,500	1,600,000	-222,500	652,500
	3	6	10	2	6	1	25	6,956,618	1,380,000	76,000	1,680,000	224,000	876,500
	4	5	2	8	5	2	22	6,255,674	1,610,000	163,000	1,680,000	-93,000	783,500
	5	10	4	6	5	2	27	6,382,044	1,620,000	186,000	1,764,000	-42,000	741,500
	Total	**47**	**23**	**23**	**26**	**6**	**125**	**25,419,870**	**7,085,000**	**497,500**	**8,324,000**	**741,500**	

TABLE 5 Allocation of Alternatives to Counties by Various Strategies

Model	Year	New					Carry-Over					Total				
		County 1	County 2	County 3	County 4	Total	County 1	County 2	County 3	County 4	Total	County 1	County 2	County 3	County 4	Total
Economic Competitiveness	1	8	9	6	12	35	0	0	0	0	0	8	9	6	12	35
	2	8	7	5	8	28	8	9	6	12	35	16	16	11	20	63
	3	8	6	3	4	21	11	10	6	9	36	19	16	9	13	57
	4	5	8	6	6	25	12	8	8	9	37	17	16	14	15	62
	5	7	4	3	10	24	9	11	10	9	39	16	15	13	19	63
	Total	**36**	**34**	**23**	**40**	**133**	**40**	**38**	**30**	**39**	**147**	**76**	**72**	**53**	**79**	**280**
Equity in Opportunity	1	2	1	2	2	7	0	0	0	0	0	2	1	2	2	7
	2	5	5	7	6	23	2	1	2	2	7	7	6	9	8	30
	3	6	3	6	4	19	7	6	8	7	28	13	9	14	11	47
	4	6	4	6	5	21	10	7	10	9	36	16	11	16	14	57
	5	4	4	3	5	16	12	8	11	11	42	16	12	14	16	58
	Total	**23**	**17**	**24**	**22**	**86**	**31**	**22**	**31**	**29**	**113**	**54**	**39**	**55**	**51**	**199**
Equity in Outcome	1	5	6	7	5	23	0	0	0	0	0	5	6	7	5	23
	2	7	7	7	7	28	5	6	7	5	23	12	13	14	12	51
	3	5	7	7	6	25	7	8	9	7	31	12	15	16	13	56
	4	5	6	5	6	22	10	11	12	9	42	15	17	17	15	64
	5	5	9	7	6	27	11	7	11	11	40	16	16	18	17	67
	Total	**27**	**35**	**33**	**30**	**125**	**33**	**32**	**39**	**32**	**136**	**60**	**67**	**72**	**62**	**261**

Table 5 shows that for economic competitiveness case in the first year, eight alternatives are allocated to County-1, nine to County-2, six to County-3, and 12 to County-4. The first year alternatives are carried over to the second year because of multiple year service life of alternatives. For the second year, eight alternatives are allocated to County-1, seven to County-2, five to County-3, and eight to County-4. Similar allocation of projects by county for the economic competitiveness case is shown in Table 5.

Equity Model (EQ-RAM) Results

While the earlier model (EC-RAM) maximizes total benefit over the five-year period, it does not guarantee that all the locations will receive equitable number of alternative during the planning cycle. Equity in opportunity model employs a threshold value θ of three (in equation 8), implying that no county may receive more than three alternatives unless all other counties have received at least one during the planning period. Table 4 illustrates the results from "Equity in opportunity" in seven alternatives at a cost of $515,000, with $1.18 million benefits and surplus of $1.08 million in the first year. Equity in opportunity row of Table 5 shows analytic principle of the constraint, as no county received more than two alternatives, (or multiples of thereof) unless all other counties received at least one alternative. Although, this policy may prevent inequity in number of alternatives allocated to each county, it does not provide maximum benefits and has highest unused total surplus at the end of planning period.

Table 6 shows benefit distribution across counties over the years. For the economic competitiveness based model (EC-RAM) County-1 received $5.3 million and County-4 received $7.55 million benefits over the planning period. Whereas, County-2 and County-3 received $2.94 million and $1.46 million respectively. EC-RAM results in about 5.71 times economic benefits in County-4 compared to County-3. Similar observations can be made for carry-over and total benefits for County-4 and County-3 for the economic competitiveness case. Thus, validating that economic competitiveness leads to inequity. Further, Table 6 also shows similar inequitable distribution of benefits for equity in opportunity case, since this equity is based on number of alternatives allocated. This limitation is overcome by introducing "Equity in Outcome" policy option.

For modeling "Equity in Outcome" policy option, value of τ (in equation (9)) is assumed to be three, such that it represents that no county should receive 3 times more benefit than other counties. Model user can use any value based on policy. Table 4 shows that equity in outcome resulted in $25.41 million with a capital cost of $7.08 million and O&M cost of $497,500. A total of 125 new improvements are implemented in the equity in outcome scenario. Table 5 suggests that all counties received almost similar number of alternatives/projects every year during the planning period. Table 5 shows 136 alternatives are carried over to the future years after implementation. Including new and carried over a total of 261 alternatives were in effect during the planning period for equity in outcome scenario. However, as proposed, equity in outcome results in Table 6 shows that no county have received benefits more than three times in any given year in the planning period. The results from equity in outcome shows the equal distribution of benefits results in trade-off of total benefits received, i.e. $25.42 million (Table 6).

Hence, this section answers the second key research question (as stated in the introduction), the economic competitiveness results in different fund allocations for safety alternatives in a region. Further, it can be concluded that economic competitiveness provides the maximum benefits by compromising equity in benefits and alternative allocation in a region. Whereas, the equity in opportunity and equity in outcome options leads to relatively low total benefits with higher unused total surplus. State agencies can employ both these models to strategise the investement in the safety alternative across various counties in the region. Although authors believe selection of strategy should be based on objective state planning agency is trying to achieve, the agencies will be better off in terms of derived safety and economic benefits by employing economic competitiveness based resource allocation model (EC-RAM).

TABLE 6 Distribution of benefits across counties according to employed strategies

Model	Year	New (Million $)					Carry-Over (Million $)					Total (Million $)				
		County 1	County 2	County 3	County 4	Total	County 1	County 2	County 3	County 4	Total	County 1	County 2	County 3	County 4	Total
Economic Competitiveness	1	0.90	0.72	0.29	0.96	2.86	0.00	0.00	0.00	0.00	0.00	0.90	0.72	0.29	0.96	2.86
	2	1.12	0.48	0.54	2.05	4.19	0.90	0.72	0.29	0.96	2.86	2.02	1.20	0.83	3.01	7.05
	3	1.66	0.63	0.18	0.78	3.25	1.12	0.48	0.54	2.05	4.19	2.78	1.10	0.73	2.83	7.44
	4	0.54	0.90	0.33	1.63	3.40	1.34	0.63	0.18	0.78	2.93	1.87	1.53	0.51	2.41	6.33
	5	1.08	0.22	0.12	2.13	3.55	0.54	0.90	0.33	1.63	3.40	1.62	1.12	0.45	3.76	6.95
	Total	**5.30**	**2.94**	**1.46**	**7.55**	**17.25**	**3.89**	**2.73**	**1.34**	**5.42**	**13.38**	**9.18**	**5.67**	**2.81**	**12.97**	**30.63**
Equity in Opportunity	1	0.42	0.09	0.17	0.51	1.19	0.00	0.00	0.00	0.00	0.00	0.42	0.09	0.17	0.51	1.19
	2	1.41	1.00	0.24	1.70	4.35	0.42	0.09	0.17	0.51	1.19	1.83	1.09	0.41	2.21	5.54
	3	1.05	0.59	0.41	1.26	3.31	1.41	1.00	0.24	1.70	4.35	2.46	1.59	0.65	2.96	7.66
	4	0.98	0.51	0.49	1.42	3.40	0.70	0.59	0.41	1.26	2.97	1.68	1.11	0.90	2.68	6.37
	5	1.55	0.30	0.05	1.70	3.59	0.98	0.51	0.49	1.42	3.40	2.54	0.81	0.53	3.12	7.00
	Total	**5.42**	**2.49**	**1.36**	**6.59**	**15.85**	**3.51**	**2.19**	**1.31**	**4.89**	**11.91**	**8.93**	**4.68**	**2.66**	**11.48**	**27.75**
Equity in Outcome	1	0.21	0.27	0.26	0.31	1.04	0.00	0.00	0.00	0.00	0.00	0.21	0.27	0.26	0.31	1.04
	2	1.29	0.78	0.46	1.21	3.74	0.21	0.27	0.26	0.31	1.04	1.50	1.05	0.72	1.52	4.78
	3	0.98	0.38	0.61	1.25	3.22	1.29	0.78	0.46	1.21	3.74	2.27	1.15	1.07	2.46	6.96
	4	1.24	0.70	0.31	0.86	3.11	0.90	0.38	0.61	1.25	3.14	2.14	1.08	0.93	2.11	6.26
	5	0.88	1.04	0.49	0.86	3.27	1.24	0.70	0.31	0.86	3.11	2.13	1.74	0.80	1.71	6.38
	Total	**4.60**	**3.16**	**2.13**	**4.49**	**14.38**	**3.64**	**2.13**	**1.64**	**3.63**	**11.04**	**8.24**	**5.29**	**3.78**	**8.12**	**25.42**

CONCLUSION

This paper presents a set of innovative and generic policy analysis tools, founded on scalable simultaneous crash prediction and resource allocation model. The model accounts for stochastic nature of crashes at intersections and optimally allocates preventative measures at critical locations in a planning period. The integrated model is robust in its formulation; and maximizes total benefits from allocation of safety improvement alternatives, within a set of optional policy constraints satisfying budgetary requirements. The proposed model is scalable in multiple-dimensions: number of counties, number of safety alternatives, planning period (years), policy options and budget (yearly or overall). The multi-year feature allows the users to effectively utilize the year-end savings in subsequent period, thereby deriving the most benefit from the available resources. Incorporation of policy constraints allows the analyst the flexibility of selectively adding required constraints to the resource allocation problem.

Model application on signalized intersections data from the Southeast region, Michigan, demonstrates differences between economic competitiveness (EC-RAM) and equity (EQ-RAM) based resource allocation models. Although economic competitiveness based model provides highest benefits by utilizing the funds appropriately, it leads to inequity in distribution of economic benefits and alternatives among counties. On the contrary, equity based models leaves higher unused surplus and low economic benefits. The equity in opportunity policy constraint while ensures somewhat equitable distribution of preventative alternatives, it results in least number of total alternatives, higher surplus and moderate benefits. Similarly, Equity in outcome model allocates benefits in an equitable manner between all counties at the cost of relatively least total benefits compared to other options and unused total surplus.

The contribution of this study to research and practice is three fold. First, this research proposed development of an integrated model that simultaneously selects mutually exclusive alternatives in the optimization process satisfying the budgetary and other constraints. Second, the policy constraint application allows flexibility to analyse various policy options for the state agencies. Third, scalability of model in multiple dimensions and generic nature of model can be leveraged for seamless application and inclusion of any input data and other factors. Future research should analyze prohibition of deficits during any year in the planning period in the allocation model as often agencies have limitation to borrow funds from future years.

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APPENDIX: Notations

Variables	Explanation
b_n	Allocated budget ($) in the analysis year *n*
c^f	Cost of fatal crash (*f*)
c^m	Cost of injury crash (*m*)
c^p	Cost of property damage (*p*)
f_i^n	Expected number of fatal crashes for location *i* in analysis period *n*
I_ε	Set of intersections at county *ε*
l_j	Service life of the alternative *j*
m_i^n	Expected number of injury crashes, *m*, for location *i* in analysis period *n*
$o_{i,j}^n$	Operation and maintenance cost for alternative *j* implemented in location *i* in the analysis year *n*
p_i^n	Expected number of property damage only crashes, for location *i* in analysis period *n*
$r_{i,j}^p$	Crash reduction factor for property damage, *p*, alternative *j* chosen for location *i*
$r_{i,j}^f$	Crash reduction factor for fatality *f*, alternative *j* chosen for location *i*
$r_{i,j}^m$	Crash reduction factor for injury *m*, alternative *j* chosen for location *i*
$x_{i,j}^n$	A binary decision variable equal to 1 if alternative *j* is implemented at location *i* in year *n*
$y_{i,j}^{n,n'}$	A binary decision variable equal to 1 if alternative *j* implemented at location *i* in year *n* is still active in *n'* year
δ	Subscript used for a county
$\pi_{i,j}^n$	Capital cost for alternative *j* implemented in location *i* in the analysis year *n*
i	Location in the study area
I	Total number of locations
I'	A subset of *I*
j	Alternative proposed to be have potential for crash reduction
J	Total number of alternatives
j'	Alternative selected for installation in addition to an existing alternative *j* already in place for
n	Planning period under consideration
N	Total planning period
Z	Objective function, dollar benefit of crashes saved for the analysis period *n*
γ_i^n	Active alternatives at location *i* during the analysis year *n*
θ	Equity in opportunity threshold, a constant that ensures maximum number of alternatives for a county
τ	Equity in outcome threshold, a constant number that ensures maximum cost for a county