Optimal Positioning of Dynamic Wireless Charging Infrastructure in a Road Network for Battery Electric Vehicles

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ABSTRACT

Dynamic wireless charging (DWC) offers a plausible solution to extending Battery Electric Vehicle (BEV) driving range. DWC is costly to deploy and thus its locations need to be optimized. This raises a question often encountered in practice for infrastructure investment: how to determine the optimal locations of DWC facilities in a network. In this paper, we propose a sequential two-level planning approach considering the objectives of both the public infrastructure planning agency and the BEV users. Two different planners' objectives namely, total system travel time and total system net energy consumption are considered. Besides these objectives, constraints such as agency budget, range reassurance, and equity in resource distribution are also addressed at the planner's level. For each objective, BEV drivers respond by choosing their preferred route based on the location of DWC facilities implemented by the planner. An effective solution algorithm is utilized that has the capability of solving relatively large-scale real-world networks within a reasonable computational time. The numerical experiment and case study results provide useful insights on optimally positioning DWC infrastructure to minimize societal cost and energy.

Keywords: Battery electric vehicle; dynamic wireless charging; travel time; equity in resource distribution

1. INTRODUCTION

Despite recent developments in battery technology, the driving range, which is the furthest distance Battery Electric Vehicles (BEVs) can travel without the need for refueling, is substantially small compared to Internal Combustion Engine Vehicles (ICEVs). Given the average fuel capacity of 20 gallons and fuel economy of 23 miles per gallon, an ICEV can drive up to 460 miles without the need for refueling. On the other hand, the average driving range of BEV can only reach to 190 miles (Bomey, 2018; Hwang et al., 2018; USDOE, 2018). This limitation can lead to range anxiety for BEV drivers where they are worried that whether they can reach their destination with the battery's remaining in the state of charge (Agrawal et al., 2016).

To overcome these disadvantages, researchers have developed induction based dynamic wireless charging (DWC). Although this technology is still evolving recent research in this domain indicates it has an edge over the conventional plug-in charging (Lukic and Pantic, 2013; Panchal et al., 2018). DWC facility can be embedded under a road and it will dynamically charge the BEV moving above. Due to this feature, DWC does not require BEV to experience charging downtime. This concept is followed by a number of studies focusing on the technical aspects of DWC (Budhia et al., 2013; Miller et al., 2015a, 2015b; Pelletier et al., 2016). Recently, researchers have also discussed the development of wireless charging BEV in relation to the commercialization of BEV (Jang et al., 2016, 2015; Ko et al., 2015; Ko and Jang, 2013). If implemented properly, DWC can extend the driving range of a large fraction of BEV trips. This would satisfy the range requirement of benefiting BEVs and help to relieve the range anxiety of BEV drivers. Lin et al., (2014) found a significant increase in BEV adoption even if only 5% of the network is implemented with DWC since the technology can address the customer range anxiety problem. Therefore, the DWC Facility Location Problem (FLP) needs to be planned adequately to reap the maximum benefit of this evolving technology. DWC Facility will constitute an important piece of the infrastructure required to allow and promote the use of BEVs and pave the way for electric autonomous vehicles.

There have been several studies in the past devoted to locating refueling facilities, and in particular, recharging infrastructure for BEV. A review of these studies will be presented deliberately in Section 2 and from which, we identify the gaps and features that distinguish our research from others (see Section 2.3 for details). The aim of this paper is to extend the research on DWC-FLP by including five important considerations, which to the best of our knowledge have not been considered simultaneously in past studies. In particular, we *develop an enhanced planning framework for optimally locating dynamic wireless charging facility considering comprehensively and simultaneously the system level network user costs, travel patterns of individuals, a reassurance that network users have enough range augmentation from DWC to get to their final destination, equity in resource distribution between sub-regions, and total budget availability from the public agency to support the needs of BEVs*. The remainder of the paper is organized as follows: Section 2 presents a summary of related literature in the domain of optimal location for refueling facilities and BEV driver behaviors under the DWC implemented network. In Section 3 we present the modeling approach and solution algorithm proposed in this study. This is followed by a numerical experiment as proof of concept in Section 4. The case study in Section 5 presents the DWC-FLP considering the traffic network dataset in Montgomery County, Maryland USA. Section 6 presents conclusions, limitations of this study, and avenues for future research.

2. LITERATURE REVIEW

We next present a summary of past researches in this domain. Past studies are summarized into two sub-sections. First, we present past efforts towards the determination of charging locations, and then we summarize the literature on BEV drivers' behavior in a road network with recharging facilities. Next, the contribution of this study in light of existing literature is highlighted.

2.1. Determination of BEV Charging Locations

Considering several studies in literature on addressing the problem of charging (either static or dynamic) FLP for BEVs, the review presented herein is not intended to include all the past research but to provide a review of selected researches in the domain of planning framework for BEV charging infrastructure for BEVs. In Table 1, we present a summary of these studies on five main elements which are Study Aspect, Objective Function, Constraints, Approach, and Additional Features. Some important methods considered were the flow-refueling location model (Kuby and Lim, 2007, 2005; Lim and Kuby, 2010), flow-based set covering model (Wang and Lin, 2009) and maximal covering location model (Farahani et al., 2013, 2012).

2.2. BEV Driver Behavior in a Road Network with DWC Facilities

The public agency decides where to implement refueling facilities and in response to that plan, the drivers choose the route that maximizes their utility (or minimizes disutility). However, in the case of a DWC implemented network, a BEV driver may also account for an increase in driving range for his vehicle and hence it should be considered in disutility or cost function. A driver's disutility is also impacted by others' route choice decision as well. These decisions collectively affect the traffic flow as well as travel times in a network. In literature, these decisions are often attributed to the lower level problem representing network user perspective and several studies have been devoted to network flow estimation under given charging facility and range constraint. Past studies have proposed both deterministic approach and stochastic approach under range uncertainty for estimating flows of BEVs. Kitthamkesorn and Chen (2017) solved the combined modal split and traffic assignment problem by using a nested weibit model on the mode choice level and suggest that path-size weibit model on the route choice level since it performs better than the traditional logit model. Liu et al. (2016) developed a model for better fuel economy estimation for electric vehicles by customizing a realistic driving cycle based on the GPS data of drivers in California. Strehler et al. (2017) determined the shortest path for battery electric and hybrid vehicles by creating a model that accounts for several factors that are not usually recognized in the ICEV shortest path problem such as extended recharging time, the balance between speed and range, and regenerative braking. Xie et al. (2017) developed a path-constrained traffic assignment for electric vehicles subject to stochastic driving ranges. Their research focuses on the tour or trip chain rather than the normal trip level where customer range anxiety is more likely to occur. When recharging time is concerned, electric vehicles' battery-charge level may be a non-linear function of recharging time in contrast to ICEV gasoline level which is a linear function of fueling time. To solve this problem, Montoya et al. (2017) proposed a hybrid metaheuristic for solving electric vehicle routing problem that takes into account components considered in ICEV studies and specifically designed components reflecting the non-linear behavior of BEV recharging.

Table 1
Past Studies on BEV Charging Infrastructure

Authors	Study Aspect	Objective Function	Selected Constraints	Approach	Additional Features
Fuller (2016)	Locating DWC in a network consist of selected highways in California to support tour-based between cities trip	Minimizing the capital cost of implementing DWC	Range Constraint	Linear Programming	Sensitivity analysis of vehicle starting range and charging power
Liu and Wang (2017)	Locating multiple types of charging facilities considering public social cost, users' car ownership choice, and users' route choice	Minimizing Weighted sum of travel cost and penalty fee for failed trips	Budget Constraint; Users' route choices follow Wardrop's first principle	Tri-Level Programming	MSA solution algorithm for lower-level user equilibrium
Chen et al. (2016)	Determining the optimal location of DWC	Minimizing total social cost	Budget Constraint	Active-Set Based Approach	New User Equilibrium Model for a DWC implemented network
Sathaye and Kelley (2013)	Determining the location of publicly-funded static charging stations in the Texas Triangle Megaregion	Minimizing total cost	Budget Constraint; selected highway corridors only	Continuous facility location models	Considers both existing charging station built by private institutes and demand uncertainty
Dong et al. (2014)	Locating multi-level of static recharging stations in the greater Seattle area	Minimizing user range anxiety as measured by the number of interrupted trips and missed vehicle miles	Budget Constraint	Genetic algorithm-based optimization	Activity-based approach for simulating driver travel and recharging pattern based on GPS travel survey
Riemann et al. (2015)	A Bi-Level approach to optimally locate DWC from a set of selected facilities	Maximizing the amount of traffic flow re-fueled by the facilities	Covering constraint formulated for the AC-PC flow refueling location model	Mixed-integer nonlinear program	Lower-level network flow problem solved by a Multinomial Logit model based on Stochastic User Equilibrium principle
Liu and Song (2017)	Sequentially determining the optimal location of DWC and optimal battery sizes for electric buses	Minimizing the capital cost of implementing DWC	Power transfer, supply and demand	Deterministic and robust optimization	Considers both (1) a deterministic model ignoring uncertainty in energy consumption and travel time and (2) an affinely adjustable robust counterpart model

Table 1 Continued

Authors	Study Aspect	Objective Function	Selected Constraints	Approach	Additional Features
Chen et al. (2017)	Studying the implementation of different types of charging considering driver's choice of charging facilities	Minimizing social cost as measured by the normalized sum in terms of monetary units of capital cost, charging time, electricity cost, and total driving time	Trip completion assurance;	Mathematical Formulation	Explores the competitiveness of DWC over Static Charging under both public and private provision scenarios; Charging prices follow either Nash equilibrium in private provisions or revenue-neutral in public provision
Xi et al. (2013)	Locating static charging of either Level 1 or 2 for BEVs in the central Ohio region	Maximizing the summation of energy recharged of the entire system	Mutually exclusive charging location; Fixed tour schedule	Linear Integer Programming	Overall service levels are less sensitive to optimization criterion as in contrast to optimal DWC location
Xu et al. (2017)	Studying the factors affecting user choice of charging mode (normal/fast; home/public) and the location of charging facilities in Japan by using users' preference data	Maximum likelihood	None	Mixed Logit Model	Battery capacity, midnight indicator, the initial state of charge, and the number of past fast charging events are important factors in the users' decision-making process
Huang et al. (2015)	Deployment of alternative refueling stations in the transportation network	Minimizing the capital cost of implementing DWC	Charging characteristics	Mixed-integer programming	Utilizes multiple deviations of paths between O-D pairs instead of the shortest path
He et al. (2015)	Locating refueling stations for BEV in a road network using a tour-based approach	Minimizing the capital cost of implementing DWC	Range Constraint	Bi-Level Programming	Considers drivers' spontaneous adjustments, the relation between travel and recharging decisions, and risk-taking behavior
Zhang et al. (2017)	Locating static supercharging for BEVs	Maximizing total flow coverage	Capacitated Flow	Arc Cover-Path Cover	Includes demand dynamics resulting from newly implemented DWC

2.3. Contribution and Significance of this Study

There is a rich literature on the refueling FLP for ICEV but only a few focusing on BEV and even less concerning DWC instead of static charging. Furthermore, in the context of DWC-FLP, the majority of the studies focus on only one level of the bi-level problem which is either optimal DWC facility plan or BEV traffic assignment. Studies considering these two levels simultaneously are minimal and thus are preferable because of the strong interdependency between two levels. However, typically these studies have restricted to small size networks owing to the expense of computational complexity. In addition, past studies using the bi-level approach generally choose their decision variable in the form of a binary variable representing whether or not to implement DWC on the entire length of a link under consideration (Chen et al., 2016; Liu and Wang, 2017). However, this choice makes the model less flexible and it would lead to a sub-optimal result under budget constraint especially in the case of a network containing long links e.g. highways. To be more specific, implementing DWC on an entire length of a highway would be an inefficient use of resources. The problem can be partially addressed by considering the highway as a collection of multiple smaller links. However, even this solution may not be optimal because it raises the question about how to segment the highway and what should be the optimal segment length. Therefore, a continuous variable representing a fraction of link length would be more appropriate for the optimization model. Besides the choice of decision variables, in past studies, the network is treated as a whole which raises problems in practical implementation especially when there are variations in funding priority among sub-regions raising equity concerns.

This study endeavors to bridge these gaps in the literature stated above. We propose a sequential two-level planning approach considering the objectives of both the planner and road users. In the Upper-Level, two different planner objectives namely, total system travel time and total system net energy consumption are considered along with three distinctive elements. First, a trip completion reassurance constraint is used in the planner level to avoid costly failed trips which are important to overcome the range anxiety problem. Second, the proposed approach divides the network into sub-regions (different from traffic analysis zones or TAZ) and adds a constraint representing equity in resource distribution in the upper level to address the differences in funding priority between regions. Third, the model formulation adopts continuous decision variables to provide flexibility in DWC implementation as opposed to binary variables used in past studies. In the Lower-Level, we present a mathematical programming (MP) formulation for a single class BEV static deterministic user equilibrium problem representing users' route choices. The user route cost function takes into account the normalized negative cost incurred due to the recharging of BEV's battery through DWC as the user travels along their preferred path. For solving the BEV user equilibrium, an effective algorithm using a slope-based path shift propensity approach is deployed because of its capability to solve large-scale network in reasonable time.

3. METHODOLOGY

3.1. Modeling Approach

In general, a government agency decides FLP under a macro perspective such as maximizing the social benefits resulting from the facilities (e.g. implementation of DWC) while ensuring that required resources for the implementation would not exceed the agency budget. Hence, one may argue that FLP can be decided based on link flows to benefits a large fraction of network users. The government agency can get information on link flows under the current condition by a variety of methods such as using sensors (e.g. microwave or infrared sensors), traffic cameras, loop detectors, or through the four-step transportation planning. Based on the existing data of traffic flow, a typical approach for implementing DWC facilities would be to locate it on links having higher flow so that more cars can be recharged. However, this approach cannot encapsulate the likely micro interpretation of the network users for whom the facility is planned. In particular, the BEV drivers are likely to choose path by factoring in both DWC implementation and travel time. Given the range constraint of BEVs and range anxiety of BEV drivers, they may prefer the DWC implemented roads. Therefore the roads with high volume will likely be loaded with more traffic and result in high congestion and extended travel time, which is not ideal. Therefore, the approach based on the existing link volume, which does not account for the changes in traffic flow in the network due to DWC, does not yield the optimal result as initially intended by the planning agency. Therefore, for selecting an optimal DWC plan, an analytical framework is warranted that takes into account the network users' response to the DWC plan.

Thinking about a single network user's perspective, he/she chooses the best possible route that minimizes his/her disutility. It is practical to assume that the route which yields the minimum generalized cost (computed by factoring in both travel time and DWC charging) would be selected. The aggregate responses of BEV drivers leading to an equilibrium traffic flow after a DWC plan need to be determined. Therefore, one-level mathematical programming is not appropriate for solving this DWC-FLP since there are two interdependent levels of decision making that is difficult to be modeled seperately. We define these two levels of optimization as an Upper-Level (UL) government agency's DWC implementation decision-making process and a Lower-Level (LL) network users' choosing route process.

The UL and LL problems will interact through a feedback mechanism. The relationship between the UL and LL are shown in Fig. 1. In the UL, with information on current travel time and traffic flow data, government agency defines the length (what fraction of link length) and location (on which links) of DWC implementation with the objective of minimizing total societal cost. The DWC facility implementation plan will consequently affect the network's user path choices leading to changes in the traffic flow pattern and hence travel time of the links which are estimated in the LL.

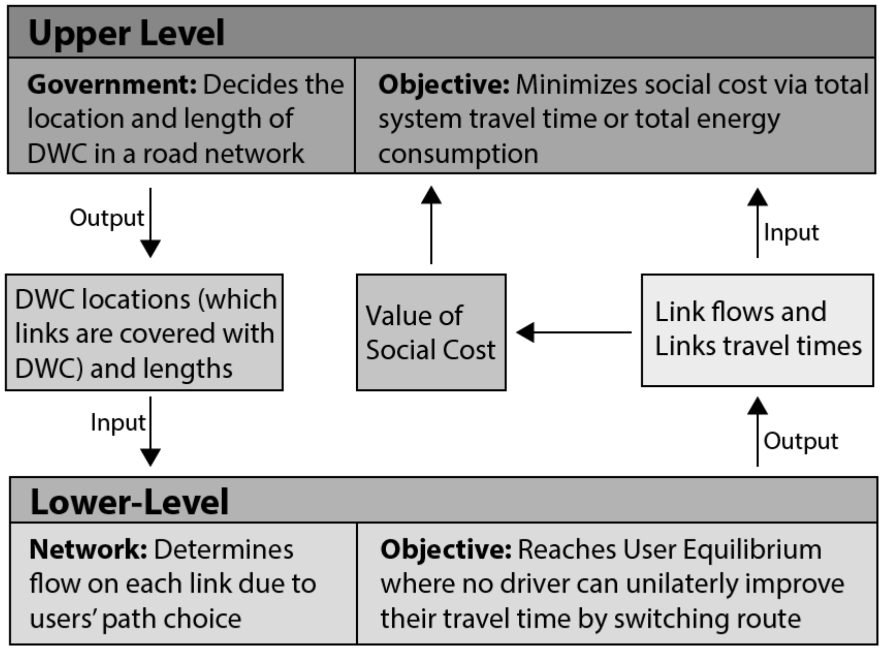


Fig. 1. Schematic representation of the interaction between upper and lower level

We next introduce the notations used in this paper, then UL and LL formulations are presented. Following are the notations used in the paper:

Notations

Set

A Set of links

W Set of Origin-Destination pairs

P_w Set of used paths for O-D pair w

D Set of regions

Parameters

b Cost of implementing dynamic wireless charging facilities (in \$/mile)

θ Agency budget

r Additional recharging miles per miles traveled on DWC charging facilities (in mile/mile)

ψ Power transfer rate (in kWh/mile)

η Cost of one unit of electricity (in \$/kWh)

τ Value of time (in \$/h)

E Upper Limit for equity in resource distribution among sub-regions constraint (Unitless)

l_a Length of link a

μ_a Negative cost experienced by the driver due to DWC recharging along link a (in travel time units, i.e. minutes)

l_a^e Length of link a having DWC charging facility (in mile)

cap_a Capacity of link a

t_a^0 Free flow travel time on link a

g_a Generalized cost for traveling on link a

G_p^w Generalized cost for traveling on path p between an O-D pair w

α_a Coefficient for link a for the link cost function

β_a Coefficient for link a for the link cost function

ζ Average fuel efficiency (in kWh per miles)

s_p Vehicle initial range at the start of the trip using path p

δ_{ap}^w Link-path incident parameter, which takes the value 1 if link a belongs to path p of O-D pair w and 0 otherwise

q_w Travel demand for O-D pair w

f_p^w Flow on path p of the O-D pair w

ε_d Predefined constant for representing funding priority in area d

C_d Preferred resource allocated to area d, reflecting the area's funding priority and road miles

Decision Variables

y_a Length of DWC facility on link *a* as a percentage of the length of link *a*

v_a Flow on link *a*

t_a Travel time on link *a*

γ_a Energy consumption of traveling on link *a* (in kWh)

3.2. Upper-Level of Government Agency Decision Making

While deciding the location of DWC facilities or network improvement, the government agency typically has an objective to minimize the total societal cost. We propose two different metrics to quantify the societal cost. The first metric is Total System Travel Time (TSTT) addressing traffic condition and the second metric is Total System Net Energy Consumption (TSNEC) addressing energy efficiency. The first term TSTT can be calculated by taking the aggregate sum among all links within the network of its flow multiplied by its travel time. TSTT is an important metric to evaluate transportation network performance and thus, it is selected as an objective in the domain of network infrastructure investment in many studies (Marcotte, 1983; Abdulaal and LeBlanc, 1979; Mathew and Sharma, 2009; Chiou, 2005; Konur and Geunes, 2011; Chow et al., 2011; Gao et al., 2011; Hajibabai et al., 2014; FHWA, 2015; Chen et al., 2016, 2017; Jing et al., 2017; Liu and Wang, 2017). Consistent with past literature this study also uses TSTT as an objective at The Upper-Level. The second term TSNEC is determined by taking the aggregate sum over all links in the network of the product between link flow and its corresponding average energy consumption by BEVs traversing on that link. These two objectives namely, TSTT and TSNEC are incorporated in Model 1 and Model 2 respectively.

Model 1: Minimizing total system travel time (TSTT)

Objective Function:

$$\min z_1 = \sum_{a \epsilon A} v_a t_a \tag{1}$$

Subject to:

$$b \sum_{a \epsilon A} y_a l_a \leq \theta \tag{2}$$

$$s_p + \sum_{a \in p} (r y_a l_a - l_a) \geq 0 \quad \forall p \in P_w, w \in W \tag{3}$$

$$0 \leq \ y_a \leq 1 \quad \forall a \in A \tag{4}$$

$$t_a = t_a^0 \left[1 + \alpha_a \left(\frac{v_a}{cap_a} \right)^{\beta_a} \right] \ \forall a \in A \tag{5}$$

$$v_a = f(y_a) \tag{6}$$

$$g_a = t_a + \mu_a \geq 0 \qquad \forall a \in A \tag{7}$$

$$y_a, v_a, t_a, \gamma_a \geq 0 \qquad \forall a \in A \tag{8}$$

Equation (1) represents the objective function of Model 1, which minimizes the total system travel time. The value of the first term, traffic flow (v_a) depends on the UL decision variable, which is the DWC plan represented by the vector (y_a). The second term, travel time (t_a) depends on the traffic flow (v_a). Although the UL decision variable, which is the DWC plan (y_a), is not explicitly present in the objective function, it fundamentally affects the objective function value. Equation (2) states that the accumulation of the cost of implementing DWC within the network must not exceed the agency budget. Equation (3) is a trip completion reassurance constraint designed to avoid a costly failed trip and to overcome range anxiety. We make a simplifying assumption that all vehicles selecting a path between an O-D pair have the same starting range s_p. Based on this assumption, Equation (3) ensures that DWC facilities are implemented in such a way that every vehicle can get sufficient additional range to complete their trip by traveling over the DWC facilities implemented on the links along their path. The second term in Equation (3) represents the additional range obtained from recharging through the DWC facilities. In the commercial market, DWC facility power transfer is measured in kW and by multiplying it with the traveling time of a BEV over the facility, we get the amount of energy in terms of electricity (kWh) transferred to the vehicle's battery. However, in Equation (3), other parameters' units are in terms of distance or miles. We divide the electricity energy by average BEV electricity consumption rate (Wh/mile) to convert it to equivalent range. To simplify the process, we introduce a coefficient r representing the additional range (in mile) per miles of travel over the DWC facility. With a DWC facility power transfer rate of 4 kWh recharged per miles traveled and a 400 Wh/Mile average fuel economy of BEV, the value of r is 10 miles recharged/mile traveled. It implies (based on this example) that if a BEV travels one mile over the DWC facilitated part of a link, it will gain 10 miles of range while losing a single mile of range in traversing that part of the link, hence resulting in net 9 miles of gain in range. Equation (4) implies that the decision variable is a continuous variable representing the length of the DWC facility of a link as a fraction of that link length (hence a value between 0 and 1). Equation (5) is the link cost function (we use BPR function developed by Bureau of Public Roads) where link travel time (t_a) is a monotonically increasing function of link flow (v_a). In Equation (5), t_a^0 is the free flow travel time and α_a, β_a, cap_a are the parameters of link cost function specific to link a. Equation (6) signifies that v_a is the function of the DWC plan (y_a) and given the input (y_a), the LL traffic assignment task generates the output of link flows in the state of user equilibrium. Equation (7) ensures that no link has negative generalized cost that can otherwise promote circular paths by vehicles to gain extra driving range (it is also required for the feasibility of LL problem). The quantity μ_a in Equation (7) is defined by Equation (17) and (18) presented later. Equation (8) represents the non-negativity characteristic of the following decision variables: traffic flow (v_a), DWC plan (y_a), travel time (t_a), and energy consumption (γ_a).

We add an equity in resource distribution constraint to address the differences in funding priority between sub-regions. Typically, a larger geographic region (i.e., state or county) consists of smaller sub-regions and available capital budget in practice is distributed based on funding priority of the region. In addition to funding priority, sub-regions also vary greatly in their area and specifically in the context of DWC-FLP, road land miles. Therefore, the money distributed to a sub-region must reflect both its funding priority and total road lane miles. To illustrate this constraint, let D_d, represents a sub-region, of study area *D* having *d* sub-regions, and these sub-regions are mutually exclusive to each other:

$$D_1 \cup D_2 \cup D_3 \ldots D_m \cup D_n \ldots \cup D_d = D, \quad D_m \cap D_n = \emptyset \quad \forall D_m, D_n$$

The equity constraint is described as follows:

$$\sum_{d \in D} \left(b \sum_{a \in D_d} y_a l_a - C_d \right)^2 \leq E \tag{9}$$

$$C_d = \theta \frac{\varepsilon_d \sum_{a \in D_d} l_a}{\sum_{d \in D} (\varepsilon_d \sum_{a \in D_d} l_a)} \quad \forall d \in D \tag{10}$$

Equation (9) works on the basis of the sum of square of the differences between two terms. The first term is the total DWC implementation cost in a sub-region D_d and the second term C_d is reflecting the preferred resource allocated to sub-region D_d. This sum of square must be less than a predefined constant E, which is empirically determined. Equation (10) states that C_d reflects both the funding priority coefficient ε_d and total road lane miles of sub-region D_d and all C_d sum up to the total budget θ over the entire area D. Equation (9) is based on the assumption that a link is part of only one sub-region C_d. However, this is not a restrictive assumption, and if there are long links in a network that extends to more than one sub-region, then those links can be divided into multiple links each spanning in one sub-region. This will be typically the case for long arterials and interstate highways. Equation (9) is geared toward being an incentive constraint rather than a restricted one. Sub-regions can exceed its preferred budget allocated C_d in ways of improving the UL objective function value, as long as the sum of square of the difference between the preferred budget and the actual DWC cost over the entire region *D* does not exceed E.

Model 2: Minimizing total system net energy consumption (TSNEC)

Objective Function:

$$\min z_2 = \sum_{a \epsilon A} v_a \gamma_a \tag{11}$$

Subject to:

$$\gamma_a = l_a \zeta - r y_a l_a \zeta \quad \forall a \in A \tag{12}$$

and constraints represented by Equations (2)-(10)

Equation (11) represents the objective function of the Model 2 which minimizes the total system net energy consumption by BEVs. The objective function value (in terms of Vehicle.kWh) is the summation among all links of the product between traffic flow and net energy consumption. Equation (12) represents the calculation of net energy consumption. The net energy consumption is computed as the required electricity (in kWh) to traverse link *a* minus the energy recharged through DWC (in kWh) while traveling along link *a*. Model 2 is also subject to the constraints (Equation (2) through (10)) listed in Model 1.

3.3. Lower-Level Network User Equilibrium

The Lower-Level problem aims to estimate the network flows resulting from the network users (BEV drivers) route choices in response to the government's DWC Implementation Plan. With an assumption that drivers are rational in their decision-making process, they will choose the path, among a set of available paths for their trip, which yields a minimum value for the normalized travel time. The generalized travel cost represents the aggregate of the following elements: (1) summation of travel times along the links included in the chosen path; (2) the aggregate benefit derived from DWC facility by BEV as they are driven along the chosen path. In this study, we assume that link costs (travel times) are separable and link travel time of a link depends on the flow of that link only.

The task of deciding the flows of paths/links based on the aggregate of network users path choice decisions is often referred to as a traffic assignment problem. Traffic assignment can be categorized as either static traffic assignment (STA) or dynamic traffic assignment (DTA). Both STA (Jiang et al., 2012; Xie and Jiang, 2016) and DTA (Agrawal et al., 2016) models have been used to characterize the route choice behavior of BEV drivers in the past. STA assumes that traffic is in a steady-state and hence flows and travel times of links can be represented using average conditions. The DTA models can capture the traffic flow dynamics more accurately compared to STA models due to the presence of temporal dimension in the model. Therefore, DTA can be utilized to accurately estimate the energy consumed by BEV, analyze the effectiveness of an operational strategy (e.g., signal coordination), and improve the traffic flow estimation. However, DTA models are characterized by inherent mathematical intractability (Peeta and Ziliaskopoulos, 2001) and deploying them in practical context entails simulation of the time-dependent traffic flow which is computationally expensive. It is difficult to design an efficient solution algorithm for a network design problem (e.g., optimal DWC location problem) that requires estimation of network flows numerous times using a DTA model. Therefore, due to these limitations of DTA, the STA is preferred in transportation planning context and is usually applied for network design problems (see e.g., Kumar and Mishra, 2018; Mishra et al., 2016). Taking the above factors into consideration, we seek to develop a static user equilibrium traffic assignment model to characterize the route choice behavior of BEV drivers in a network with DWC facility.

Wardrop's User equilibrium (UE) principle is mostly used for finding the network flows in a transportation network. It states that the journey times in all routes actually used are equal and less than those that would be experienced by a single vehicle on any unused route. UE is achieved when drivers cannot improve their travel time (cost) unilaterally by switching routes. According to Sheffi (1985), under non-negative monotonically increasing separable link cost function and non-negative demand, the UE-STA problem can be formulated as a convex optimization problem. In the context of this study, we present an MP formulation for single class BEV static deterministic user equilibrium (BEV-UE) problem in a network with DWC facility. The BEV-UE needs to incorporate changes in link cost functions due to DWC investment decisions. The BEV-UE problem is formulated as follows:

BEV-UE:

Objective Function:

$$\min z_3 = \sum_{a \in A} \left(\int_0^{v_a} t_a(x_a)\, dx + \mu_a v_a \right) \tag{13}$$

Subject to:

$$\sum_{p} f_p^w = q_w, \qquad \forall\, w \in W \tag{14}$$

$$f_p^w \geq 0, \quad \forall\, p \in P_w, w \in W \tag{15}$$

The definitional constraints:

$$v_a = \sum_{w \in W} \sum_{p \in P_w} \delta_{ap}^w f_p^w\,, \quad \forall\, a \in A \tag{16}$$

$$\mu_a = -l_a^e \psi \eta \left(\frac{60}{\tau}\right), \forall\, a \in A \tag{17}$$

$$l_a^e = y_a l_a, \forall\, a \in A \tag{18}$$

Equations (13)-(18) represents the BEV-UE formulation under the DWC facility proposed in this study. Equation (13) represents the minimization of the objective function. Equation (14) is the flow conservation constraint. Equation (15) ensures that path flows are non-negative. Equations (16)-(18) are definitional constraints. Equation (16) defines the relationship between link and path flows. Equation (17) defines the negative cost experienced by BEV drivers due to DWC charging. Equation (18) determines the length of the link covered with DWC facility and connects UL decision variables to the LL problem. Next, we prove the equivalency of above MP formulation with user equilibrium of BEVs in a DWC facilitated network.

Proposition. Under the assumption of monotonically increasing separable link cost function, the MP formulation presented by Equations (13)-(18) is equivalent to Wardrop User Equilibrium of BEV drivers defined as below:

BEV-UE in a DWC facilitated network is achieved when generalized cost of all used paths between an O-D pair are equal which is less than or equal to generalized cost of any unused paths.

Proof: The Lagrangian of the minimization problem represented by Equations (13)-(18) can be formulated as:

$$\mathcal{L}(\boldsymbol{f}, \boldsymbol{\sigma}) = z_3(\boldsymbol{f}) + \sum_{w \in \boldsymbol{W}} \sigma_w \left[q_w - \sum_{p \in \boldsymbol{P}_w} f_p^w \right] \tag{19}$$

where, σ_w is the Lagrange multiplier associated with equality (flow conservation) constraint represented by Equation (14). Note that definitional constraints do not enter in the Lagrange function $\mathcal{L}(.)$. At the stationary point of Lagrangian, the following conditions need to hold:

$$f_p^w \frac{\partial \mathcal{L}}{\partial f_p^w} = 0, \forall p \in \boldsymbol{P}^w, w \in \boldsymbol{W} \tag{20}$$

$$\frac{\partial \mathcal{L}}{\partial f_p^w} \geq 0, \forall p \in \boldsymbol{P}^w, w \in \boldsymbol{W} \tag{21}$$

$$\frac{\partial \mathcal{L}}{\partial \sigma_w} = 0, \forall w \in \boldsymbol{W} \tag{22}$$

In addition to conditions (20)-(22), non-negativity constraints (15) of path flows need to be satisfied. Condition (22) simply states that the flow conservation condition needs to hold. Now for notational simplicity, we focus on a single O-D pair $w \in \boldsymbol{W}$. However, the derived results will be valid for all O-D pairs. The partial derivatives of Lagrangian $\mathcal{L}(.)$ with respect to path flow variable is given as:

$$\frac{\partial \mathcal{L}}{\partial f_p^w} = \frac{\partial}{\partial f_p^w} z_3(\boldsymbol{f}) - \sigma_w \tag{23}$$

Using the diagonal rule, the partial derivatives of z_3 with respect to f_p^w is given as:

$$\frac{\partial z_3}{\partial f_p^w} = \frac{\partial z_3}{\partial x_a} \frac{\partial x_a}{\partial f_p^w} \tag{24}$$

Noting the fact that the partial derivate of x_a with respect to f_p^w is δ_{ap}^w, the partial derivatives of z_3 with respect to f_p^w is given as:

$$\frac{\partial z_3}{\partial f_p^w} = \sum_{a \in A} (t_a + \mu_a) \delta_{ap}^w \tag{25}$$

Note that $t_a + \mu_a = g_a$ is the generalized cost of traveling on a link a. Using Equation (25) partial derivatives of z_3 with respect to f_p^w is the generalized cost of path p represented as G_p^w:

$$\frac{\partial z_3}{\partial f_p^w} = G_p^w \tag{26}$$

Therefore, using Equation (23), the partial derivatives of $\mathcal{L}(.)$ with respect to $f_p^w$is given as:

$$\frac{\partial \mathcal{L}}{\partial f_p^w} = G_p^w - \sigma_w \tag{27}$$

Now, using Equations (20), (21) and (27) we get:

$$f_p^w(G_p^w - \sigma_w) = 0, \forall p \in \boldsymbol{P}^w, w \in \boldsymbol{W} \tag{28}$$

$$G_p^w - \sigma_w \geq 0, \forall p \in \boldsymbol{P}^w, w \in \boldsymbol{W} \tag{29}$$

The Equations (28) and (29) together imply that either flow on a path f_p^w is zero or its generalized cost G_p^w is equal to Lagrange multiplier σ_w. In addition, the Equation (29) implies that Lagrange multiplier σ_w of a given O-D pair is less than or equal to the generalized cost of all paths connecting this O-D pair. Now considering σ_w as the minimum generalized path cost for the O-D pair w, this is equivalent to the condition of Wardopian User Equilibrium. This proves that the MP formulation presented by Equations (13)-(18) is equivalent to Wardrop's User Equilibrium for BEV drivers in a network with DWC facility.

3.4. Solution Algorithm

The DWC-FLP problem is modeled as a Bi-Level Programming with a computationally heavy objective function. There are two main reasons for this difficulty as follows: (1) the value of the objective function cannot be explicitly calculated by the UL decision variable (i.e. DWC plan) alone; and (2) it requires an additional sub-level optimization model (i.e. BEV-UE) to compute the components (i.e. travel time and energy consumption) and ultimately the objective function value (OFV) and thus demands heavy computational time. In order to solve this problem, we utilize and extend an algorithm called Constrained Local Metric Stochastic Response Surface (ConstrLMSRS) developed by Regis (2011) to solve the Bi-level problem.

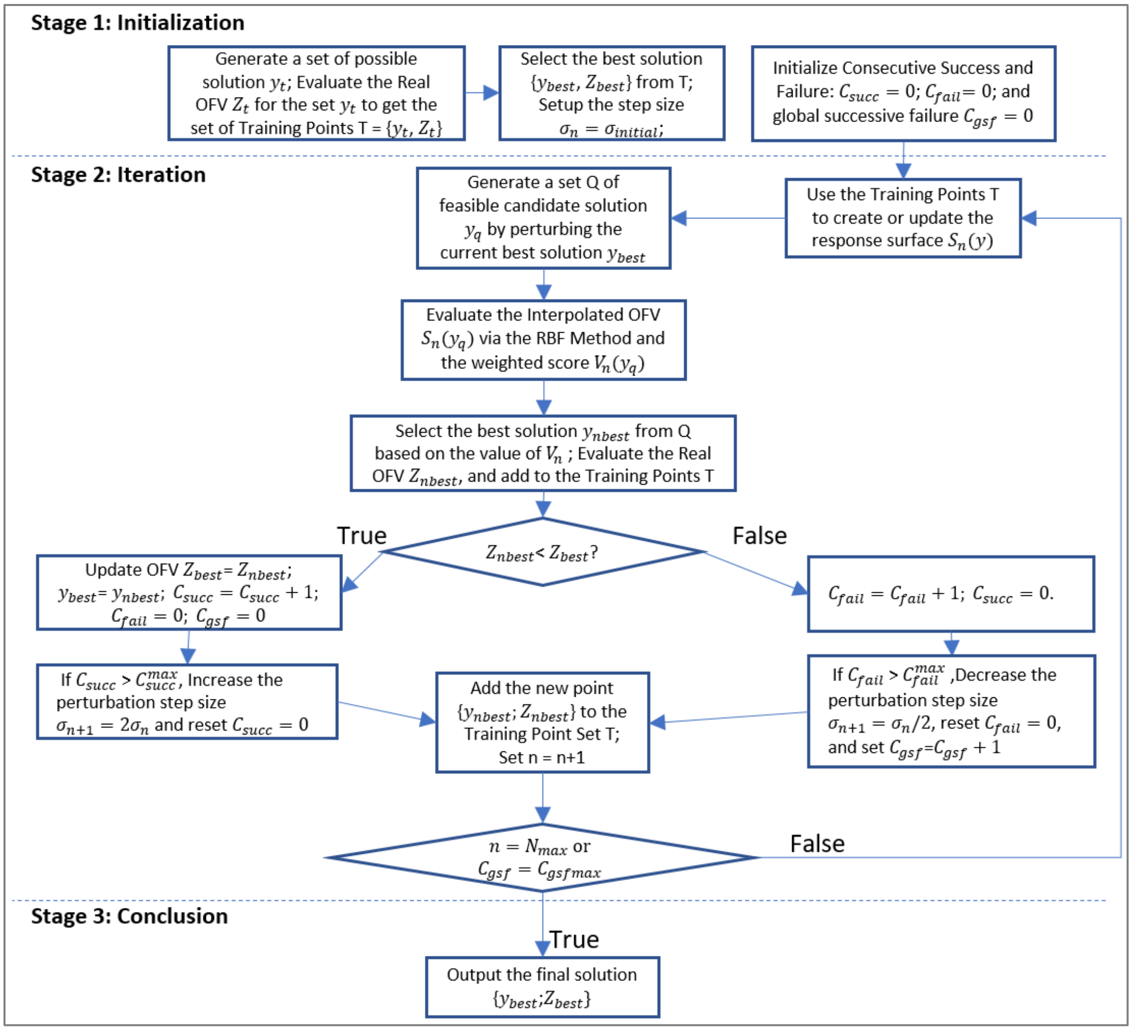


Fig. 2. Modified ConstrLMSRS Algorithm Flowchart

The algorithm works as a feedback loop until the termination criteria are met. It consists of three stages: initialization, iteration, and conclusion as shown in Fig. 2. We have presented the pseudo-code of the UL solution algorithm in the appendix.

At each iteration, a large number of candidate feasible solutions (we choose 20,000 in our numerical experiment and case study) are generated. Each candidate solution requires running the traffic assignment task for the LL BEV-UE to get the objective function value. Thus, the process of performing this task for the set of candidate solutions is an expensive task. In order to address this problem, while estimating the objective function value for each candidate solution, the Radial

Basis Function (RBF) Interpolation method is utilized in substitution of performing the BEV-UE traffic assignment. The following section discusses briefly describes this method.

Radial Basis Function (RBF) Interpolation Method

The RBF interpolation was introduced by Powell (1992) and used by Regis (2011) in solving an optimization problem with an expensive objective function. The method can be processed with small computational cost. Here we present a brief overview of this method. For the full description of the radial basis function interpolation, please refer to (Powell, 1992).

Given a set of T training points, of which OFV are known : $T = \{y_t, Z(y_t)\}$ we can construct a response surface model: $S(y) = \sum_t \omega_t \phi \, \|y - y_t\| + l(y)$ to interpolate the objective function value. Note here that y_t is a variable with a dimensions.

Where:

$\phi(r)$: A cubic form function $\phi(r) = r^3$

$\|.\|$: Euclidean norm

$l(y)$: A linear polynomial function in a variables to be determined, with a coefficient c which has $(a+1)$ dimensions

ω_t: A coefficient to be determined which has t dimensions

The two coefficients ω_t and c can be calculated as follows:

$$\begin{pmatrix} \lambda & H \\ H^T & 0_{(a+1)\times(a+1)} \end{pmatrix} \begin{pmatrix} \omega_t \\ c \end{pmatrix} = \begin{pmatrix} Z^T(y_t) \\ 0_{a+1} \end{pmatrix}$$

Where

λ: A matrix with $t \times t$ dimension, calculated as: $\lambda_{ij} = \phi\|y_i - y_j\|$ with $i, j = 1,2 \dots, t$

H: A matrix with $t \times (a+1)$ dimension, where the t^{th} row is $[1, y_t^T]$.

By solving this set of equations, we acquire the value of the coefficient $\omega_t = (\omega_1, \omega_2 \dots, \omega_t)^T$ and $c = (c_1, c_2 \dots, c_{a+1})^T$. By plugging these coefficients back, the response surface model $S(y)$ is constructed and utilized to interpolate the OFV of a candidate solution y.

The main advantages of ConstrLMSRS are wider searching range, faster evaluation of the objective function. For the first advantage, in each iteration, a large set of candidate points is generated via perturbing the current best solution. The perturbation step size is selected as a continuous variable to sufficiently cover all the possible solutions. Other approaches such as Active-Set Based Algorithm (Chen et al., 2016) only consider one feasible solution at a time. As a result, the searching region can cover a wide range of possible solutions without compensating computational time and power by evaluating them via the RBF method. For the second advantage, instead of running a traffic assignment task for each candidate point generated within each iteration, only one traffic assignment task is computed for the best candidate point per iteration, which results in a significantly fewer computing step, complexity, and time. It is important to mention that the study uses ConstrLMSRS method as it is able to deal with real-world size network with moderate computational time but other heuristic algorithms can also be used for this purpose such as Memetic Algorithm (Pishvaee et al., 2010), Differential Evolution (Koh, 2007), Evolutionary Algorithms (Lau et al., 2009) and Hill climbing (Los and Lardinois, 1982).

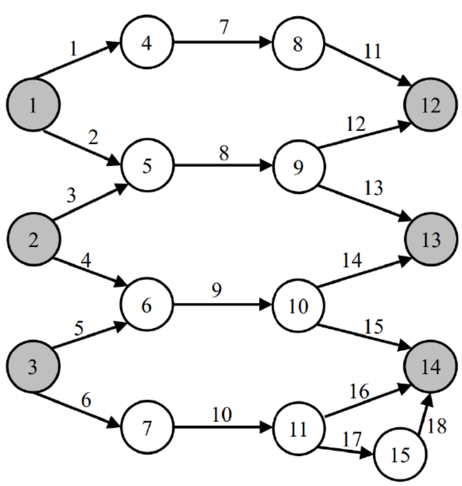
Lower Level BEV-UE Solution Algorithm

The LL problem (BEV-UE) is solved by customizing the SPSA algorithm developed by Kumar and Peeta (Kumar and Peeta, 2014). The SPSA flow update mechanism was used with the modified cost function and has been implemented in this study through a C++ script. Modified cost function includes the travel time and negative cost due to DWC charging. The SPSA yields a UE link flows and link travel times which is feedback to the UL. The SPSA implementation steps are not presented here for brevity (The readers can refer to Kumar and Peeta, (2014) for SPSA implementation details).

4. NUMERICAL EXPERIMENTS

4.1. Small Test Network

Numerical experiments are first conducted using a small size test network to obtain insights before conducting detailed analysis. The topology of the test network is shown in Fig. 3. The network consists of 15 nodes, 18 links, three origins, and three destinations. Three origins are represented as nodes 1, 2, and 3. Similarly, three destinations are nodes 12, 13, and 14. The number inside the circle represents node number and the number beside the link represents the link number. The travel demand for various O-D pairs and paths in the form of a sequence of links is also shown in Fig. 3. There are six O-D pairs with non-zero travel demand. In addition, we divide the network into 8 sub-regions, each has its own set of links and funding priority coefficient ε_d as shown in Fig. 3. Table 2 presents the links parameters of the test network.



O-D Demand

O\D	12	13	14
1	10000	4000	0
2	0	10000	3000
3	0	60000	10000

O-D	Path Number	Path as the link sequence
1-12	1 2	1-7-11 2-8-12
1-13	3	2-8-13
2-13	4 5	3-8-13 4-9-14
2-14	6	4-9-15
3-13	7	5-9-14
3-14	8 9 10	5-9-15 6-10-16 6-10-17-18

Sub-Regions	Links	ε_d
1	1,2	1
2	3,4	1
3	5,6	1
4	7,8	1
5	9,10	1
6	11,12	1
7	13,14	1
8	15,16,17,18	1

Fig. 3. Small 18 Link Test Network

Table 2
Link Properties of Test Network

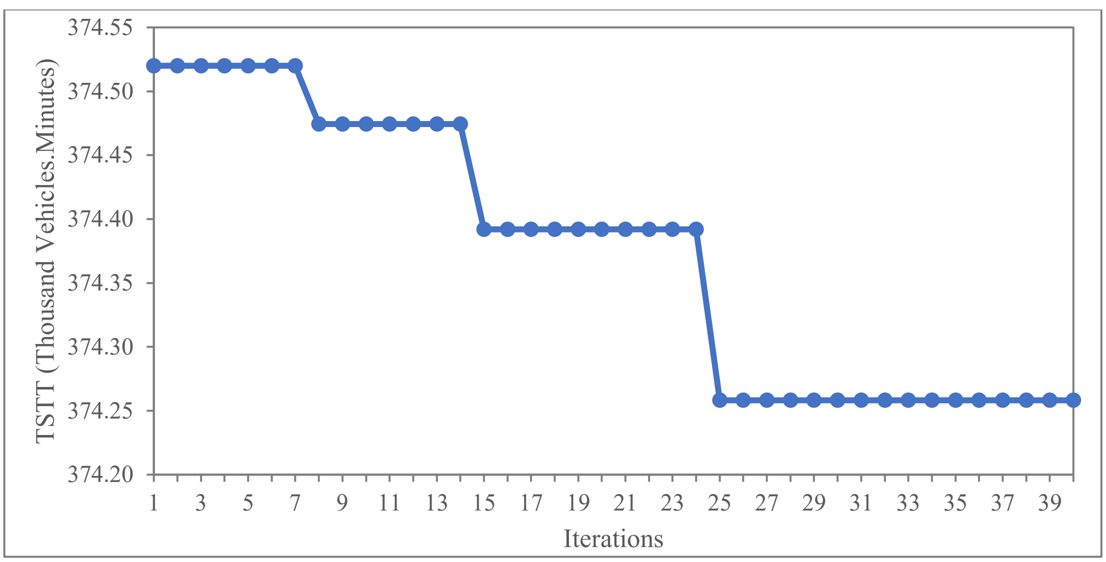
Link Number	From Node	To Node	c_a	t_a^0	α_a	β_a	l_a
1	1	4	3000	1.25	0.15	4	1.3
2	1	5	4000	1.25	0.13	4.1	1.3
3	2	5	5000	1.25	0.1	3.9	1.3
4	2	6	3000	1.25	0.12	3.8	1.3
5	3	6	7000	1.25	0.13	3.5	1.3
6	3	7	6000	1.25	0.125	3.2	1.3
7	4	8	3500	1.25	0.128	3.3	1.3
8	5	9	8000	1.25	0.127	3.4	1.3
9	6	10	9000	1.25	0.13	3.9	1.3
10	7	11	2500	1.25	0.132	4.2	1.3
11	8	12	3500	1.25	0.133	4.6	1.3
12	9	12	4000	1.25	0.134	4.2	1.3
13	9	13	4500	1.25	0.136	3.3	1.3
14	10	13	5000	1.25	0.139	3.8	1.3
15	10	14	4000	1.25	0.138	3.2	1.3
16	11	14	3800	1.25	0.14	3.6	1.3
17	11	15	3800	1.00	0.14	3.6	1.1
18	15	14	3800	0.25	0.15	3.2	0.3

Assumptions

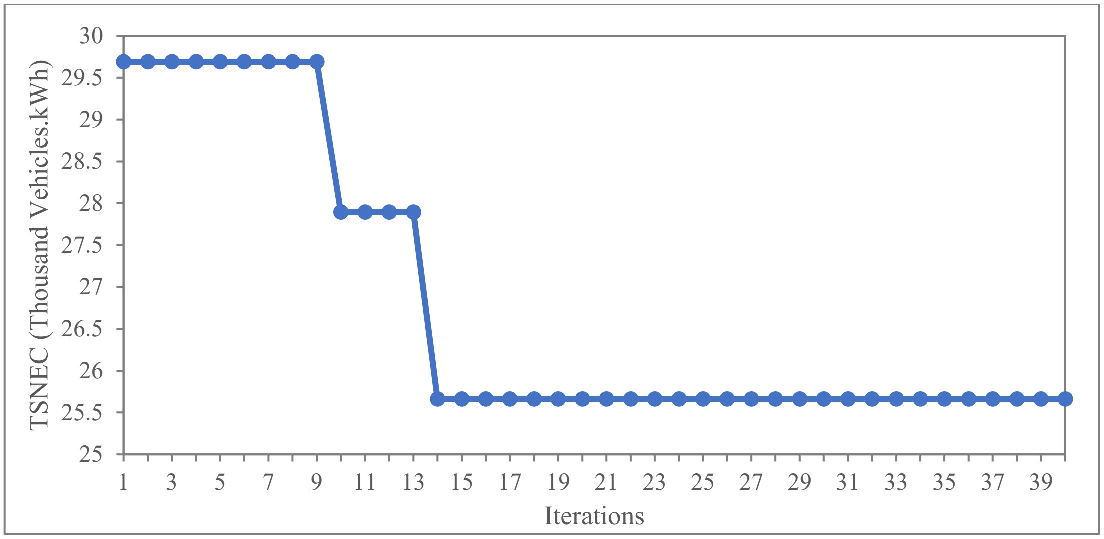
The study makes some assumptions for conducting numerical experiments which include: (1) the cost of implementing DWC is $4 million per lane per mile, (2) all vehicles using the network are BEVs and have the capability to be charged with DWC, (3) % DWC refers to inductive charging available as a percentage of the link and in case of multiple lanes, only one lane is implemented with DWC facility, (4) the problem considered is an un-capacitated refueling model which indicates that there is no limitation on the number of vehicles being charged at the same facility (in our case a given section of link) at the same time, and (5) the public agency has $3.6 million in budget and this budget scenario is herein referred to as the Base Scenario to distinguish itself with other budget scenario mentioned in Section 4.3 Budget Sensitivity Analysis.

4.2. Numerical Results and Insights

To assess the model convergence, the UL objective functions value within each iteration are stored for performance assessment purpose. Fig. 4(a) shows the TSTT objective function value with the progress of iterations. The objective function value starts at 374,519 and decreases further with iterations. There are significant drops in the objective function value at the 8th, 15th, and 25th iteration, and it reaches the minimum value 374,258 after the 25th iteration. The algorithm terminates at the 40th iteration of. We see that the objective function value is not improving after the 25th iterations. This is due to the fact that the set of training points already covered most of the "peaks" and the iteration does not need to "search" any further. At the end of iterations, the TSTT value represents the objective function corresponding to the final best solution for the DWC plan.



(a) TSTT objective function convergence with iterations



(b) TSNEC objective function convergence with iterations

Fig. 4. TSTT and TSNEC convergence with iterations

Similar to Fig. 4(a) for TSTT, Fig. 4(b) shows the TSNEC objective function value with increasing iterations. In iteration 1, the TSNEC value was 29,690 which reduced to 27,894 in the 10th iteration and reached the minimum value of 25,662 after the 14th iteration. The TSNEC model reaches convergence sooner than the TSTT model. The result DWC plans for both TSTT and TSNEC in the Base Scenario are presented later in Section 4.3.

To validate the benefit of DWC in Model 2, we calculate the changes in TSNEC as compared to the Do-Nothing scenario and total energy recharged under various user route choice scenarios. The scenarios are developed by modifying the convergence criteria of the SPSA algorithm (Kumar and Peeta, 2014). The percentage of difference (nue_frac) between a route choice scenario and the base user equilibrium is computed as follow:

$$nue\_frac = \frac{1}{|A|}\sum_{a\in A}\frac{|v_a^{ue} - v_a^{nue}|}{v_a^{ue}}$$

Where:

$|A|$ Cardinality of set A

nue_frac: Percentage difference in route choice from UE

v_a^{ue}: Flow of link a resulting from drivers on UE path

v_a^{nue}: Flow of link a resulting from drivers on non-UE paths

A used path is considered as non-UE if its generalized cost is higher than minimum cost path of the O-D pair by more than 1% margin. Algorithm at the lower level was terminated as the value of nue_frac falls below various threshold levels (e.g. 10%, 20%). Table 3 shows the percentage of decrease in TSNEC as compared to the Do-Nothing scenario and the total energy recharged for various user route choice scenarios (represented by percentage difference from UE, nue_frac). The DWC plan is taken from the result of Model 2 minimizing TSNEC under a budget of $3.6 million. The benefit from DWC drops as BEV users deviate from the user equilibrium state but only by a marginal margin. At the base user equilibrium, the percentage decrease in TSNEC and total energy recharged are 62% and 41,489 (Vehicle.kWh) respectively. The numbers drop noticeably in the 10% Difference scenario at only 55% and 36,636 (Vehicle.kWh). However, the decreasing rate flattens out as the percentage of difference increases. At the 60% Difference scenario, the percentage decrease in TSNEC and total energy recharged remains at a high level of 47.14% and 31,631 (Vehicle.kWh) respectively.

Table 3
Benefit from DWC implementation at Different Route Choice Scenario

User route choice scenario	% Decrease in TSNEC (compared to Do-Nothing)	Total energy recharged (Vehicle.kWh)
Base (UE)	61.88%	41,489
10% Difference	54.69%	36,636
20% Difference	53.92%	36,149
30% Difference	53.72%	36,168
40% Difference	53.45%	35,973
50% Difference	47.32%	31,872
60% Difference	47.14%	31,631

We also attempted to validate the RBF Interpolation performance since a poor estimation of the objective function would result in an incorrect best candidate point. Fig. 5 shows the objective function value in both TSTT and TSNEC (plotted on the secondary axis) as estimated by the RBF Interpolation method (shown dotted) and by using modified SPSA (BEV-UE solution) method (shown as a solid line). The performance of RBF Interpolation is positive with a root mean square error between the predicted value and the actual value for the TSTT and TSNEC models as 465.17 and 51.62 respectively.

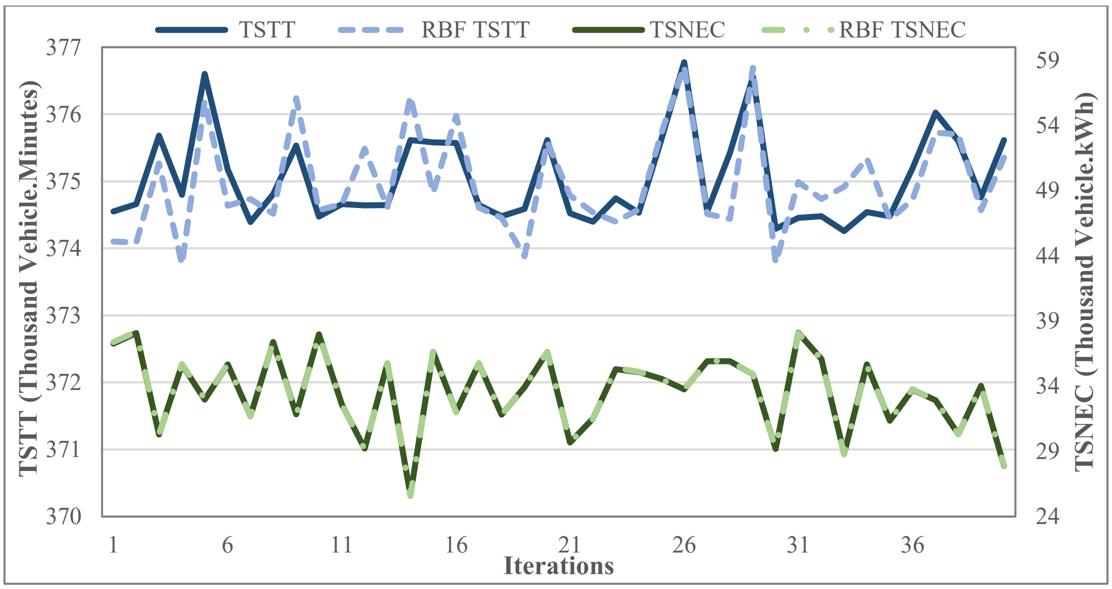


Fig. 5. Values of TSTT and TSNEC via RBF and Traffic Assignment Approaches

4.3. Budget Sensitivity Analysis

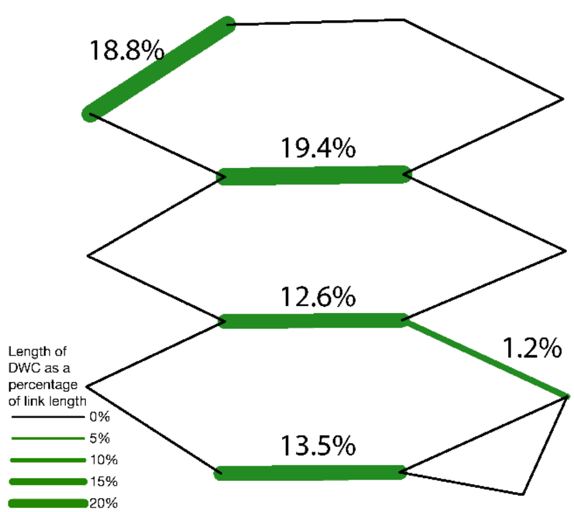
A sensitivity analysis with respect to budget was performed. The results of budget sensitivity analysis in terms of optimal values of TSTT and TSNEC are shown in Table 4. For a budget of \$3.6 million, Model 1 of TSTT minimization results in an optimal TSTT value of 374,258 and its TSNEC is computed as 28,828. Similarly, Model 2 of TSNEC minimization results in an optimal TSNEC value of 25,612 and its TSTT is computed as 375,714. Two other budget scenarios were considered to assess model performance. One lower budget of \$3.4 million (i.e., 5% less than the base budget of \$3.6 million), and one higher budget of \$3.8 million (i.e., 5% more than the base budget) are used as two more budget scenarios. Optimal and computed values for TSTT and TSNEC respectively for a budget of $3.4 and $3.8 million are also presented in Table 4.

Table 4
Result of Sensitivity Analysis with Respect to Budget

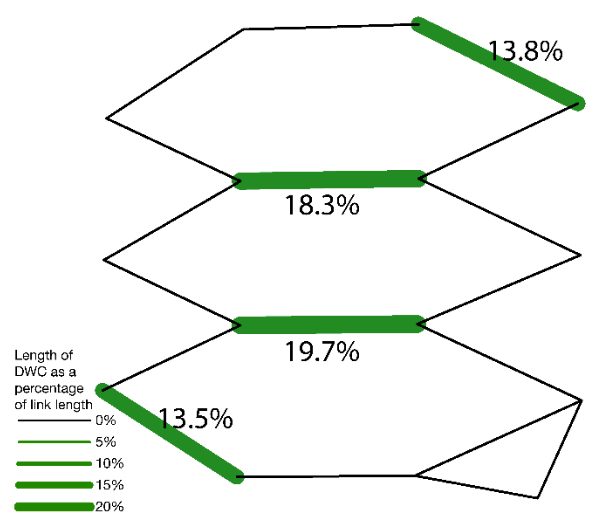
Objective Function	Budget	Budget Scenario	TSTT Value	TSNEC Value
TSTT	$3.6 million	Base	374,258*	28,828
TSNEC	$3.6 million	Base	375,714	25,612*
TSTT	$3.8 million	1.05 x Base	374,455*	32,695
TSNEC	$3.8 million	1.05 x Base	375,607	24,308*
TSTT	$3.4 million	0.95 x Base	374,250*	31,021
TSNEC	$3.4 million	0.95 x Base	375,550	27,582*

*Note: * shows optimal objective function value*

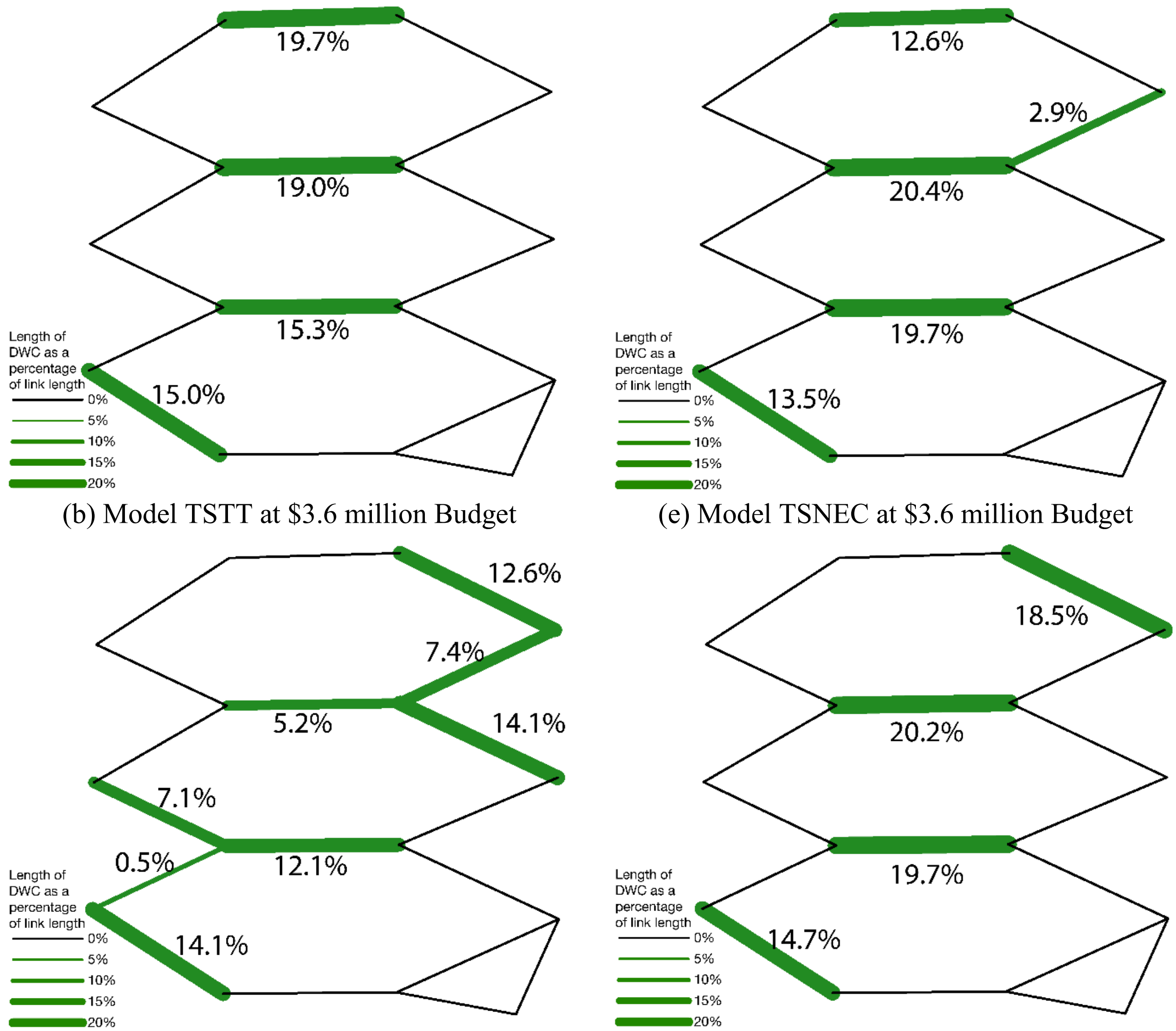
Fig. 6(a)-(c) shows the percentage of DWC implemented in the 18-link network with a budget of $3.4, $3.6, and $3.8 million respectively with the objective function TSTT. Similarly, Fig. 6(d)-(f) shows the percentage of DWC implemented in the 18-link network with a budget of $3.4, $3.6, and $3.8 million respectively when the objective function is TSNEC. Overall, six scenarios were analyzed considering three budget levels for each objective function TSTT and TSNEC as summarized in Table 4. These numerical experiments provide some useful insights and are presented next.



(a) Model TSTT at $3.4 million Budget



(d) Model TSNEC at $3.4 million Budget



(b) Model TSTT at \$3.6 million Budget (e) Model TSNEC at \$3.6 million Budget

(c) Model TSTT at \$3.8 million Budget (f) Model TSNEC at \$3.8 million Budget

Fig. 6. DWC Plan under Different Budgets

The first observation from Fig. 6 is that if two links are in the same sub-region (please refer to Fig. 3 for sub-region layout), both Model 1 and 2 tend to apply DWC on only one link and the other would not receive any. However, the net gain in range due to DWC recharging among various path for a given OD pair do not suffer too much from this since each path has at least one link covered with DWC. This conforms to the constraint of equity in resource distribution, and in this case, all sub-regions are treated equally with the same funding coefficient ε_d.

The second observation is that links 8 and 9, which are in the middle of the network, receive DWC treatment in all budget scenarios and models. In general, links which are traversed by multiple paths (or shared link) are prioritized for DWC. Both links 8 and 9 are part of three used paths, which is higher than any other links. As a result, an investment on DWC on links 8 and 9 can be considered more cost-effective than others since those links provide services to multiple paths. The rationale behind this prioritization is in a tight budget scenario, if DWC facilities are implemented on links serving only one path, there would not be sufficient facilities to ensure all vehicles completing their trip without battery depletion. However, the amount of DWC implemented on links 8 and 9 should take into account the planner objective (i.e. TSTT and TSNEC). In contrast, links 16, 17, and 18 do not have any DWC treatment since those links constitute only one path.

The third observation from Fig. 6, is that even under the same budget scenario, two different objective functions leads to two distinctly different results (set of y_a) implying that the optimal location of DWC facility will differ based on agency's objective (TSTT versus TSNEC). The TSNEC model favors the centralized approach which is shown in Fig. 6(d) and Fig. 6(f) where only 4 links out of 18 links are selected for DWC. The TSNEC objective function incentivizes the DWC plan to recharge as much vehicle as possible. One can expect higher traffic flows on links implemented with longer length of DWC which ultimately enhances the objective function value of TSNEC. In the example of links 8 and 9, the amount of DWC implemented is considerably higher than the others in all budget scenarios. In contrast, the TSTT model prefers a disperse approach toward DWC Implementation because in this case, travel time is a concern in the objective function. One disadvantage in the viewpoint of DWC facilities concentrating on selected links is that those links will attract more users leading to an increase in traffic flow and ultimately to higher travel time. If DWC facilities are implemented in a sprawling approach, users will have multiple choices for travel while gaining DWC benefits and network flow will be distributed more evenly to suit the planner's objectives of TSTT. In particular, looking at coverage (what percentage of link length) of DWC suggested by proposed models on links 8 and 9, we observe that for every budget scenario, TSTT based model tend to suggest smaller coverage of DWC on these two links compared to that suggested by TSNEC based model. This is due to the fact that TSTT favors the sprawling approach for DWC to avoid congestion and thereby provides multiple DWC enabled routes to BEV drivers. The difference between these two models is further reinforced numerically by Table 4, which indicates that for each budget scenario, the TSTT model results in a lower TSTT value and higher TSNEC value compared to the TSNEC model and vice versa.

5. CASE STUDY

5.1. Montgomery County Network

The proposed framework is applied to the Montgomery County network in Maryland as the case study to attest to the applicability of the proposed approach for real-size networks in practice. Montgomery is the most populous county in the state with a population close to one million, 400,000 households, and 600,000 employment. The County boundary and transportation network are presented in Fig. 7. The County contains parts of the heavily traveled roadways in the Washington DC-Baltimore region (Washington DC is referred to as Washington in the remainder of the paper). The County has an extensive highway network with the Capital Beltway (or Interstate-495), which surrounds Washington, passing through Montgomery County. Interstate-270 forms one leg of an interstate triangle between Washington DC, Baltimore City, and Frederick city. The County also contains a portion of route 29, one of the major state routes, which traverses the Washington and Baltimore beltways. The Montgomery network consists of 4,420 links, 1,752 nodes, 225 of which are Origin-Destination nodes, and 34,187 Origin-Destination pairs with non-zero demand. The demand in the morning peak hour period is 3,564,993 vehicles. Montgomery

County has an extensive continuous emission monitoring (CEM) program and the mission is to examine emission reduction strategies. This paper is geared towards this mission by proposing an emission reduction strategy using the proposed DWC implementation model to facilitate the adoption of BEVs. However, the proposed methodology can be extended to other regions as well.

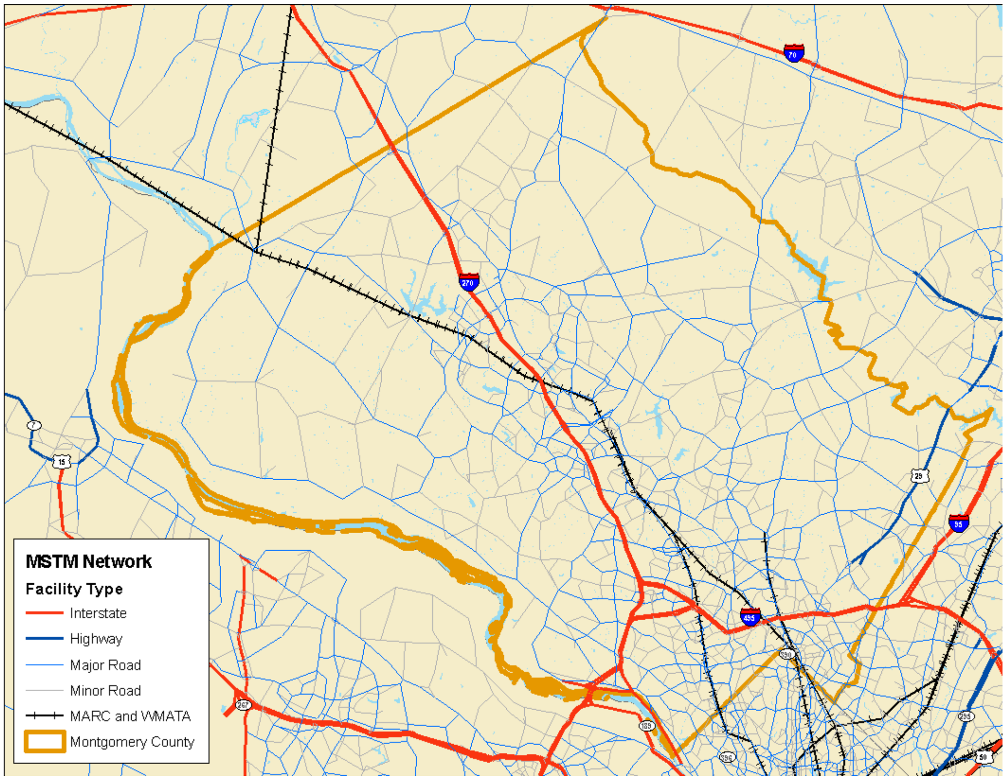
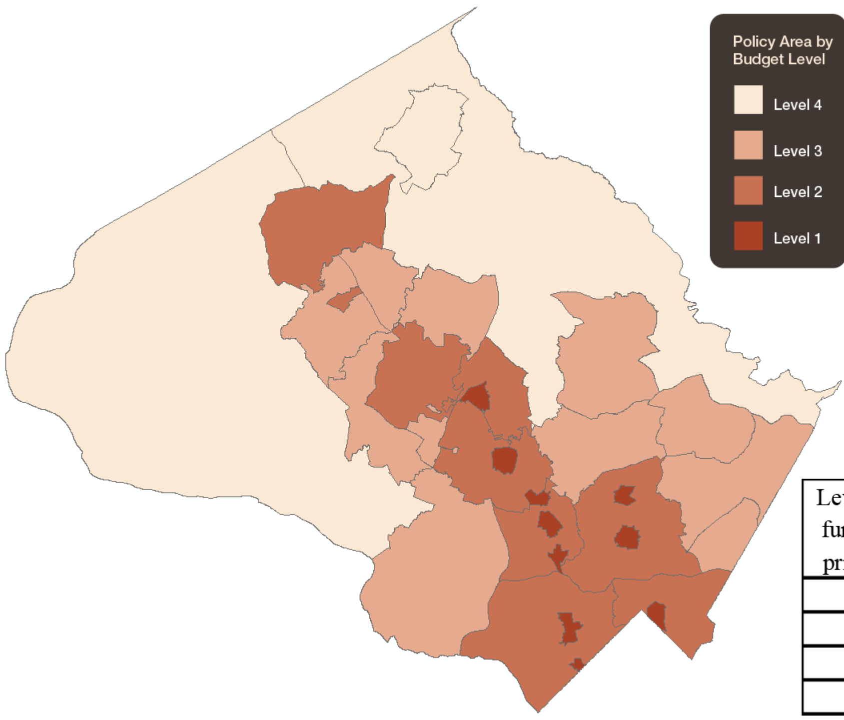


Fig. 7. Montgomery County Transportation Network

5.2. Equity in Transportation Funding

The Montgomery County area is divided into smaller areas for the purpose of allocating resources which are called Transportation Policy Regions. The resources of each Policy Region are meant for the investment into the road exclusively confined within that area. In addition, the Montgomery County planning commission defines four levels of funding priority for each Transportation Policy Region. These four levels of funding raise the problem of equitable distribution of resources, which restrains the optimization model. The division of The Transportation Policy Region and its level of funding priority are shown in Fig. **8**.



Level of funding priority	ε_d
4	0.85
3	0.9
2	0.95
1	1

Fig. 8**. Transportation Policy Region by Funding Level**

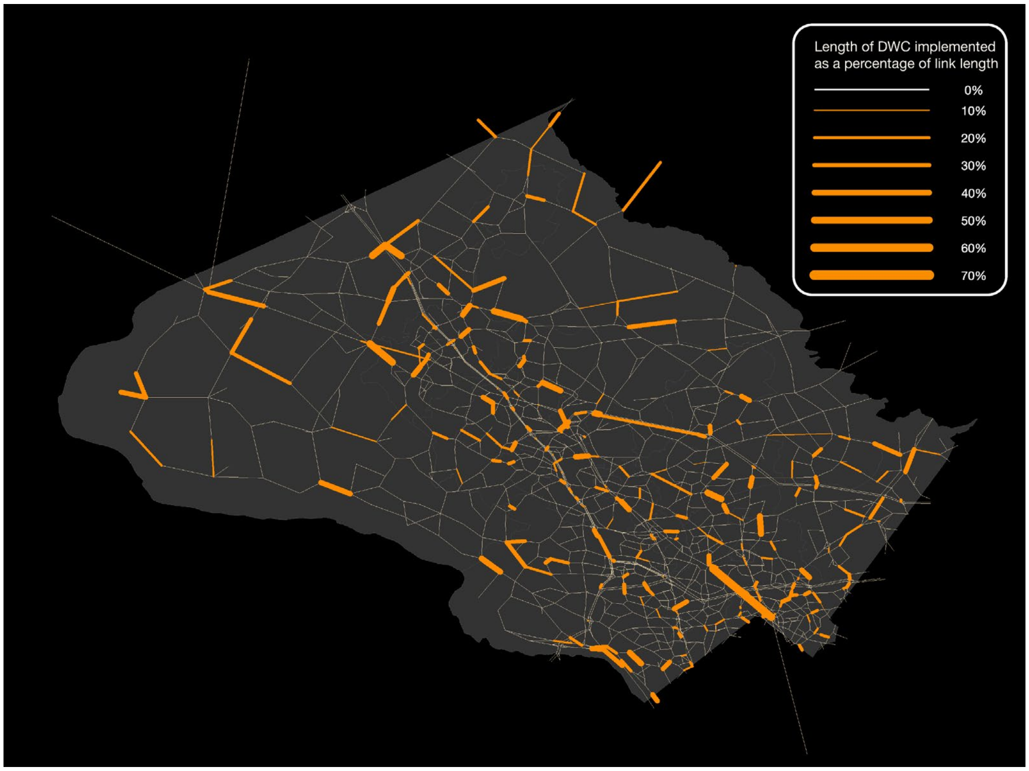
The higher-ranking Policy Region tends to be closer to District of Columbia and along Interstate 270 such as Silver Spring CBD or Bethesda CBD. These areas tend to be quite small. In contrast, other areas that are on the lower side of ranking are located in remote areas. They are characterized by a larger area and are responsible for longer road land miles. In the model, these Transportation Policy Regions and funding priority are treated as sub-regions while incorporating the equity in resource distribution constraint in Equation (9).

5.3. DWC Implementation Plan for the Study Area

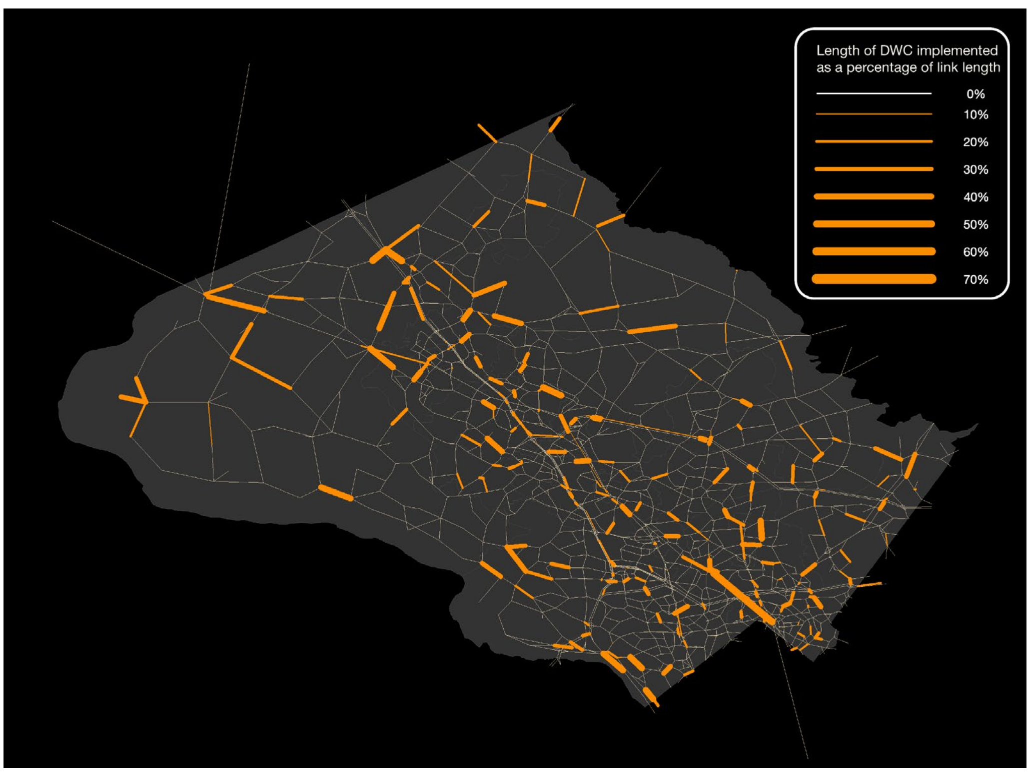
Model 1 and Model 2 have been implemented to the Montgomery County network for deciding the DWC implementation plan for two objectives namely, minimizing the TSTT and TSNEC. The optimization model is implemented based on the assumption of 100 million dollars budget and 60 minutes of initial recharging time or 30 miles in initial range for BEVs in all routes. The results of

DWC facility location plan for TSTT and TSNEC minimization scenarios are presented in Fig. 9(a) and Fig. 9(b) respectively.

Insights from the numerical experiments of the two scenarios presented in Fig. 9(a) and Fig. 9(b) emphasize the importance of Interstate 270 as the models suggest a high percentage of DWC implementation along this highway. However, only several intermittent segments of Interstate 270 are suggested for DWC in both scenarios, which indicates that both scenarios prefer the non-contiguous segments for DWC. In particular, suggested DWC implemented segments on this highway are located where on and off-ramp movements of several traffic paths coincide with each other rather than on the non-weaving portion of Interstate highway. The scattered approach of implementing DWC can be more efficient compared to the continuous approach as it leads to larger network coverage under a restricted budget. Other important roads are Georgia Avenue, which connects Silver Spring to the Capital Beltway and Wheaton CBD, and Maryland State Route 200. The area on the southwest of the County such as Potomac does not receive the same treatment of DWC facilities compared to other areas which can be explained by its smaller number of trips (generated from or attracted to) as well as the shorter traveling distance.



(a) DWC Plan for TSTT Model



(b) DWC Plan for TSNEC Model

Fig. 9. Final DWC Implementation Strategy for Montgomery County

The objective function values of the two scenarios were compared with the Do-Nothing scenario (no DWC Implementation). Model 1 (TSTT minimization) experiences a 0.0055% decrease in total system travel time and Model 2 (TSNEC minimization) experiences a 28% decrease in total system net energy consumption. By using the optimal plan from TSTT and TSNEC model, the total system travel time is lowered by 998 million (Vehicles-Minutes) and 400 million (Vehicle-Minutes) respectively. In addition, the total energy recharged by BEVs through DWC under TSTT and TSNEC models are 5.35 million and 5.44 million (Vehicles-kWh) respectively. This indicates that the power requirement for electrifying the county's transportation system will be huge and may demand adequate planning for power generation and distribution.

6. CONCLUSIONS

In this paper, we propose a modeling framework for optimally positioning induction-based DWC facilities in a transportation network for BEVs. The framework aims to support transportation planners and engineers in local agencies. A bi-level modeling framework is proposed considering both the different objectives of the planner and network users. In the Upper Level, Total System Travel Time (TSTT) and Total System Net Energy Consumption (TSNEC) are two objectives of the planner considered. In the Lower Level (LL), the user's route choice is modeled subjected to the DWC infrastructure provided by the planner. As a proof-of-concept, an example 18 link network is tested under different budget scenarios to demonstrate the model performance in the TSTT and TSNEC minimization. Results showed that suggested DWC infrastructure investment is different for TSTT and TSNEC minimization, even though there is some commonality between two cases. Upon successful implementation of the 18 link network, the model is applied to a real-world Montgomery County network from Maryland, USA. The results of the real-world network were intuitive, as model results suggest DWC on major highways and arterials in an intermittent fashion.

The insights from this research will enable planners and policymakers in making informed decisions and for devising plans and policies that are not only optimal from road network perspective but also from the perspective of power grids, transmission losses, and energy efficiency. The analysis results show that for the Montgomery County Case Study, with an assumption of 60 minutes recharging time yielding 30 miles of initial range for the user, a 100 million dollar expense in DWC is required to sufficiently recharge all BEV within the network. The optimal DWC plan of the TSTT model can lower the total system travel time by 0.0055% and the TSNEC model can lower the total system net energy consumption by 28%. Future avenues of research include analysis of DWC network in a mixed environment of conventional vehicles and BEVs; consideration or estimation of power availability from neighborhood electric grids; and induced demand because of DWC implementation.

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Appendix A

Pseudocode: Modified Constrained Local Metric Stochastic Response Surface

Stage 1. Initialization

Step 1.1. Generate a set of initial *t* training points (DWC implementation plan) which satisfies all constraints of the optimization problem $Y_0 = \{y_1, y_2 \dots y_t\}$. These points do not necessarily yield the optimal results of the optimization. Each training points y_t has a dimension of *d*.

Step 1.2. Evaluate the objective function of each training points using the expensive objective function $Z = \{\mathrm{f}(y_1), \mathrm{f}(y_2) \dots \mathrm{f}(y_n)\}$. Sort for the minimum value of the: $Z_{best} = \min(Z)$ at y_{best}.

Set 1.3. Setup the initial step size $\sigma_n = \sigma_{initial}$; Consecutive Success and Failure: $C_{succ} = 0$;$C_{fail} = 0$; and global successive failure $C_{gsf} = 0$

Stage 2. Iteration. While the termination condition ($n > N_{max}\ or\ C_{gsf} > C_{gsfmax}$) is not satisfied

Step 2.1. Using the training points $T = \{(y_{1,}\mathrm{f}(y_1)), (y_{2,}\mathrm{f}(y_2)) \dots (y_{t,}\mathrm{f}(y_t))\}$ create or update the response surface $S_n(y)$

Step 2.2. Generate q candidates points for each iteration *n*: $C_n = \{y_{n,1}, \dots y_{n,q}\}$ as follow: For j =1…q:

Generate *d* uniform random numbers $w_1, w_2 \dots w_n$ in the range [0,1]. Let $I_{pert} = \{i: w_i < p_{sltc}\}$. If $I_{pert} = \emptyset$, then select j from the set [1,...,*d*] and set $I_{pert} = \{j\}$

Generate j-th candidate solution by: $y_{n,1} = y_{best} + \Delta_{n,j}$ where $\Delta_{n,j}^{i}$=0 for all $i \notin I_{pert}$ and $\Delta_{n,j}^{i}$ is a normal random variable with mean 0 and standard deviation σ_n for all $i \in I_{pert}$

Step 2.3. For each $y_{n,j} \in C_n$

If the candidate point $y_{n,j}$ satisfy all constraints within the optimization,

Evaluate the objective function $S_n(y_{n,j})$ by using the response surface model.

Let $S^{min} = \min\{S_n(y_{n,j}), y_{n,j} \in C_n\}$ and $S^{max} = \max\{S_n(y_{n,j}), y_{n,j} \in C_n\}$. Compute the score for each $y_{n,j} \in C_n$ for the response surface: if $S^{max} \neq S^{min}$ then $V_n^S = (S_n(y_{n,j}) - S^{min})/(S^{max} - S^{min})$, else $V_n^S = 1$.

Evaluate the minimum distance from the candidate $y_{n,j}$ to training points by

$D_n(y_{n,j}) = min_{1\le i\le n} \left|\left|y_{n,j} - y_i\right|\right|, y_i \in Z$. The symbol $||.||$ represents the Euclidean norm. Let $D^{min} = \min\{D_n(y_{n,j}), y_{n,j} \in C_n\}$ and $D^{max} = \max\{D_n(y_{n,j}), y_{n,j} \in C_n\}$.

Compute the score for distance criterion score for each candidate: if $D^{max} \neq D^{min}$ then $V_n^D = (D_n(y_{n,j}) - D^{min})/(D^{max} - D^{min})$ else $D_n^S = 1$.

Step 2.4. Determine the weighted score for each candidate points: $V_n = w_n^S V_n^S + w_n^D V_n^D$. The coefficient w_n^S, w_n^D can be determined as follow:

$w_n^S \begin{cases} v_{mod(n-n_0,k)} & if\ mod(n-n_0,k) \neq 0 \\ v_k & otherwise \end{cases}$ and $w_n^D = 1 - w_n^S$ where k is an integer and

v_k is a series of weights in ascending order within the range of [0,1]. Select y^* within the set of candidates points C_n that yields the highest weighted score V_n.

Step 2.5. Evaluate the expensive objective function for the solution y^* to get the value $Z_{n=}$f(y^*) and add the point {y^*, f(y^*)} to the training points poll T.

Step 2.6. If $Z_n < Z_{best}$ update the current best solution Z_{best}= Z_n, update the consecutive success and failures: $C_{succ} = C_{succ} + 1;\ C_{fail} = 0;\ C_{gsf} = 0$ otherwise $C_{fail} = C_{fail} + 1;\ C_{succ} = 0$.

Step 2.7. Adjusting the step size and counters:

If C_{succ} exceeds the maximum number of success C_{succ}^{max}, set $\sigma_{n+1} = 2\sigma_n$ and reset $C_{succ} = 0$

If C_{fail} exceeds the maximum number of success C_{fail}^{max}, set $\sigma_{n+1} = \sigma_n/2$, reset $C_{fail} = 0$,

and set C_{gsf}=$C_{gsf} + 1$

Set $n = n+1$

End the while iteration.

Step 3. Conclusion

Return the optimal objective function value Z_{best} and the vector of decision variable $\{y_{best}\}$ when stopping criterion is met. The stopping criterion adopted in the Montgomery Case Study is either the iteration reaches 40 iterations or global successive failure reaches 10.