Incorporating Uncertainty and Risk in Transportation Investment Decision Making

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Abstract
The authors present a framework for addressing uncertainty and risk for large-scale transportation investments involving public-private participation. Demand, fare/toll, and demand responsive costs are considered in the uncertainty analysis. Uncertainty analysis provides information on economic feasibility of the project. A set of relaxation policies is proposed to form various Ownership, Tenure, and Governance (OTG) strategies reflecting the nature and level of participation by the public and private entity. A Monte Carlo Simulation based Value at Risk (VaR) is used to quantify risk. Finally, a methodology is proposed to integrate uncertainty and risk. The framework is tested on the proposed multibillion dollar international river crossing entitled as the Detroit River International Crossing (DRIC) connecting the cities of Detroit in the U.S with Windsor in Canada. The analysis provides insights to probable outcomes for this transportation infrastructure investment under different OTG scenarios.

Key words: uncertainty, value at risk, monte carlo simulation, investments

1. Introduction
Transportation infrastructures are integral parts of a nation’s network connectivity. Transportation projects represent major public investments in construction, operation, and maintenance of facilities over an extended period such as: mass-transit systems, freeways, subways, bridges, tunnels, etc. Typically, these investments are irreversible in nature and require long-term commitment by taxpayers relative to their utilization, maintenance, and operation.

A recent report suggests that projected federal, state and local highway revenues are not sufficient to meet estimates of future highway requirements in the U.S (USDOT 2006). Lack of
capital funds to meet the needs of the country may result in increased private participation in transportation infrastructure projects (Roth 1996). The potential of such projects to produce economic benefits has become an increasingly important factor in investment decision making. Such investments may involve the private enterprise in the construction, operation and maintenance process along with the federal, state and local governments.

In traditional economic analysis, future cash flows are assumed to be fully deterministic in nature. Thus, these are not designed to account for any risk and uncertainty in the assessment of future returns. In reality, many of these infrastructure projects are associated with significant uncertainties stemming from lack of knowledge about future cost and revenue streams. The term “risk” refers to situations where the decision maker can assign mathematical probabilities to the randomness relative to future outcomes. In contrast, the term “uncertainty” refers to situations when this randomness cannot be expressed in terms of mathematical probabilities (Knight 1921).

Significant research on investment decision making under uncertainty and risk is reported in the fields of economics and financial management. These include theoretical approaches on capital investment considering the irreversibility of investment decisions and uncertainty of economic environment (Dixit and Pindyck 1994), actual experience of users and policy makers in an agent-based approach reflecting network management and financing policies (Zhang et al. 2008), investment decision making for a Build Operate and Transfer (BOT) problem using a multi-objective genetic algorithm and a mean-variance model for BOT scheme under demand uncertainty (Chen et al. 2003, Chen et al. 2006), capacity expansion using demand uncertainty and simulated annealing (Sun and Turnquist 2007) and decision making under uncertainty for highway development using real options approach (Zhao and Kockelman 2006).
2. Problem Statement

The problem investigated in this research relates to the need for a unified approach in incorporating uncertainty and risk in transportation investment decision making. This paper presents an analytic framework to explore the implications of a joint ownership of a transportation infrastructure project, when cost/revenue and demand estimates are subject to significant uncertainties. A case study is also presented to demonstrate the application of the framework.

The framework explores various forms of joint ownership associated with the public and private enterprise. There are a number of reasons for the growing trend of private participation in public projects. These include, the scarcity of fiscal resources at the public sector level, the perception that the private sector is more efficient in managing large projects, and the advantage of the public and the private sector jointly sharing risks and uncertainties, thereby reducing exposure levels to financial losses for both entities. Joint ownership has become increasingly popular in Europe, Australia and more recently in Asia, as it allows a part or the whole of the capital funds from private resources in exchange of future revenues (Garber and Hoel 2002, Khasnabis et al. 2010). Joint ownership is generally associated with three terms: Ownership, Tenure and Governance (OTG). An OTG strategy can be looked upon as a mechanism to plan, design, implement, operate, and maintain a project by developing various combinations of ownership, tenure, and governance procedures, where:

- The term ‘Ownership’ has embedded in it, the concept of ‘possession’ and ‘title’ related to the property in question. Depending upon the nature of the joint project, its ownership may belong to the public entity, private entity, or both during the concession period. Ownership may also change at the end of the concession period (Merna and Njiru 1998).
‘Tenure’ refers to the status of holding a possession of a project for a specific period, ranging from few days to a number of years. For most joint ownership projects, tenure is likely to coincide with the concession period; however, exceptions to this general rule may be encountered.

‘Governance’ refers to management, policy and decision making pertaining to an organization with the intent of producing desired results.

The objective of this research is to propose a framework to incorporate uncertainty and risk, and to evaluate the proposed framework with a real-world case study. The methodology proposed in the paper is designed for major transportation infrastructure projects involving public private participation. The case study is applied to an international toll bridge.

3. A Combined Framework for Uncertainty and Risk Analysis

The proposed framework to incorporate the concept of investment decisions under uncertainty and risk is illustrated in Figure 1 and is categorized into three steps;

1. Step 1: Uncertainty Analysis
2. Step2: Risk Analysis
3. Step 3: Integration of uncertainty and risk

**Step-1: Uncertainty Analysis**

Uncertainty analysis is further divided into three sub-steps:

Step - 1.1: Policy Options

Step - 1.2: Bi-level Programming for uncertainty analysis

Step - 1.3: Feasibility Analysis
Step 1.1 is an examination of the investment policy options recommended by the relevant public agencies relating to new transportation projects that may represent various combinations of rights and responsibilities of public and private agencies (FHWA 2010). At one end of the spectrum, the public entity may have all the major responsibilities, with the private agency playing a minor role. At the other end, the roles may be reversed. Various other combinations may form the intermediate range.

In Step 1.2, a bi-level approach is considered for evaluation of the proposed policy options. The policy maker (upper level) is assumed to have some understanding of the road users’ likely response (lower level) to a given strategy. However, the strategy set by the policy maker can only influence (but not control) the road user behavior relative to choice of routes, modes, etc. In other words, policy options and choice decisions can be represented as a bi-level program, where, the upper level involves the policy maker’s decision to determine the toll value, while road users are assigned to the proposed facility at the lower level. Further, the upper level may involve three entities (1) private investor, (2) public investor, (3) road user. While the designed toll value for the three perspectives maybe different at the upper level, a user equilibrium assignment process is introduced at the lower level with an elastic demand feature designed to consider uncertainty in travel pattern.

In Step 1.3 economic and financial feasibility of various policy options are examined. Policy regulations such as construction cost subsidy, concession period extension, etc. can be considered if necessary. The relaxations are embedded in a set of OTG strategies discussed later. The viability of the project under different strategies can be tested using a set of pre-specified criteria.
Step 2: Risk Analysis

Three sub-steps are proposed in the risk analysis:

Step 2.1: Identification of risk variables

Step 2.2: Setting up the simulation process

Step 2.3: Estimation of Value at Risk (VaR)

Variables associated with different investment options are identified in step 2.1. For example, in transportation investments, possible risk variables are related to demand, fare, and costs. Probabilities are assigned to the risk variables in a simulation model. A number of simulation approaches and risk measures are presented in the literature (Jorion 1997). In step 2.2, various iterations of the simulation cycle are recorded. In step 2.3, a measure of risk is determined. One such measure is “Value at Risk” (VaR), that can be used to denote the maximum expected loss over a given horizon at a given confidence level for a specific policy option. This step is designed to enable the decision maker avoid risky policy options, and to focus more on those options with modest risk exposure.
FIGURE 1 Proposed Methodology for Uncertainty and Risk Analysis
Step 3: Integration of Uncertainty and Risk

In this step, Measures of Effectiveness (MOE) of uncertainty and risk analyses are combined. Policy options to address the implications of uncertainty and risk can be proposed for further consideration.

3.1 Uncertainty Analysis

Sources of uncertainty in the transportation infrastructure investment can arise from lack of knowledge about future costs and revenues. Bulk of the cost element is from construction cost incurred before the facility is opened to traffic; other cost elements such as operation and maintenance costs (regular and periodic) depend on future travel demand. Revenue is directly dependent on travel demand and toll. Thus, uncertainties related to cost and revenue is primarily related to travel demand.

Investments in major transportation infrastructure are often complex, with a mix of public and private finance with the respective agencies having different missions and motivations. The public sector may consist of national, state and local agencies with a social welfare perspective. Public and private entities are interested in exploring optimal tolling strategies that may yield different solutions (Hyman and Mayhew 2008, Wong et al. 2005, Palma et al. 2006, Rouwendal and Verhoef 2006). While the public entity’s primary interest is to maximize consumer surplus\(^1\) (social welfare), the private entity is interested in maximizing profit. Since the public sector is the eventual owner and operator of the facility, it must ensure that the facility attracts users and serves the needs of the community (Yang and Meng 2000). Thus, the optimal toll must be viable to the

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\(^1\) The additional value or benefit received over and above the expenses actually made is known as consumer surplus.
ultimate end users. Hence, in the investment decision making process, three entities’ perspectives should be considered: (1) the private, (2) the public, and (3) the user.

*Private Investor’s Perspective*

The objective of the private investor is to maximize profit. The annual profit for demand uncertainty is the difference between benefit and cost and is presented as following (Chen and Subprasom 2007):

\[
P_n \left( \tau, x(\tau, \epsilon) \right) = B_n - C_n
\]

Where, \(P_n\) is the profit generated in year \(n\), which is a function of the demand \((x)\) and toll \((\tau)\). \(B_n\) and \(C_n\) are corresponding revenue and cost for year \(n\) respectively. The revenue generated is a function of uncertain demand and toll, while the cost can be presented in the form of capital and operation and maintenance cost.

*Public Investor’s Perspective*

The proposed framework is based on the premise that the primary objective of the public entity is to maximize consumer surplus, typically measured as the additional monetary value over and above the price paid (Wohl and Hendrickson 1984). There are other social benefits such as improved traffic flow, environmental benefits, higher safety etc., that may be derived from major infrastructure projects. These are not incorporated in the proposed framework and the classical approach of maximization of consumer surplus was used as the only public benefit.
Road User’s Perspective

Ideally, project benefits should be uniformly distributed over the entire study area. If the project only benefits a small section of travelers in the study area, then the distribution of such benefits cannot be considered equitable. Theil’s index, one of the commonly used measures of inequality distribution was used in this study because of its flexible structure (Theil 1967). The Theil’s Index is considered as a minimization function and is based on the distribution of trips among TAZs in the study area. Lower values of Theil’s index represent increasing levels of equity in the distribution of benefits, and vice versa. A zero value of the index indicates a perfect equity of distribution of benefits across the study area.

While the upper level program determines the toll for various perspectives considered, the lower level determines the route choice of users for a designed toll value subjected to uncertain demand. The lower level problem is a user equilibrium traffic assignment with elastic demand (Sheffi 1985).

Necessary algorithms for maximization of profit for the private sector, for maximization of consumers’ surplus for the public sector, and minimization of Theil’s Index for the road user are discussed in detail in the project report (Khasnabis and Mishra 2009). A summarized version of the formulation is presented in Appendix A.

Demand Elasticity and Uncertainty

Addition of new links or improvement of the road network will reduce the travel cost between origin and destination. This improvement can result in increasing demand between the corresponding OD pairs. An exponential demand function can be used to estimate the annual demand (Sheffi 1985).
\[ q_{rs}^n = q_{rs}^n \exp\left(-\lambda \pi_{rs}^n\right) \]  
(2)

Where, \( q_{rs}^n \) is the random potential demand between \( r-s \), \( \pi_{rs}^n \) is the minimum travel cost between \( r-s \) which includes the designed toll value, \( \lambda \) is a positive constant, and \( q_{rs}^n \) is the realized travel demand for year \( n \) between the OD pair \( r-s \).

Uncertainty in travel demand is incorporated through a random sampling approach with a predefined mean and variance. Random numbers are generated with a predefined probability distribution function (i.e. normal distribution). This is performed exogenously from the lower level traffic assignment (Chen and Subprasom 2007)

\[ q_{rs}^n = \overline{q}_{rs}^n + z\sigma_{rs}^n \]  
(3)

Where, \( \overline{q}_{rs}^n \), \( \sigma_{rs}^n \) are the mean and standard deviation of random potential demand for OD pair \( r-s \), and \( z \) is a random variable generated from normal distribution with mean zero and unity variance. The link travel time used in the lower level traffic assignment problem is the Bureau of Public Roads function, denoted as (Sheffi 1985):

\[ t_a^n(x_a^n) = t_a^0 \left(1 + 0.15 \left( \frac{x_a^n}{G_a^n} \right)^d \right) \]  
(4)

where, \( t_a^0 \) and \( G_a \) is the free flow travel time and capacity for link \( a \).

3.2 Risk Analysis

Risk is often defined as the probability of occurrence of an undesirable outcome (Jorion 1997). Risk analysis consists of simulating the various inputs over the life of the project and finding
the present value. This process is repeated a number of times using Monte Carlo Simulation (MCS) to incorporate risks from multiple sources, both on revenues and on costs. The Measure of Effectiveness (MOE) thus obtained would reflect the effect of risk.

In the proposed risk analysis, the simulation model employs pre-defined realizations of toll and traffic volume to analyze the effect of indecisive inputs on the output of the modeled system. Risk can be quantified and measured in different ways (Mun 2006). Value at Risk (VaR) is one such that can be defined as the maximum expected loss (or the lower value of MOE) over a target horizon, with a given level of confidence (Jorion 1997). VaR describes a quantile of the projected distributions of gains and losses over the target horizon. If $\alpha$ is the selected confidence level, VaR corresponds to the $(1-\alpha)$ lower tail level. For example, for 90 percent confidence level, VaR should be such that it exceeds 10 percent of the total number of observations in the distribution.

4. Case Study

A proposed international bridge between the city of Detroit in the U.S and the city of Windsor in Canada is selected as the case study. Surface trade between Southwestern Ontario and Southeastern Michigan exceeded 200 billion in 2004 and is expected to increase by two-fold by the year 2030 (MDOT 2003). 70 percent of trade movement between the US and Canada is by trucks. Approximately 28 percent of surface trading is by trucks for the crossings between Southeast Michigan and Southwest Ontario (MDOT 2008). Majority of the trade is for the crossings in the Detroit River area, connecting the city of Detroit in the U.S and the city of Windsor in Canada. This large trade volume has a significant positive effect on the local, regional and national economies through cross-border employment opportunities.
The Central Business Districts (CBD) of the cities of Detroit and Windsor are currently connected by four crossings: (1) The Ambassador Bridge (AB), (2) The Detroit Windsor Tunnel (DWT), (3) a Rail Tunnel (RT), and (4) The Detroit Windsor Truck Ferry (DWTF). Both AB and DWT are across the Detroit river, built during the late 1920s. AB is a privately owned four-lane suspension structure, while DWT is a two-lane facility with height restriction, jointly owned by the two cities and operated by a private corporation. The RT and DWTF, both constructed under the Detroit river, carry cargo between two cities. The Blue Water Bridge (BWB) across the St. Clair River (100 km north of Detroit) that connects the cities of Port Huron in the U.S and Sarnia in Canada. BWB is a six lane arch structure built in 1938 and renovated in 1999; and is jointly owned by the two cities.

The Canada–U.S–Ontario–Michigan Transportation Partnership Study (Partnership Study) attempted to develop long-term strategies to provide safe and efficient movement for people and goods between Michigan and Ontario (MDOT 2008). Even though the current capacities of the Ambassador Bridge and the Detroit-Windsor tunnel adequately serve the traffic needs during most hours, on specific days during peak periods, the systems operates at full capacity. Considering long-term traffic growth and the overall importance of the Detroit River crossings on the regional economy, the need for a new crossing seems immensely justified. As a result of number of recent studies, the Michigan Department of Transportation (MDOT), and the Ontario Ministry of Transportation have identified a bridge known as X-10(B) as the most preferred alternative to build in the vicinity of the Ambassador Bridge (MDOT 2008). The alternative has been referred to as the Detroit River International Crossing (DRIC) in the case study (Figure 2). DRIC, with some minor adjustments, has recently been renamed as the New International Trade Crossing (NITC) (Detroit Free Press, 2012). For most up to date information on the case study please refer to the
FIGURE 2 Network of Study Area (Mishra 2009)
project website\textsuperscript{2}.

5. Results

Two alternative bridge structures are proposed for X-10(B); (1) a suspension bridge, or (2) a cable-stay bridge. The preliminary cost estimates of the bridges along with associated infrastructures are $1,809 million and $1,814 million respectively. The cost of $1,814 million includes construction cost of the bridge, toll plaza, access highway with a new interchange, cost of property acquisition, planning and design (all for the US part of the project). By the same token, all the toll revenue compiled to assess the benefits reflects the fare collected at the Detroit end of the bridge that is expected to be open to traffic in the year 2015.

5.1 Travel Demand Uncertainty

Travel demand data for the study area in the form of origin-destination (OD) matrices for the study area are obtained from MDOT for the years 2015, 2025, and 2035. There are a total of 960 Traffic Analysis Zones (TAZ) in the Detroit (U.S) side of the border and 527 TAZs in the Windsor (Canada) side of the border. Including 23 external TAZs, the study area consists of a total of 1510 TAZs, making for a total of 1510 x 1510 possible interchanges in the travel demand matrix. The analysis period for the case study is considered as 35 years (2015-2050). The OD matrix for the year 2045 was projected by considering the growth trends for each TAZ. A coefficient of variation\textsuperscript{3} of 0.15 is

\textsuperscript{2} https://www.michigan.gov/builthisbridge

\textsuperscript{3} The coefficient of variation (COV) is the ratio of the standard deviation and the mean. For this research a COV of 0.15 is assumed by observing the variation in demand over time for ten years.
considered to incorporate variance in travel demand. A potential\(^4\) OD matrix was not available. The base and horizon year projected OD matrices were increased by ten percent to obtain the potential OD. The standard deviation of the OD matrix was obtained from the coefficient of variation and the expected demand of the OD matrix.

**Solution Approach for Demand Uncertainty**

A Monte Carlo Simulation (MCS) procedure was used to simulate the OD matrix. The potential OD matrix and the variance OD matrix served as the input to the MCS. The OD matrices were subjected to 200 realizations and each realization was recorded (Equation 20). From the distribution of OD matrix, the median matrix was chosen for further analysis. However, one can use any percentile from the distributed OD matrix. This procedure was followed for all the horizon years. The resulting OD matrix from MCS incorporates the variation and resulting uncertainties in travel demand, which is used in the elastic traffic assignment procedure.

The proposed traffic assignment model was calibrated for the base year 2004. Actual toll values for cars and trucks for the year 2004 are utilized to determine the assigned volume on the existing river crossings in the network. The proposed elastic traffic assignment model and the potential OD matrix for the year 2004 are utilized to determine the assigned volume for cars and trucks. The observed car and truck volumes are obtained from MDOT (MDOT 2003). The close correspondence between the assigned and observed volumes at the respective crossings demonstrates the calibration of the model. Results of

\(^4\) The potential OD matrix contains the maximum possible trips that can be made if the travelers are not sensitive to the user cost. In elastic traffic assignment the potential OD matrix is used to test the sensitivity of demand with respect to the user cost (both travel time and travel cost).
the calibration are not presented in the paper for the sake of brevity. The details of calibration of the model are discussed in the project report (Khasnabis and Mishra 2009).

5.2 Single Entity Perspective Decision Making Under Uncertainty
The objectives of the three entities from investment viewpoint are different, as discussed earlier. Three entity objectives are used at the upper level and ridership is determined at the lower level. The bi-level process is solved in TransCAD (Caliper 2008). A GISDK script is written to solve the bi-level model in TransCAD. The output of the upper level (toll value and the entity-specific objective function) served as the input to the lower level (ridership estimation). The bi-level process can be viewed as a non-linear problem reflecting the nature of the objective functions at the upper and the lower level. The elastic traffic assignment problem is solved by user equilibrium method using Frank Wolfe Algorithm (Sheffi 1985).

5.3 Results of Travel Simulation
Results of the travel simulation are presented for the three entities for different horizon years in Table 1. For the private entity, the objective is profit maximization with the assumption that the total cost (capital, operation and maintenance cost) will be borne by the private entity. Revenue is considered as a surrogate for profit in this paper. As explained earlier, the optimization processes for the three entities were solved by the bi-level approach. Toll values are set at the upper level and ridership is determined at the lower level. The toll values are obtained in an iterative manner with directional search\(^5\) to

\(^5\) Directional search is a technique for finding the optimal value of a unimodal function by successively narrowing from the possible range of values.
obtain the optimum value of the objective function for profit maximization, consumer surplus maximization and inequality minimization.

For example, in the profit maximization strategy, optimal toll values of $2 per car and $14 per truck resulted in annual revenue\(^6\) of $68.54 million in the year 2015. For the same toll values the consumer surplus and Theil’s index are estimated to be $346.07 million and 0.86 respectively for the year 2015. The last two numbers are not, however, optimal values as explained below.

When objective of the public entity is considered, the optimal toll is $0.5 per car and $4.33 per truck (year 2015, second row, Table 1) that resulted in an optimal consumer surplus of $730.36 million, which is higher than the estimated consumer surplus for profit maximization. The consumer surplus allows more travelers\(^7\) to use the facility in lowering the difference between willingness to pay and what the travelers actually pay. The revenue and Theil’s index for toll value of $0.5 for cars and $4.33 for trucks are estimated to be $25.78 million and 0.79 respectively (Non-optimal).

Similarly, when the perspective of the users is considered (year 2015, third row, Table 1) the optimal toll values obtained are $0.25 per car and $1.04 per truck, resulting in a Theil’s index of 0.70 (minimum of the three Theil’s index values) for the year 2015. For the toll value of $0.25 per car and $1.04 per truck the corresponding revenue and consumer surplus are estimated at $7.41 and $258.62 million respectively (Non-optimal).

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\(^6\) Revenue is defined as the monetary benefit obtained by the toll/fee collection only and for a specific year where as profit is for the analysis period.

\(^7\) It should be noted that more travelers using the facility do not necessarily increase the revenue, because revenue is the product of toll value and the corresponding ridership.
### TABLE Travel 1 Simulation Results (25)

<table>
<thead>
<tr>
<th>Year</th>
<th>Car Toll ($)</th>
<th>Truck Toll ($)</th>
<th>Annual Revenue (Million $)</th>
<th>Annual Consumer Surplus (Million $)</th>
<th>Theil’s Inequality Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>2015</td>
<td>Private Perspective</td>
<td>2$\textsuperscript{8} \quad 14$\textsuperscript{9}</td>
<td>68.54\textsuperscript{10}</td>
<td>346.07</td>
<td>0.86</td>
</tr>
<tr>
<td></td>
<td>Public Perspective</td>
<td>0.5\textsuperscript{11} \quad 4.33\textsuperscript{12}</td>
<td>25.78</td>
<td>730.36\textsuperscript{13}</td>
<td>0.79</td>
</tr>
<tr>
<td></td>
<td>User Perspective</td>
<td>0.25\textsuperscript{14} \quad 1.04\textsuperscript{15}</td>
<td>7.412</td>
<td>258.62</td>
<td>0.70\textsuperscript{16}</td>
</tr>
<tr>
<td>2025</td>
<td>Private Perspective</td>
<td>3</td>
<td>15</td>
<td>118.22</td>
<td>550.98</td>
</tr>
<tr>
<td></td>
<td>Public Perspective</td>
<td>0.78</td>
<td>5.28</td>
<td>43.65</td>
<td>1091.91</td>
</tr>
<tr>
<td></td>
<td>User Perspective</td>
<td>0.52</td>
<td>2.06</td>
<td>19.53</td>
<td>352.60</td>
</tr>
<tr>
<td>2035</td>
<td>Private Perspective</td>
<td>4.5</td>
<td>19</td>
<td>199.30</td>
<td>681.45</td>
</tr>
<tr>
<td></td>
<td>Public Perspective</td>
<td>1.28</td>
<td>6.75</td>
<td>73.70</td>
<td>1343.04</td>
</tr>
<tr>
<td></td>
<td>User Perspective</td>
<td>0.86</td>
<td>3.35</td>
<td>40.02</td>
<td>464.08</td>
</tr>
<tr>
<td>2045</td>
<td>Private Perspective</td>
<td>6.00</td>
<td>21.00</td>
<td>281.95</td>
<td>802.24</td>
</tr>
<tr>
<td></td>
<td>Public Perspective</td>
<td>1.75</td>
<td>7.41</td>
<td>105.42</td>
<td>1594.95</td>
</tr>
<tr>
<td></td>
<td>User Perspective</td>
<td>1.26</td>
<td>4.52</td>
<td>68.13</td>
<td>565.78</td>
</tr>
<tr>
<td>2050</td>
<td>Private Perspective</td>
<td>8.73</td>
<td>22.25</td>
<td>330.63</td>
<td>936.19</td>
</tr>
<tr>
<td></td>
<td>Public Perspective</td>
<td>1.93</td>
<td>7.82</td>
<td>125.19</td>
<td>1664.37</td>
</tr>
<tr>
<td></td>
<td>User Perspective</td>
<td>1.60</td>
<td>5.70</td>
<td>96.22</td>
<td>685.32</td>
</tr>
</tbody>
</table>

Three distinct toll values are obtained for three different entities, each of which represents the optimum values for the three objective functions defined in equations 6, 12, and 15 in Appendix A. The highest toll value resulted for profit maximization and the least

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8 Represents the Optimal value of car toll from the Private Perspective  
9 Represents the Optimal value of truck toll from the Private Perspective  
10 Represents the maximum value of Revenue from the Private Perspective  
11 Represents the Optimal value of car toll from the Public Perspective  
12 Represents the Optimal value of truck toll from the Public Perspective  
13 Represents the maximum value of Consumer Surplus from the Public Perspective  
14 Represents the Optimal value of car toll from the User Perspective  
15 Represents the Optimal value of truck toll from the User Perspective  
16 Represents the minimum value of Theil’s value from the User Perspective
toll value for Theil’s Index, thereby demonstrating that the objectives of the private investor and the users are satisfied. Additionally, the toll value for the public entity perspective is lower than that for the private perspective for all the years. Similar trends are observed for the other horizon years during the analysis period presented in Table 1. Increased travel demand in future years resulted in higher toll values, higher revenue and higher consumer surplus in succeeding years. The same is generally true for Theil’s Index, although there are some exceptions.

5.4 Ownership, Tenure and Governance Strategies
The authors’ initial work on the concept of OTG scenarios was presented at the World Conference on Transport Research at the University of California, Berkeley in 2007 (Khasnabis et al. 2007). Though single entity participation in large transportation projects is important, their involvement with other entities is likely to increase the overall viability of the project. Ownership, Tenure and Governance (OTG) are the three principal components of joint ownership (Mishra et al. 2013; Mishra et al. 2014).

A number of OTG strategies are considered to represent varying levels and types of public-private participation in the DRIC project. The strategies vary in the degree of participation by the public and the private entity. Five types of OTG strategies are considered:

1. OTG-1: Exclusive Private Participation
2. OTG-2: Major Private Participation
3. OTG-3: Moderate Private Participation
4. OTG-4: Major Public Participation
5. OTG-5: Exclusive Public Participation
The transition from OTG-1 to OTG-5 is marked by decreasing levels of private and increasing levels of public participation, with OTG-1 and OTG-5 representing exclusive private and public participation respectively. A number of relaxation policies are also considered to encourage joint ownership.

**OTG-1**

For OTG-1, the total capital cost is borne by the private entity. The objective of the strategy is profit maximization. After construction of the facility, the private entity is authorized to collect toll, operates and maintains the facility throughout the concession period. The eventual owner of the facility is the public entity, even though the private entity is responsible for all the expenditures and toll collection during concession period.

The cumulative cash flow and Internal Rate of Return (IRR) are the two MOEs plotted in Figure 3. The negative cost elements for 2004-2014 represent the planning and construction of the facility. When the facility is opened to traffic, the cumulative negative value of cash flow decreases, as the toll charges are collected and the break even period occurs in the year 2034 (Figure 3). The IRR\textsuperscript{17} for OTG-1 strategy is 4.61\% over the 35 years of concession period. A Minimum Attractive Rate of Return (MARR)\textsuperscript{18} of six percent was assumed for the case study. The IRR being lower than the MARR lends the project economically infeasible for the strategy (OTG-1) tested.

\textsuperscript{17} IRR provides an estimate of the return or yield of the investment, given a set of expenditure and revenue data along with their expected dates over the life of the project. IRR is defined as the interest rate at which the Net Present Worth (or Net Annual Worth or Net Future Worth) of the investment is equal to zero.

\textsuperscript{18} MARR is the rate of return below which the investment proposal is to be deemed unacceptable.
FIGURE 3 Cumulative Cash Flow and IRR: Exclusive Private Participation (OTG-1)

(Note: There is no IRR value till the end of 2025 as the cost is much higher than the benefit received. The IRR at the end of 2030 is -1.82)

Other OTG’s

A complete description of the four other OTG’s is provided in the project report (Khasnabis and Mishra 2009). Essentially, the transition from OTG-1 to OTG-3 is marked by higher levels of subsidy to the private entity either by reduced cost, or by extension of the concession period, or by a combination of the two. These relaxation policies adopted to encourage private participation will obviously reflect on higher financial responsibility for the public sector. OTG-4 represents major public participation with the private entity playing a minor role in sharing costs and revenues. OTG-5 represents a completely public undertaking, encompassing all financial, management, operational and maintenance responsibilities.
TABLE 2 OTG Strategies, Relaxation Policies and IRR’s

<table>
<thead>
<tr>
<th>OTG Strategy</th>
<th>Explanation</th>
<th>Relaxation Policy</th>
<th>Entity Objective</th>
<th>IRR (%) (Private)</th>
<th>IRR (%) (Public)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OTG-1</td>
<td>Exclusive Private</td>
<td>1. No Relaxation</td>
<td>Profit Maximization</td>
<td>4.61</td>
<td></td>
</tr>
<tr>
<td>OTG-2</td>
<td>Major Private Participation</td>
<td>2(a). Toll Plaza Cost Subsidy 2(b). Toll Plaza, Interchange, and Inspection Plaza Cost Subsidy 2(c). Construction Cost Subsidy</td>
<td>Profit Maximization</td>
<td>5.14</td>
<td>5.89</td>
</tr>
<tr>
<td>OTG-5</td>
<td>Exclusive Public</td>
<td>No Relaxation</td>
<td>Consumer Surplus Max.</td>
<td>3.51**</td>
<td></td>
</tr>
</tbody>
</table>

Note: *: Private entity is only responsible for a part of the construction cost and receives all the benefits throughout the concession period. Lesser investment and higher return for the private entity has resulted in relatively larger IRR. This OTG strategy is considered as an attractive option for the private entity.**: IRR for the public entity (the remainder of the IRR are for the private entity).
Synthesis of Results for OTG Strategies

The objective of OTG strategy analysis is to assess the fiscal impact of varying levels of joint ownership scenarios on the public and the private entities. OTG-1 and OTG-5 represent exclusive private and public projects respectively (with no role by the other agency) as the two ends of the joint participation spectrum. OTG-2 and OTG-3 represent various levels of relaxation policies designed to provide higher levels of subsidies to the private agency with all the revenue committed to the agency. OTG-4 is a major public program with minor participation by the private agency both in cost and revenue sharing. Results of this analysis are presented in Table 2 and can be summarized as follows:

- For exclusive private participation (OTG-1), the project is not financially viable, as the IRR generated (4.61 percent) is lower than the MARR (6.0 percent). Further, varying degrees of relaxation are proposed in (OTG-2 and OTG-3) to encourage private participation. None of the three relaxation policies within OTG-2 resulted in IRR values higher than the MARR (6%). On the other hand, all of the four relaxation policies in OTG-3 resulted in financially viable solutions with IRR values exceeding the MARR (6%). The exceedingly high IRR (22.97 percent) for OTG-3(d) is caused by a very large construction cost subsidy to the private agency. For major and exclusive public participation (OTG-4 and OTG-5), the project is not financially viable, with none of the IRR values exceeding the 6 percent mark. OTG-4(a) and 4(b) represent small amounts of costs and revenue sharing by the private agency with the public agency having the major financial responsibility. OTG-5, on the other hand, is exclusive public participation and is not financially viable.
• In summary, the OTG strategies representing different joint ownership scenarios generate varying returns ranging from a low of 4.61 percent to a high of 22.97 percent to the private agency, and from 3.27 percent to 3.69 percent for the public agency. With an assumed MARR of 6 percent, all the four options within OTG-3 are found financially viable for the private agency. Unfortunately, none of the two options in OTG-4, and the single option in OTG-5 is financially viable for the public agency.

• The implication of the above finding (that the project is not financially viable, with the public agency sharing most of the fiscal responsibility using the MARR criterion) needs additional discussion, even though this is somewhat irrelevant to the focus of the paper, incorporation of uncertainty and risk in investment decision. Recalling that the only revenue considered in the analysis is the toll collection, one might argue that the real benefit of such a project to the public at-large manifests itself in the form of “intangibles” such as: increased mobility, improved safety, higher levels of comfort and reduced emissions. Further, for most highway projects supported by taxpayers, no such revenue is ever collected, and only toll roads are an exception in this regard, because of the funding mechanism. For large-scale transit projects funded by tax dollars, the fare-box revenue typically covers a portion of the operating cost, and there is no “payback” program for the capital cost. Thus, one might further contend that an IRR of 3.51 percent for OTG-5 based upon toll collection alone, should be considered an acceptable return, even though it is lower than the MARR of 6 percent. Alternatively, the above mentioned “intangibles” should be duly incorporated in the analysis. This topic can be a
subject of future research.

5.5 Risk Analysis

Results of OTG analysis serve as major input to risk analysis. Toll values for the horizon years were determined from the uncertainty analysis. The upper and lower limit of the toll values are set using an assumed coefficient of variation of ten percent. MCS techniques were used to obtain the simulated cumulative cash flow for design years. The random values are instrumental in developing ridership estimates resulting from elastic traffic assignment, and the corresponding operation and maintenance cost. For each random toll value, and the appropriate ridership, operation and maintenance cost changes, the IRR value is estimated. 10,000 such iterations are performed, and the corresponding IRR’s are recorded. The authors have used the results of OTG-3(b) to demonstrate risk analysis and integration of uncertainty and risk. The distribution of all realizations of IRR for OTG-3(b) is plotted in Figure 4.

Procedure for Obtaining VaR

VaR is measured in absolute and relative terms. Absolute VaR is defined as the maximum expected lower value of IRR at a given level of confidence. Relative VaR is defined as the difference between 50\textsuperscript{th} percentile and the absolute VaR. Figure 4 shows the mean value of IRR on the X-axis, the frequency on primary Y-axis, and the cumulative probabilities on secondary Y-axis for the OTG-3(b), concession period extension. The 5\textsuperscript{th} percentile IRR of the distribution of OTG-3(b) is 5.99 percent, indicating that only five times out of 100, IRR is likely to be less than 5.99 percent (or, 95 times out of 100, IRR is likely to exceed 5.99 percent). Thus the VaR for OTG-3(b) is 5.99 percent at 95 percent level of confidence. In other words, the minimum expected IRR for five percent of the times can not be lower
than 5.99 percent. The 95th percentile relative VaR is the difference between the 50th percentile IRR and 5th percentile IRR (or VaR at 95th percentile), i.e. 6.04-5.99 = 0.05 percent. The 95th percentile relative VaR suggests that the maximum difference in IRR at 95th percentile level of confidence can not exceed 0.05 percent for the OTG-3(b). Similarly, the 90th percentile VaR can be determined. The 95th and 90th percentile absolute and relative VaR for all the strategies are presented in Table 4. Relative VaR creates a benchmark for comparison between alternatives. Hence, the authors feel that relative VaR should be used as a measure of risk when comparing alternatives with different means.

Note: 50% IRR is 6.04%

**FIGURE 4 Value at Risk for OTG-3 Concession Period Extension**
### TABLE 4 Risk Analysis Summary

<table>
<thead>
<tr>
<th>OTG Strategy</th>
<th>Relaxation Policy</th>
<th>Mean IRR (%</th>
<th>95% VaR (%)</th>
<th>90% VaR (%)</th>
<th>95% Relative VaR (%)</th>
<th>90% Relative VaR (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OTG-1</td>
<td>No Relaxation</td>
<td>4.66</td>
<td>4.58</td>
<td>4.59</td>
<td>0.08</td>
<td>0.07</td>
</tr>
<tr>
<td>OTG-2</td>
<td>2(a). Toll Plaza Cost Subsidy</td>
<td>5.19</td>
<td>5.10</td>
<td>5.11</td>
<td>0.09</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>2(b). Toll Plaza, Interchange, and Inspection Plaza Cost Subsidy</td>
<td>5.95</td>
<td>5.86</td>
<td>5.88</td>
<td>0.08</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>2(c). Construction Cost Subsidy</td>
<td>5.90</td>
<td>5.81</td>
<td>5.83</td>
<td>0.08</td>
<td>0.07</td>
</tr>
<tr>
<td>OTG3</td>
<td>3(a). Construction Cost Subsidy</td>
<td>6.19</td>
<td>6.10</td>
<td>6.12</td>
<td>0.09</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>3(b). Concession Period Extension</td>
<td>6.04</td>
<td>5.99</td>
<td>6.00</td>
<td>0.05</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>3(c). Construction Cost Subsidy and Concession Period Extension</td>
<td>7.24</td>
<td>7.18</td>
<td>7.19</td>
<td>0.06</td>
<td>0.05</td>
</tr>
<tr>
<td>OTG-4</td>
<td>4(a). Partly Construction Cost by Private Entity</td>
<td>23.66</td>
<td>23.19</td>
<td>23.27</td>
<td>0.46</td>
<td>0.39</td>
</tr>
<tr>
<td></td>
<td>4(b) Operation and Maintenance Cost –Public Entity</td>
<td>3.83</td>
<td>3.74</td>
<td>3.76</td>
<td>0.09</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>4(c). Construction Cost Subsidy-Public Entity</td>
<td>4.10</td>
<td>4.01</td>
<td>4.02</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td>OTG-5</td>
<td>No Relaxation</td>
<td>3.65</td>
<td>3.55</td>
<td>3.59</td>
<td>0.10</td>
<td>0.07</td>
</tr>
</tbody>
</table>
6. Integration of Uncertainty and Risk

The concepts of uncertainty and risk from an investment decision perspective were addressed in an earlier section of this paper. An approach to integrate uncertainty and risk for exploring different OTG options is presented in this section, with IRR serving as the MOE for uncertainty analysis, and VaR for risk analysis. The MOE’s are presented in Figure 5(a) and 5(b) to demonstrate both uncertainty and risk for different OTG strategies on the X and Y-axes respectively.

Figure 5(a) shows all the OTG strategies considered in uncertainty and risk analysis. However, OTG-4(a) appears to be one outlier from rest of the strategies. Figure 5(b) shows integration of uncertainty and risk without OTG-4(a) to better visualize the effect of all OTG’s considered.

FIGURE 5(a) Integration of Uncertainty and Risk (Note: The outlier in the graph represents higher IRR and VaR)
A favorable OTG strategy is the one with IRR higher than the MARR (six percent in this case) and a relative VaR at the lower end. While MARR represents a threshold value, no such threshold value on relative VaR is established in the literature. Table 3 shows that four OTG strategies resulted in IRR values higher than six percent. The highest IRR (22.97%) resulted for the OTG-3(d) strategy for the private entity. For the same OTG strategy, the relative VaR is also the highest (0.46%, at 95 percent level of confidence), making this strategy most vulnerable to future risks (also shown in Figure 5(a)). From the remaining feasible strategies, the combination of construction cost subsidy and concession period extension (OTG-3(c)) resulted in an IRR of 7.24% and a relative VaR is 0.06% at 95 percent level of confidence. However, all the OTG-3 strategies appear to be feasible with higher IRRs and lower relative VaR (Figure 5(b)).
7. Conclusions

The primary objective of this study is to develop a framework to incorporate the concept of uncertainties and risks in transportation investment decisions; and to apply the framework on a real-world case-study to augment the decision making process. The entities often involved in, or affected by such investment decision are enlisted as: private, public, and users, each with different set of objectives and expectations; profit maximization, consumer surplus maximization and inequality minimization, respectively. A procedure for a single entity uncertainty analysis is presented as a bi-level process. The upper level constitutes the preference of the policy maker, and the lower level determines the user’s response to the policy. The output of uncertainty analysis serves as an input to risk analysis. IRR and VaR are considered as the MOE’s for uncertainty and risk analysis respectively and determined using MCS techniques. The objective of each entity, when subjected to uncertainty, is considered in assessing the optimal demand and toll estimates.

Typically, economic analysis of infrastructure projects are based upon the assumption of deterministic cash flows using the premise that all future costs and revenues are fully “known”. In reality, there are significant uncertainties stemming from the lack of knowledge about the future cash flow streams. The picture is further complicated by the fact that transportation projects are typically long-term in nature, and that “the longer the project, the higher the uncertainty”. The proposed framework is an attempt by the authors to address the effect of uncertainty and risk on transportation investment decisions.

Uncertainty analysis can be used to determine the optimal value of the entity-specific objective function (profit maximization or welfare maximization or inequality minimization) in a joint ownership project in the face of variable travel demand, toll,
operation and maintenance costs. With no prior information on demand, toll, and cost, a deterministic analysis can result into misleading conclusions, whereas uncertainty analysis determines the optimal results from each entity perspective. Further, risk analysis considers inputs from uncertainty analysis to determine the maximum expected lower value of MOE at a given level of confidence.

If the single entity uncertainty analysis does not result in feasible solutions, relaxation policies are proposed. Relaxation policies may include extension of the concession period and financial support from the other entities involved in the decision making process leading to the formulation of different OTG strategies to reflect multi-entity operation of the proposed facility and various relaxation policies.

IRR is considered as the measure of feasibility for uncertainty analysis. Similarly, VaR is recommended as a measure of risk analysis. A methodology for integrating uncertainty and risk is proposed. It is observed that projects producing higher IRR may also be associated with higher VaR. The integration of uncertainty and risk allows the decision maker to choose from a set of alternative investment strategies to minimize uncertainty and risk, subject to the IRR meeting the MARR criterion. The concept of integrating uncertainties and risks demonstrated the need to consider not only the return on the investment (IRR) in the decision making process, but also the risk factor (VaR). The final decision should be based upon the joint consideration of both factors. Ideally, the alternative with the highest IRR (provided it is higher than the MARR), and with the minimum relative VaR is the most desired one.

The framework is applied to study the investment decision making of DRIC connecting two countries US, and Canada; a project in the planning stage for over ten years.
Results of the case study indicate that the framework presented is viable; however additional research is needed to integrate the perspectives of all the entities into a multi-objective framework. The case study presented clearly demonstrates that a strategy considered economically viable under deterministic scenario, may not be so when risk factors are incorporated in the analysis. As another future task, the effect of changes in the toll structure of the competing bridges on DRIC can be incorporated into the uncertainty and risk analysis framework. The proposed procedure may be used by transportation and financing professionals involved in infrastructure investment decisions. Such professionals include: engineers/planners/economists, investment and cost analysts involved both in private and public financing of infrastructure projects.
Acknowledgement

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Appendix

*Private Investor’s Perspective*

If $P^n$ is the profit generated in year $n$, which is a function of the demand ($x$) and toll ($\tau$). $B^n$ and $C^n$ are corresponding revenue and cost for year $n$ respectively. The revenue generated is a function of uncertain demand and toll, while the cost can be presented in the form of capital and operation and maintenance cost. The revised equation for the private investor perspective is:

$$P^n(\tau, x(\tau, \varepsilon)) = \sum_{a \in A} \gamma x^n_a(\tau) \tau^n_a - C^n_{a,c} - O^n_a(x^n_a)$$

where, $\gamma$ is a parameter which converts hourly link flows to annual link flow, $x^n_a$, $\tau^n_a$, $C^n_{a,c}$, $O^n_a$ are the demand, toll charge, construction cost and operation and maintenance cost for year $n$ on link $a$ respectively. $N$ is the analysis period and $\widetilde{A}$ is a set of newly implemented links subjected to toll. The objective function for profit maximization can be formulated as:

$$\text{max. } \sum_{n \in N} P^n(\tau, x(\tau, \varepsilon))$$

subject to: $\tau^n, x^n(\tau, \varepsilon) \geq 0$

where, $x^n(\tau, \varepsilon)$ is determined from the lower level program and suggests that the toll value and the volume cannot be negative.
Public Investor’s Perspective

The consumer surplus can be mathematically represented as,

$$
\phi_{rs}^{n} = \int_{0}^{q_{rs}^{-1}(\omega)} d\omega - q_{rs}^{n} \pi_{rs}^{n}
$$

(8)

where, $\phi_{rs}^{n}$ is the consumes surplus for the O-D pair r-s for the year n, $q_{rs}^{n}$ is the demand between the Origin-Destination (O-D) pair r-s for year n, $q_{rs}^{-1}(\omega)$ is the inverse demand function for the O-D pair r-s, and $\pi_{rs}^{n}$ is the minimum travel cost between the O-D pair r-s. The first term of the equation 8 represents the user willingness to pay to travel from r-s and the second term is the amount user actually paid (or minimum travel cost to travel from r-s). The consumer surplus is a measure from the public entity perspective used in a number of studies in transport network design (Ukkusuri and Patil 2009, Chen and Subprasom 2007, Yang and Meng 2000, Zhang and Ge 2004, Zhang and Kumaraswamy 2001, Zhao and Kockelman 2006).

Consumer surplus for an O-D pair r-s for an improved case is given by (Ukkusuri and Patil 2009):

The consumer surplus for the total network can be represented as:

$$
\sum_{rs} \phi_{rs}^{n} = \sum_{rs} \int_{0}^{q_{rs}^{-1}(\omega)} d\omega - \sum_{rs} q_{rs}^{n} \pi_{rs}^{n}
$$

(9)

The annual consumer surplus in monetary terms can be represented as:
\[ \sum \theta_{n}^\alpha = \frac{\gamma}{\Theta} \left[ \sum_{n} \int_{0}^{\varphi_{n}} q_{n}(\omega) d\omega - \sum_{n} q_{n}^\alpha \sigma_{n}^{\omega} \right] \]  

(10)

where, \( \theta \) is a parameter which converts time value to monetary terms, \( \gamma \) is the parameter that converts hourly to annual demand. The savings in consumer surplus can be defined as the difference between the consumer surplus and the cost of the project ((Chen and Subprasom 2007, Yang and Meng 2000). This can be represented as:

\[ \psi^\alpha(\tau, x(\tau, \varepsilon)) = \varphi^\alpha - C^\alpha \]  

(11)

where, \( \psi^\alpha \) is the savings in consumer surplus. A higher consumer surplus is better for the public investor. The public entity perceives the user benefit equivalent to a value which travelers expect to receive from making trips as measured by the gross amount paid by the travelers in making a trip. The objective function for consumer surplus maximization can be formulated as:

\[ \text{max. } \sum_{n} \psi^\alpha(\tau, x(\tau, \varepsilon)) \]  

(12)

subject to: \( \tau, x(\tau, \varepsilon) \geq 0 \)  

(13)

where, \( x(\tau, \varepsilon) \) is determined from the lower level program.

**Road User’s Perspective**

Theil’s index, in its simplest form to measure road user’s perspective and can be estimated as (Theil 1967):

\[ \sum \theta_{n}^\alpha = \frac{\gamma}{\Theta} \left[ \sum_{n} \int_{0}^{\varphi_{n}} q_{n}(\omega) d\omega - \sum_{n} q_{n}^\alpha \sigma_{n}^{\omega} \right] \]  

(10)
\[ T^n_b = \sum_r \left( \sum_s q^n_{rs} \right) \left( \sum_s \phi^n_{rs} \right) \ln \left( \sum_s \phi^n_{rs} \right) \]  \hspace{1cm} (14)  

where, \( q^n_{rs} \) is the travel demand of the OD pair \( r-s \) in the \( n^{th} \) year, \( q^n \) is the total demand (i.e. \( \sum_r \sum_s q^n_{rs} \) ) for the whole network, \( \phi^n_{rs} \) is the consumer surplus improvement for the OD pair \( r-s \) in the \( n^{th} \) year, \( \phi^n \) is the total consumer surplus improvement (i.e. \( \sum_r \sum_s \phi^n_{rs} \) ).  

If every zone has same benefit then the Theil’s index is zero (perfect equality), and if the benefit is concentrated at one (perfect inequality) zone then the Theil’s index is \( \ln q^n \). The lower the Theil’s index, the more equitable is the project. The objective function for user inequality (between groups) minimization can be formulated as:

\[
\min \sum_n T^n_b (\tau, x(\tau, \varepsilon)) \]  \hspace{1cm} (15) 

subject to: \( \tau, x(\tau, \varepsilon) \geq 0 \)

where, \( x(\tau, \varepsilon) \) is determined from the lower level program.

**Route Choice Behavior**

While the upper level program determines the toll for various perspectives considered, the lower level determines the route choice of users for a designed toll value subjected to uncertain demand. The lower level problem is a user equilibrium traffic assignment with elastic demand (Sheffi 1985).
\[
\min_{x \in \mathbb{R}^{(r,s)}} \sum_{a \in (A \setminus \lambda)} \int_0^x t_a(w) \, dw + \sum_{a \in \lambda} \int_0^x \left( t_a(w) + \theta \tau \right) \, dw - \sum_{rs} \int_0^{q_{rs}'}(w) \, dw
\]  

(16)

subject to:

\[
\sum_f^r = q_{rs}
\]  

(17)

\[
f_k^r \geq 0
\]  

(18)

\[
q_{rs} \geq 0
\]  

(19)

\[
x_a = \sum_{r} \sum_{s} \sum_{k} f_k^r \delta_{a,k}^r
\]  

(20)

\[
\delta_{a,k}^r = \begin{cases} 
1 & \text{if link } a \text{ is on path } k \text{ between O-D } r - s \\
0 & \text{Otherwise}
\end{cases}
\]  

(21)

The objective function in expression 16 minimizes the travel time of the network till equilibrium is achieved. The first two terms are the link performance function of all non-tolled and tolled links in the network respectively. The third term is the inverse demand function associated with the OD pair \( r-s \), which is a decreasing function of the OD travel times. Expression 17 is a flow conservation constraint to ensure that flow on all paths connecting each OD pair has to equal the trip rate. Expression 18 and 19 are non-negativity constraints to ensure that the flow cannot be negative. The definitional relationship of link flow from path flows is presented in expression 20 and 21. The minimization problem in expression 16 consists of toll value \( \tau \) which is a function of a set of link flows \( x_a(r,s) \) and a set of OD demands \( q_{rs}(r,s) \). Flow in lower level is a function of toll in the upper level (recall three policy perspectives specified in upper level).
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