Preserving an Aging Transit Fleet: An Optimal Resource Allocation Perspective Based on Service Life and Constrained Budget

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ABSTRACT

Local, county and state level transit agencies with large fleets of buses and limited budgets seek a robust fund allocation mechanism to maintain service standards. However, equitable and optimal fund allocation for purchasing, operating and maintaining a transit fleet is a complex process. In this study, we develop an optimization model for allocation of funds among different fleet improvement programs within budget constraints over the planning period. This is achieved by minimizing the Net Present Cost (NPC) of the investment within the constraint of a minimum level of fleet quality expressed as a surrogate of the remaining life of the fleet. Integer programming is used to solve the formulated optimization problem using branch and bound algorithm. The model formulation and application are demonstrated with a real world case study of transit agencies. It is observed that minimizing NPC provides a realistic way to allocate resources between different program options among different transit agencies while maintaining a desired quality level. The proposed model is generalized and can be used as a resource allocation tool for transit fleet management by any transit agency.

Key Words: transit fleet, net present cost, integer programming, branch and bound algorithm

INTRODUCTION

Transit agencies with limited resources depend on federal support for up to 80 percent of the capital cost of buses in the United States *(1)*. The remaining share is provided by state and local governments. These funds are to be judiciously used to meet the dual purpose of replacing and/or rehabilitating aging vehicles. Hence, most transit agencies (local, county and/or state level) need a robust fund allocation mechanism to operate and maintain the aging fleet within budget constraints. Ideally, a bus that completes its service life needs to be replaced. Many states in the U.S do not have the matching funds needed to procure new buses for their constituent agencies; hence they use different rebuilding alternatives. The rebuild option, however, is not a permanent solution, as it only postpones the replacement of a bus. Therefore, the decision regarding replacement and rehabilitation of a fleet becomes a critical aspect of transit fleet management. While replacing the aging fleet is the most desirable option from a quality point of view, budgetary constraints require transit agencies to use a combination of new and old buses to provide services for their customers. Thus the challenge before the agency lies in finding an optimum combination of new and old buses by partially replacing and partially preserving the existing fleet.

A number of studies conducted between 1980 and 2000 explored the economics of purchasing new buses versus rebuilding of existing buses. These studies found that up to certain limits, it is cost-effective to rebuild an existing bus, thereby extending its effective life by a few years at a fraction of its replacement cost. The topic of optimally allocating resources between new buses and rebuild options was initiated at Wayne State University in 2000 as a part of a studies sponsored by the Michigan Department of Transportation *(2)* and the U.S. Department of transportation *(3)*. The later study resulted in the development of a two-stage linear programming model to allocate resources among different improvement programs *(4)*. A number of studies conducted between 2007 and 2010 attempted to improve upon the original model by suggesting both structural and methodological changes *(5-6)*. These studies attempted to maximize the quality of the bus fleet by optimizing different surrogates of Remaining Life (RL)*.

The research presented in this paper represents further modifications to these models by minimizing the investment cost, as opposed to maximizing RL (or a surrogate thereof).Initial attempts to formulate this problem resulted in maximizing the Total Weighted Remaining Life (TWARL) defined as:

$$TWARL = \sum_i \frac{\sum_j f_{ij}^m r_{ij}^m}{\sum_j f_{ij}^m} \quad (1)$$

where, f_{ij}^m is the number of buses for an agency *i* with remaining life of *j* years on m^{th} planning year; r_{ij}^m is the remaining life of *j* years for an agency *i* on m^{th} planning year for a corresponding bus; *i* is the agency, *j* is the remaining life, and *m* is the planning year in consideration. Mathew et al. (2010) reformulated the model by maximizing total system weighted average remaining life (TSWARL) defined as the sum of TWARL over the planning period in consideration, i.e. $\sum_m TWARL$, where:

$$TSWARL = \sum_m \sum_i \frac{\sum_j f_{ij}^m r_{ij}^m}{\sum_j f_{ij}^m} \quad (2)$$

Both TWRL and TSWARL can be looked upon as surrogates of the quality of the fleet. Research presented in this paper is based upon an alternative approach of cost minimization, and essentially builds upon the work reported by Mathew et al (2010).

The prime impetus behind this paper is exploring the feasibility of using an alternative approach of minimizing the amount of investment, as measured by net present cost (NPC), as opposed to maximizing quality (TWARL or TSWARL). Since the objective function of the earlier models was maximization of TWARL and TSWARL, the NPC-value was obtained only as a by-product in the Mathew et.al model. This paper seeks to re-formulate this problem with the objective of minimizing NPC, with appropriate constraints. As discussed later in the paper, the proposed model may have several dimensions of impact on the previously developed models and the status of current practice.

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LITERATURE REVIEW

Literature review on transit fleet management is organized into three areas: (1) resource allocation models, (2) measures of effectiveness, (3) modeling approaches. The review is not intended to be exhaustive, but to highlight some of the general trends in addressing the allocation problem.

Resource Allocation Models

Transit fleet management problems can be broadly classified into fleet maintenance and fleet replacement programs. Several studies are reported on the maintenance planning and management of transit system. Some prominent bus maintenance management studies include: bus maintenance programs for cost-effective reliable transit *(7-8)*, a generalized framework for transit bus maintenance operation *(9)*, manpower allocation for transit bus maintenance program *(10)*, fleet maintenance programs covering all aspects of repair and preventive maintenance *(11)*, a simulation model for comparing a bus maintenance

*RL can be defined as the difference between the minimum normal service life (MNSL) and the age of the bus. The MNSL of a medium-sized bus, the subject matter of this study is taken as seven years per guidelines of the U. S. Department of Transportation.

system's performance under various repair policies *(12)*, and performance indicators for maintenance management *(13)*. These problems primarily cater to an operator concerned with the day to day operation and maintenance of its fleet. A closely related problem, but addressing the need of a state transit planner is the replacement and/or rebuilding of buses *(14-15)*. Most of the resource allocation problems are characterized by a very specific formulation, stated objectives and constraints, as opposed to a standard formulation and solution methodology. These problems and their solutions demonstrate the benefit derived from a proper mathematical modeling.

Measures of Effectiveness

The most commonly adopted measure of effectiveness (MOE) for the resource allocation is in terms of monetary units. The performance measures frequently used in literature are maximization of revenue, return, or profits, benefit to cost ratio, internal rate of return, pay off period, cost effectiveness *(16-22)*. NPC is a widely understood and used MOE in transportation decision making. Minimization of NPC has been used as a MOE for evaluation of transit level of service *(23)*; for evaluation of rail transit investment priorities *(24)*; for finding the optimal bus transit service coverage in an urban corridor *(25)*; for modeling the timing of public infrastructure projects *(26)*; for a decision support tool for evaluating investments in transit systems fare collection *(27)*; for analyzing induced demand with introduction of new transit system *(28)*; for analyzing transportation impacts on economic development *(29)*; for analyzing externalities associated with light rail investment *(30)*; for resource allocation among transit agencies *(6)*; and for project selection problem under uncertainty *(31)*.

Modeling Approaches

Similar to the diverse objectives and requirements of the allocation problem, the modeling and solution approaches also vary. However, the most common modeling approach is optimization, with linear programming as the most popular tool because of its faster convergence feature *(4, 32)*. Non-linear programming can be used to model different systems realistically. However, convergence to unique solution is computationally intensive *(18, 33)*. Some of these problems require complex non-convex formulation; and Branch and Bound Algorithm (BBA) has been used to solve specifically large scale allocation problems. Examples include forecasting of energy consumption in multi-facility locations *(34)*, generating signal timing plans to maximize bandwidth for traffic networks *(35)*, a single-track train timetabling problem to minimize the total train travel time *(36)*, and journey planning procedures for multi modal passenger transport services *(37)*, and optimum allocation of resources for replacement and rehabilitation of transit fleet *(5-6)*.

Summary

The review of literature shows that (i) transit resource allocation models have focused on maximizing service life or minimizing cost, (ii) monetary units such as NPC, Benefit to Cost (B/C) ratio, Internal Rate of Return (IRR), and pay-off period are used as measures of effectiveness, (iii) although there is no common framework for the resource allocation problem, the most promising one seems to be an optimization model, (iv) allocation criteria could be based on some suitable performance measure specific to the problem domain; (v) the most common form of optimization is LP because of its fast convergence feature, but non-linear optimization models with mixed integer programming formulation have been used successfully; and (vi) Branch and Bound Algorithm (BBA) is used for integer programming problems and has the scalability to address large problems.

MODEL FORMULATION

The model is formulated as an optimization problem where the objective is to minimize the NPC of the total investment of the fleet for all the agencies over the entire planning period, subject to budget, demand, rebuild, and non-negativity constraints. This formulation is given below followed by an explanation of notations:

Mathematical Construct	Explanation	Eq.#
$Minimize \quad Z_x = \sum_{m=1}^{N} \sum_{i=1}^{A} \sum_{k=1}^{P} \frac{x_{i,k}^{m} * c_k^m}{(1+\emptyset)^m}$	*Objective function*: net present cost of the transit fleet resource allocation	(3)
subject to:		
$\sum_{m=1}^{N} \sum_{i=1}^{A} \frac{\sum_{j=1}^{J} (r_{i,j}^{m} + x_{i,j}^{m}) * j}{\sum_{j=1}^{J} (r_{i,j}^{m} + x_{i,j}^{m})} \geq TSWARL$	*Constraint*: Sum total of the weighted average remaining life of the fleet of all the constituent agencies for the whole planning period should be greater than predetermined value of total system weighted average remaining life	(4)
$\sum_{m=1}^{N} \sum_{i=1}^{A} \sum_{k=1}^{P} y_{ik}^{m} * c_k^m < \sum_{m=1}^{N} b_m, \forall m$	*Constraint*: Total cost of improving the buses for different improvement schemes, agencies and over a planning period should not exceed budget for the planning period	(5)
$\sum_{m=1}^{N} b_m = B$	*Constraint*: Planning period budget is equal to the sum available budget for each year, where budget is *a priori*.	(6)
$\sum_{k=1}^{P} y_{ik}^{m} = r_{ij}^{m}, \forall i, m, j$	*Constraint*: The buses that are improved under improvement scheme *k* are the ones that have completed their minimum normal service life and have remaining life *j*	(7)
$y_{i\gamma}^{m} = \sum_{\alpha\beta} \delta_{i,(\alpha\beta)}^{m} + \delta_{i,(\gamma)}^{m} \forall i, m, j$ $\forall \alpha, \beta \in \{2,3\}, \gamma\{4\}$	*Constraint*: The buses that have been rehabilitated twice or remanufactured once will be replaced	(8)
$y_{i\gamma}^{m} > 0$	*Constraint*: Non-negativity constraint. Number of buses chosen for improvement should be greater	(9)

	than 0	
$x_{ij}^{m} = \begin{cases} y_{ik}^{m}, if\ j = l_k, l_k \in \{2,3,4,7\} \\ 0\ , \quad Otherwise \end{cases}$	*Constraint*: The life of the buses is improved by either two, three, or four years for a re-built bus and by seven years for a new bus	(10)
$\delta_{i,(\alpha\beta)}^{m} = \begin{cases} min\left\{y_{i\alpha}^{m-(\alpha+\beta)}, y_{i\beta}^{m-\alpha}\right\}\ if\ m \geq \alpha+\beta, \forall\ m > \alpha+\beta, \\ 0\ , \quad Otherwise \end{cases}$	*Constraint*: Auxiliary constraint of Eq. (7), represents replacement option after $\alpha+\beta$ years (REHAB)	(11)
$\delta_{i,(\gamma)}^{m} = \begin{cases} y_{i\alpha}^{m-\gamma}, \forall\ m > \gamma, \\ 0\ , \ Otherwise \end{cases}$	*Constraint*: Auxiliary constraint of Eq. (7), represents replacement option after γ years(REMANF)	(12)

The objective function shown in equation (3) represents the NPC or Z_x of the transit fleet resource allocation. The decision variable x_{ij}^{m} is defined in equation (10) with the help of an auxiliary variable y_{ik}^{m}. This definitional constraint in equation (10) ensures that the life of the buses is improved by either two, three, or four years for a re-built bus and by seven years for a new bus. Other buses in the system will have no additional years added. The constraint (4) represents the sum total of the weighted average remaining life of the fleet of all the constituent agencies for the whole planning period, designated as TSWARL, which is determined previously. The choice of TSWARL is defined by the user. A lower value of TSWARL suggests low cost improvement options are chosen, and vice versa. Equation (5) represents the constraint of a fixed budget for the seven-year planning horizon with the planner having the budget flexibility across the years. Equation (6) represents the planning period budget being equal to the sum available budget for each year. Equation (7) ensures that all the buses that have completed their Minimum Normal Service Life (MNSL) requirements will be eligible for improvement as per Federal Highway Administration (FHWA) standards. MNSL can be defined as the number of years or miles of service that the vehicle must provide before it “qualifies” for federal funds for rehabilitation, remanufacturing and replacement.

Equation (8) represents policy constraints which ensure that the buses that have been rehabilitated twice or remanufactured once will be replaced. The two terms in this constraint are defined in equations (11) and (12). These three constraints are specific to the case study presented in this paper, and can be revised at the discretion of the user. Thus, equations (8) and (11) ensure that a bus that was rebuilt twice (each time its life is increased by α or β years is replaced. Obviously, this policy is applicable only after $\alpha + \beta$ years. Similarly, a bus that is remanufactured resulting in an increase in life by γyears must be replaced (equations 8 and 12) and is applicable only after γ years. This constraint presented in equations (8, 11, and 12) is specific to the case study presented in this paper, and can be revised at the discretion of the user. Equation (9) is a non-negativity constraint to ensure that the number of buses chosen for improvement is never negative. The formulation involves non-linear functions, non-differentiable functions, step functions, and integer variables. Although the step function can be generalized to linear forms, the formulation will require additional variables which may result in variable explosion rendering the model unsuitable for large/real world problems.

The notations are given below.

Variables		**Explanation**
b_m	:	budget available for m^{th} planning year
c_k^m	:	cost of implementation of the improvement program k on m^{th} year
$f_{i,j}^m$	:	Number of buses for an agency i with remaining life of j years on m^{th} planning year
l_k	:	additional year added to the life of the bus due to improvement program k, $l_k \in \{2,3,4,7\}$
r_{ij}^m	:	number of existing buses with remaining life of j years for an agency i on m^{th} planning year
x_{ij}^m	:	number of buses which received remaining life of j years for an agency i on m^{th} planning year due to the improvement program
y_{ik}^m	:	number of buses chosen for the improvement program k adopted for an agency i on m^{th} planning year
$\delta_{i,(\alpha,\beta)}^m$	:	number of buses already improved by α, β years due to rehabilitation in the m^{th} planning year for agency i, $(\alpha, \beta \in \{2,3\})$
$\delta_{i,(\gamma)}^m$	:	number of buses already improved by γyears due to remanufacture in the m^{th} planning year for agency i, $(\gamma \in \{4\})$
$\emptyset$		The interest rate used for NPV
A	:	total number of agencies
B	:	total budget available for the project for all planning years
i	:	1, 2, …,A, the subscript for a transit agency
j	:	1, 2, …,Y, the subscript for remaining life
k	:	1,2,…., P the subscript used for improvement program
m	:	1, 2, …,N, the subscript used planning year

N : number of years in the planning period

P : number of improvement programs

REHAB1 : the first improvement program- rehabilitation of bus yielding α *(=2)* additional years

REHAB2 : the second improvement program- rehabilitation of bus yielding β *(=3)* additional years

REMANF : the third improvement program- rehabilitation of bus yielding γ (=4)additional years

REPL : the last improvement program-replacement of bus yielding 7 additional years

TSWARL : Total System Weighted Average Remaining Life , $TSWARL = \sum_m TWARL$

TWARL : Total Weighted Average Remaining Life= $TWARL = \sum_i WARL_i$

$WARL_i$: Weighted Average Remaining Life for agency *i*=$WARL_i = \frac{\sum_j f_{ij}^m r_{ij}^m}{\sum_j f_{ij}^m}$

Y : minimum service life of buses

Z_x : The objective function as minimization of NPV for the resource allocation in the planning period

SOLUTION APPROACH

A Branch and Bound Algorithm (BBA) is used in this paper because of the integer nature of decision variables as called for in the case study. The BBA approach is applied in three steps. The first step involves coding of the decision variable cells. The second step involves model initialization, where the convexity and the size of the problem in terms of number of variables, integers, bounds and surface nature are determined. A diagnosis of the model is performed to check the nature of the desired model (linear, quadratic, conic, non-linear, etc.). Finally, the third step involves the development of constraint coded cells. Budget constraints (Equation 5 and 6), mandatory replacement constraints (Equation (8)), and REBUILD constraints are coded. The BBA approach is explained below.

Let y_{ik}^m is the number of buses to be added to a fleet when it reaches a zero remaining life for *k* type of improvement for agency *i*, on m^{th} year. If y_{ik}^m is not an integer, we can always find an integer $[y_{ik}^m]$ such that:

$$[y_{ik}^m] < y_{ik}^m < [y_{ik}^m] + 1 \quad (13)$$

Equation (12) results in the formulation of two sub problems, with an additional upper bound constraint

$$y_{ik}^m \leq [y_{ik}^m] \quad (14)$$

and another with lower bound constraint

$$y_{ik}^m < [y_{ik}^m] + 1 \quad (15)$$

If the decision variables with integer constraints already have integer solutions, no further action is required. If one or more integer variables have non-integer solutions, the Branch and Bound method chooses one such variable and creates two new sub-problems where the value of that variable is more tightly constrained. These sub problems are solved and the process is repeated, until a solution is found where all of the integer variables have integer values (to within a small tolerance). The complete methodology is implemented in a VBA based solver platform *(38-39)*, on an Intel (R) Xeon (R) Core 2 Quad, 4GB memory, 2.0 GHz under Windows 7 operating system. A precision value of 1.0E-6 is used to determine how closely the estimated constraints match with the given values. Each optimization run requires approximately 20,000 iterations to find the optimal value. Each iteration requires 0.10 seconds, and one complete optimization run requires approximately 33 minutes.

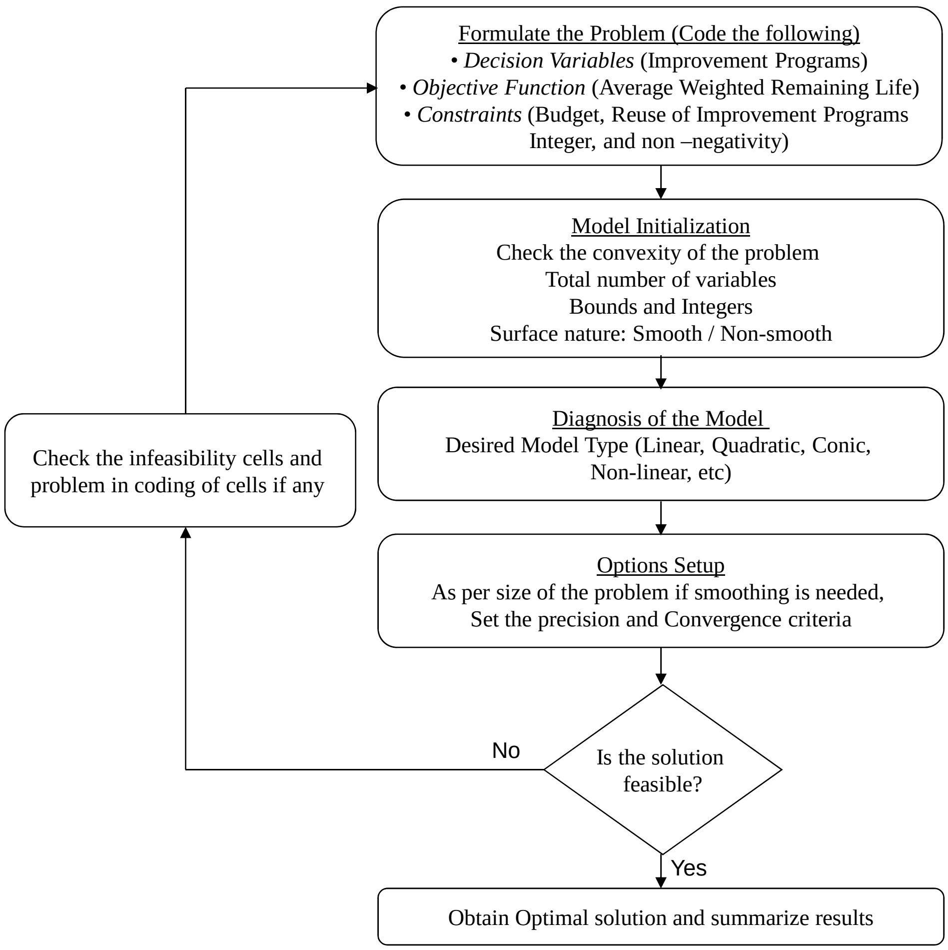

FIGURE 1 Flowchart to the proposed solution methodology

CASE STUDY

The case study includes a fleet of 720 medium sized buses operated by 93 transit agencies with the capital funding program administered by the Michigan Department of Transportation (MDOT). The following improvement options are used in the case study (Khasnabis et al. 2004):

- Replacement (REPL)—process of retiring an existing vehicle and procuring a completely new vehicle. Buses proposed to be replaced using federal dollars are expected to be at the end of their MNSLs, as described above. (Life expectancy: seven years)
- Rehabilitation (REHAB)—process by which an existing bus is rebuilt to the original manufacturer's specification. The focus of rehabilitation is on the vehicle interior and mechanical

systems, including rebuilding engines, transmission, brakes, and so on. Two types of rehabilitation: REHAB1 and REHAB2 with moderate to higher levels of engine rebuilds are considered in this study. (Life expectancy: two to three years for medium duty, medium sized buses)

- Remanufacturing (REMANF)—process by which the structural integrity of the bus is restored to original design standards. This includes remanufacturing the bus chassis as well as the drivetrain, suspension system, steering components, engine, transmission, and differential with new and manufactured components and a new bus body. (Life Expectancy: 4 years)
- Further, it was assumed that a vehicle may be rehabilitated (REHAB1 or REHAB2) only up to two consecutive terms, and then must be replaced (REPL) with a new bus. A vehicle with REHAB1 and REHAB2 (or vice versa) in two consecutive terms also should be replaced. A vehicle may be remanufactured (REMANF) only one time, and then must be replaced (REPL) with a new bus. A vehicle rehabilitated (REHAB1 and REHAB2) once can be eligible for remanufacturing (REMANF) before it is replaced (REPL).

Case Study Overview

A Public Transportation Management System (PTMS) database developed by Michigan Department of Transportation (MDOT) containing actual fleet data is used for the case study demonstration. This database is used because it permits a real application and direct comparison of results with Mathew et al. (2010). To ensure compatibility between the results, the basic model parameters (e.g. budgets, policy constraints, extended life values associated with different improvement options, etc.) and critical assumptions and constraints are kept same as in the above model. The distribution of the Remaining Life (RL) in years of the fleet for a few of the 93 agencies for the base year (2002) is shown in Table 1. Only a fraction of the table is presented for the sake of brevity. Table 1 shows the distribution of fleet size by their remaining life (RL) for each agency. For example, agency 1 has one bus with zero years of RL, 2 buses with seven years of RL and so on, for a total fleet size of 3.The last row of the table shows that the total fleet is of 720 buses, of which 235 buses have zero years of RL, and need replacement. The last column of the Table 1 gives the weighted average remaining life (WARLi) for each agency, computed from the distribution of RL for the agency. For example, the WARLi of the first agency is calculated as (0x1+1x0+…+7x2)/3 =4.67. The base year total weighted average remaining life of the entire fleet (TWARL) is 225.23 years.

TABLE 1: Base year distribution of remaining life (RL), fleet size, and weighted average of remaining life of sample agencies before allocation of resources for the case study

Agency	**Distribution of Remaining Life**								**Total**	**WARLi**
	0	1	2	3	4	5	6	7	**Fleet**	**(years)**
1	1	0	0	0	0	0	0	2	3	4.67
2	1	0	0	0	0	0	0	0	1	0
3	1	0	0	0	0	0	0	0	1	0
.	.	.	.	.	.	.	.	.	.	.
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91	0	1	0	0	0	0	1	0	2	3.5
92	2	1	0	0	1	1	0	3	8	3.88
93	2	0	0	0	1	3	1	0	7	3.57
Total	**235**	**122**	**44**	**23**	**63**	**77**	**78**	**78**	**720**	**225.23**

Case Study Problem

The budgets available for each year and the unit cost for each improvement options are shown in Table 2. A seven year planning period is considered conforming to the MNSL requirement of medium sized buses. Replacing all the 236 buses with zero years of RL would require $19,161,900 (235 x $81,540) of investment which exceeds the first year budget. Similarly, in the second year, 122 buses which had one year of RL in the base year will qualify for improvement.

TABLE 2: Budget Scenario

Year	Budget	Improvement Options and Costs			
		REPL (X1= 7Years)	REHAB1 (X2= 2 Years)	REHAB2 (X3=3 Years)	REMANF (X4= 4 Years)
2002	5,789,000	81,540	17,800	24,500	30,320
2003	9,130,000	81,540	17,800	24,500	30,320
2004	6,690,000	88,063	19,220	26,400	32,750
2005	2,025,449	88,063	19,220	26,400	32,750
2006	13,969,324	95,108	20,740	28,500	35,370
2007	6,600,000	95,108	20,740	28,500	35,370
2008	13,970,880	102,720	22,400	30,780	38,200
2009	6,880,000	102,720	22,400	30,780	38,200
Total	**65,054,653**				

Replacing all these buses with remaining life 1 year would require $9,947,880 (122x$81,540), which also exceeds the second year budget and so on for other years. Moreover, if the replacement process is continued from year 2002 through 2009, when the buses reach their MNSL, it will cost $88,488,688 (i.e. 235*81,540+122*81,540+……+235*102,720) to maintain the fleet size of 720 buses throughout the planning period. However the total available budget is only $65,054,000[†] (Table 2). Therefore, there is a need for a mechanism to identify improvement options for each agency, so that the NPC is minimized with a user defined TSWARL.

Earlier Model

Mathew et al. proposed an optimization model with an objective function of maximization of TSWRL (Mathew et al. 2010). They solved the case study using two different algorithms named Genetic Algorithm (GA) and BBA (Mathew et al. 2010), with a total budget constraint over the planning period ($65.05 million), as opposed to individual annual budgets. It was assumed for the purpose of the model, that the transit agency has the flexibility to “borrow” funds from future years, and to “transfer” funds saved from past years to future years. The BBA model produced better results that are shown in Table 3.

[†] The budget is based upon original estimates by MDOT with additional modifications, and explained in Mathew et al. (2010).

TABLE 3: Model Results with TSWARL as the Objective Function *(Source: Mathew et al. 2010)*

Case (1)	Year (2)	REPL X1 7 YEARS (3)	REHAB1 X2 2 YEARS (4)	REHAB2 X3 3 YEARS (5)	REMANF X4 4 YEARS (6)	Total Assigned Fleet (7)	TWARL (years) (8)	Amount Committed ($) (9)	NPC ($) at 6% annual interest (10)
Single-stage Three Dimensional Model –BBA	2002	150	85	0	0	235	466.19	13,744,000	12,915,380
	2003	110	12	0	0	122	444.36	9,183,000	9,384,792
	2004	127	2	0	0	129	431.58	11,222,441	11,006,817
	2005	35	0	0	0	35	360.37	3,082,205	1,700,606
	2006	61	0	0	4	65	328.99	5,943,068	4,470,876
	2007	60	0	7	10	77	297.10	6,259,680	4,267,136
	2008	29	12	3	34	78	258.24	4,638,820	2,922,005
	2009	57	82	42	47	228	410.11	10,780,000	6,826,213
	Total	**629**	**193**	**52**	**95**	**969**	**2996.94***	**64,853,214**	**53,493,825****

*The objective function, **: Model by-product

Table 3 shows the allocation of fleet by different improvement options from each year (Column2 thru 7), and the amount committed each year, along with their NPC at an annual interest rate of six percent. The last row of Table 3 also shows that this allocation resulted in a total TSWARL of 2996.94 (amount maximized by the objective function), with a total commitment of $64,853,214 for a NPC of $53,493,825. It should be noted that inn this model, NPC is estimated by post-processing, and is not a part of the optimization procedure for this model.

Proposed Model

Table 4 shows the allocation of different improvement options by using the NPC minimization approach, along with a total budget constraint of $65.05 million. Table 4, developed in the same format as Table 3, shows that the NPC derived by the proposed model (the object of optimization; $52.09 million) is lower than that presented in Table 3 ($53.49 million). The TSWARL value attained is 2966.99 years, and is marginally lower than the corresponding figure of 2996.99 years presented in Table 3 (the object of optimization). A lower and an upper bound of TSWARL of 2500 and 3000 years respectively were provided as constraints (Expression 4) to the optimization model. Each model run requires a threshold value of TSWARL. The minimum TSWARL may be derived by allocating the least cost improvement option to all buses as they reach MNSL. Similarly, the upper bound of TSWARL can be obtained by allocating highest cost alternative all buses as they reach MNSL. The lower and upper bound TSWARL values of 2496.72 and 3105.61 were obtained by allocating REHAB1 (lowest cost alternative) and REPL (highest cost alternative) respectively to all the buses reaching MNSL. In this case, the higher value of was referred to TSWARL of 2996.94 which was obtained from Mathew et al. (2010) and used in the remainder of the paper for comparison purposes.

By comparing Tables 3 and 4, it is found that the proposed model, when compared to the previous model results in a savings of $1.40 million (2.60%), attained at a reduction of TSWARL value of 29.65 (1.00%) years. Thus, the proposed model results in a small reduction in cost at the expense of a small reduction in the quality of the fleet, as measured by TSWARL. While it is hard to designate the outcome of the proposed model as an improvement, it provides a viable approach for solving the same problem with a different optimization function that considers the "time value of money" in the decision-making process. Other differences in the allocation of the improvement options by the two methods can be observed by comparing the two tables.

TABLE 4 : NPC Minimization Model Resource Allocation Results for Case Study

Case (1)	Year (2)	REPL X1 7 YEARS (3)	REHAB1 X2 2 YEARS (4)	REHAB2 X3 3 YEARS (5)	REMANF X4 4 YEARS (6)	Total Assigned Fleet (7)	TWARL (years) (8)	Amount Committed ($) (9)	NPC ($) at 6% annual interest (10)
NPC Minimization (Proposed Model)	2002	174	61	0	0	235	483.79	15,273,760	15,273,760
	2003	122	0	0	0	122	466.10	9,947,880	9,384,792
	2004	98	7	0	0	105	427.72	8,764,714	7,800,564
	2005	23	0	0	0	23	350.71	2,025,449	1,700,606
	2006	46	0	0	24	70	317.39	5,223,848	4,137,777
	2007	42	0	0	35	77	281.91	5,232,486	3,910,018
	2008	15	15	0	48	78	238.50	3,710,400	2,615,686
	2009	50	121	0	81	252	400.89	10,940,600	7,276,124
	Total	**570**	**204**	**0**	**188**	**962**	**2966.99******	**61,119,137**	**52,099,32***

*The objective function, **: Model by-product

The primary reason of including the budgetary constraint of $64.85 million in the proposed model presented in Table 4 is to ensure compatibility of the results with those presented in the earlier study (Mathew et al. 2010). However, the NPC minimization problem can also be solved without any budget constraint. Results are shown in Table 5. A comparison of the two tables shows that the results are very similar, with minor differences. The case without budget constraint resulted in a lower NPC ($52.07 million versus $52.09 million) compared to its counterpart; but the amount committed, $61.23 million is higher than $61.11 million committed in earlier the case. In both cases, the differences are marginal. The small savings in the NPC is achieved at the expense of a small loss in the value of TSWARL, 2965.43 versus 2966.99.

From a modeling point of view, budget need not serve as a mandatory constraint for NPC minimization. As explained above, for comparison purposes the authors have used budget as a constraint so that amount committed does not exceed a specified value. For the remainder of the paper the results presented are using the budget constraint (Table 4).

TABLE 5: NPC Minimization Model Resource Allocation Results for Case Study (Without Budget Constraint)

Case (1)	Year (2)	REPL X1 7 YEARS (3)	REHAB1 X2 2 YEARS (4)	REHAB2 X3 3 YEARS (5)	REMANF X4 4 YEARS (6)	Total Assigned Fleet (7)	TWARL (years) (8)	Amount Committed ($) (9)	NPC ($) at 6% annual interest (10)
NPC Minimization (Proposed Model)	2002	120	77	38	0	235	484.72	12,086,400	12,086,400
	2003	122	0	0	0	122	467.04	9,947,880	9,384,792
	2004	121	0	0	0	121	417.99	10,655,623	9,483,467
	2005	61	0	0	0	61	354.01	5,371,843	4,510,303
	2006	46	0	0	17	63	319.84	4,976,258	3,941,662
	2007	40	0	0	37	77	283.81	5,113,010	3,820,739
	2008	13	21	0	44	78	238.91	3,486,560	2,457,887
	2009	48	67	0	83	198	399.11	9,601,960	6,385,852
	Total	**571**	**165**	**38**	**181**	**955**	**2965.43****	**61,239,534**	**52,071,102***

*The objective function, **: Model by-product

It is possible to derive a set of feasible solutions by varying the input (minimum) value of TSWARL, and using the proposed model to obtain the minimum NPC. The relationship between TSWARL and NPC is depicted with a total of 22 data points obtained by as many runs of the proposed model. Figure 2 shows a set of four curves each representing the four programs, (REPL, REHAB1, REHAB2, and REMNF) consisting of 22 data points over a range of NPC values from $43 million to $53.03 million. Figure 2 shows that larger investments in fleet (as reflected by increased NPC values) are generally associated with increased new purchases, (REPL) and reduced number of REHAB1 buses The number of REHAB2 buses is generally not affected by changes in investment levels, and appears to be the "least preferred" improvement option. Clearly, the marginal increase in cost from REHAB1 to REHAB2 of $6,700 is not justified by the marginal increase in life improvement of one year. The number of REMANF buses increases with increase in NPC up to $50 million, beyond which very little change is observed. Overall, these trends appear logical and reasonable.

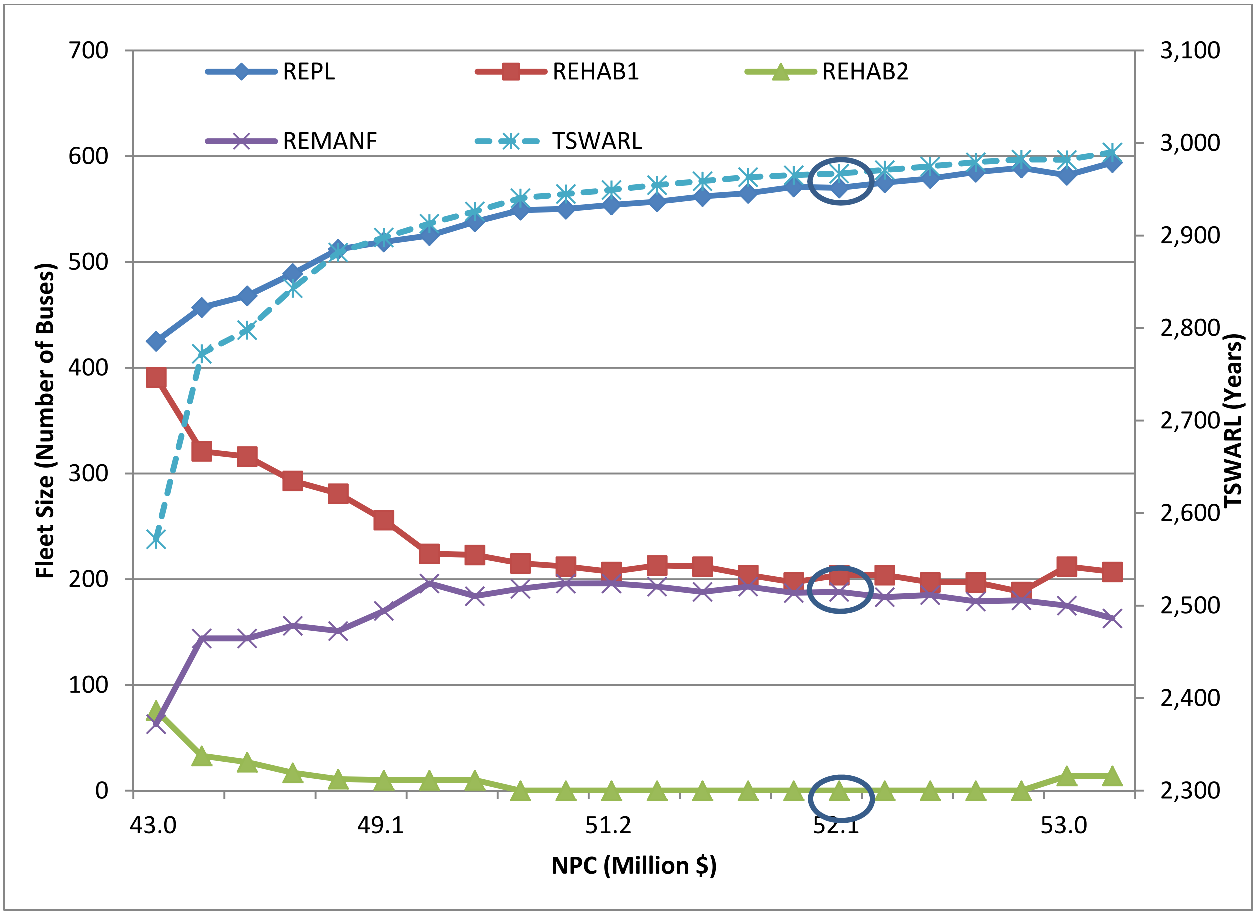

FIGURE 2 Fleet Size Distribution for Various NPCs and Corresponding TSWARL *(Note: The circled point on TSWARL solution is presented in Table 3)*

Figure 2 also shows that increased investment in fleet (as reflected in higher NPC values) is associated with higher quality of fleet (as reflected in higher TSWARL values), as expected. TSWARL has a higher rate of increase with increasing NPC up to the value of $50 million. Beyond that point, the slope gets flatter. A year-by-year analysis for the 22 solutions is presented in Figure 3.Three MOEs viz. NPC, TWARL, and TSWARL are shown on the x-axis, primary y-axis, and secondary y-axis respectively. It should be noted that both NPC and TSWARL are measured over the planning period (seven years), whereas TWARL is an annual measure that is depicted for all seven years over the planning period. When one solution point for TSWARL is considered, its corresponding TWARL for all years can be found by drawing an imaginary vertical line on the specific curve, and projecting the point on the left-hand vertical axis.

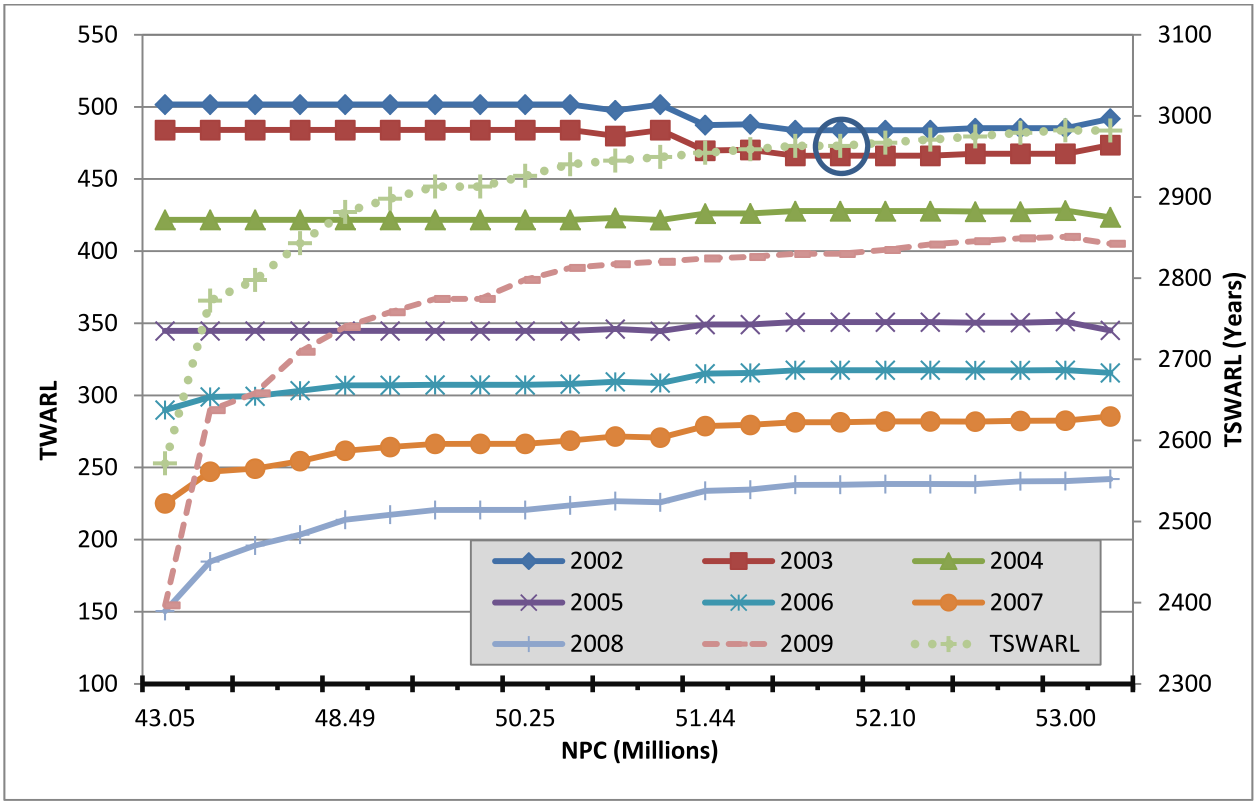

FIGURE 3 NPC, TSWARL and Individual Year TWARL *(Note: The circled point on TSWARL solution is presented in Table 3)*

The trends in Figure 3 show that the TWARL values for the years 2002 through 2006 remain approximately the same irrespective of increase in NPC and TSWARL. TWARL for 2002 and 2003 are virtually parallel, and show little variation with change in NPC, except one point where both curves have a little dip. For the years 2007 through 2009 TWARL increases with increase in NPC and TSWARL. Significant increase in TWARL is observed for the year 2009 with increase in NPC. Among all the years, the maximum TWARL is observed for the year 2002 for all model runs. TWARL decreases as the year's progress from 2002 onwards except 2009. Figure 4 also shows that the relationship between TWARL and NPC is not linear. (Note: The data points depicted in Figure 3are specifically identified in Figures 2.). Lastly, a sample allocation of resources among constituent agencies is shown in Table 6. A complete allocation among the 93 agencies is not shown for the purpose of brevity. Table 6 is self-explanatory.

Table 6: Sample Resource Allocation Among Agencies Over Planning Period

		Distribution of remaining life (years)										
Year	**Agency**	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**Fleet Size**	**TWARLi**	**NPC**
2002	1	0	0	0	0	0	0	0	3	3	7.00	81,540
	2	0	0	0	0	0	0	0	1	1	7.00	81,540
	.	.	.	.	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.	.	.	.	.
	92	0	1	2	0	1	1	0	3	8	4.38	35,600
	93	0	0	2	0	1	3	1	0	7	4.14	35,600
Sub-total		***0***	***122***	***105***	***23***	***63***	***77***	***78***	***252***	***720***	***483.79***	***15,273,760***
2003	1	0	0	0	0	0	0	3	0	3	6.00	0
	2	0	0	0	0	0	0	1	0	1	6.00	0
	.	.	.	.	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.	.	.	.	.
	92	0	2	0	1	1	0	3	1	8	4.25	76,925
	93	0	2	0	1	3	1	0	0	7	3.14	0
Sub-total		***0***	***105***	***23***	***63***	***77***	***78***	***252***	***122***	***720***	***466.10***	***9,384,792***
.	**.**	**.**	**.**	**.**	**.**	**.**	**.**	**.**	**..**	**.**	**.**	**.**
.	**.**	**.**	**.**	**.**	**.**	**.**	**.**	**.**	**.**	**..**	**..**	**.**
2008	1	0	3	0	0	0	0	0	0	3	1.00	0
	2	0	1	0	0	0	0	0	0	1	1.00	0
	.	.	.	.	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.	.	.	.	.
	92	0	3	2	3	0	0	0	0	8	2.00	0
	93	0	0	0	5	1	1	0	0	7	3.43	26,929
Sub-total		***0***	***252***	***161***	***133***	***71***	***46***	***42***	***15***	***720***	***238.50***	***2,615,686***
2009	1	0	0	0	0	3	0	0	0	3	4.00	76,216
	2	0	0	0	0	0	0	0	0	2	7.00	68,315
	.	.	.	.	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.	.	.	.	.
	92	0	2	6	0	0	0	0	0	8	1.75	44,692
	93	0	0	5	1	1	0	0	0	7	2.43	0
Sub-total		***0***	***161***	***254***	***71***	***127***	***42***	***15***	***50***	***720***	***400.89***	***7,276,124***
Total											**2966.99**	**52,099,32**[*]

CONCLUSION

The model proposed in this paper is the result of continuing research on the topic of resource allocation between different fleet life improvement options (rehabilitation, remanufacturing, or replacement) among a number of constituent agencies over a planning period in an equitable manner. The objective of the optimization model is minimization of investment cost, expressed as NPC, with TSWARL, and budget as the primary constraints. The cost minimization approach is not to be considered as inferior or superior to its predecessor, TSWARL maximization (Mathew et al. 2010), but simply as a complementary means for resource allocation for investing in and preserving transit fleet. The fundamental difference between the two approaches is the minimization of cost (NPC) as opposed to the maximization of the quality of the fleet (TSWARL). The primary impetus for this model is the perception that cost is more easily identified as a decision making tool compared to quality that is often regarded as somewhat abstract in nature. Because NPC is affected by the time of the investment within the planning period, an additional degree of complexity is introduced in the algorithm through the incorporation of interest factors. For a transit agency, the desired goal will be to identify improvement options that result in the lowest NPC with an acceptable TSWARL. The proposed approach is designed to assist the user accomplish this objective.

The proposed model provides a set of solutions each depicting a minimum NPC for a specified value of TSWARL, so that the user has a choice of selecting a solution that meets his/her quality requirement for which the NPC is minimized. For the case study presented, the solutions provide a trend suggesting that irrespective of NPC, replacement options receive the highest number of buses in the analyzed planning period followed by rehabilitation, and that the remanufacturing option is the least preferred option and is not affected by the investment level. Further, higher NPC's are associated with larger replacement options. The presented set of solutions shows how the agency can choose the optimum set of investment options to minimize the NPC for a specified TSWARL. Curves similar to Figures 3 and 4 can be developed by a state to allocate funds among a set of improvement programs to over a planning horizon to minimize NPC to match a desired quality requirement. Table 5 can similarly developed to distribute the program-specific funds among the constituent agencies.

The set of solutions shows that TSWARL increases with increasing NPC, signifying that the quality of the fleet is likely to improve with increasing levels of improvement. Further, the relationship between NPC and TSWARL is non-linear in nature because of the incorporation of the interest factors in computing NPC. When NPC is compared with individual year quality measure (TWARL), it is observed that initially, TWARL remains relatively constant with increase in NPC up to a certain point, beyond which TWARL increases in the later years.

The proposed NPC minimization model has several dimensions of significant impact and contribution to practice. First, the proposed model provides a new dimension of NPC, a cost measure to be aware of while exploring different transit investments. Second, this model results in the minimum value of NPC to assist transit agencies in making critical decision using a common benchmark at policy level while maintaining a desirable TSWARL. Third, the solution results in optimal improvement strategies for the fleets with no remaining life such that NPC is minimized, in a multiple year planning period subject to budget and other constraints.

The model application is demonstrated for the medium duty, medium sized transit fleet system in Michigan. However, the methodology can be applied to other local and state agencies with different fleet age types, policy, and budget constraints. This study can be extended as a multi-objective optimization problem for solving NPC and TSWARL simultaneously to incorporate different fleet types with variant composition of improvement and budgetary options.

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