A Graph Theoretic Approach for Public Transit Connectivity in Multi-Modal Transportation Networks

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1   Abstract

2   Connectivity plays a crucial role as agencies at the federal and state level focus on expanding the public
3   transit system to meet the demands of a multimodal transportation system. Transit agencies have a need to
4   explore mechanisms to improve connectivity by improving transit service. This requires a systemic
5   approach to develop measures that can prioritize the allocation of funding to locations that provide greater
6   connectivity, or in some cases direct funding towards underperforming areas. In this paper, we propose
7   measures to determine connectivity from a graph theoretic approach for all levels of transit service
8   coverage integrating routes, schedules, socio-economic, demographic, and spatial activity patterns. The
9   objective of using connectivity as an indicator is to quantify and evaluate transit service in terms of
10 prioritizing transit locations for funding; providing service delivery strategies, especially for areas with
11 large multi-jurisdictional, multi-modal transit networks; providing an indicator of multi-level transit
12 capacity for planning purposes; assessing the effectiveness and efficiency for node/stop prioritization; and
13 making a user-friendly tool to determine locations with highest connectivity while choosing transit as a
14 mode of travel. An example problem shows how the graph theoretic approach can be used as a tool to
15 incorporate transit specific variables in the indicator formulations and compares the advantage of the
16 proposed approach compared to its previous counterparts. Then the proposed framework is applied to the
17 comprehensive transit network in the Washington-Baltimore region. The proposed analysis offers reliable
18 indicators that can be used as tools for determining the transit connectivity of a multimodal transportation
19 network.

20 Key Words: public transportation, connectivity, graph theory, multimodal transit network

21

22   Introduction

23 Transit networks consist of nodes and lines to represent their layout. The nodes are called stops and the
24 lines are called links or route segments. Links in a multimodal transit network have different
25 characteristics from those in a road network. While a link in a road network is a physical segment that
26 connects one node to another, a link of a multi-modal transit network is part of a transit line that serves a
27 sequence of transit stops (nodes). Since a stop can be served by different transit lines, multiple transit
28 links may exist between nodes in a multi-modal transit network. But in the case of a highway network
29 only one link exists between two nodes. Headway, frequency, speed, and capacity are critical terms that
30 define the characteristics of a route for a transit link. Similarly, transit nodes are composed of a different
31 set of characteristics than highway nodes.

32   Determining the level of service of a transit network is a difficult task. There are two principal
33 reasons. First, the number of factors related to service quality, such as walking distance, in-vehicle travel
34 time, waiting time, number of destinations served and number of transfers needed to reach destinations
35 makes the transit connectivity a multidimensional problem. Second, the transit system consists of many
36 different routes. Determining the extent to which the routes are integrated and coordinated so that the
37 transit system is connected is another task (*1*). The structure of the public transit network is critical in
38 determining performance, coverage, and service of the network. Network connectivity can be used as a
39 measure to study the performance of the transit system which will assist decision makers in prioritizing

1   transit investment and deciding which stops/lines need immediate attention in regard to
2   operation/maintenance (*2*). In this context, connectivity is one of the index measures that can be used to
3   quantify and evaluate transit performance (*3*).

4   Measures of transit connectivity can be used for a number of purposes. First, in a public or quasi-
5   public agency, connectivity can be used as a measure in public spending to quantify transit stop and route
6   performance and to evaluate the overall system performance. Second, in a rural or suburban area where
7   exact information on transit ridership, boardings, and alightings are not available (which are generally
8   obtained from a comprehensive and well-designed transit assignment in a travel demand model or from
9   an advanced transit system where smart cards are used to keep track of revenues) to obtain a measure of
10   performance for developing service delivery strategies. Third, to serve as a performance measure in a
11   large scale urban multi-modal transit network containing local buses, express buses, metro, local light rail,
12   regional light rail, bus rapid transit, and other transit services which serve both urban and rural areas,
13   where transit services are provided by different public and private agencies with little coordination (an
14   example being Washington DC-Baltimore area, with more than 18 agencies providing services). Fourth,
15   to provide an assessment of effectiveness and efficiency of a transit system with quantifiable measures
16   that can be used to prioritize the nodes/links in a transit system, particularly in terms of emergency
17   evacuation. Fifth, to assist transit agencies with the development of a set of tools for the potential transit
18   users to assess the level and quality of transit service at their place of residence or work.

19   This paper proposes a unique approach to measuring transit connectivity, particularly for
20   applications where the use of transit assignment models or ridership tracking tools is not available. This
21   method incorporates a graph theoretic approach to determine the performance of large-scale multimodal
22   transit networks to quantify the measures of connectivity at the node, and line level. This is achieved
23   through an assessment of connectivity that incorporates the unique qualities of each transit line and
24   measures of accessibility. By combining these criteria in a single connectivity index a quantitative
25   measure of transit performance is developed that goes beyond the traditional measure of centrality. The
26   new connectivity index significantly extends the set of performance assessment tools decision makers can
27   utilize to assess the quality of a transit system.

28   The next section presents the literature review indicating the use of connectivity in past research,
29   followed by the objective of research showing the scope of improvement in the existing literature. The
30   methodology section describes a step by step process of obtaining the transit connectivity. An example
31   problem is then presented demonstrating various connectivity indexes. A case-study shows how the
32   concept can be applied in real-world applications. The next section shows results of the case study.
33   Finally, findings of the study are discussed in the conclusion section.

34

Literature Review

36   Centrality measures are well studied in the literature. However, their application to public transit is rare.
37   Table 1 represents a summary of connectivity index measures (or derivatives thereof) found in the
38   literature. The first measure in Table 1 is degree of centrality. The total number of direct connections a
39   node has to other system nodes is defined as the degree centrality. Equation (1) suggest that the degree of
40   centrality of a node $D_c(n)$ in a larger network "*N*" is the sum of number of links originated from "*p*"

1   number of nodes crosses through node "*n*" (δ_{np}), where *p* represents all nodes except n (i.e. $p \in (N -$
2   $n)$). This measure is then normalized by dividing by the total number of system nodes *N* minus 1.
3   Equation (2) represents a conditional statement to support the degree centrality, where δ_{np} represents a
4   binary indicator variable which takes the value 1, if node "*p*" is incident upon node "*n*", and 0 otherwise.
5   The Degree centrality is the most widely used measure of connectivity in the literature which ranges from
6   transportation to computer science to epidemiology ((*4*),(*5*),(*6*), (*7*), (*8*)).

7   The degree centrality $D_c(n)$ simply counts the number of direct connections a node has to other
8   nodes in the network, but does not account for the quality of the connection or indirect accessibility to
9   other nodes. Eigenvector centrality acknowledges that not all connections are equal. It assigns relative
10 'scores' to all nodes in the network based on the principle that connections to high-scoring nodes
11 contribute more to the score of the node in question than equal connections to low-scoring nodes. The
12 eigenvector centrality $D_e(n)$ of node *n*, in the network *N* (*n*, *l*), is defined in equation (3), which is the
13 multiplication of degree centrality to δ_{np}, and scaled by the eigenvalue λ. Degree centrality ($D_c(n)$) is the
14 eigenvector in equation (3). The eigenvector centrality succeeded the development of degree centrality
15 and is used for a number of studies.

16   As defined by Freeman (1979), a node's closeness centrality is the sum of graph-theoretic
17 distances from all other nodes, where the distance from a node to another is defined as the length (in
18 links) of the shortest path from one to the other. Equation (4) shows the formulation for closeness
19 centrality. Nodes with low closeness scores have short distances from others, and will   tend to be more
20 accessible. While higher closeness score represents farther distance from other nodes and are not easily
21 accessible. In topology and related areas in mathematics, closeness is one of the basic concepts in a
22 topological space.

23   Betweenness centrality is defined as the share of times that a node n_1 needs a node *n* (whose
24 centrality is being measured) in order to reach a node n_2 via the shortest path. Equation (5) shows the
25 formulation for betweenness centrality. Alternatively, betweenness centrality basically counts the number
26 of geodesic paths that pass through a node *n*. The denominator exists to address the case where there are
27 multiple geodesics between n_1 and n_2, and node *n* is only along some of them. Hence, betweenness is
28 essentially *n*'s share of all paths between pairs that utilize node *n*—the exclusivity of *n*'s position.

29   Previous node indexes did not take into account transit characteristics. Park and Kang (2011)
30 introduced the transit characteristics into the node centrality measures and proposed the connectivity
31 index as a true measure of a transit node (*38*) . The Connectivity index of a node can be defined as the
32 sum of connecting powers of all lines crossing through a node *n*. The connectivity index is shown in
33 equation (6). The total connecting power of a node is the multiple of connecting power of a line at node *n*
34 ($P_{l.n}^t$). The conditional value of presence of a line is represented by a binary indicator variable ($\mu_{l,n}$),
35 which takes the value 1 if line *l* contributes to the connectivity at node *n*, and 0 otherwise. The
36 characteristics of a link contain the performance of a series of nodes in that link. A link is a part of the
37 transit route, which, in turn is a function of the speed, distance, frequency, headway, capacity,
38 acceleration, deceleration, and other factors. Since a route will contain both in-bound and out-bound, the
39 line performance will, in part depend upon the directionality of the transit route, that is, whether the line is
40 circular or bidirectional. The total connecting power of line *l* at node n is the average of outbound and
41 inbound connecting power and can be defined as:

1

$$P_{l,n}^t = \frac{P_{l,n}^o + P_{l,n}^i}{2} \qquad (6.1)$$

2

3   The outbound connecting power of a line l, at node n can be defined as (*38*)

$$P_{l,n}^o = \alpha C_l \times \beta V_l \times \gamma D_{l,n}^o \qquad (6.2)$$

4

5   where, C_l is the capacity of line l, V_l is the speed of line l, and $D_{l,n}^o$ is the distance of line l from
6   node n to the destination. The parameter α is the scaling factor coefficient for capacity, β is the scaling
7   factor coefficient for speed, and γ is the scaling factor coefficient for distance. Similarly, the inbound
8   connecting power of line l can be defined as

$$P_{l.n}^i = \alpha C_l \times \beta V_l \times \gamma D_{l,n}^i \qquad (6.3)$$

9   where, $P_{l.n}^i$ is the inbound connecting power of line l at node n. While the outbound connecting
10   power of a transit line at a certain transit stop represents connectivity from the stop to the downstream
11   stops of the transit line, the inbound connecting power measures connectivity from the upstream stops of
12   the transit line to the stop under consideration.

13

Problem Statement and Objectives

15   Many measures of transit service and accessibility have been put forth in the literature, but few offer a
16   metric to measure the quality of service and performance of a large multi-modal regional transit system.
17   The literature that does purport to offer such insight requires significant amounts of data not only about
18   the transit system, but also of the complete demographics of the service area. Other methods require a full
19   transportation demand and transit assignment models, tools that are prohibitively expensive for many
20   localities.

21   Measuring transit system performance and the level of service at many different levels is vital to
22   funding decisions. Agencies with the objective to improve the transit system using external funds must
23   make the case that the project will make worthwhile improves to the system. At the same time, agencies
24   in the quest for investigating the potential effect of removing a stop, group of stops or transit line from
25   service must know the potential effect it will have on the performance of the system. In the absence of
26   complex transportation demand models, this information is nearly impossible to obtain. A methodology
27   that reduces the need for heavy data inputs, yet provides important information on system performance is
28   critical to the decision-making process. Transit planning agencies may also be interested in applying such
29   an index to determine the best use of land surrounding well connected transit nodes. Beyond Transit
30   Oriented Development (TOD) style plans, the connectivity index provides a way for planners to measure

TABLE 1 Literature on Centrality And Connectivity Measures In Social Networks And Transportation

Measure	Mathematical Construct	Eq. No.	Definition	Application
Node Measure-Degree Centrality	$D_c(n) = \frac{\sum_{p \in N} \delta_{np}}{n-1}$, where, $\delta_{np} = \begin{cases} 1 \ if \ p \ is \ dependet \ on \ n, \forall p \in (N-n) \\ 0 \qquad\qquad\qquad\qquad\quad Otherwise \end{cases}$	(1) (2)	Normalized score based on total number of direct connections to other network nodes	Network and Graph Theory (*3*) (*9*) (*10*) (*11*) (*4*); computer and information science (*5*) (*12*) (*6*) (*13*); gene-disease (*14*) (*7*) (*15*); shortest path (*3*) (*16*) (*17*); transportation (*18*) (*8*) (*19*)
Node Measure-Eigenvector Centrality	$D_e(n) = \frac{\sum_{p \in N} \delta_{np} \times D_c(n)}{\lambda}$	(3)	Assigns relative 'scores' to all nodes in the network based on the principle on connections	Network and Graph Theory (*20*) (*21*) (*22*)(*21*); Social Science (*17*) (*23*) (*24*) (*25*) (*26*) (*27*)
Node Measure-Closeness Centrality	$D_{cc}(n) = \frac{\sum_{n_1 \in N} L_{n,n_1}}{N-1}, \forall N > 2$	(4)	Sum of graph-theoretic distances from all other nodes	Network and Graph Theory Shortest path (*17*) (*28*) (*29*) ; Computer science (*30*) (*5*) (*6*)
Node Measure-Betweeness Centrality	$D_b(n) = \sum_{n_1} \sum_{n_2} \frac{\delta_{n_1,n_2}(n)}{\delta_{n_1,n_2}}, \ n_1 \neq n \neq n_2$	(5)	Sum of the number of geodesic paths that pass through a node *n*	Network and Graph Theory (*30*) (*31*) (*32*) (*29*) ; computer and information science (*5*) (*6*) (*33*) (*34*); shortest path (*17*) (*35*)
Node Measure-Connectivity Index	$\theta_n = \sum_{l \in L} P_{l,n}^t \mu_{l,n}$	(6)	Sum of connecting powers all lines crossing through a node *n*	Transportation (*1*) (*2*) (*36*) (*37*) (*38*) Network and Graph Theory (*39*) (*40*) (*41*) (*42*) (*43*) (*44*)
Line Measure-Connecting Power	$P_{l.n}^t = \frac{P_{l.n}^o + P_{l.n}^i}{2}$	(7)	Connectivity power of a line which is a function of transit characteristics	Transportation and Other applications (*17*) (*28*) (*36*) (*38*)
Line Measure-Connectivity Index	$\theta_l = \frac{1}{|S_l| - 1} \sum_{n \in S_l, n \neq n_0} \theta_n$	(8)	Sum of connecting powers all nodes in a line	Transportation and Other applications (*17*) (*28*) (*32*) (*29*) (*38*)

passenger acceptance rates and accessibility for a single node based on its access within an entire multi-modal regional transportation network.

The objectives of this paper are several fold, with the overall goal of providing a strong measure of system performance with the lowest possible data requirements. This paper will first seek to construct a list of node and link based commonly encountered flow processes and define them in terms of a few underlying characteristics; second, to determine and propose the best suited measures in terms of transit connectivity; third, to examine these measures by running simulations of flow processes and comparing the results in a real-world case study; and fourth, to suggest the best practices which can be adopted for decision-making. All the aforementioned problems require development of a tool to quantify connectivity of a public transportation system. The proposed methodology is presented in the next section and the notations are shown in Appendix-I.

Methodology

The methodology presented in this paper is for transit systems at different levels. As the very nature of nodes, and lines as each require a unique formulation, the description below explains the mathematical construct of these transit levels in a step-by-step manner.

Node Connectivity
The proposed methodology consists of better representations of transit node index measures. In the proposed formulation, we consider the congestion effects achieved because of lane sharing of transit lines of buses, light rail, bus rapid transit, and other similar transit facilities. We have redefined the connecting power of a transit line as other measures have not incorporated the transit attractiveness as per the land use and transportation characteristics of the area the transit line is passing through. As discussed previously, the connecting power of a transit line is a function of the inbound and outbound powers, as the connecting power may vary depending on the direction of travel. The inbound and outbound connecting power of a transit line can be redefined as follows.

$$P_{l,n}^o = \alpha(C_l \times \frac{60}{F_l} \times H_l) \times \beta V_l \times \gamma D_{l,n}^i \times \vartheta A_{l,n} \tag{9}$$

$$P_{l,n}^i = \alpha(C_l \times \frac{60}{F_l} \times H_l) \times \beta V_l \times \gamma D_{l,n}^i \times \vartheta A_{l,n} \tag{10}$$

The addition in equation (9) is a term for activity density of transit line "l" at node "n", and ϑ is the scaling factor for the variable. The density measurement represents the development pattern based on both land use and transportation characteristics. The literature defines the level of development a number of ways, but for simplification purposes we have considered it to be the ratio of households and employment in a zone to the unit area. Mathematically, activity density (equation (11)) is defined as:

$$A_{l,n} = \frac{H_{l,n}^z + E_{l,n}^z}{\Theta_{l,n}^z} \tag{11}$$

The connectivity index measures aggregate connecting power of all lines that are accessible to a given node. However not all lines are equal; nodes with access to many low quality routes may attain a connectivity index score equal to a node with only a couple very high quality transit lines. This means that while both nodes are able to provide good access, the node with the fewest lines provides the most access with the lowest need to transfer. To scale the index scores based on the quality of individual lines, that is, scaling for the least number of transfers needed to reach the highest number and quality of destinations, the node scores are adjusted by the number of transit lines incident upon the node. The inbound and outbound connecting power of a transit line can be further refined as:

$$P_{l,n}^o = \alpha(C_l \times \frac{60}{F_l} \times H_l) \times \beta V_l \times \gamma D_{l,n}^i \times \vartheta A_{l,n} \times \varphi T_{l,n} \tag{12}$$

$$P_{l,n}^i = \alpha(C_l \times \frac{60}{F_l} \times H_l) \times \beta V_l \times \gamma D_{l,n}^i \times \vartheta A_{l,n} \times \varphi T_{l,n} \tag{13}$$

This equation adds the number to transit lines "l" at node "n", and φ is the scaling factor for the number of transit lines. The transfer scale is simply the sum of the connectivity index scores for each of the transit lines that cross a node divided by the count of the number of lines that are incident upon the node. The transfer scaled index (equation (11)) is defined as:

$$T_{l,n} = \frac{\sum P_{l,n}^t}{\Theta_l^n} \tag{14}$$

Line Connectivity

The total connecting power of a line is the sum of the averages of inbound and outbound connecting powers for all transit nodes on the line as presented in Equation (6.1) scaled by the number of stops on each line. The scaling measure is used to reduce the connecting score of lines with many stops like bus lines to properly compare to lines with only a few stops like rail. The line connectivity can be defined as following:

$$\theta_l = (|S_l| - 1)^{-1} \sum P_{l,n}^t \tag{15}$$

Case Study

The proposed framework is applied to a comprehensive transit network in the Washington-Baltimore region. The complete transit network is adapted from Maryland State Highway Administration data. The transit database consists of two largest transit systems namely, Washington Metropolitan Area Transit Authority (WMATA), and Maryland Transit Administration (MTA). WMATA is a tri-jurisdictional government agency that operates transit service in the Washington, D.C. metropolitan area, including the Metrorail (rapid transit), Metrobus (fixed bus route) and MetroAccess (paratransit), and is jointly funded by the District of Columbia, together with jurisdictions in suburban Maryland and northern Virginia. There is approximately $300 million spent in the WMATA capital, operating and maintenance cost of

which is $150 million per year of Federal funds available that are required to be matched by $50 million in annual contributions from DC, Northern Virginia and suburban Maryland, each for ten years.

WMATA has the second highest rail ridership in the US with over 950,000 passengers per day. This is second only to New York. The WMATA Metro provides an extensive heavy rail system with 106.3 route miles. The WMATA bus system also serves an extensive ridership of over 418,000 unlinked daily trips. Figure 1(a) shows the WMATA network at Union Station.

Figure 1(a) Thematic of the transit lines in Washington DC

Figure 1(b). Thematic of the transit lines in Baltimore

On the other hand, MTA is a state-operated mass transit administration in Maryland. MTA operates a comprehensive transit system throughout the Baltimore-Washington Metropolitan Area. There are 77 bus lines serving Baltimore's public transportation needs. The system has a daily ridership of nearly 300,000 passengers along with other services that include the Light Rail, Metro Subway, and MARC Train. The Baltimore Metro subway is the 11th most heavily used system in the US with nearly 56,000 daily riders. Nearly half the population of Baltimore lack access to a car, thus the MTA is an important part of the regional transit picture. The system has many connections to other transit agencies of Central Maryland: WMATA, Charm City Circulator, Howard Transit, Connect-A-Ride, Annapolis Transit, Rabbit Transit, Ride-On, and TransIT. Figure 1(b) shows MTA network around Camden yard station in downtown Baltimore. Both the WMATA Metro rail system and the Baltimore transit system are connected by the MARC commuter rail system. This system has a daily ridership of over 31,000. In the next section, results of the proposed methodology are discussed (APTA 2011). The complete methodology is integrated in a Geographic Information System (GIS) user-interface using ArcInfo (ESRI 2010).

Results

The results reported in the following sections are based on the application of methods developed in this paper on a large-scale multi-modal network of Washington DC and Baltimore region. Table 2 (a) provides a summary of Baltimore/Washington regional transit system. The system represents one of the

largest and most heavily patronized transit systems in the county. The application of the methodology to this complex network provides a demonstration of public transit performance in the Baltimore/Washington Region.

Node Level

The Washington/Baltimore region has a significant number of transit nodes, each of which provide a varying degree of connectivity to the network. Determining network connectivity and funding prioritization is a highly complex task in a multi-modal network. Funding prioritization is additionally aided by the connectivity index by providing decision makers with a tool to measure network resilience. As with any network, transit systems are designed to interact with many different nodes, while remaining functional in the event that a particular node becomes inaccessible. Additionally, resiliency tests based on connectivity can reveal if there is an over concentration of connections which rely on a given node, line, or region.

Table 2 (a) Summary of the Transit System

Attribute	Bus	Rail
Number of Lines	949	33
Route Miles	11,827	1,121
Nodes	7,713	208
Average Speed (Free Flow)	22	47

Table 2(b) Network Resiliency

Rank/*Attribute*	NODES	SMZ	SCORE
1	5841	64	28.50
2	5857	64	25.52
3	5853	64	20.86
4	5840	64	20.23
4	5854	64	20.23
6	5846	64	18.33
7	5845	64	17.99
8	6865	64	17.00
9	5849	64	16.84
10	48658	1188	14.17
Top 10 Connectivity			199.67
Total Regional Connectivity			4283.46
Connectivity without top nodes			4083.79
Reduction in regional connectivity			-4.89%
Representation of total nodes			0.14%

Table 2(b) shows the top ten nodes in the network. This table presents a potential problem for the regional transportation network. The results show that nine out of the ten most connected nodes are located in the same zone. These zones are less than a few blocks from each other; thus, it is feasible that an event could occur that would remove these nodes from service. If all ten of the nodes were to be removed from service, regional network connectivity would be reduced from a score of 4,283 to 4,083 or by about 5%. This is remarkable in that these nodes represent less than 5% of connectivity and less than 0.1% of the total system nodes, yet the system connectivity is heavily reliant of these few connections. A similar comparison can be made for all the nodes in the network.
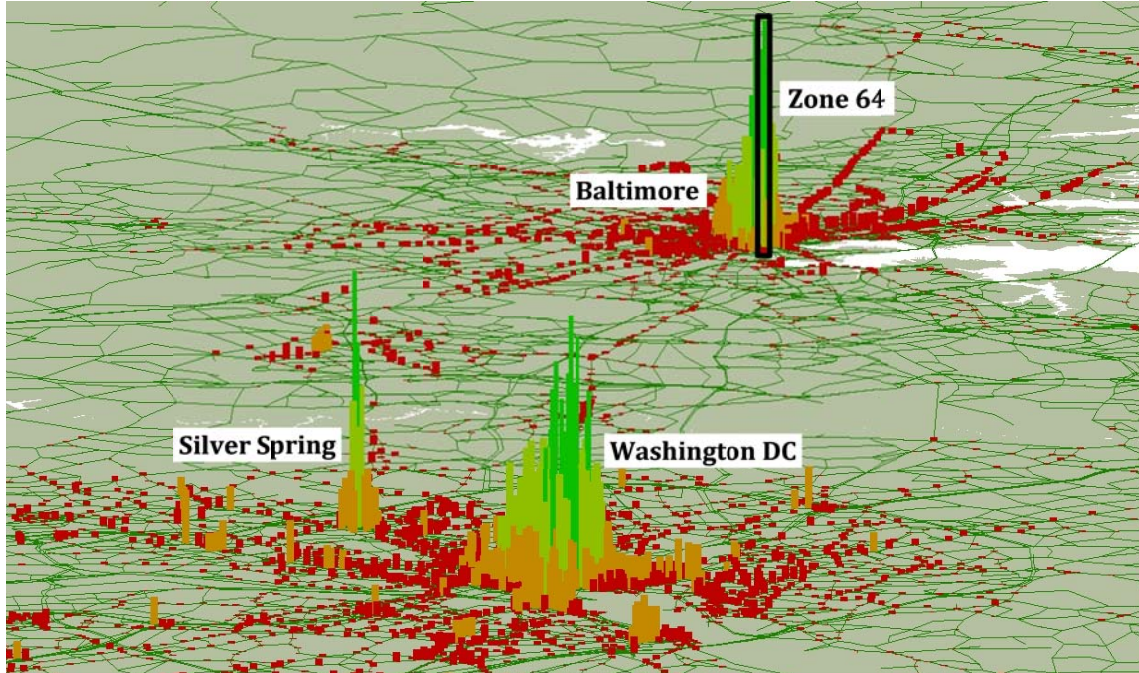
Figure 2 Regional Node Connectivity

Figure 2 is a three-dimensional graph of node connectivity in the Baltimore/Washington region. The map shows the extent of connectivity for the three major transit areas, Washington DC, Baltimore and Silver Spring. The figure also illustrates the location of zone 64 which has the highest concentration of well-connected transit nodes.
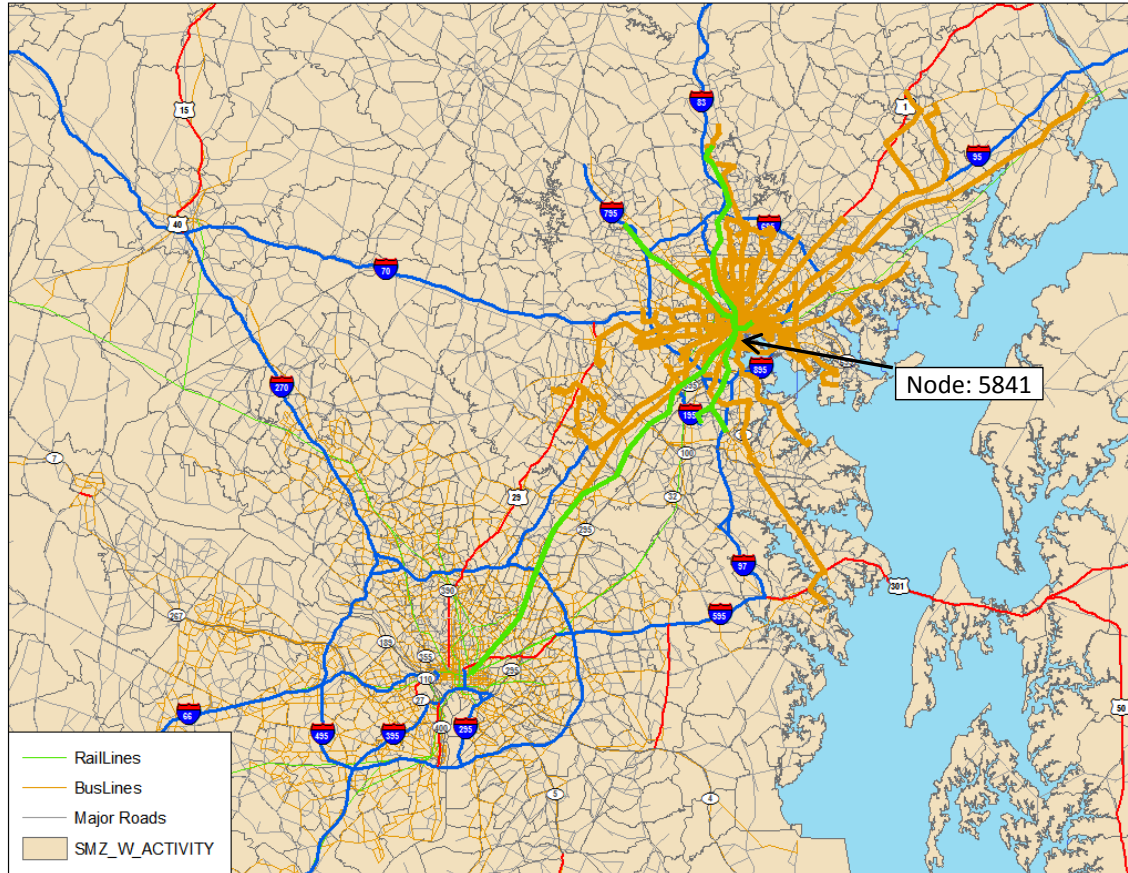
Figure 3 Node Connectivity

Figure 3 plots the lines (rail in green and bus in orange) that the best connected node (node number 5841 and zone number 64) in the region can reach within a single transfer. While other nodes in the system provide access to as many locations and lines as possible, this node is able to move riders from the origin to all of the locations shown in Figure 4 with the fewest resources and lowest transfer times. Additionally, a review of this site shows that land use can be improved to capitalize on the regional connectivity of this node. To the north of the node is the Baltimore City Hall,the US Post Office and Court house. To the south of the node is a parking structure and a surface parking lot. Since this node can be reached from most of Baltimore in a single transfer and much of Washington DC in two transfers, the city could opt to zone the area for higher density and encourage development. This would likely not significantly increase congestion around the site if transit usage could be encouraged.

Line Level

The quality of connectivity for a transit line is determined by several factors. First, the line needs to provide access to at least some dense development; second, the line should provide access to desirable locations with the fewest number of transfers; third the line must connect to other modes to maximize connectivity. The line connectivity index is applied to the Baltimore/Washington regional transportation network. The region provides both rail and bus services. The rail services analyzed in this paper include

WMATA's Metro, Baltimore's light rail and metro system, and the regional MARC commuter lines jointly operated by AMTRAK and CSX. All significant local and regional bus services were included in the analysis. Not included in the study were national bus and rail services, like Greyhound and AMTRAK. While these services do provide a level of connectivity, the primary concern of this paper is how local and regional systems work to create regional connectivity that local and state decision makers can influence.

Figure 4 shows the line connectivity index for the federal triangle area in the vicinity of Washington, DC. The map clearly shows that there is a concentration of highly connected lines that are near the Farragut, McPherson Square, and Metro North transfer centers. On the other hand, there are very few lines with a high connectivity index that are in close proximity to Union Station.
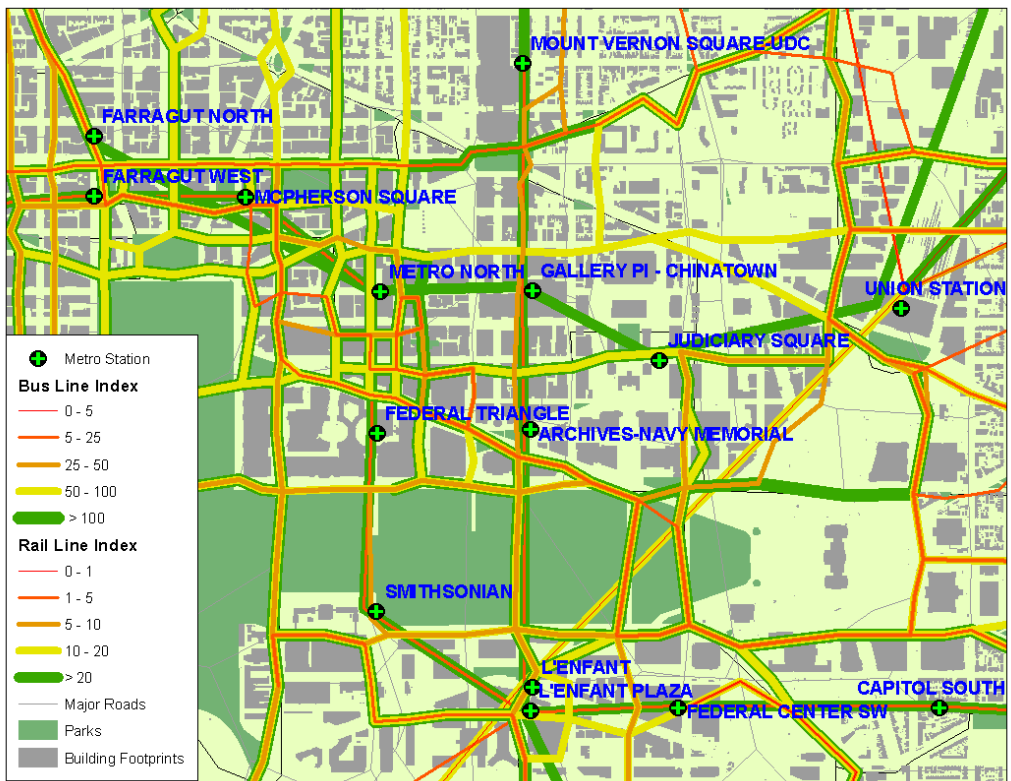

Figure 4 Washington Transit Line Index

Figure 5 shows the connectivity of transit lines in downtown Baltimore. There are several very linear transit lines which provide a high degree of connectivity. These lines serve as the backbone for transit service, enhancing connectivity for all lines that intersect them. This configuration results in a high level of connectivity with fewer resources.
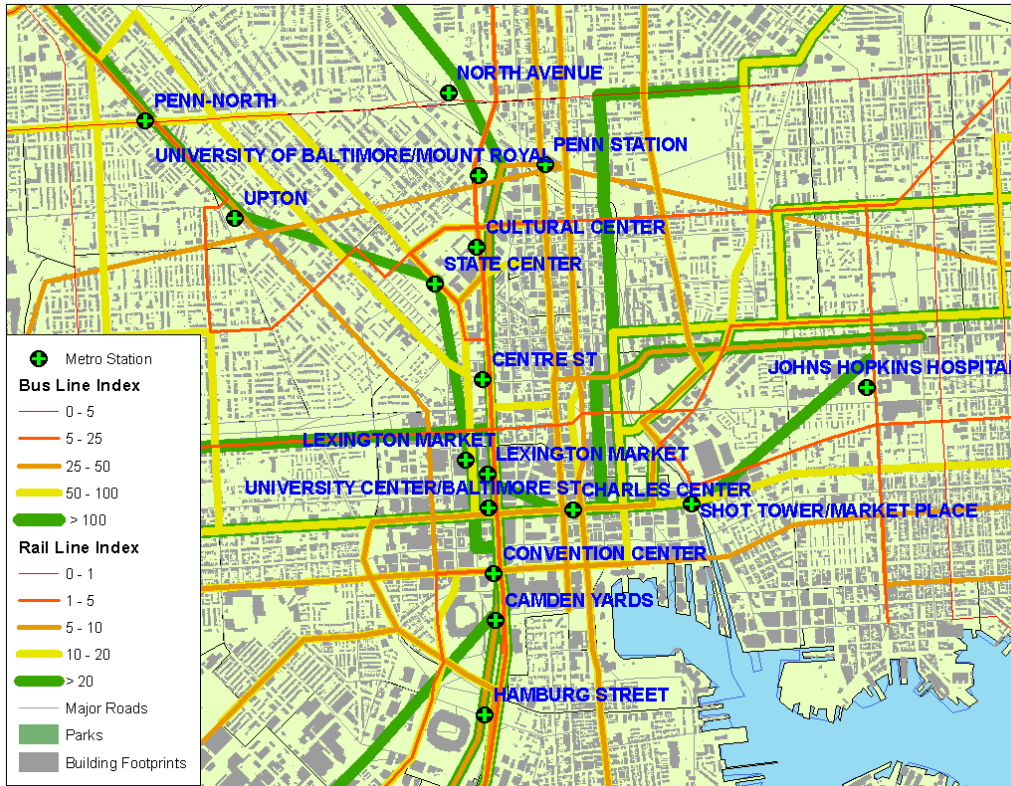
Figure 5 Baltimore Transit Line Index

Rail Service

The analysis shows, somewhat intuitively, that the two metro systems, one by WMATA and the other by MTA, provide the highest level of service in terms of line connectivity, followed by Baltimore's light rail system. WMATA provides the highest level of line connectivity along its red and blue lines. The Yellow and Orange lines have the lowest level of connectivity (Figure 6). In Baltimore, the MARC (commuter rail) line has the highest level of connectivity, followed by the Metro (subway) line then the yellow line (light rail). Not surprisingly, commuter rail lines that pass through mainly suburban and rural areas provide the least amount of connectivity, as they typically only connect with bus nodes at the beginning and end of the lines.
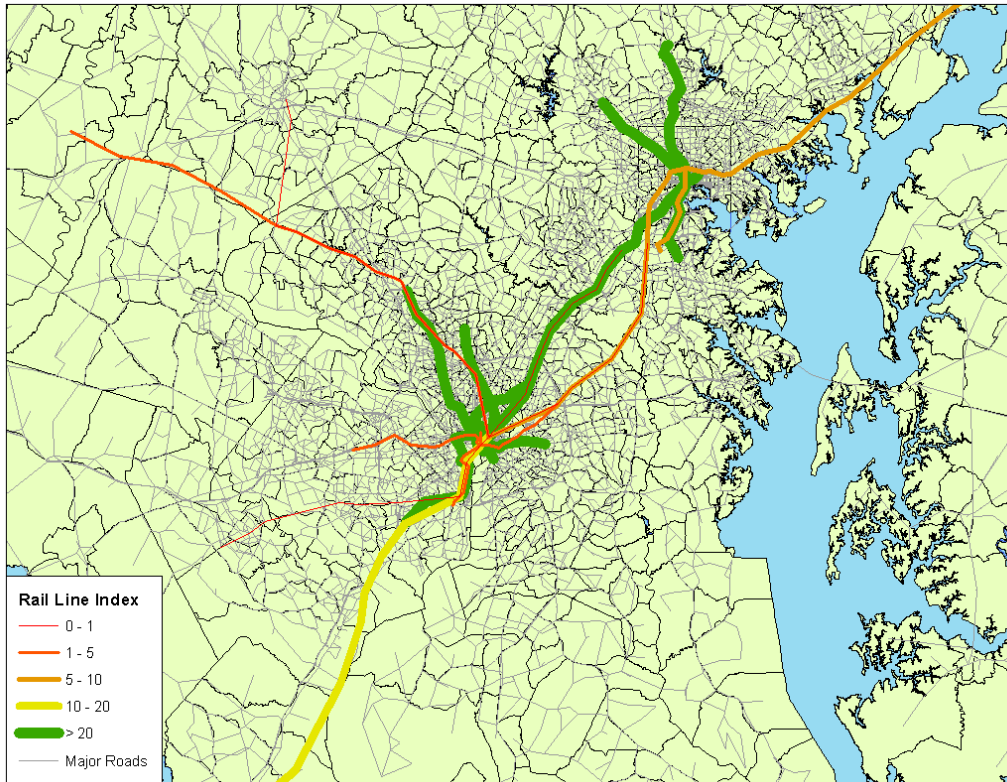
Figure 6 Regional Rail Index

The results provide some insight on how future investments in rail and bus should be prioritized. Heavy rail systems that provide a backbone service for bus connections have the highest potential for regional connectivity. Commuter rail systems provide connectivity for moving passengers between metropolitan areas, but provide a lower level of connectivity overall. When bus and heavy rail service is coordinated with commuter rail service, line and system connectivity are enhanced.

Bus Service

Figure 7 shows the regional connectivity bus index. The line with the highest connecting power is MTA's route 5 which passes through the node with the highest connectivity score in the system. The line has a combination of local and express buses that run through Downtown Baltimore and connects to many other major bus routes as well as all three rail modes. The bus line with the second best connectivity is MTA's express route 150 which has rail connections and provides a transfer point to Howard County bus service. Surprisingly, the best connected bus lines in the Baltimore/Washington region are in Baltimore, despite the fact the WMATA has an extensive route system that relies on the metro service to serve as the backbone of the bus service. Perhaps it is the reliance on a second mode which limits the number of direct bus routes WMATA offers that reduces the connectivity of its bus lines. Most of the major routes on Washington are radial, in that they are meant to feed a central rail station. The suburban lines that are less centralized suffer from low connectivity because they typically serve residential areas and provide access to a Metro stations rather than dense employment and shopping areas. Baltimore's bus lines offer more connectivity with fewer resources by structuring their bus (and rail) service as a network rather than a radial system.
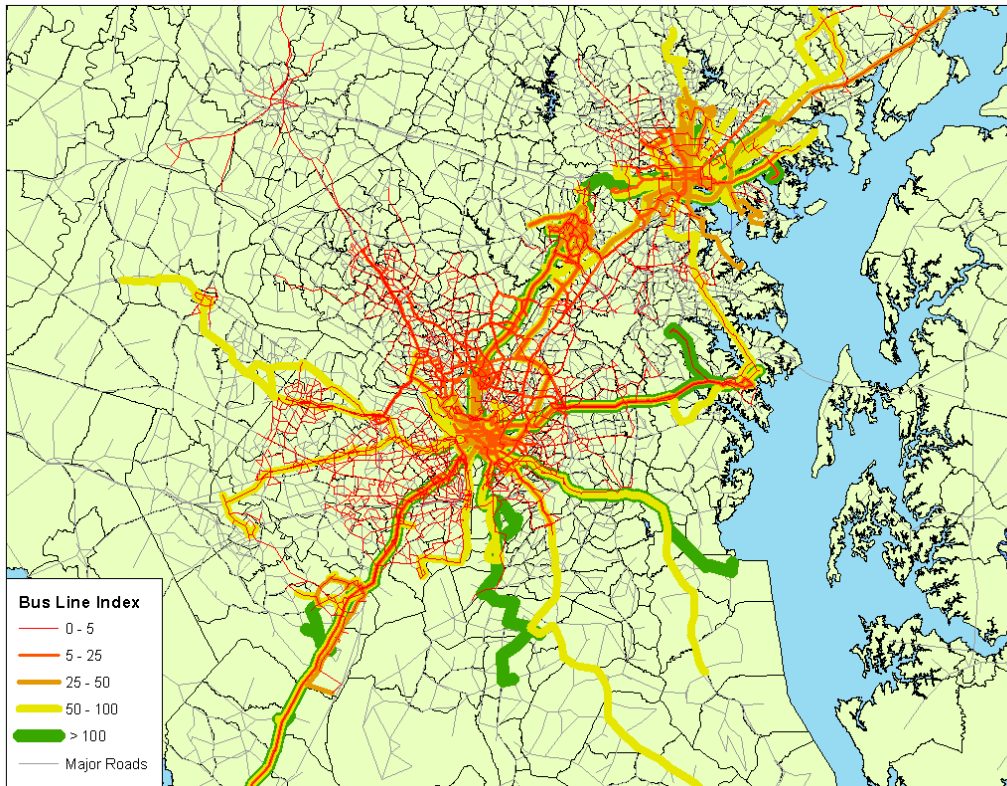
Figure 7 Regional Bus Index

Visualizing in a 3-Dimensional Plane

Each of the connectivity elements can be represented within a three-dimensional network to better aide visualization. Figure 8(a) provides a vertical overview of the Baltimore transit network with 3D nodes scaled to their relative connectivity. In figure 8(b) the same Baltimore network is represented, but this time tilted to give a better view of the effect of connectivity on nodal height. In Figures 8(c) and 8(d) the Washington DC transit network is provided in the same way. The representation shows the difference in how connectivity is distributed in the network. For Baltimore, there is a high degree of connectivity in the city center, but to some degree, connectivity is also spread among several major transit lines. In DC, almost all of the connectivity is concentrated in the center of the city, with a few hubs dotted around the periphery.
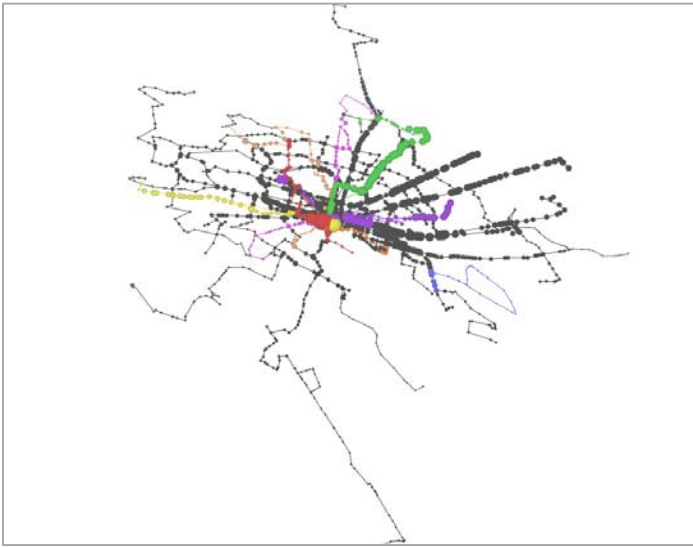
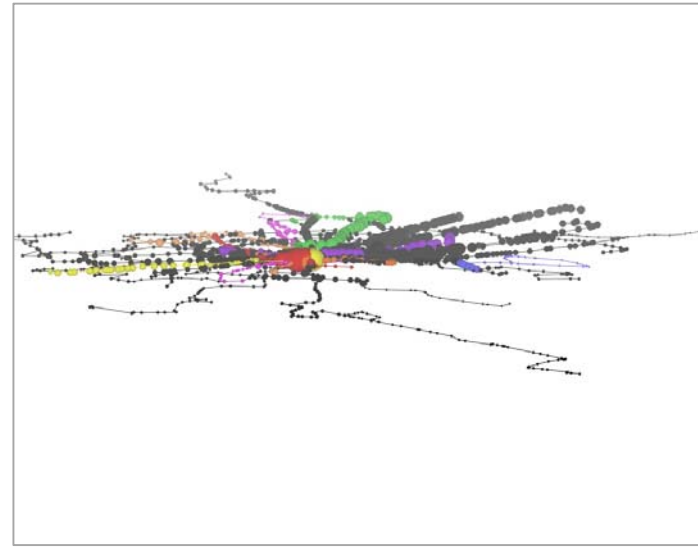Figure 8(a) Top View of Baltimore Transit Network
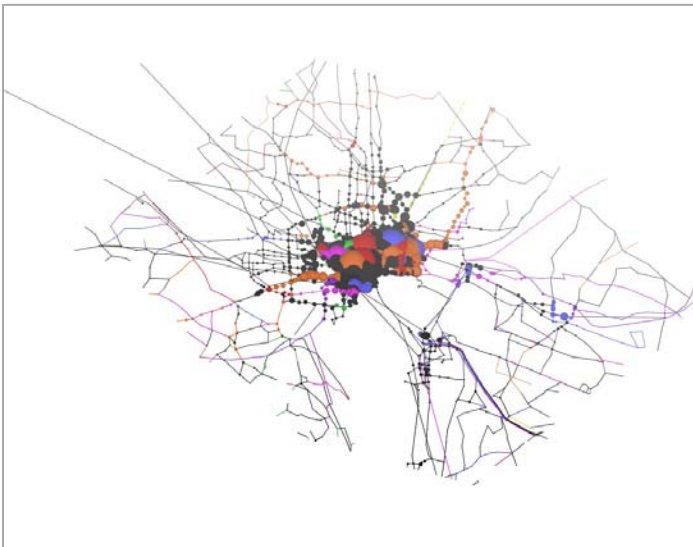

Figure 8(b) Front View of Baltimore Transit Network
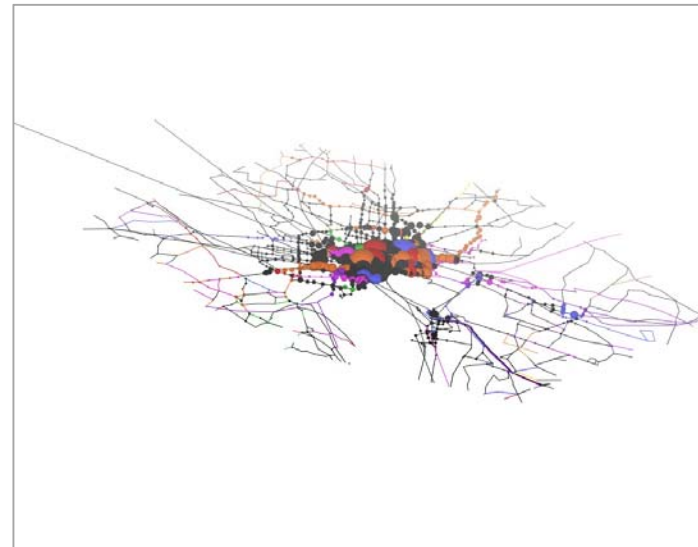

Figure 8(c) Top View of Washington DC Transit Network

Figure 8(d) Front View of Washington DC Transit Network

1

2   **FIGURE 8: Visualization in a Three Dimensional Plane**

Conclusion

The objective of this paper is to develop connectivity indicators to represent the potential ability of a transit system encompassing comprehensive clustered development in a multimodal transportation network. Connectivity defines the level of coordination of the transit routes, coverage, schedule, speed, operational capacity, urban form characteristics, and is an influential element of the image of any transit network. Though the concept of connectivity is used in social networks and partly in transportation engineering, its application in transit analysis has been limited. The difficulty for the development of connectivity indicators lies in the complex interacting factors embedded in a multimodal transit network encompassing various public transportation modes with different characteristics, such as buses, express buses, subways, light rail, metro rail, commuter and regional rail. In addition, multimodal transit networks, like road networks consist of nodes and links. However, links in a multimodal transit network have different characteristics from those in a road network as link in a multimodal transit network are part of a transit line that serve a sequence of transit stops (nodes) and a stop can be served by different transit lines; multiple links may exist between nodes in a multimodal transit network. The indicator development process is further complicated as connectivity varies by urban form with differences among geographical, land use, highway and trip pattern characteristics between regions. The performance indicator should include all the aforementioned complexities and should be quantified to portray connectivity of the multimodal transportation network.

In this paper, first the connectivity indexes used for different purposes in the social networks are reviewed. Then a new set of indicators are developed to reflect the transit mode, network, and zonal characteristics. A set of connectivity indexes is developed at node, and link level. The node connectivity index includes the transit lines passing through it, and their characteristics such as speed, capacity, frequency, distance to destination, activity density of the location, and degree centrality. The link connectivity index is the sum of connectivity indexes of all stops it passes through and normalized to the number of stops.

The major contributions of the paper include: (1) extending the graph theoretic approach to determine the performance of the multimodal transit network; (2) quantifying the measures of connectivity at the node, and line level; (3) applying the methodology to demonstrate the proposed approach in a simplified example problem; (4) examining the transit network performance of Washington-Baltimore region; (5) providing a comprehensive framework for analyzing connectivity, and efficiency of transit networks for agencies that do not have access to well-developed travel demand and transit assignment models, and (6) integrating the complete methodology in a GIS user interface to enhance visualization, and interpretation of the results. Further this study can be extended to analyze changes in the performance measure with changes to the transit network as a sensitivity analysis; incorporating other attributes to the current formulation, and extending the proposed research for prioritizing locations in the case of transit emergency evacuation.

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Appendix-I: Notations for Transit Connectivity

Notation		Explanation
$D_c(n)$	:	Degree of centrality of node n
$D_{cc}(n)$	:	Closeness Centrality
$D_e(n)$	:	Eigenvector centrality of node n
D_l^i	:	Inbound distance of link l
D_l^o	:	Outbound distance of link l from node n to destination
L_{n,n_1}	:	Shortest distance between node n_1 to n
$P_{l,n}^i$		Inbound connecting power of link l
$P_{l,n}^o$	:	Outbound connecting power of link l
$P_{l,n}^t$	:	Total connecting power of line l at node n
S_R	:	Set of stops in region R
S_l	:	Set of stops in line l
S_σ	:	Set of stops in region center σ
V_l	:	Average Speed of link l
n_0	:	Initial stop
$t_{n_1,n}$	:	Transfer time from n_1 to n
δ_{n_1,n_2}	:	Total number of paths between n_1 and n_2
$\delta_{n_1,n_2}(n)$	:	Number of paths exist between n_1 and n_2 those pass through n
δ_{np}	:	A binary indicator variable for determining the degree centrality, which takes the value of 1 when node p is dependent on n, and 0 otherwise
θ_R	:	Connectivity index for region R
θ_l	:	Connectivity index for line l
θ_n	:	Connectivity index for node n
$\rho_{n_1,n}$	:	Passenger acceptance rate from node n_1 to n
ρ_R	:	Density measure for region R
a	:	Parameter for passenger acceptance rate
b	:	Parameter for passenger acceptance which is sensitive to travel time
L	:	link
N	:	Node
N	:	Network system
P	:	Node dependent on n
α	:	Scaling factor coefficient for Capacity of line l
β	:	Scaling factor coefficient for Speed of line l
γ	:	Scaling factor coefficient for distance of line l
λ	:	Eigenvalue
$A_{l,n}$	:	Activity density of line l, at node n
ϑ	:	Scaling factor for activity density
$H_{l,n}^z$	:	Number of households in zone z containing line l and node n
$E_{l,n}^z$	:	Employment for zone z containing line l and node n
$\Theta_{l,n}^z$	:	Area of z containing line l and node
Θ_l^n	:	Number of lines l at node n