OPTIMAL EMISSION PRICING MODELS FOR CONTAINING CARBON FOOTPRINTS DUE TO VEHICULAR POLLUTION IN A CITY NETWORK

By

Sushant Sharma, Ph.D.
Research Associate
Nextrans Center,
Regional University Transportation Center
Purdue University,
West Lafayette
P: +1-765-496-9768
Email: sharma57@purdue.edu

Sabyasachee Mishra, Ph.D.
Faculty Research Associate
National Center for Smart Growth Research and Education
University of Maryland
College Park, MD 20741
P:+1-301-405-9424
Email: mishra@umd.edu

Word Count:
Number of Tables: 4
Number of Figures: 6
Total Count: = 5,487 + (10 x 250) = 7,987
Date Submitted: October 2010

Submitted for Peer Review and for Compendium of Papers CD-ROM at the Annual Meeting of the Transportation Research Board (TRB) in January 2011, and for Publication in the Journal of TRB

ABSTRACT

This study proposes nine different models to reduce vehicular green house gas emission by designing optimal emission pricing in a given transportation system. All the models are formulated as a bi-level problem, i.e. upper level as planner's policy variable and the lower level as road user's response to the strategies set by the planner. The model is solved using genetic algorithm at the upper level and Frank-Wolfe algorithm at the lower level. The developed models are tested on a small hypothetical test network and a real medium sized network of Mumbai city in India. The performance of all the proposed models is compared to the Base-Case (do nothing) and reductions in emissions shows efficacy of the models. The study makes two major contributions, first it proposes a new set of models for planners to design emission pricing for emission reduction considering possible constraints in the field and second it realistically models both planner's decision and user's response to the decision to achieve minimal value of objective. Although the proposed models are solved for CO_2 only, the methodology can be used for analysis of policy variables for any pollutant.

1. INTRODUCTION

Sustainability is concerned with attainment of goals through a variety of policy instruments, given not only the transportation network and environmental parameters but also the travel behavior *(1)*. The travel behavior of road users can be influenced by imposing optimal impedance so as to achieve objectives like minimal emission and reduction of carbon footprint due to vehicles on the road network. In this context emission pricing and congestion pricing can be seen as options that can modify the traffic flows in a transportation system so as to achieve minimal emissions. The lack of efficient methods for minimizing emissions using suitable policy variables can be attributed to the traditional perception within the transportation community that believes the minimization of travel times will concurrently result in associated reductions in the undesirable environmental by-products of vehicular movement. However, recent research findings point to the fact that travel time variables are affected differently from air quality and fuel consumption variables, due to various traffic flow improvement strategies like capacity expansion (*2-4*).

In U.S., all federal and state agencies are constructively working towards identifying and addressing environmental issues and designing policies to develop a sustainable and livable environment. The vehicular pollution is being studied in various contexts from reducing pollutants emitted from a vehicle using a new technology, to develop emission pricing so as to curtail the present emission levels. The international concern towards green house gases (GHGs) being at the center of all emission reduction issues. Almost 28% of GHGs are produced by transportation sector (*5*) and a major portion of it is attributed to emission from private vehicles. With more emphasis in reducing the carbon footprints from various sources the state Department of Transportation's (DOTs) and Metropolitan Planning Organization's (MPOs) are also looking to regulate the CO_2 emissions from vehicles. At present there are two types of feasible methods one long term solution to improve public transport such that the mode shift can cause a large emission reduction and other short term solution of changing the traveler behavior by imposing emission pricing such that there is minimal emission produced in the system. While improvement of public transport is the ideal scenario, for long term sustainable solution, the emission pricing needs more careful analysis before deciding the additional cost to the user so as to achieve minimal emissions. In this study we develop various models to reduce total system emission (in terms of CO_2) in a given network by shifting the traffic flows on different links/routes by imposing optimal emission pricing. The shift in traffic flows causes changes in the average speed of the links leading to change in emissions. Further, emission factor is a function of average speed. While, it is equally important to quantify emissions also as a function of acceleration, deceleration and idling of vehicles but that is used in operational models rather than planning models which are macroscopic in nature. Moreover availability of such micro-scopic level data is a challenge in itself. The developed models consider a planner as the policy designer whose sole objective is either minimizing congestion or emission or both with variety of constraints in the real world. The methodology developed in this paper is generic and can be applied to any pollutant. However we show the application of proposed models in terms of reducing CO_2, a major GHG from transportation sector.

2. LITERATURE REVIEW

In this section we introduce bi-level problem and some of the studies that considered minimizing emission as one of the objective in a transportation system.

In general, the bi-level problem can be expressed as follows: the leader or system manager (referred as planner in remainder of the paper) wishes to determine an optimal policy as a function of his/her control variable (x) and the users respond (y) to these policy decisions. The user response generally takes the form of a network traffic flow. The planner then seeks to minimize both x and y, where some constraints may be imposed upon x as well as the fact that y should be a user equilibrium flow, parameterized by the control vector, x. The network users, after and with complete knowledge of the planners decision, make route choice decisions in an attempt to minimize their travel cost, resulting in an aggregate network flow pattern. A complete description of bi-level problem can be found in Yang and Yagar, 1994 *(6)*.The optimal traffic flow is known by solving traffic assignment problem. The process of allocating given set of trip interchanges to the specified transportation system is usually referred to as traffic assignment. The fundamental aim of the traffic assignment process is to reproduce on the transportation system, the pattern of vehicular movements which would be observed when the travel demand represented by the trip matrix, or matrices to be assigned is satisfied. In this paper we use bi-level model to capture users response to the planners policy variable (optimal emission pricing) for achieving his/her goal of minimal emissions.

Some of the initial studies in this domain considered only traffic assignment while modeling and quantifying the emissions. Tzeng and Chen, 1996 investigated traffic assignment as a multi objective decision model with system optimum conditions to consider the environmental parameters (*7*). Bendek and Rilett, 1998, formulated a system equitable traffic assignment which uses generalized environmental cost as the objective function (8). A multiple user class equilibrium assignment algorithm was formulated by Venigalla et. al., 1999, to determine the vehicle trips and vehicle miles of travel in various operating modes on highway links (*9*). A specialized equilibrium assignment algorithm was used for finding emissions. Nagurney, 2000a, with the help of three distinct paradoxical phenomena tested on a hypothetical small road network proved that the so-called improvements to the transportation network may result in increased emissions (*10*). Further, Nagurney, 2002, considered a multi criteria traffic network model with emissions in the objective function (*11*). Sugawara and Niemeier, 2003, explored theoretical emissions-optimized trip assignment model to estimate the maximum carbon monoxide reduction under varying congestion levels on a hypothetical network (*12*). The experimental results indicated moderate reductions in system-level vehicle emissions under emissions-optimized trip assignment as compared to the conventional user-equilibrium and system optimum models. The solutions were also compared with Bendek and Rilett, 1998, and Venigalla et. al., 1999. Recent research related to emission minimizing in the networks include imposing emission pricing as one of the solution. Yin and Lu, 1999 studied the traffic equilibrium problems with environmental concerns, and proposed minimal traffic emission model (MTE) (*13*). Later, Yin and Lawphongpanich, 2006 studied congestion and emission pricing such that it allows decision makers to trade-off between two conflicting objectives, alleviating congestion versus reducing traffic emissions (*14*). However, no pre specified constraints were considered in the model. Sharma and Mathew, 2007 studied transportation network design in a bi-level problem when user is conscious about emission, in terms of emission cost (*15*). This was modeled in traffic assignment stage by a generalized cost function; a convex combination of travel time function and emission function.

Although most of these studies have tried to understand emission reduction either formulating it as objective in the traffic assignment problem or making improvement (i.e. capacity expansion and toll) to the network while minimizing the total emissions. However, there is a need to model the optimal emission pricing value that reduces the emission while road user behavior is captured and different planners' perspectives can be accounted for, in terms of various objectives. In this study we attempt to find the optimal emission price value for a network such that it reduces the overall emissions and associated objectives for the planner. Variety of constraints has been designed to be fit in the model as needed for planning and analysis by the planner.

3. MODEL FORMULATION AND SOLUTION METHODOLOGY

In this study, the optimal emission pricing model is formulated as a bi-level problem with a number of constraints. The upper level is the planner's perspective i.e. either minimizing total system emission (TSEM) or total system travel time (TSTT) or both objectives simultaneously by determining a set of optimal emission pricing subjected to some constraints. The lower level of the model represents the road user's behavioral reaction towards the planner's policy decisions (optimal emission pricing vectors) subject to the classical deterministic user equilibrium conditions. The deterministic user equilibrium is well known as static traffic assignment and is commonly used to model the road user behavior in transportation planning.

3.1 Upper Level

In this study we formulate one Base-Case (do nothing) and nine different categories of models to augment planners decision making procedure. The models have been developed to incorporate various objectives of the planner either single or in combination at the upper level. The lower level is same for all the models as it captures the user's response towards planner's policy at the upper level. Table 1 represents the structure of the proposed models, their objectives and constraints at upper level and lower level.

Model-1 demonstrates the planner's objective to minimize total system emission while obtaining optimal emission pricing. The total emission 'TE_e' is the sum of product of traffic flow 'x_a' and emission factor '$ef_a(v_a)$' as function of average speed 'v_a' on link 'a' and length of the link 'l_a'. The emission pricing value 'e_a' for each link acts as an additional cost for a road user given by $c_a(x_a, e_a)$ as shown in equation (3). Thus different values of 'e_a' lead to change in travel cost and hence variation in the flows throughout the network. The real value variable 'e_a' is chosen such that it is within the value of 1 (i.e. maximum increase in travel cost is 100%) and 0 (i.e. no emission pricing at all). The change in flows because of emission pricing further causes changes in travel time which varies the average speed on the link and further emission factor (see equation 1) and hence total emissions.

Model-2 represents the planner's goal to estimate total system emission by obtaining optimal emission pricing such that total system travel time 'TT_e' i.e. time spent by users in transportation network remains minimum. It is given by sum of product of flow 'x_a' on link 'a' and travel time $t_a(x_a\ )$ as a function of flow on the link 'a' (equation 2).

Model-3 depicts planner's objective to minimize the total system emission subject to a threshold 'TT_B' on total system travel time. 'TT_B' acts as a constraint, since the total system travel time may get sacrificed in order to minimize total system emissions.

Model-4 is a case when planner minimizes total system travel time while keeping a constraint on total emissions produced in the network. The constraint is written as total emission budget 'TE_B'.

Model-5 is when planner has to constraint the emission produced on a particular link. This case is relevant when particular route or link passes through a residential zone and planner attempts to reduce emissions on that link to some extent while imposing emission pricing on the network. The main objective of reducing total system emission is the same as model-4.

Model-6 employs a very different constraint of minimum volume and capacity ratio. This is relevant if traffic flows from the longest route may get shifted to large extent on other links on the same route due to emission pricing. For this constraint the minimal threshold for traffic flow on a link can be decided by planner based on his/her experience.

Model-7 is a multi-objective model in which both objectives of total system travel time and total system emissions are being minimized simultaneously. Since in multi-objective problems, there is no best solution with respect to both objectives as a best solution for one may be worse off at the cost of the other. Therefore, there usually exists a set of solutions and these are called pareto optimal solutions. This model results a set of pareto optimal solutions and each solution set has different values of policy variable.

Model-8 is also multi-objective model with emission produced on a link as a constraint whereas Model-9 contains the constraint of volume capacity ratio. The multi-objective models (Model-7, Model-8 and Model-9) are different from single objective models (Model-1 through Model-6) since they consider both objectives simultaneously and offer variety of solutions to choose from.

The notations used in the models are given below:

TE_e : *is the total system emission with emission pricing vector "e"*
TT_e : *is the total system travel time with emission pricing vector "e"*
x : *is the vector equilibrium link flows,* $x = [x_a]$.
e : *is the vector of emission pricing,* $e = [e_a]$.
TT_B : *is the maximum threshold for total system travel time fixed by planner.*
TE_B : *is the maximum threshold for total system emission fixed by planner.*
E_a : *is the maximum accepted emission of a pollutant on link "a".*
VC_a : *is the minimum required value of Volume Capacity ratio on link "a".*
$ef_a(v_a)$: *is the speed dependent emission factor for link "a" (gm/miles) where* v_a *is link speed.*
l_a : *is the length of link a (miles).*
t_a^0 : *free flow travel time.*
$t_a(x_a)$: *travel time as a function of flow* x_a.
$c_a(x_a, e_a)$: *travel cost as a function of flow* x_a *and emission pricing* e_a.
$f_k^{r,s}$: *is the flow on path k between OD pair r s.*
$\delta_{a,k}^{rs}$: *is 1 if route k between OD pair r, s uses link a, and 0 otherwise.*
A : *is the set of links in the network.*
Ω : *is the set of OD pairs.*
q : *is the vector of fixed OD pair demands,* $q^{rs} \in q$.
K : *is the set of paths or routes between OD pair r and s.*

TABLE 1 Planner Based Models For Emission Reduction

Scenario*	UPPER LEVEL		LOWER LEVEL	
	OBJECTIVE	CONSTRAINT	OBJECTIVE	CONSTRAINT
Base-Case	-	-	$\sum_a \int_0^{x_a} t_a(x_a)$	$\sum_{\forall k} f_k^{rs} = q^{rs}$ $x_a = \sum_r \sum_s \sum_k \delta_{a,k}^{rs} f_k^{rs}$ $f_k^{rs} \geq 0$ $x_a \geq 0$ $k \in K;\ a \in A$ $r, s \in \Omega$
Model-1	$TE_e = \sum_a (x_a ef_a(v_a) l_a)$	$0 \leq e_a \leq 1$	$\sum_a \int_0^{x_a} c_a(x_a, e_a)$	
Model-2	$TT_e = \sum_a (x_a t_a(x_a))$	$0 \leq e_a \leq 1$		
Model-3	$TE_e = \sum_a (x_a ef_a(v_a) l_a)$	$\sum_a (x_a t_a(x_a)) \leq TT_B$ $0 \leq e_a \leq 1$		
Model-4	$TT_e = \sum_a (x_a t_a(x_a))$	$\sum_a x_a ef_a(v_a) l_a \leq TE_B$ $0 \leq e_a \leq 1$		
Model-5	$TE_e = \sum_a (x_a ef_a(v_a) l_a)$	$x_a ef_a(v_a) l_a \leq E_a$ $0 \leq e_a \leq 1$		
Model-6	$TE_e = \sum_a (x_a ef_a(v_a) l_a)$	$x_a / c_a > VC_a$ $0 \leq e_a \leq 1$		
Model-7	$TE_e = \sum_a (x_a ef_a(v_a) l_a)$ $TT_e = \sum_a (x_a t_a(x_a))$	$0 \leq e_a \leq 1$		
Model-8	$TE_e = \sum_a (x_a ef_a(v_a) l_a)$ $TT_e = \sum_a (x_a t_a(x_a))$	$x_a ef_a(v_a) l_a \leq E_a$ $0 \leq e_a \leq 1$		
Model-9	$TE_e = \sum_a (x_a ef_a(v_a) l_a)$ $TT_e = \sum_a (x_a t_a(x_a))$	$x_a / c_a > VC_a$ $0 \leq e_a \leq 1$		

Note : * The “Base-Case” scenario is solved as a simple UE assignment method (Lower Level only)

The emission function $ef_a\ (v_a)$ typically has a polynomial form with an average link speed ‘v_a’ as the dependent variable and is given as

$$ef_a(v_a) = b_1\, v_a^2 + b_2\, v_a\ + b_3 \tag{1}$$

where: b_1, b_2, and b_3 are the coefficients to be calibrated from the observed vehicular emission data. In this paper we consider the pollutant as CO_2, a major GHG and adopt a polynomial function from El-Shawarby et. al. *(16)*.The reason for considering only one pollutant is present focus of agencies and policy makers on minimizing the GHGs from vehicles as discussed in the introduction.

3.2 Lower Level

The lower level of the bi-level formulation assigns the trip matrix into the network using the route choice algorithm. A user equilibrium assignment based on Wardrop's first principle is proposed, which denotes that “no user can experience a lower travel time by unilaterally changing routes” *(17)*. However, it assumes that the user has perfect knowledge of the travel cost and flows are present simultaneously on all the links. In simple terms the equilibrium is achieved when the travel cost on all used paths is equal. This principle is behaviorally robust, computationally efficient, and possesses unique solution *(18)*. The formulation for the user equilibrium assignment in the form of an optimization problem is shown in second column of Table 1. The travel time function $t_a(.)$ is specific to a given link 'a' and the most widely used model is Bureau of Public Roads (BPR) function given by

$$t_a(x_a) = \; t_o\left(1 + \alpha_a\left(\frac{x_a}{C_a}\right)\right)^{\beta_a} \tag{2}$$

where $t_a(.)$ is free flow time on link 'a', and α_a and β_a are link specific constants, normally calibrated using the observed field data. The BPR function is a monotonically increasing convex function. The emission price variable e_a changes to travel time into travel cost such that φ is value of time in monetary terms ($/hr).

$$c_a(x_a, e_a) = \varphi\,(1 + e_a\,)\,t_a(x_a) = \varphi\,(1 + e_a)\,t_o\left(1 + \alpha_a\left(\frac{x_a}{C_a}\right)\right)^{\beta_a} \tag{3}$$

Constraint shown in Table 1 for lower level are flow conservation equation, states that the flow on all paths connecting each O-D pair has to be equal to the O-D trip rate. In other words, all trips have to be assigned to the network. The next constraint is a definitional constraint relating the link flows 'x_a'and path flows 'f_k^{rs}'. The remaining two constraints are non-negativity conditions that are required to ensure that the solutions are physically meaningful.

3.3 Solution Algorithm

The overall solution algorithm is presented in Figure 1. The upper level is solved using genetic algorithm (GA) since its efficacy in solving bi-level problems of large real sized network has been proved in the literature *(19,20)*, which is our final objective to make the model realistic and applicable. The lower level has been solved by using traditional Frank-Wolfe algorithm; the detail algorithm is available in Sheffi, 1985 *(18)*.

The algorithm starts with the upper level by reading all the inputs including network details, demand matrix, constraints, link cost functions, travel time function, investment function and emission cost functions. Inputs on constraints include total system travel time threshold (TT_B), total emission threshold (TE_B), Volume capacity threshold (VC_a) or maximum emission on a link (E_a). A population of link emission pricing vector is created and randomly initialized. These trial links emission pricing vectors are then translated into the current travel cost. The lower level algorithm is then invoked with the current link capacity vector where the demand matrix is assigned into the network using the formulation presented in Table 1. The lower level is solved using Frank-Wolfe Algorithm. The output of the lower level is the link flow vector which is used to compute link travel time using the BPR function and travel cost. Since BPR equation, is a monotonically increasing convex function and hence the travel cost is also convex function. The travel cost on the link 'a' depends on the flows on that link alone, the lower level formulation is convex. Therefore, there is a unique global solution and can be computed by any efficient convex combination method like Frank-Wolfe algorithm. The Frank-Wolfe algorithm, used in this study, is extensively reported in literature and has been elaborately discussed in Sheffi, 1985 (*18*). Then TSTT is computed as the sum of the product of the link travel time and link flows in the network. The average speed on each link is computed from the length and the travel time on that link. The average speed on the link is used to derive emission factor based on the equation 1. After calculating speed dependent emission factors, total emissions generated for each pollutant is computed.

The emission of each pollutant is a cumulative sum of the product of the link lengths, the traffic flow of particular mode and emission factor of a particular pollutant and mode. Thus, the total system travel time and the total emissions computed will form the objective function values of the current generation. Once the values of objective functions are obtained, solutions are checked for constraint violation and fitness function is computed. If the current generation is greater than the pre-specified maximum generations then algorithm is terminated. Solutions are reported in the form of total system travel time, total emissions, emissions on each link, optimal emission pricing vector, and link travel times. Otherwise, a new set of solutions are obtained using the genetic algorithm. This process is repeated till number of generations is completed.

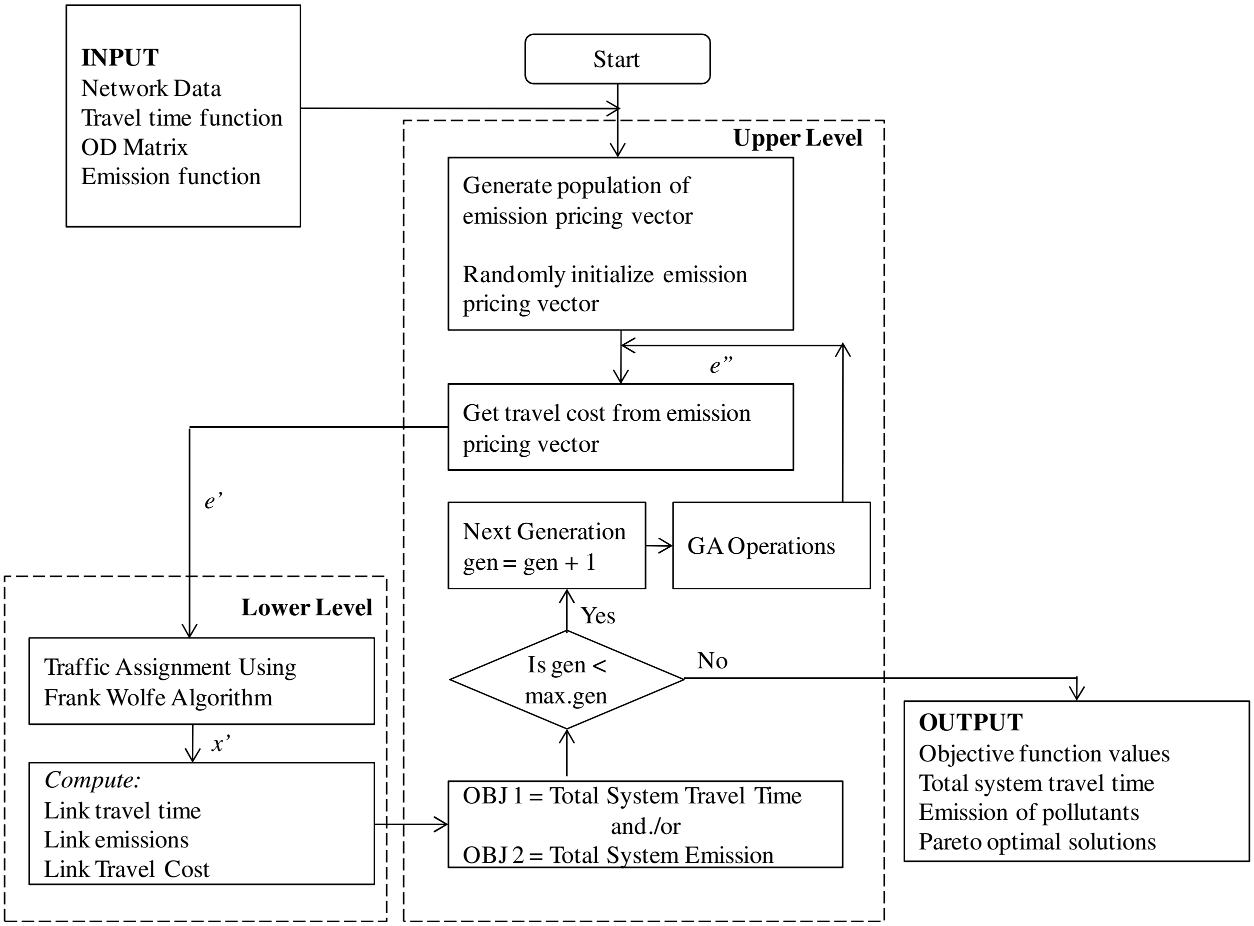

FIGURE 1 Flowchart demonstrating solution methodology for proposed model

4. TEST NETWORK

To explore the applicability of the model, a test network consisting of four nodes and five links is considered (Figure 2). The length (*l*), capacity (*C*), Free Flow Speed (FFS),α, and β of each link is also presented in Figure 2. The demand from node 1 to node 4 is taken as 4,000 vehicle/hour. For all single objective models (Base-Case and model 1 through 6), link level solution is presented in Table 2. The link level result of each model is shown in form of link emissions, link flow, link speed, link v/c ratio and optimal pricing for each link. The variation in each link attribute with different pricing options is shown in Table 2.

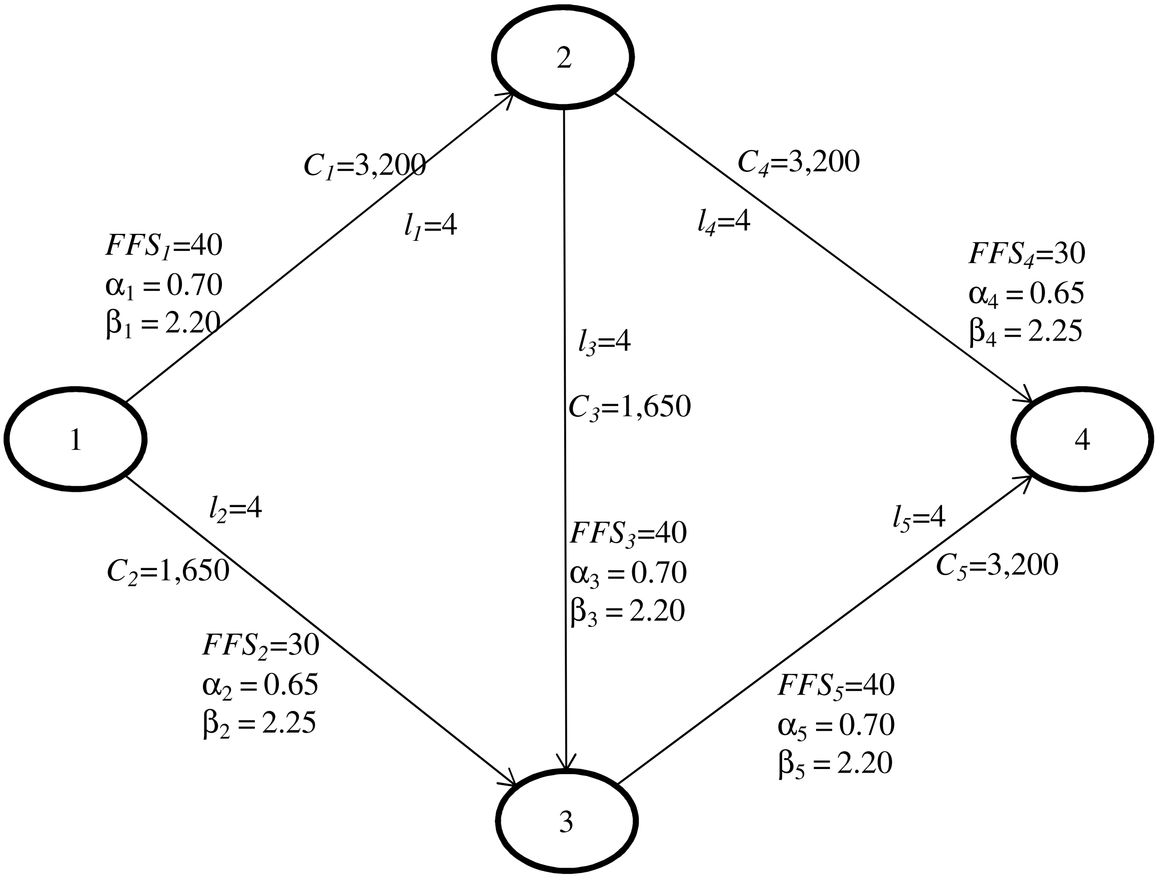

FIGURE 2 Small Test network

In Table 2, the results for each link are different for Model-1 compared to Base-Case. All the links are subjected to pricing thereby increasing the user cost such that the traffic flow is dispersed so as to minimize total system emissions. For example, for link 1, the optimal travel cost is 0.457 times the Base-Case travel cost (or Base-Case travel cost * 1.457). Similarly, pricing of 0.921 is highest for link 3 in Model-1. These optimal travel cost values act as impedance for road users such that shift in the traffic flows on various links result in the minimum value of objective function.

TABLE 2 Link level results for the test network

Model	Link	Emission (gm)	Flow (veh/hr)	Speed (mi/hr)	v/c ratio	Pricing
Base-Case	*a1*	2,685,662	2,691	27	0.841	-*
	a2	1,746,202	1,309	21	0.793	-
	a3	929,875	1,380	36	0.431	-
	a4	1,748,856	1,311	22	0.795	-
	a5	2,683,558	2,690	27	0.841	-
Model-1	*a1*	1,973,520	2,084	31	0.651	0.457
	a2	2,751,689	1,916	16	1.161	0.016
	a3	105,876	164	40	0.051	0.921
	a4	2,758,747	1,920	16	1.164	0.134
	a5	1,969,025	2,080	31	0.650	0.764
Model-2	*a1*	2,727,533	2,726	27	0.852	0.291
	a2	1,690,567	1,274	22	0.772	0.307
	a3	983,798	1,453	36	0.454	0.236
	a4	1,688,157	1,273	22	0.771	0.457
	a5	2,729,462	2,727	27	0.852	0.520
Model-3	*a1*	2,202,179	2,286	30	0.714	0.236
	a2	2,404,236	1,714	18	1.039	0.063
	a3	352,331	543	39	0.170	0.811
	a4	2,454,064	1,743	17	1.057	0.165
	a5	2,168,452	2,257	30	0.705	0.504
Model-4	*a1*	2,478,846	2,522	28	0.788	0.465
	a2	2,013,908	1,478	20	0.896	0.213
	a3	719,226	1,086	38	0.339	0.220
	a4	1,945,171	1,436	20	0.870	0.472
	a5	2,530,097	2,564	28	0.801	0.850
Model-5	*a1*	1,861,298	1,982	32	0.619	0.598
	a2	2,930,494	2,018	15	1.223	0.008
	a3	546,786	835	39	0.261	0.811
	a4	1,497,795	1,147	23	0.695	0.677
	a5	2,885,836	2,853	26	0.891	0.417
Model-6	*a1*	2,467,979	2,513	28	0.785	0.874
	a2	2,028,624	1,488	20	0.902	0.386
	a3	663,561	1,006	38	0.314	0.087
	a4	2,059,999	1,507	20	0.913	0.024
	a5	2,444,933	2,493	28	0.779	0.252

Note: No pricing is performed for Base-Case.

The system level results are presented in Table 3. In the second, third, and fourth column TSEM[1], TSTT[1], and vehicle miles travelled (VMT[1]) for each model is presented. In comparison to Base-Case, there is 2.4% reduction in TSEM and 16.33% increase in TSTT is observed for Model-1. The objective of Model-1 is minimization of TSEM, which shows the efficacy of

---

[1] In remainder of the paper total system emission is denoted as TSEM, total system travel time is denoted as TSTT, and vehicle miles travelled is denoted as VMT.

model in reducing TSEM, but this leads to increase of TSTT. Moreover the VMT decreases by 12.51% compared to Base-Case. Similarly Model-2 results show reduction in TSTT of 0.13%.

The Model-3 has an additional constraint of TSTT to Model-1 considering planners limiting traffic congestion in form of pre-specified threshold for time spent in the network by users. The threshold for TSTT can be pre-specified based on planner's experience. However , the value of TSTT was assumed as the average from Model-1 and Model-2 i.e. 98,368 min ((105,858+90,878)/2). The Model-3 showed a reduction of 10.69% of VMT compared to Base-Case (the reduction is slightly less than Model 1; i.e. 12.51%). There is 2.17% reduction in TSEM compared to base case, while TSTT is increased by 7.96% (as opposed to 16.33% in case of Model 2). Clearly, Model 3 performed better on both TSEM and TSTT compared to Model 1. Model-4 represents a case where planner has a pre-determined target of reduction of emissions for a system. This value can be anything like percentage reduction in emissions from current or base case emission scenario. The difference between Model-2 and Model-4 is addition of upper bound of TSTT as a constraint. The constraint value was chosen similar to Model-3. Model-4 shows a reduction of 1.09% in TSEM and increase of 1.18% of TSTT compared to Base-Case. This model is useful when planner tries to simultaneously minimize congestion using pricing and emissions in the system. Model-5 is constructed as an added layer of information from the planner's perspective. An additional constraint of threshold of emission on a particular link is introduced in Model-5. This constraint makes sense from planner's perspective as it is possible that one link passes through residential neighborhood, and it is desirable to reduce emission on the specific link/route. In the test network one link is considered while multiple links can be easily integrated in case of a real world network. The link emission constraint was considered as 1,500,000 gm of emission threshold on link 4 in the test network (sixth row, Table 3). The results show reduction of 0.73% of TSEM and increase of 10.64% of TSTT compared to the Base-Case. Moreover VMT reduced by 9.28% for Model 5 in comparison to Base-Case. Model-6 represents planner's strategy towards containing a flow (i.e. v/c ratio) on a particular link. Since pricing might result in shift of large flow on a particular link, this model results in optimal pricing such that the flow on particular links is maintained to specified threshold v/c value. In the test network the v/c ratio of 0.30 for link 3 is added as a constraint in Model-6 (seventh row, Table 3). The results show reduction of TSEM by 1.32%, an increase of TSTT of 1.8% and 8.46% reduction of VMT compared to the Base-Case. While various models were presented in this section, all the models considered only one objective at a time at upper level, the next section explores the consideration of more than one objective and solution for multiple objectives simultaneously.

4.1 Multi Objective Optimization Results for Test Network

While it is imperative to minimize TSEM and TSTT individually, from planner's perspective, it is desirable to consider both or consider a significant value of TSEM and TSTT as per the planning need. Multi-Objective (MO) Optimization is suitable for considering more than one objective function in the planning process. Three scenarios of MO optimization problems are analyzed for the test network:

- Minimization of TSEM and TSTT with no additional constraint (Model-7)
- Minimization of TSEM and TSTT with emission on a link as an additional constraint (Model-8)
- Minimization of TSEM and TSTT with v/c ratio on a link as an additional constraint (Model-9)

The results from these models are presented in Figure 3. Unlike single objective optimization results (Model-1 through Model-6), the results from Model-7 are series of pareto optimal solutions satisfying both the objective functions (TSEM and TSTT) to varying degrees. Each point on Figure 3 represents a unique value of TSTT and TSEM and contains a solution vector of optimal pricing strategies for the network. For instance, two extremes of results on the pareto optimal curves are: maximum emission with least TSTT, and maximum TSTT with least emission (i.e. both ends of the pareto optimal curve). On the other hand, the pareto optimal solution represents a spectrum of trade off solutions between the two extremes. Model-7 resulted in minimum TSEM of 9,558,480 grams, and minimum TSTT of 90,878 minutes that concur with the optimal solution obtained from Model-1 and Model-2.

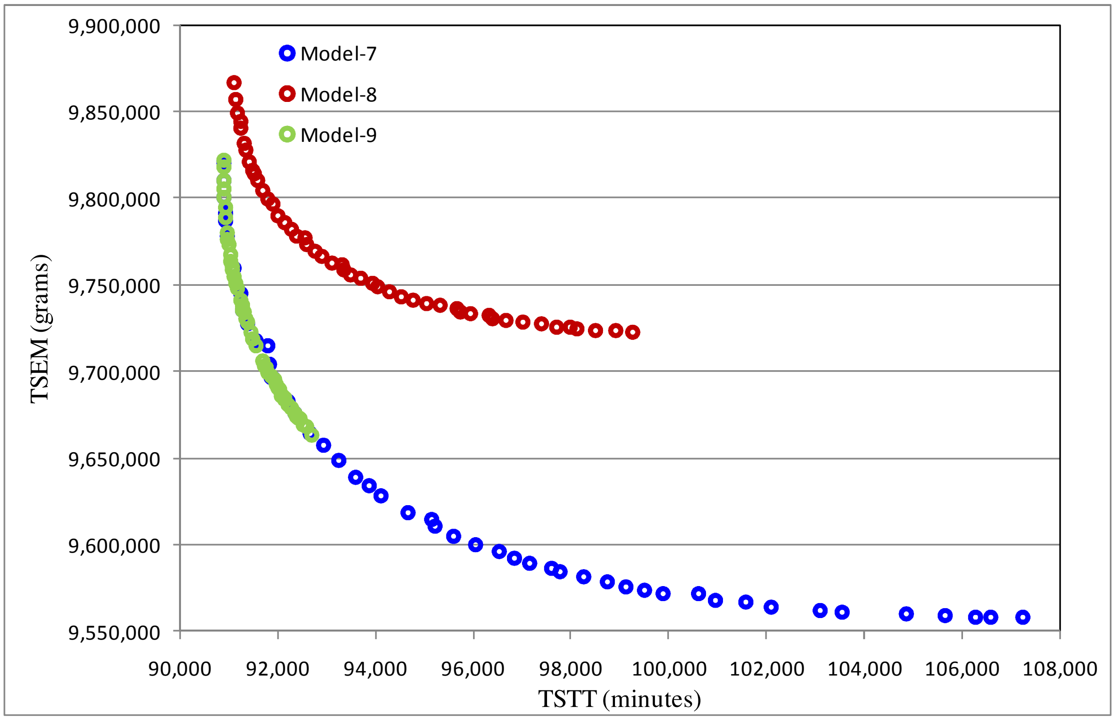

FIGURE 3 Pareto optimal solutions from Model-7, Model-8 and Model-9.

The next MO model is Model-8, which is a modified version of Model-7 with emission on particular link as an added constraint. Figure 3 shows the set of pareto optimal solutions generated for Model-8. The results show minimum TSEM of 9,722,392 grams, and minimum TSTT of 91,113 minutes.

TABLE 3 Network level results for the test network

				Constraint				% Improvement**		
Model	TSEM (gm)	TSTT (min.)	VMT	TSEM (gm)	TSTT (min.)	Link Emission (gm)	Link v/c ratio	TSEM	TSTT	VMT
Base-Case	9,794,156	91,000	41,524	-	-	-	-			
Model- 1	9,558,858*	105,858	36,328	-	-	-	-	-2.40%	16.33%	-12.51%
Model- 2	9,819,517	90,878*	38,907	-	-	-	-	0.26%	-0.13%	-6.30%
Model- 3	9,581,261*	98,240	37,086	-	98,368	-	-	-2.17%	7.96%	-10.69%
Model- 4	9,687,247	92,077*	38,172	9,689,188	-	-	-	-1.09%	1.18%	-8.07%
Model- 5	9,722,208*	100,683	37,670	-	-	1,500,000	-	-0.73%	10.64%	-9.28%
Model- 6	9,665,096*	92,637	38,011	-	-	-	0.31	-1.32%	1.80%	-8.46%

Note: *: Objective function; **: % improvement = (Subject model - Base-Case) *100 / Base-Case

The higher value of TSEM and TSTT of Model-8 compared to Model-7 can be attributed to an additional constraint of emission threshold on link 4. In Model-9 the constraint is with v/c ratio on particular link (link 3 in this case). Similar to other solution we can see the set of solutions in Figure 3. However most of the solution points overlap with Model-7 but because of the additional constraints the range of solutions is smaller. The minimum TSEM value is 9,663,635 grams and TSTT is 90,878 minutes. The pareto optimal solutions provided options for the planner to consider a desired solution from series of alternative solutions which cannot be obtained by single objective optimization (Model-1 to Model-6).

4.2 Synthesis of Test Network Result

Synthesis of the test network result for all models in the form of TSEM and TSTT is presented in Figure 4. A total of 13 data points are presented. One Base-Case, six single objective optimization models, and two subset of the pareto optimal results for three MO optimization solutions (2x3). The two subsets of MO include minimum of objective 1 and objective 2. These data points for multi-objective optimization is an indicative of array of pareto-optimal solutions, where as one can choose any other desired data points. The test network results can be summarized as follows:

- Minimum TSEM is achieved by Model-1. Similar result is also achieved by Model-7, at the minimum of objective 1. The robustness of the multi-objective optimization (in Model-7) is demonstrated with realization of similar TSEM to Model-1.
- Minimum TSTT is achieved by Model-2. Similar TSTT is also achieved by Model-8, at the minimum of objective 2.
- Model-3 produced second-best TSEM (first best being Model-1), with improved TSTT. Along with the single objective optimization, the multi-objective optimization provided a range of options to select for the decision makers.
- Model-4 through Model-6 and other multi-objective optimization solution points produced intermediate solutions of TSEM and TSTT. These solution points can serve as tradeoff between the two spectrum of minimum TSEM and TSTT.

5. CASE STUDY

The Central Business District (CBD) of Mumbai, India commonly referred as "Fort Area" is considered as the case study in this paper. All the links in the Fort Area network carries heavy traffic during peak hours on weekdays. The topography of the Fort Area is presented in Figure 5. Traffic flow data is for evening peak hours (between 5:00 p.m. to 7:00 p.m.) of working days. The road network has 17 highway nodes and 56 highway links. Various traffic flow and network parameters such as OD matrix, mode split, α_a, β_a, free flow speed, and capacity are reported in Sharma and Mathew *(15)*. The original OD matrix was increased by employing a growth factor of 1.2 to represent the present demand on the network. The peak period trips in the network are 37,317 vehicles. The link characteristics of the network have remained unchanged as reported in

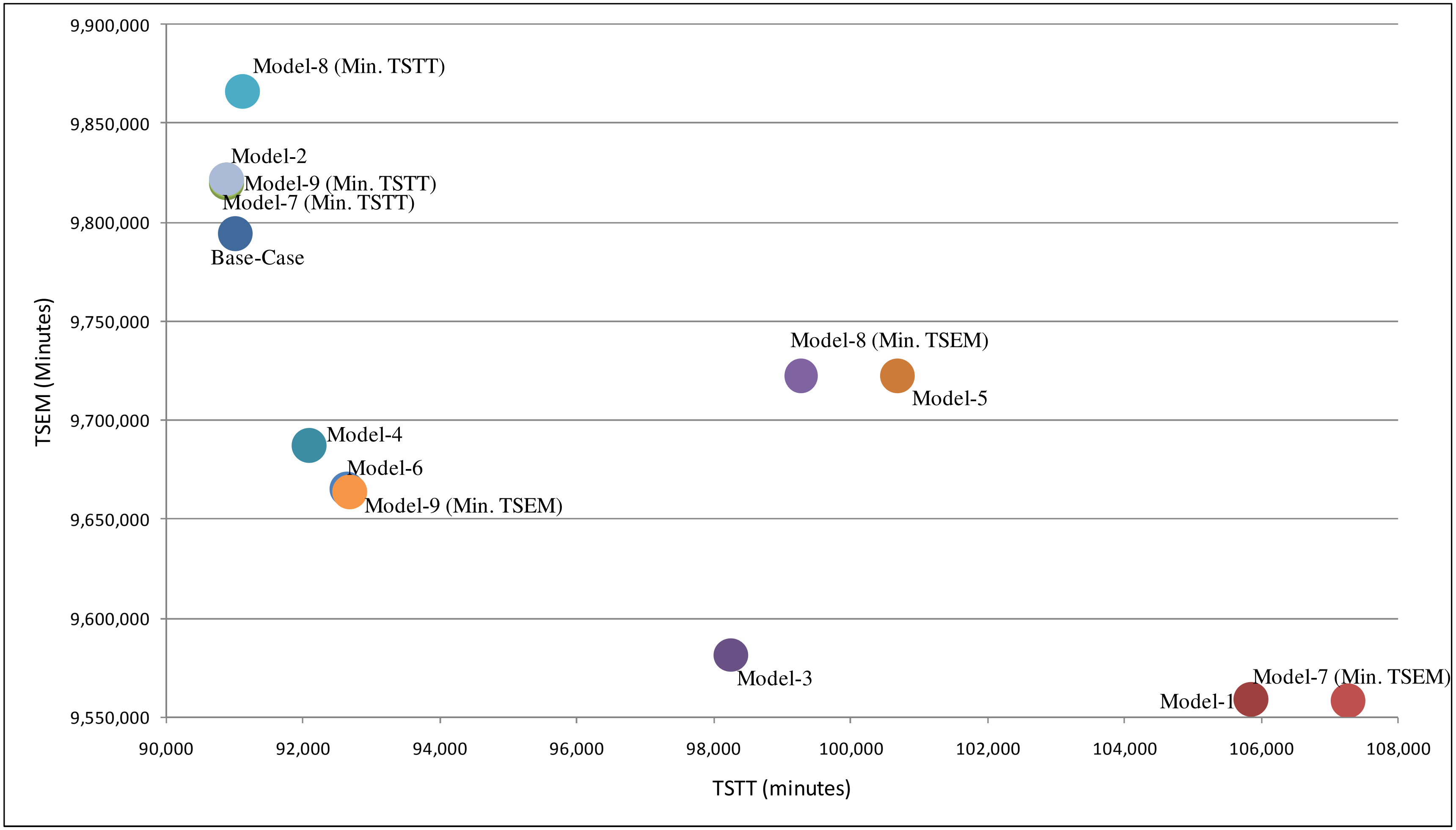

FIGURE 4 Synthesis of Test Network Results

Sharma and Mathew (*15*). The GA parameters were chosen after performing a sensitivity analysis to obtain the best solution.

5.1 Case Study Results and Discussion

For the Fort Area network following results are presented.

- Base-Case
- TSEM minimization (Model 1)
- TSTT minimization (Model 2)
- TSEM and TSTT minimization (Model 7)

The other models (presented in Table 1) can also be solved using the proposed methodology and adding the threshold value of various desired objectives as constraints. We are presenting the working of only three basic proposed models for sake of brevity. The results from Base-Case, Model-1, and Model-2 are presented in Table 4. Compared to Base-Case, Model-1 resulted in decrease in 2.38% of TSEM, while TSTT is increased by 8.45% (Second row, Table 4). Although this improvement is small it should be noted that the reduction in TSEM is only for peak hour of the day since loaded demand is for peak hour. The overall reduction in CO_2 for a complete day and over the entire life of the network will be substantial. Further the amount of reduction in the emissions may vary among different networks based on network topology and demand. In this case study, the network is heavily congested (V/C>0.9) and lack of efficient alternative routes may not cause substantial reduction in emissions. For Model-1 the VMT is decreased by 2.82% (Second row, Table 4). The reduction in amount of pollutant also depends on the relation of emission factor with the speed. The more sensitive the emission factor of a pollutant to average speed, more reduction can be achieved by containing traffic flow (and hence speed) by emission pricing.

Model-2 resulted in marginal increase and decrease in TSTT and increase in TSEM, and VMT. This can be attributed to congestion level on the case study network. Had it been less congested the reduction in TSTT would have been more. MO optimization (Model-7) was also performed for the Fort Area network. The pareto optimal solutions for Model-7 are presented in Figure 6. Minimum TSEM of 12,469,310 grams and minimum TSTT of 167,610 minutes are resulted from Model-7. The set of solutions shows the capability of model to provide a large number of choices to the planners. Results of Model-8 and Model-9 for fort area are not presented in this paper for brevity.

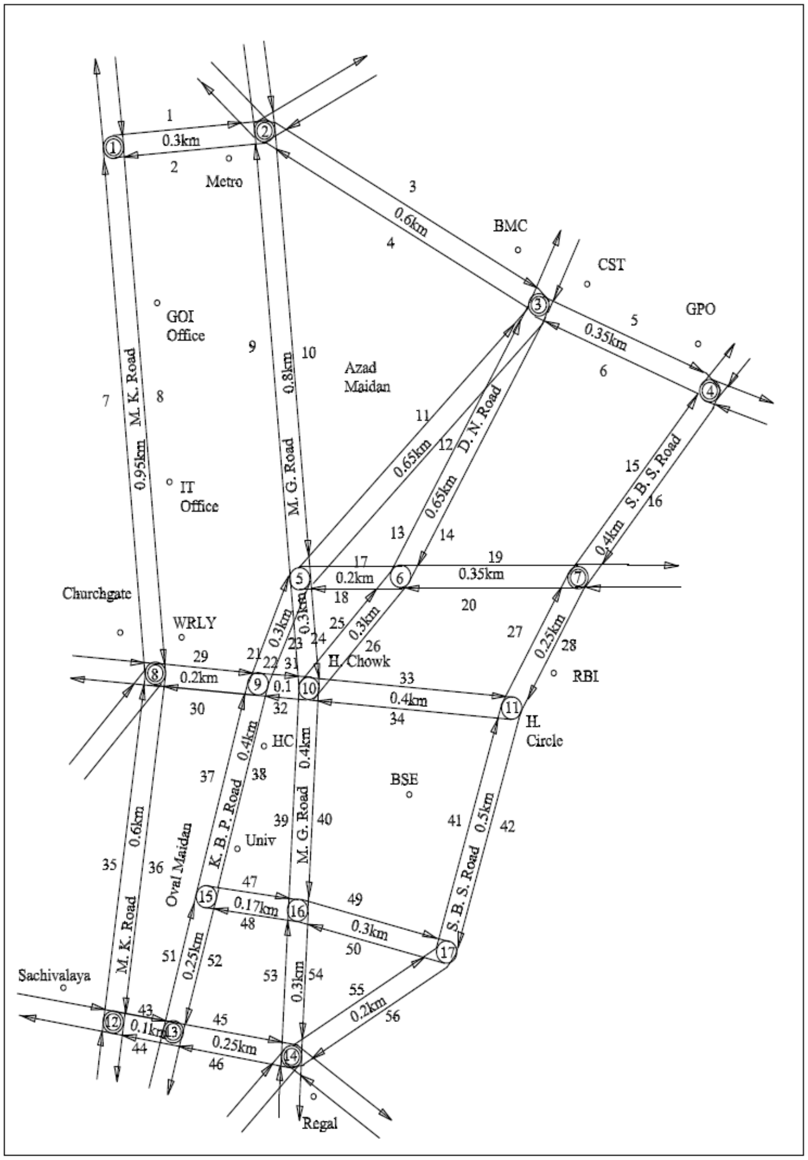

FIGURE 5 Network of Fort Area, Mumbai, India.

TABLE 4 Emission Pricing Results for the Fort Area Network

Model	TSEM	TSTT	VMT	% Improvement**		
				TSEM	TSTT	VMT
Base-Case	12,769,641	168,045	29,192			
Model-1	12,465,742*	182,252	28,370	-2.38%	8.45%	-2.82%
Model-2	12,832,822	167,685*	29,585	0.49%	-0.21%	1.35%

Note: *: Objective function; **: % improvement = (Subject model - Base-Case) *100 / Base-Case

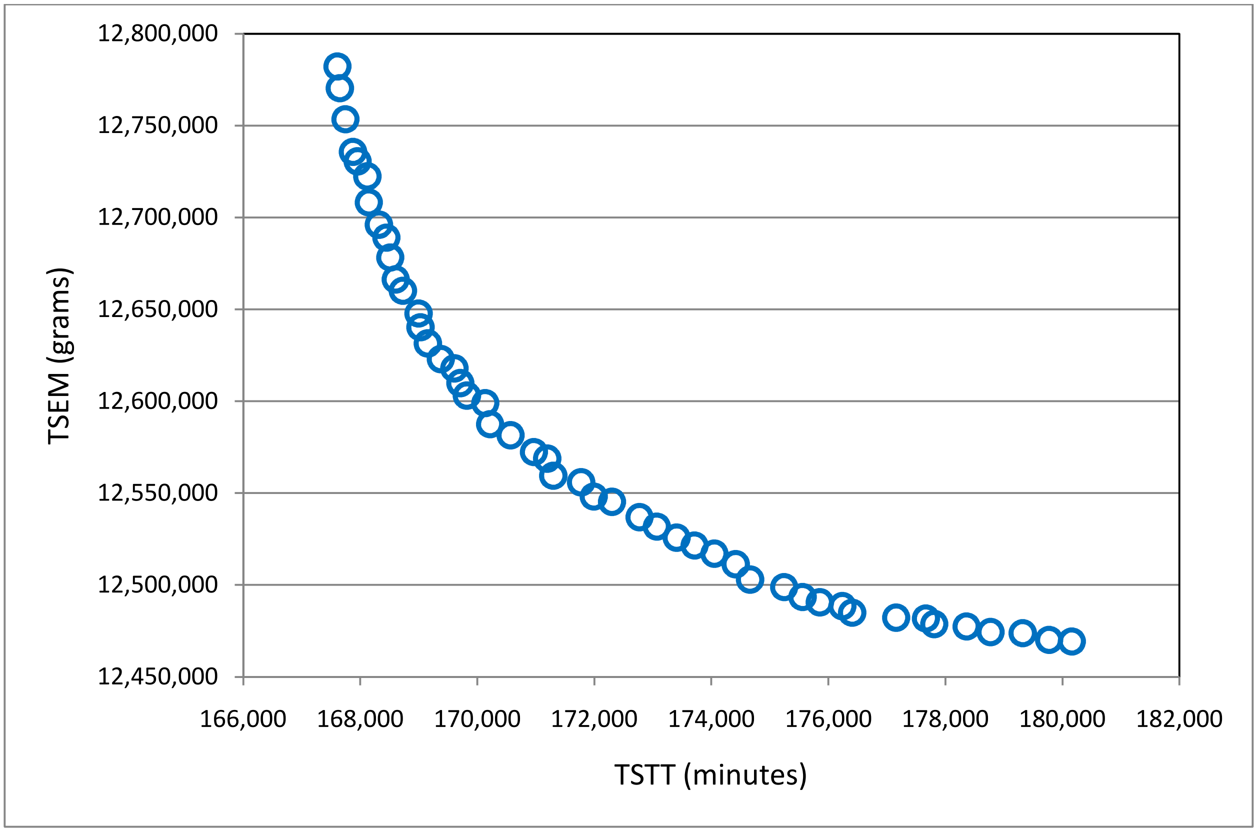

FIGURE 6 Pareto optimal solutions for Fort Area Network

6. CONCLUSION

The paper presents a series of alternative approaches for planners to minimize emission considering a number of options such as emission pricing, link specific emission and flow constraints. A Base-Case and six single objective optimization models are presented. The objective is kept as minimization of TSEM or TSTT. The functionality and significance of each model is examined with the help of a test network. Improvement of specific measures such as TSEM, TSTT, and VMT are compared to the Base-Case. In each of the six models, either TSEM or TSTT is minimized subjected to a set of emission pricing options. In addition to TSEM and TSTT minimization, threshold for maximum acceptable emission, system travel time and link flows and emission are considered as constraints. To minimize and consider both objectives (TSEM and TSTT) simultaneously multi-objective optimization models were proposed. As opposed to single objective optimization, multi-objective optimization models provided a set of pareto-optimal solutions to act as tradeoffs between TSEM and TSTT to account for the planner's desired objectives.

The transportation network in the CBD of Mumbai, India was considered in the case study. Single objective models produced better TSEM and TSTT based on their corresponding objective function when compared to the Base-Case. In addition, the multi-objective optimization model produced a set of solutions to choose considering both TSEM and TSTT. All the proposed models offer strategies to minimize emission with a number of insights to the other network parameters such as VMT and average travel time. The proposed models can serve as a set of useful tools to minimize emission, travel time and both. An insight from the study is minimizing total system travel time does not reduce the total emissions produced in the transportation system. The robustness of the proposed models is examined with the case study, and the framework can be used to solve medium to large scale city networks. Although only CO_2 has been studied in this paper as it being a GHG and pollutant of immediate concern, the proposed models are generic and applicable for various other pollutants. However, the amount of reduction in emissions from the proposed models depends on the network characteristics and pollutant type. This study can be further extended by incorporating multi-modes and interaction among these modes in modeling.

REFERENCES

1. Nagurney, A. *Sustainable Transportation Networks.* Glos,UK., 2000.
2. Noland, R B, and M A Quddus. Flow improvements and vehicle emission: Effects of trip generation and emission control technology. *Transportation Research Part D,* Vol. 11, 2006, pp. 1–14.
3. Noland, R B. Relationships between highway capacity and induced vehicle travel. *Transportation Research Part A,* Vol. 35, 2001, pp. 47–72.
4. Yu, L. Collection and evaluation of modal traffic data for determination of vehicle emission rates under certain driving conditions. Technical Report, *Centre of Transport Training and Research, Texas Southern University*, 3100 Cleburne Avenue Houston, , Texas, 1997.
5. EPA. 2008 *Inventory of U.S. Green House Gas Emissions and Sinks:1990-2006*, Washington D.C.
6. Yang, H. and Yagar, S. Traffic assignment and traffic control in general freeway arterial corridor systems, Transportation Research Part B , Vol. 28, 1994, pp. 463-486.
7. Tzeng, G., H. and Chen, C., H. Multiobjective decision making for traffic assignment", IEEE Transactions on Engineering Management, Vol. 40(2). (1993)
8. Bendek, C M, and Rilett, L., R. Equitable traffic assignment with environmental cost functions. *Journal of Transportation Engineering, ASCE,* Vol. 124,No. 1,1998.
9. Venigalla, M., Chatterjee, M., A., and Bronzini, M., S. A specialized equilibrium assignment algorithm for air quality modeling. *Transportation Research Part D,* Vol. 4, 1999, pp. 29–44.

10. Nagurney, A. Congested urban transportation networks and emission paradoxes. *Transportation Research Part D* Vol. 5, 2000, pp. 145–151.
11. Nagurney, A. and Dong, J. A multiclass, multicriteria traffic network equilibrium model with elastic demand, Transportation Research Part B, Vol. 36, 2002, pp. 445-469.
12. Sugawara, S, and Niemeier, D., A. How much can vehicle emissions be reduced ?: Exploratory analysis of an upper boundary using an emissions optimized trip assignment. *Transportation Research Record,* Vol. 1815, 2003, pp. 29–37.
13. Yin, Y., and Lu, H. Traffic equilibrium problems with environmental concerns. *Journal of the Eastern Asia Society for Transportation Study* , Vol. 3, 1999, pp. 195-206.
14. Yin, Y. and Lawphongpanich, S. Internalizing emission externality on road networks." *Transportation Research Part D,* Vol. 11, 2006, pp. 292–301.
15. Sharma, S., and Mathew, T., V. Transportation network design considering emissions as bilevel optimization problem. *TRB 86th Annual Meeting Compendium of Papers CD-ROM.* Washington D.C.: Transportation Research Board, 2007.
16. El-Shawarby, I., Ahn, K., and Rakha, H. Comparative field evaluation of vehicle cruise speed and acceleration level impacts on hot stabilized emissions. Transportation Research Part D, Vol.10, 2005, pp. 13-30.
17. Wardrop, J. Some Theoretical Aspects of Road Traffic Research. *Proceedings of the Institution of Civil Engineers*, Part 2, 1952, pp. 325-378.
18. Sheffi, Y. *Urban Transportation Networks: Equilibrium Analysis with Mathematical Programming Methods.* New Jersey: Prentice-Hall, Englewood Cliffs, 1985.
19. Yin, Y. Genetic algorithms based approach for bi-level programming models. *Journal of the Transportation Engineering, ASCE,* Vol.126, No. 2, 2000, pp.115-120.
20. Mathew, T. V, and Sharma, S. Capacity expansion problem for large urban transportation networks. ASCE, Vol. 125, No. 1, 2009.