A Multi-Objective Optimization Model for Transit Fleet Resource Allocation

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Abstract

State and local transit agencies require government support to preserve their aging transit fleet. With passage of time, transit fleet gets older and requires maintenance cost to keep it operational. To provide services at a desired level, transit agencies require maintaining a minimum fleet size. Two imperative considerations from transit planning viewpoint are (1) remaining life of the total fleet, and (2) cost required to maintain the fleet size. When the former is a quality measure indicating the health of the fleet, the latter is an economic measure requiring minimum expenditure levels. Ideally, the agencies would like to maximize the total remaining life of the fleet and minimize the total cost required to maintain the fleet size. In this paper, the authors propose a multi-objective optimization model (MO) to simultaneously incorporate both objectives when subjected to budget and a number of operational constraints. The MO problem is solved by using classical weight sum approach by employing Branch and Bound Algorithm (BBA) that has proven to be better than other solution methodologies. The MO resulted in pareto optimal solutions with the possible trade-off between the two objectives. The model is applied to a real large scale transit fleet system in the state of Michigan, U.S. The case study results demonstrate, the proposed model is compact, efficient, robust and suitable for long range planning with multiple solutions to choose from a pareto optimal frontier. The correlation between decision variables and objective functions has been investigated in-depth and provides important insights. The proposed model can act as a tool for resource allocation for transit fleet among agencies for state and local agencies.

Keywords: transit fleet, multi-objective optimization, branch and bound algorithm, pareto optimal

1. Introduction

A number of state Departments of Transportation (DOT) and their local transit agencies are concerned about the escalating costs of new buses and lack of funds to keep up with their replacement needs. The cost of replacing the aging transit fleet in the US to maintain current performance levels is estimated to exceed one billion dollars annually *(1, 2)*. The addition of new buses to the existing fleet of any transit agency is a capital intensive process. In the US, the federal government provides a bulk of the capital funds needed to replace the aging transit fleet, with the requirement of a minimum matching support (usually 20%) from non-federal sources.

A bus that completes its service life should ideally be replaced. However, lack of capital funds often prevents state DOTs from procuring new buses for their constituent agencies. Several rebuilding alternatives to bus replacement are available to the transit industry that can be classified under two generic categories: bus rehabilitation and bus remanufacturing. A number of studies conducted between 1980 and 2000 explored the economics of replacement of buses versus rebuilding of existing buses. Most of these studies found that up to certain limits, it is cost-effective to rebuild an existing bus thereby extending the life by a few years with a fraction of the procurement cost of a new bus. The problem addressed is typical to a state DOT in the US that supports the fleet management of its constituent transit agencies. While replacing the aging fleet is the most desirable option from a quality point of view, budgetary constraints require transit agencies to use a combination of new and old buses to provide services for their customers. Thus the challenge before the agency lies in finding an optimum combination of new and old buses by partially replacing and partially preserving the existing fleet. Two imperative and conflicting measures in the resource allocation are (1) fleet quality measure; and (2) economic measure. These two measures are quantified in the following section.

<u>Fleet Quality Measure</u>

A quality measure can be considered as the remaining life of the fleet system. An agency's objective is to maximize the quality measure. Recent studies attempted to improve upon the original model by suggesting both structural and methodological changes *(3, 4*). These studies attempted to maximize the quality of the bus fleet by optimizing different surrogates of Remaining Life (RL)[1]. In this context, Total Weighted Average Remaining Life (TWARL) for one agency can be defined as:

$$TWARL = \sum_i \frac{\sum_j f_{ij}^m r_{ij}^m}{\sum_j f_{ij}^m} \tag{1}$$

where, f_{ij}^m is the number of buses for an agency *i* with remaining life of *j* years on m^{th} planning year; r_{ij}^m is the remaining life of *j* years for an agency *i* on m^{th} planning year for a corresponding bus; *i* is the agency, *j* is the remaining life, and *m* is the planning year in consideration. Mathew et al. (2010) reformulated the existing model *(3)* by maximizing Total System Weighted Average Remaining Life (TSWARL) defined as the sum of TWARL over the planning period in consideration *(4)*, i.e. $\sum_m TWARL$, where:

$$TSWARL = \sum_m \sum_i \frac{\sum_j f_{ij}^m r_{ij}^m}{\sum_j f_{ij}^m} \tag{2}$$

Both TWARL and TSWARL can be looked upon as surrogates of the quality of the fleet and need to be *maximized*. In the remainder of this paper, we will be using abbreviations of these quality measures. Further, research presented in this paper is also based upon an alternative approach of cost minimization as another economic measure.

Economic Measure

In terms of economic measure the agency has scarcity in funding to manage the fleet so that the fleet size is maintained with minimum cost. Net Present Cost (NPC) is used in a number of studies to measure the expenditure. For the proposed resource allocation problem, NPC is defined as the sum of spending in improvement of transit options when the dollar value of the expenditure is brought to its present value using appropriate interest rate throughout the planning period. If $x_{i,k}^m$ is the number of buses chosen for agency *i*, for improvement option *k*, and for time period *m*; c_k^m is the cost of improvement option k in year m; and $\emptyset$ is the interest rate/discount factor then NPC can be measured as

$$NPC = \sum_m \sum_i \sum_k \frac{x_{i,k}^m * c_k^m}{(1+\emptyset)^m} \tag{3}$$

The agency's objective is to *minimize* the NPC while maintaining the quality measure to minimum standards.

Measure Preference

If the agency's objective is to maximize the quality measure, that maximum value may not be attainable within a given budget constraint. Both of the above mentioned measures are conflicting, and an agency would like to invest minimum amount (in terms of net present cost) over planning period to obtain the best fleet quality measure possible. In a true sense both objectives (to maximize fleet quality and to minimize the NPC) should be considered simultaneously. In this paper, we present a multi-objective optimization (MO) framework rather minimizing or maximizing each objective separately. MO results in a set of pareto optimal solutions as opposed to just one solution, and it allows the agency to investigate a

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[1]RL can be defined as the difference between the minimum normal service life (MNSL) and the age of the bus. The MNSL of a medium-sized bus, the subject matter of this study is taken as seven years per guidelines of the U. S. Department of Transportation.

trade-off between the two objectives. Pareto-optimal solutions are those in which it is impossible to make one objective better off without necessarily making the other objective(s) worse off *(5)*. The remainder of the paper is organized as follows. A literature review is presented in the next section along with the need of the research followed by methodology. The case study section shows the structure of a real world data. Results and discussion section includes the findings of the research. Lastly, the summary of research and future steps are presented in the conclusion section.

2. Literature Review

Literature review section is organized into three areas: (1) transit fleet improvement options, (2) quantitative measures for fleet management, (3) multi-objective optimization applications for transit. The review is not intended to be exhaustive, but to highlight some of the general trends in addressing the allocation problem.

<u>Transit Fleet Improvement Options</u>

Transit improvement options drew significant attention in the 1980s, and received renewed research interest in the late 1990s. A number of studies found replacement, rehabilitation, and remanufacturing are the preferred options (*6-9*). The literature review clearly showed that remanufacturing and rehabilitation of buses, if done properly, can be a cost-effective option. The studies mentioned above stressed the importance of proper preventive maintenance as a primary factor contributing to the success of rehabilitation programs. These studies emphasized that rehabilitation, "if properly done," can be a successful strategy, clearly referring to the quality of maintenance and steps taken by the agency to prevent major breakdowns in machine components or bus body infrastructure. For the purpose of this paper, the following terms are adapted from the literature.

i. Replacement (REPLACE): Process of retiring an existing vehicle and procuring a completely new vehicle. Vehicles replaced using federal dollars must have completed their MNSL requirements.

ii. Rehabilitation (REHAB): Process by which an existing vehicle is rebuilt to the original manufacturer's specification, with primary focus on the vehicle interior and mechanical system.

iii. Remanufacturing (REMANF): Process by which the structural integrity of the vehicle is restored to original design standards. This includes remanufacturing the body, the chassis, the drive train, and the vehicle interior and mechanical system.

Note: The generic term 'REBUILD' has been used in this paper to mean Rehabilitation and or Remanufacturing.

<u>Quantitative Measures for Fleet Management</u>

Service life maximization is the most commonly adopted measure of effectiveness (MOE) for the resource allocation in conjunction with budgetary constraints. Some prominent bus maintenance management studies include: bus maintenance programs for cost-effective reliable transit service (*10-11*), a generalized framework for transit bus maintenance operation (*12*), manpower allocation for transit bus maintenance program (*13*), framework for evaluating a transit agency's maintenance program (*14*), a simulation model for comparing a bus maintenance system's performance under various repair policies (*15*), and performance indicators for maintenance management (*16*). These problems primarily cater to an operator, who is concerned with the day to day maintenance for an efficient fleet operation.

Another commonly adopted MOE for transit resource allocation is in terms of monetary units. The performance measures frequently used in literature are maximization of revenue, return, or profits, benefit to cost ratio, internal rate of return, pay off period, cost effectiveness (*17-23*). NPC is a widely understood and used MOE in transportation decision making. Minimization of NPC has been used as a MOE for evaluation of transit level of service (*24*); for evaluation of rail transit investment priorities (*25*); for finding the optimal bus transit service coverage in an urban corridor (*26*); for modeling the timing of public infrastructure projects (*27*); for a decision support tool for evaluating investments in transit systems fare collection(*28*); for analyzing induced demand with introduction of new transit system (*29*); for analyzing transportation impacts on economic development (*30*); for analyzing externalities associated with light rail investment (*31*); for resource allocation among transit agencies(*3-4*); and for project selection problem under uncertainty (*32*).

Multi-Objective Optimization in Transit Application

Research on multi-objective optimization application for transit has been limited in the literature. Lianbo et al. (*33*) studied the train planner for urban rail transit; they proposed a multi-objective optimization model for train formation, train counts as well as operation periods considering factors such as transport capacity, the requirements of traffic organization, corporation benefits, passenger demands, and passenger choice behavior under multi-train-routing mode. Desai et al. (*34*) studied the fleet management for electric vehicles by minimizing fuel economy and pollutants (HC; CO; NOx); Mauttone and Urquhart (*35*) analyzed a MO transit network design problem and obtained non-dominated solutions representing different trade-off levels between the conflicting objectives of users and operators. Wu et al. (*36*) used MO to obtain optimal transit stops under American Disabilities Act (ADA). Among the solution methodologies, Genetic Algorithm (GA) is extensively used to model multi-objective problems. However, in this study we will be using branch and bound analysis (BBA) and not genetic algorithm (GA), as Mathew et. al. (2010) reports that GA provides inferior solution compared to BBA in solving transit resource allocation problems *(4)*.

3. Motivation

While some studies described in literature made a contribution towards formulating and solving the transit resource/fleet allocation problem and others applied MO in transit, to the best of authors knowledge the above mentioned objectives of transit fleet resource allocation have never been investigated simultaneously. Most of the similar studies in literature have following limitations:

- In literature only fleet quality measure (TWARL or TSWARL) has been maximized without considering any economic measure (like Net Present Cost (NPC)) other than available budget.
- A key gap in literature has been in terms of inability to control NPC. The previous studies have only considered NPC as a byproduct of the process of maximizing TSWARL. In practice, while considering the investment for fleet improvement, state DOTs are concerned about economic measures such as NPC for spending in a multi-year planning period, that may become a critical factor in the final decision making process. Thus, minimizing NPC rather obtaining it as a byproduct would have more significant implication to the transit agency.
- In literature there is a lack of understanding of relationship between both performance measures TSWARL and NPC. Further, the possible relationship between different improvement options such as REHAB, REMANF, and REPLACE and both measures (TSWARL and NPC) has not been studied explicitly. This paper investigates the correlation between each option and performance measures. Moreover, in this paper we also show that in a multi-year planning period how TWARL of a single year is related to either and both performance measures.

4. Methodology

As mentioned earlier, in this study we formulate a multi-objective optimization problem with the first objective of minimization of NPC of the total investment of the fleet for all the agencies over the entire planning period and the second objective of maximization of TSWARL, subjected to budget, demand, rebuild, and non-negativity constraints. For ease in understanding the complete formulation there is a simple explanation provided in front of each equation. The formulation notations are given below followed by the formulation and their explanation:

Variables		Explanation
b_m	:	budget available for m^{th} planning year
c_k^m	:	cost of implementation of the improvement program *k* on m^{th} year
$f_{i,j}^m$	:	Number of buses for an agency *i* with remaining life of *j* years on m^{th} planning year
l_k	:	additional year added to the life of the bus due to improvement program *k*, $l_k \in \{2,3,4,7\}$
r_{ij}^m	:	number of existing buses with remaining life of *j* years for an agency *i* on m^{th} planning year
x_{ij}^m	:	number of buses which received remaining life of *j* years for an agency *i* on m^{th} planning year due to the improvement program
y_{ik}^m	:	number of buses chosen for the improvement program *k* adopted for an agency *i* on m^{th} planning year
$\delta_{i,(\alpha,\beta)}^m$	:	number of buses already improved by α, βyears due to rehabilitation in the m^{th} planning year for agency i, $(\alpha, \beta \in \{2,3\})$
$\delta_{i,(\gamma)}^m$	:	number of buses already improved by γyears due to remanufacture in the m^{th} planning year for agency i, $(\gamma \in \{4\})$
$\emptyset$		The interest rate used for NPC
A	:	total number of agencies
B	:	total budget available for the project for all planning years
i	:	1, 2, …,A, the subscript for a transit agency
j	:	1, 2, …,Y, the subscript for remaining life
k	:	1,2,…., P the subscript used for improvement program
m	:	1, 2, …,N, the subscript used planning year
N	:	number of years in the planning period
P	:	number of improvement programs
REHAB1	:	the first improvement program- rehabilitation of bus yielding α *(=2)* additional years
REHAB2	:	the second improvement program- rehabilitation of bus yielding β *(=3)* additional years
REMANF	:	the third improvement program- rehabilitation of bus yielding γ (=4)additional years
REPLACE	:	the last improvement program-replacement of bus yielding 7 additional years
TSWARL	:	Total System Weighted Average Remaining Life , $TSWARL = \sum_m TWARL$
TWARL	:	Total Weighted Average Remaining Life=$TWARL = \sum_i WARL_i$
$WARL_i$	:	Weighted Average Remaining Life for agency *i*=$WARL_i = \frac{\sum_j f_{ij}^m r_{ij}^m}{\sum_j f_{ij}^m}$
Y	:	minimum service life of buses
Z_x	:	The objective function as minimization of NPV for the resource allocation in the planning period
ρ	:	Weight factor $0 \leq \rho \leq 1$

Mathematical Construct	Explanation	Eq.#
Minimize $Z = (1-\rho) * \bar{Z}_x^1 - \rho * \bar{Z}_x^2; \ \forall\ 0 \le \rho \le 1$	*Overall Objective*: weighted sum of normalized net present cost (NPC) and sum of normalized total system weighted average remaining life of the fleet (TSWARL). As we are minimizing NPC and maximizing TSWARL, the value of the objective function TSWARL is to be negative to represent the overall problem as minimization problem. The weight (ρ) considered here is between 0 and 1.	(4)
$Z_x^1 = \sum_{m=1}^{N} \sum_{i=1}^{A} \sum_{k=1}^{P} \frac{x_{i,k}^m * c_k^m}{(1+\emptyset)^m}$	*Objective function 1* : net present cost of the transit fleet resource allocation (NPC)	(5)
$Z_x^2 = \sum_{m=1}^{N} \sum_{i=1}^{A} \frac{\sum_{j=1}^{J}(r_{i,j}^m + x_{i,j}^m) * j}{\sum_{j=1}^{J}(r_{i,j}^m + x_{i,j}^m)}$	*Objective function 2 :* sum total of the weighted average remaining life of the fleet of all the constituent agencies for the whole planning period (TSWARL)	(6)
Subjected to following constraints		
$\sum_{m=1}^{N} \sum_{i=1}^{A} \sum_{k=1}^{P} y_{ik}^m * c_k^m < \sum_{m=1}^{N} b_m, \forall m$	*Constraint*: Total cost of improving the buses for different improving schemes, agencies and over a planning period should not exceed budget for the planning period	(7)
$\sum_{m=1}^{N} b_m = B$	*Constraint*: Planning period budget is equal to the sum available budget for each year, where budget is *a priori*.	(8)
$\sum_{k=1}^{P} y_{ik}^m = r_{ij}^m, \forall i, m, j$	*Constraint*: The buses that are improved under improvement scheme *k* are the ones that have completed their minimum normal service life and have remaining life *j*	(9)
$y_{i\gamma}^m = \sum_{\alpha\beta} \delta_{i,(\alpha\beta)}^m + \delta_{i,(\gamma)}^m \forall i, m, j$	*Constraint*: All the buses that have completed their minimum normal service life	(10)
$y_{i\gamma}^m = \sum_{\alpha\beta} \delta_{i,(\alpha\beta)}^m + \delta_{i,(\gamma)}^m \forall i, m, j$ $\forall \alpha, \beta \in \{2,3\}, \gamma\{4\}$	*Constraint*: The buses that have been rehabilitated twice or remanufactured once will be replaced	(11)
$y_{i\gamma}^m > 0$	*Constraint*: Non-negativity constraint. Number of buses chosen for improvement should be greater than 0	(12)
$x_{ij}^m = \begin{cases} y_{ik}^m, if j = l_k, l_k \in \{2,3,4,7\} \\ 0, \quad Otherwise \end{cases}$	*Constraint*: The life of the buses is improved by either two, three, or four years for a re-built bus and by seven years for a new bus	(13)
$\delta_{i,(\alpha\beta)}^m = \begin{cases} min\left\{y_{i\alpha}^{m-(\alpha+\beta)}, y_{i\beta}^{m-\alpha}\right\} if m \ge \alpha+\beta, \forall m > \alpha+\beta, \\ 0, \quad Otherwise \end{cases}$	*Constraint*: Auxiliary constraint of Eq. (7), represents replacement option after $\alpha + \beta$ years (REHAB)	(14)

$\delta_{i,(\gamma)}^{m} = \begin{cases} y_{i\alpha}^{m-\gamma}, \forall m > \gamma, \\ 0\,, \; Otherwise \end{cases}$	*Constraint*: Auxiliary constraint of Eq. (7), represents replacement option after γ years (REMANF)	(15)

The overall objective function is shown in equation (4), is a weighted sum of normalized NPC and TSWARL. Since we are minimizing NPC and maximizing TSWARL, the value of the objective function TSWARL is negative to represent the overall problem as minimization problem. The weight (ρ) considered here is between 0 and 1. The objective functions shown in equations (5) and (6) represent NPC and the TSWARL respectively of the transit fleet resource allocation. The decision variable x_{ij}^{m} is defined in equation (13) with the help of an auxiliary variable y_{ik}^{m}. This definitional constraint in equation (13) ensures that the life of the buses is improved by either two, three, or four years for a re-built bus and by seven years for a new bus. Other buses in the system will have no additional years added. The constraint (7) represents the sum total of the weighted average remaining life of the fleet of all the constituent agencies for the whole planning period, designated as TSWARL, which is determined previously. The choice of TSWARL is defined by the user. A lower value of TSWARL suggests low cost improvement options are chosen, and vice versa. Equation (8) represents the constraint of a fixed budget for the seven-year planning horizon with the planner having the budget flexibility across the years. Equation (9) represents the planning period budget being equal to the sum available budget for each year. Equation (10) ensures that all the buses that have completed their Minimum Normal Service Life (MNSL) requirements will be eligible for improvement as per Federal Highway Administration (FHWA) standards. MNSL can be defined as the number of years or miles of service that the vehicle must provide before it "qualifies" for federal funds for rehabilitation, remanufacturing and replacement.

Equation (11) represents policy constraints which ensure that the buses that have been rehabilitated twice or remanufactured once will be replaced. The two terms in this constraint are defined in equations (14) and (15). These three constraints are specific to the case study presented in this paper, and can be revised at the discretion of the user. Thus, equations (11) and (14) ensure that a bus that was rebuilt twice (each time its life is increased by α or β years is replaced. This policy is applicable only after $\alpha + \beta$ years. Similarly, a bus that is remanufactured resulting in an increase in life by γ years must be replaced (equations 11 and 15) and is applicable only after γ years. This constraint presented in equations (11, 14, and 15) is specific to the case study presented in this paper, and can be revised at the discretion of the user. Equation (14) is a non-negativity constraint to ensure that the number of buses chosen for improvement is never negative. The formulation involves non-linear functions, non-differentiable functions, step functions, and integer variables. Although the step function can be generalized to linear forms, the formulation will require additional variables which may result in variable explosion rendering the model unsuitable for large/real world problems.

5. Solution Approach

The solution methodology is presented in Figure 1.The first step is to initiate the solver to read the input and look up for the binary variable indices with lower bound to 0 and upper bound to 1 for each binary decision variable. Please note that the objective function consists of both TSWARL and NPC in the proposed transit fleet resource allocation model. An important consideration needs to be given to the overall objective function which cannot be a direct sum of both the objectives as it is possible that magnitude of one objective may be very high compared to other. In classical weighted some approach this is overcome by normalizing each objective function. The normalized objective function can be determined by obtaining expectation of each objective function value divided by the expected value of the objective function. The next step is to construct one empty node and create a tree by setting an initial value of objective function. In the tree we try to solve for a node by setting the binary variable bounds, and fix binary value according to the two vectors in the node. The binary variables 0 or 1 for the four improvement options REHAB1, REHAB2, REMANF, and REPLACE are considered in the optimization problem. In the optimization problem all the constraints described in equation (6) through (15) are to be considered. The best value of the objective function is to be estimated considering initial weight starting from 0.

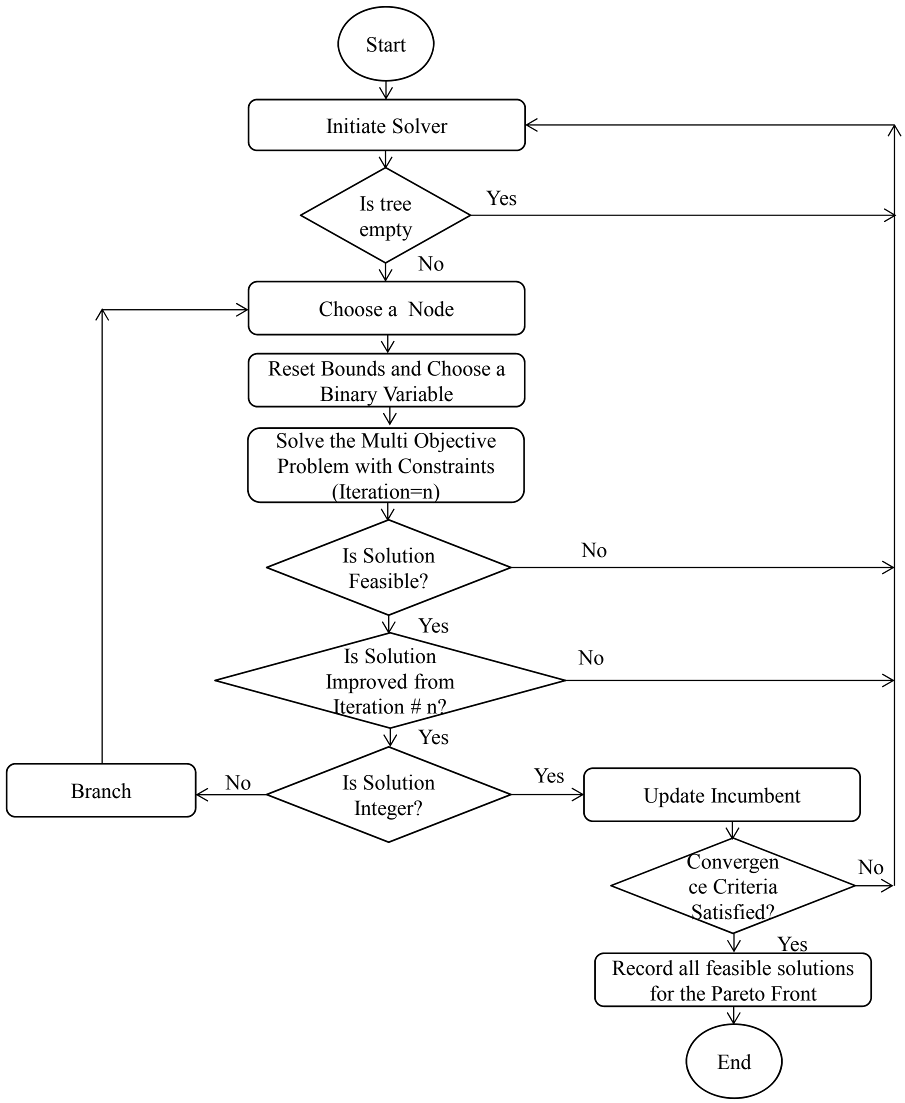


FIGURE 1: Solution Algorithm for BBA Based Multi-Objective Transit Fleet Resource Allocation Problem

In the first optimization results, the solution algorithm checks if the current result is feasible and satisfying all constraints with reasonable value of the objective function. Next, it compares the resulted objective value with the current best and check if all the binary variables are 1 or 0. Solution algorithm updates the current best objective value, i.e. if newly obtained objective is better than the current, then set current to the new one, otherwise keep the incumbent. This process is repeated up to a change of objective function value between iterations achieves a precision of 10^{-6} for all weights. Summarize all results and draw the pareto front to visualize the multi objective optimization results.

6. Data

A Public Transportation Management System (PTMS) database developed by Michigan Department of Transportation (MDOT) containing actual fleet data is used for the case study demonstration. The distribution of the Remaining Life (RL) in years of the fleet for a few of the 93 agencies for the base year (2002) is shown in Table 1. Only a fraction of the table is presented for brevity. Table 1 shows the distribution of fleet size by their remaining life (RL) for each agency. For example, agency 1 has one bus with zero years of RL, 2 buses with seven years of RL and so on, for a total fleet size of 3.The last row of the table shows that the total fleet is of 720 buses, of which 235 buses have zero years of RL, and need replacement. The last column of the Table 1 gives the weighted average remaining life (WARLi) for each agency, computed from the distribution of RL for the agency. For example, the WARLi of the first agency is calculated as (0x1+1x0+…+7x2)/3 =4.67. The base year total weighted average remaining life of the entire fleet (TWARL) is 225.23 years. The following improvement options are used in the case study:

- Replacement (REPLACE)—process of retiring an existing vehicle and procuring a completely new vehicle. Buses proposed to be replaced using federal dollars are expected to be at the end of their MNSLs, as described above. (Life expectancy: seven years)
- Rehabilitation (REHAB)—process by which an existing bus is rebuilt to the original manufacturer's specification. The focus of rehabilitation is on the vehicle interior and mechanical systems, including rebuilding engines, transmission, brakes, and so on. Two types of rehabilitation: REHAB1 and REHAB2 with moderate to higher levels of engine rebuilds are considered in this study (Life expectancy: 2 to 3 years)
- Remanufacturing (REMANF)—process by which the structural integrity of the bus is restored to original design standards. This includes remanufacturing the bus chassis as well as the drivetrain, suspension system, steering components, engine, transmission, and differential with new and manufactured components and a new bus body. (Life Expectancy: 4 years)
- Further, it was assumed that a vehicle may be rehabilitated (REHAB1 or REHAB2) only up to two consecutive terms, and then must be replaced (REPL) with a new bus. A vehicle with REHAB1 and REHAB2 (or vice versa) in two consecutive terms also should be replaced. A vehicle may be remanufactured (REMANF) only one time, and then must be replaced (REPL) with a new bus. A vehicle rehabilitated (REHAB1 and REHAB2) once can be eligible for remanufacturing (REMANF) before it is replaced (REPLACE).

TABLE 1 Base year distribution of remaining life (RL), fleet size, and weighted average of remaining life of sample agencies before allocation of resources for the case study

Agency	Distribution of Remaining Life								Total Fleet Size	WARLi (years)
	0	1	2	3	4	5	6	7		
1	1	0	0	0	0	0	0	2	3	4.67
2	1	0	0	0	0	0	0	0	1	0
.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.
92	2	1	0	0	1	1	0	3	8	3.88
93	2	0	0	0	1	3	1	0	7	3.57
Total	235	122	44	23	63	77	78	78	720	225.23

7. Case Study Problem

The budgets available for each year and the unit cost for each improvement options for each year are shown in Table 2. A seven year planning period is considered conforming to the MNSL requirement of medium sized buses. Replacing all the 235 buses with zero years of RL (last row and second column of Table 1) would require $19,161,900 (235 x $81,540) of investment which exceeds the first year budget. Similarly, in the second year, 122 buses which had one year of RL in the base year will qualify for improvement.

TABLE 2 Available Budget and Cost of Improvement Options

Year	Budget	Improvement Options and Costs			
		REPLACE (X1= 7Years)	REHAB1 (X2= 2 Years)	REHAB2 (X3=3 Years)	REMANF (X4= 4 Years)
2002	5,789,000	81,540	17,800	24,500	30,320
2003	9,130,000	81,540	17,800	24,500	30,320
2004	6,690,000	88,063	19,220	26,400	32,750
2005	5,200,000	88,063	19,220	26,400	32,750
2006	6,750,000	95,108	20,740	28,500	35,370
2007	6,600,000	95,108	20,740	28,500	35,370
2008	5,850,000	102,720	22,400	30,780	38,200
2009	6,680,000	102,720	22,400	30,780	38,200
Total	**52,889,000**				

Replacing all these buses with remaining life 1 year would require $9,947,880 (122x$81,540), which also exceeds the second year budget and so on for other years. Moreover, if the replacement process is continued from year 2002 through 2009, when the buses reach their MNSL, it will cost $88,488,688 (i.e. 235*81,540+122*81,540+……+235*102,720) to maintain the fleet size of 720 buses throughout the planning period. However the total available budget is only $52,889,000 (Table 2). Therefore, there is a need for a mechanism to identify improvement options for each agency, so that the NPC is minimized with a user defined TSWARL. The case study problem is solved using Premium Solver Platform *(37-38)*.

8. Results and Discussion

The results from the proposed model are illustrated in the Table 3 . We can see the values for a set of weights ranging from 0 to one (the value of ρ) for each objective function. If the weight is 0 the formulation is equal to single objective minimization of NPC as an objective function; whereas, if the weight is 1 it represents a case of maximization of TSWARL as the only objective. The weights in the case study were choosen to represent as many possible points in the complete the solution space, hence 60 weights were generated between 0 and 1. Only a total of seven points (including the two extreme points) are shown in Table 3.

TABLE 3 Best pareto optimal solutions along with their weights

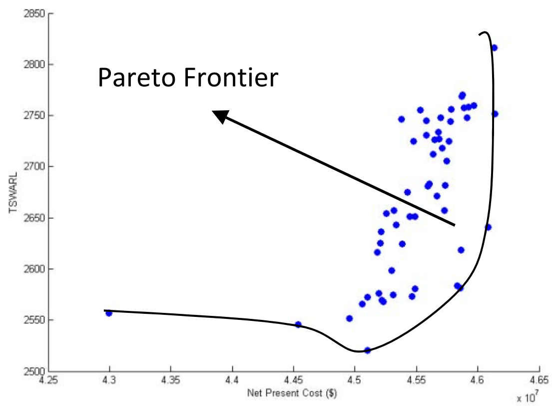
REPLACE (X1) (7 Years)	REHAB1 (X2) (2 Years)	REHAB 2(X3) (3 Years)	REMANF (X4) (4 Years)	TSWARL	NPC ($)	Weight (ρ)
424	391	77	63	2556.42	42,991,614	0.00
404	371	140	172	2520.38	45,102,062	0.03
401	356	141	164	2545.40	44,536,264	0.22
384	410	61	272	2636.52	45,213,129	0.27
452	368	41	144	2770.41	45,877,804	0.64
445	296	73	179	2768.73	45,868,090	0.92
456	321	23	183	2815.82	46,133,613	1.00

Table 3 list the best pareto optimal solution along with the number of buses allocated for each improvement option, value of objectives TSWARL and NPC and the corresponding weights. It should be noted due to space constraints we are not listing all the 60 solutions rather randomly picked solutions and extreme solutions. Table 3 shows the extereme solutions are the best solutions for each objective. However the least value of NPC objective ($42,991,614) can also be attributed to large surplus, i.e. the budget is not being fully utilized and hence a total surplus of $4,670,867 (see Table 4) leads to this value. Although the budget constraint keeps the amount committed for improvements below budget, but does not limit the minimum amount to be spent leading to large surplus in this case. The algorithm makes use of this feature leading to high surplus but at the same time minimum NPC. The Table 4 lists the year-wise allocation of resources for improvement, the two solutions listed are extreme, one for weight (ρ) = 0 and another for weight (ρ) =1. Each row shows yearly allocation of buses for each improvement option and subsequently money for those improvements in that year. All of the 60 solutions of the proposed model contain this information.

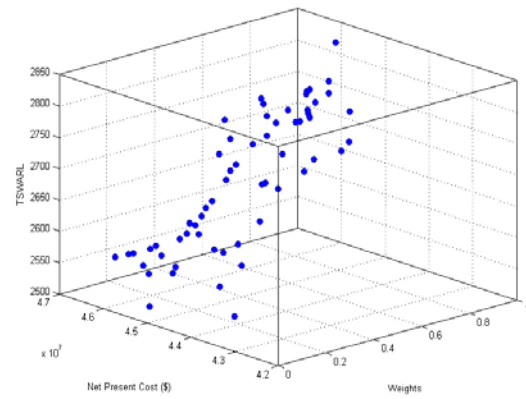
TABLE 4 Complete solutions of the proposed model at extreme weights (ρ = 0; ρ = 1)

Year	REPLACE (X1) (7 Years)	REHAB1 (X2) (2 Years)	REHAB2 (X3) (3 Years)	REMANF (X4) (4 Years)	Total Number of Buses	TWARL	NPC	Amount Committed	Surplus
	(No. of Buses)					(years)	($)	($)	($)
ρ = 0									
2002	221	14	0	0	235	500.88	18,26,9540	18,269,540	-12,480,540
2003	122	0	0	0	122	483.19	9,384,792	9,947,880	-81,7880
2004	58	0	0	0	58	421.96	4,545,794	5,107,654	1,582,346
2005	23	0	0	0	23	344.95	1,700,606	2,025,449	3,174,551
2006	0	0	0	63	63	287.14	1,765,030	2,228,310	4,521,690
2007	0	0	77	0	77	221.42	1,639,858	2,194,500	4,405,500
2008	0	78	0	0	78	146.74	1,231,707	1,747,200	4,102,800
2009	0	299	0	0	299	150.47	4,454,287	6,697,600	182,400
Total	**424**	**391**	**77**	**63**	**955**	**2556.74**	**42,991,614**	**48,218,133**	**4,670,867**
ρ = 1									
2002	235	0	0	0	235	501.61	19,161,900	19,161,900	-13,372,900
2003	122	0	0	0	122	483.93	9,384,792	9,947,880	-817,880
2004	22	22	0	0	44	416.51	2,100,593	2,360,226	4,329,774
2005	17	6	0	0	23	337.83	1,353,795	1,612,391	3,587,609
2006	26	0	0	59	85	302.01	3,611,660	4,559,638	2,190,362
2007	10	0	23	50	83	255.17	2,522,056	3,375,080	3,224,920
2008	10	48	0	20	78	204.22	2,020,699	2,866,400	2,983,600
2009	14	245	0	54	313	314.55	5,978,119	8,988,880	-2,108,880
Total	**456**	**321**	**23**	**183**	**983**	**2815.83**	**46,133,614**	**52,872,395**	**16,605**

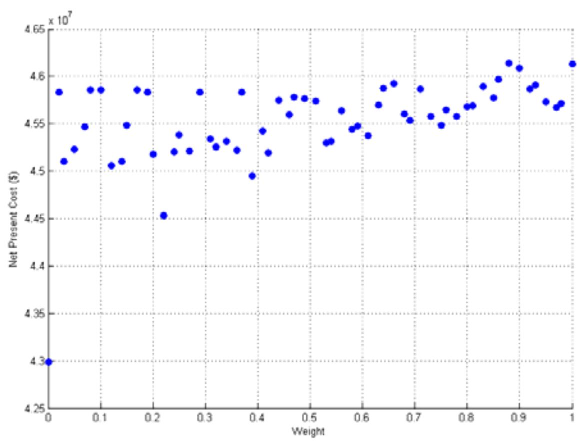
The Figure 2(a) is an illustration of the final pareto frontier solutions. We can observe diversity of solutions across the region and a pareto frontier representive quality of solutions. One can observe none of the points in the figure is the best solution representing the minimum value of NPC and the maximum value of TSWARL, these solutions are pareto optimal solutions. The Figure 2(b) illustrates all the solutions at different weights and their corresponding value of TSWARL and NPC in monetary terms, a nice spread of solutions confirms better solution quality. Figure 2(c) shows that minimum value of NPC can be achieved at weight 0, which is intuitive as at ρ=0, problem becomes single objective minimization of NPC. The value of NPC at weight 0 is so low, it may seem like outlier. The reason for this is the amount committed for improvements is very low and a reasonable amount of surplus money stays available for this particular weight (as seen in Table 2, last column). Similarly in Figure 2(d) we can observe highest value of TSWARL that we try to achieve in the problem (ρ=1) and nice spread of solutions at different weights.



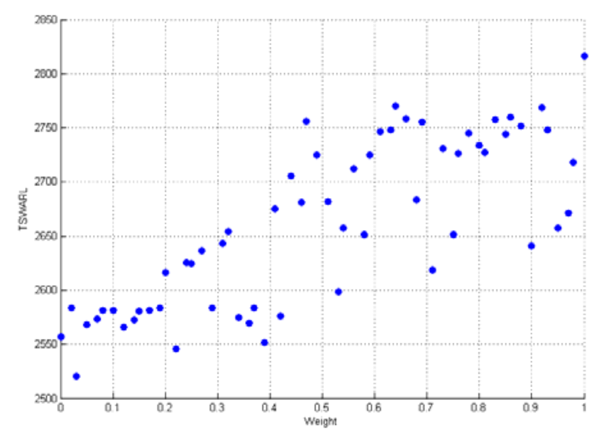
(a) Pareto Front and Pareto Optimal Solutions



(b) Plot of TSWARL, Net Present Cost and weights



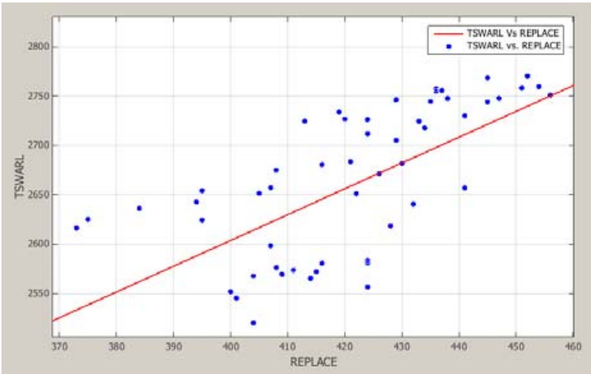
(c) Objective Net Present Cost Values at different weights



(d) Objective TSWARL values at different weights

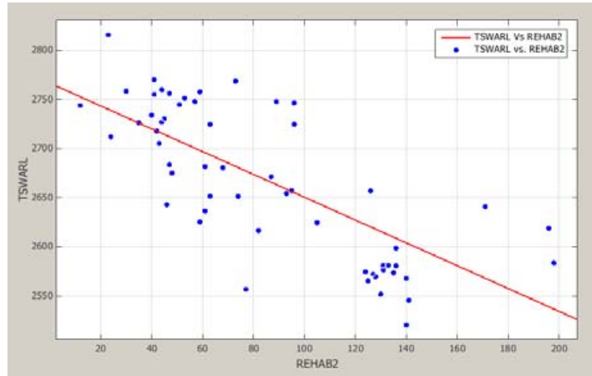
FIGURE 2 Value of each objective function with respect to the weights

The result presented in Table 4 leads to interesting questions about possible correlation between objectives (TSWARL and NPC) and number of buses improved under each option and TWARL. Relationship between the objective function and decision variables (buses under each improvement options) are shown in Figure 3. It may be noted that the intent of the figures below is to understand the relationship between TSWARL and NPC with different objective functions and not to forecast or predict the results. The results shown in Figure 3 are not intended to be used as a substitute for optimization in resource allocation problems rather has been shown to explore correlation.



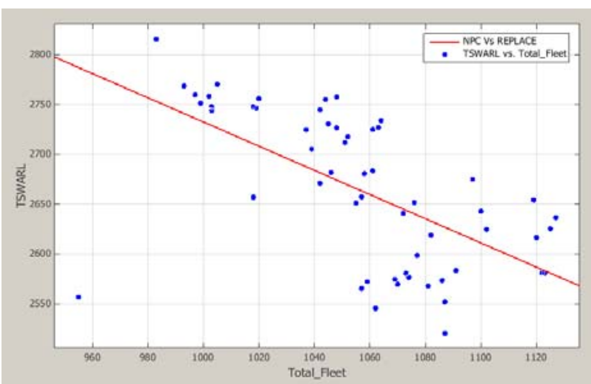
Goodness of fit: SSE: 2.061e+005 **<u>R-square: 0.4087</u>**
Coefficients:
p1 = 49.15
p2 = 2660

(a) Linear Relationship between TSWARL and Fleet for Replacement (REPLACE) option in the planning period



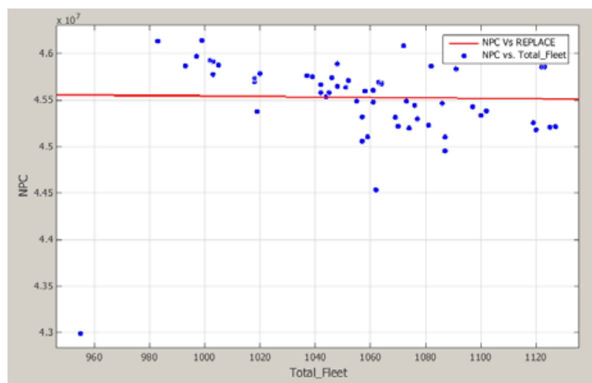
Goodness of fit: SSE: 1.451e+005 **<u>R-square: 0.583</u>**
Coefficients:
p1 = -58.73
p2 = 2660

(b) Linear Relationship between TSWARL and Fleet for Rehabilitation (REHAB2) option in the planning period



Goodness of fit: SSE: 2.129e+005 **<u>R-square: 0.3895</u>**
Coefficients:
p1 =-47.97
p2 = 2660

(c) Linear Relationship between TSWARL and Total Fleet selected for all improvements in the planning period

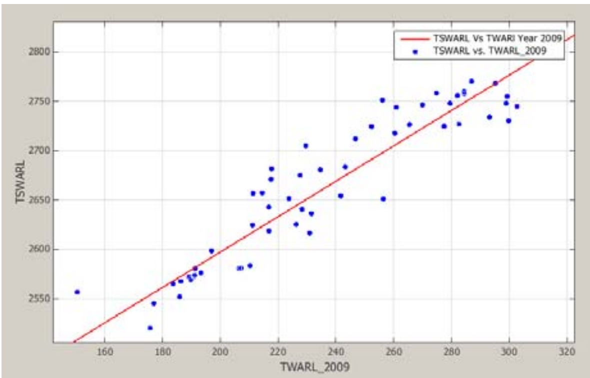


Goodness of fit: SSE: 1.249e+013 **<u>R-square: 0.00047</u>**
Coefficients:
p1 = -1.005e+004
p2 = 4.553e+007

(d) Linear Relationship between NPC and Total Fleet selected for all improvements in the planning period

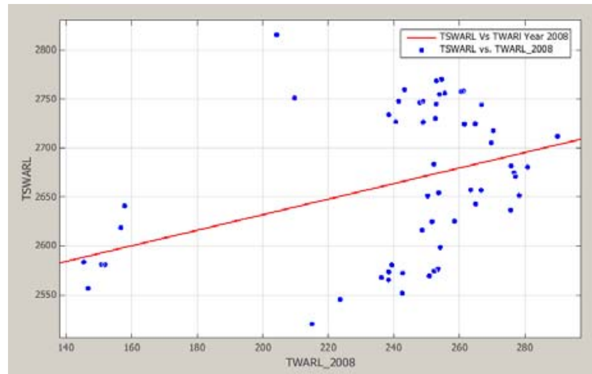
FIGURE 3 Relationship of improvement options with objective functions during planning period for all generated solutions in a linear polynomial form as f(x) = p1*x + p2

As can be observed from Figure 3, only a linear relation between TSWARL and REHAB2 option has acceptable R-square value. Further, it is an inverse relationship, i.e. the higher the number of buses in REHAB2, the lower the value of TSWARL. The relationship between TSWARL and REPLACE option is a positive one, but it does not show a strong correlation. Similarly, there is a weak correlation between TSWARL and Total Fleet for improvement, NPC, and REPLACE option (Figure 3 (d)). Other comparisons are not listed, as the relationship betweeen decision variables and objective functions is weak. It can be inferred that TSWARL is inversely impacted by buses going for rehabilitation for 3 years and that is the reason we see very small number of buses allocated for rehabilitation (REHAB2) option in the Table 4. In fact the key to maximizing TSWARL is to rehabilitate the least number of buses in the fifth year. This further leads to question of exploring each year improvements along with the objective function values. The Figure 4 (a-d) illustrates some insights in that direction.



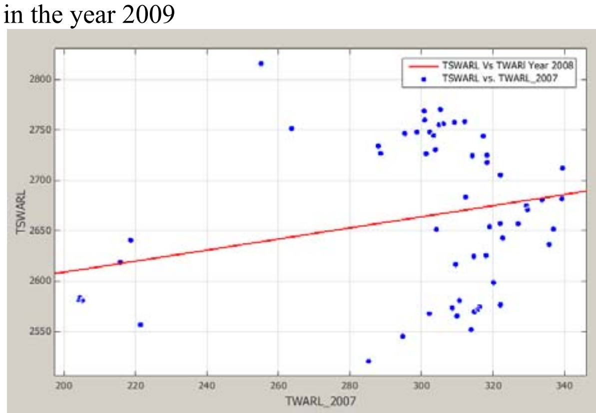
Goodness of fit: SSE: 4.234e+004 **R-square: 0.8785**
Coefficients:
p1 = 72.05
p2 = 2660

a) Linear correlation between TSWARL and TWARL in the year 2009



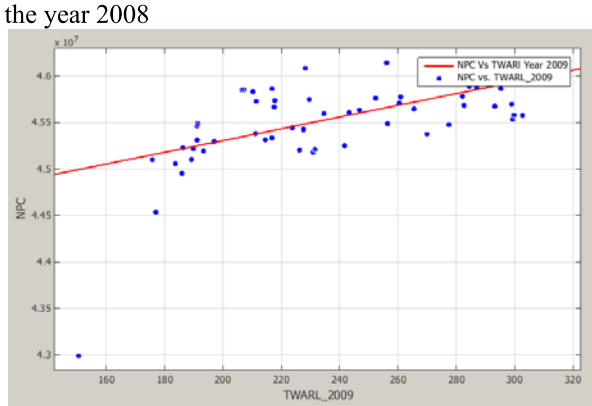
Goodness of fit: SSE: 2.83e+005 **R-square: 0.1884**
Coefficients:
p1 =33.37
p2 =2660

b) Linear correlation between TSWARL and TWARL in the year 2008



Goodness of fit: SSE: 3.181e+005 **R-square: 0.0876**
Coefficients:
p1 = 22.76
p2 = 2660

c) Linear correlation between TSWARL and TWARL in the year 2007



Goodness of fit: SSE: 8.701e+012 **R-square: 0.3039**
Coefficients:
p1 = 2.537e+005
p2 = 4.553e+007

d) Linear correlation between NPC and TWARL in the year 2009

FIGURE 4 Relationship of TWARL for year 2009, 2008, 2007 with objective function values in a linear polynomial form as f(x) = p1*x + p2

In the Figure 4(a), it can be observed that there is a strong correlation between TWARL for the year 2009 and TSWARL. This implies that the fleet chosen for the improvement in the last year of planning period significantly contributes to the high value of TSWARL. However, it should be observed that the similar relationship does not hold for TWARL values for the years 2007, 2008, years (Figures 4b and 4c) and for other years 2002-2006. Further, it is seen that the TWARL values do not have a significant effect on the value of Net Present Cost.

Budget Sensitivity

To understand the budget sensitivity, multiple runs with different weights and lower budgets were performed. It was observed that solver fails to obtain a solution without violating the budget constraints below 13% of the original budget value \$52,889,000. Therefore, in the Table 5, we present solutions obtained at lower budget value \$46,013,430 (13% reduction of \$52,889,000) and the weights of ρ = 0, 0.5 and 1. It can be observed (Table 5) that at a lower budget, the algorithm chooses lower cost, and medium age improvement options (REHAB2 and REMANF) to achieve the minimum NPC value. At the original budget and for similar weight, the algorithm prefers extreme improvement options (REHAB1 and REPLACE) with higher cost and longer life (better quality). Another interesting observation is for ρ = 1 (Maximization of TSWARL), the total number of buses for improvement between year 2002-2005 remains the same at both the budget levels (Table 4 and 5). The difference starts to appear after year 2006 where the model tries to adjust for the budget.

9. Synthesis of Results

The multi objective optimization approach presented for the transit resource allocation resulted in a pareto optimial solutions demostrating trade off between NPC and TSWARL. The optimization results show that appropriate improvement options can be choosen to achieve a specific objective function. The relationship between NPC and TSWARL is non-linear in nature because of the incorporation of the interest factors in computing NPC. When NPC is compared with individual year quality measure (TWARL), it is observed that initially, TWARL remains relatively constant with increase in NPC up to a certain point, beyond which TWARL increases in the later years. In all the solutions, a relationship between replacement (REPLACE) option and rehabilitation option (REHAB2) with TSWARL has been consistently observed. However, the relationship between TSWARL and REHAB2 is inversely proportional but strongly correlated. This represents the fleet size chosen for this improvement governs the overall objective of TSWARL.

The relationship between TSWARL and TWARL in the year 2009 is very strongly correlated compared to any other relationship between decision variables and objectives. Thus suggesting that the last year's total expected weighted remaining life plays an important role in maximizing the TSWARL objective. A limitation of the formulation is that minimum NPC can be achieved by investing relatively less in the improvements and obtaining alarge surplus (reducing expenditure from budget) as the constraint is to spend less than a particular budget value. This can be overcome by adding a constraint on minimum spending amount. A sensitivity analysis for a lower budget shows efficacy of the model to work at 13 percent lower than original budget and obtain results. A comparison between low budget and exact budget cases show variation in fleet selection for each improvement option under different budget levels.

Computational Effort

The average computational time to solve this problem for a single weight using the PSP solver platform *(37;38)* is two minutes in a Windows 7 64 bit operating system, on i7 Quad Core Processor and 6 GB RAM. The overall time taken to obtain the all the 60 solutions is approximately two hours.

TABLE 5 Complete solutions of the proposed model at lower budget and weights (ρ=0; ρ=0.5; ρ=1)

Year	REPLACE (X1) (7 Years)	REHAB1 (X2) (2 Years)	REHAB2 (X3) (3 Years)	REMANF (X4) (4 Years)	Total Number of Buses	TWARL	NPC	Amount Committed	Surplus
(No. of Buses)						(years)	($)	($)	($)
ρ = 0 (Budget=$46,013,430)									
2002	166	62	7	0	235	450.28	14810740	14810740	-9021740
2003	40	0	82	0	122	418.47	4972264	5270600	3859400
2004	86	2	18	0	106	388.92	7197453	8087058	-1397058
2005	12	11	7	0	30	345.58	1219947	1452976	3747024
2006	1	0	0	146	147	305.32	4165722	5259128	1490872
2007	1	0	102	3	106	251.78	2322641	3108218	3491782
2008	0	85	0	0	85	178.42	1342245	1904000	3946000
2009	0	243	0	1	244	161.52	3645444	5481400	1398600
Total	**306**	**403**	**216**	**150**	**1075**	**2500.29**	**39,676,456**	**45,374,120**	**7,514,880**
ρ = 0.5 (Budget=$46,013,430)									
2002	235	0	0	0	235	501.61	19161900	19161900	-1.3E+07
2003	122	0	0	0	122	483.93	9384792	9947880	-817880
2004	44	0	0	0	44	421.66	3448533	3874772	2815228
2005	15	0	8	0	23	342.43	1286418	1532145	3667855
2006	0	0	0	63	63	284.62	1765030	2228310	4521690
2007	0	0	77	0	77	218.90	1639858	2194500	4405500
2008	0	86	0	0	86	145.33	1358036	1926400	3923600
2009	0	230	0	0	230	115.92	3426374	5152000	1728000
Total	**416**	**316**	**85**	**63**	**880**	**2514.41**	**41,470,943**	**46,017,907**	**6,871,093**
ρ = 1 (Budget=$46,013,430)									
2002	235	0	0	0	235	501.61	19161900	19161900	-1.3E+07
2003	122	0	0	0	122	483.93	9384792	9947880	-817880
2004	44	0	0	0	44	421.66	3448533	3874772	2815228
2005	22	0	1	0	23	344.15	1648833	1963786	3236214
2006	0	0	0	63	63	286.35	1765030	2228310	4521690
2007	0	0	77	0	77	220.63	1639858	2194500	4405500
2008	0	79	0	0	79	146.20	1247498	1769600	4080400
2009	0	265	0	0	265	135.16	3947779	5936000	944000
Total	**423**	**344**	**78**	**63**	**908**	**2539.68**	**42,244,224**	**47,076,748**	**5,812,252**

10. Conclusion

A novel multi-objective optimization model for transit fleet resource allocation is proposed in this paper. Two conflicting objectives, maximization of TSWARL and minimization of NPC are used. TSWARL is a quality measure represents remaining life of the transit fleet that the agency would like to maximize. Further, it is equally important to determine the cost required to achieve a certain TSWARL, this in terms of present value of the cost can be referred to NPC. It being an expenditure measure the transit agency would like to minimize NPC, a premise that conflicts with maximizing TSWARL. In the single objective optimization problem either TSWARL or NPC can be analyzed only one at a time. Further, while analyzing TSWARL the single objective optimization problem is blind to the NPC, and vice versa; as each is assumed as a by-product of other. The proposed multiobjective optimization problem has the advantage of considering both objectives simultaneously and provides a series of solutions as a trade-off for the decision maker. Branch and bound algorithm (BBA) is used to solve the multi-objective formulation since it results in better optimal solutions compared to GA for such problems. The transit fleet data over an eight year period from the Michigan Department of Transportation is used as the case study. As per FTA standards, four improvement options are used to allocate the fleet approaching MNSL.

The multi objective transit fleet resource allocation model has multiple dimensions of significant contribution to research and practice. First, the proposed model provides a trade-off between two objectives TSWARL, the quality measure and NPC, the cost measure. An analysis of this trade-off has not been attempted in literature. Second, solutions to both objectives in a multi-year planning period provide the decision makers with multiple options. Third, the proposed method allows the decision makers to explore the trade-off solutions between the conflicting objectives like TSWARL and NPC to make an informed decision. The research in transit resource allocation can be further enhanced in several ways. The classical technique of weighted sum approach presented in the paper has been extensively applied in multi-objective optimization research and practice. However, recent advancement of evolutionary approaches for solving multi-objective optimization can be considered in future research. The case study demonstrated in the paper is for the medium duty, medium sized transit fleet system in Michigan. The methodology can be applied to different fleet age types, policy, and budget constraints. Another factor is fleet uncertainty because of bus breakdown, accidents or other events, that can be modeled into the problem to build a robust fleet resource allocation.

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