Multi-period Transportation Network Investment Decision Making and Policy Implications using Econometric Framework

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Abstract

Transportation infrastructure projects take numerous years of planning before they are scheduled for construction. Prioritization of such projects over a multi-period planning horizon (under a limited budget) is a computationally challenging task, as it is usually formulated as a bilevel network design problem (NDP). Although multi-period network investment is studied in the literature, its application by public agencies is limited because of lack of reliable data and the long-term nature of the problem. The contribution of this research is two-fold. First, it extends a previously published single year discrete network design formulation to a multi-period discrete network design problem (MPNDP) to capture both the spatial and temporal patterns of multi-period network investment decision. Second, using the MPNDP investment results; and the network characteristics, this research develops and evaluates a new econometric model, the Multi-Period Econometric Network Investment Model (MENIM). MENIM can be used by agencies in place of MPNDP to approximate network investments. The proposed model is calibrated and validated using medium to large scale networks. MENIM results are comparable to MPNDP results with acceptable computational time. Finally, policy implications for public agencies are provided.

Key Words: Network Design Problem, Investment; Decision Making; Econometric Framework; Transportation Planning

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1. Introduction

Decision for roadway transportation network investments either for improvement (capacity or operational) or maintenance are needed on a regular basis to ensure that travel demand is satisfied within acceptable congestion levels and safety. Planning for such improvements requires a strategic approach that considers budget availability and policy limitations over a period of multiple years, while at the same time accounting for network utilization changes by the users (due to the investment, land use changes, and population growth). Even though developing and adopting a multi-period network improvements plan is a preferred strategy, when compared to single year planning, the literature suggests that multi-period network investment research is limited (Ukkusuri and Patil 2009; Wei and Schonfeld 1994).

The roadway transportation network investment problem is typically formulated as a network design problem (NDP see Johnson et al., 1978). Depending on the definition of the decision variables the NDP can be classified as: (i) continuous or CNDP (Suwansirikul et al. 1987), (ii) discrete or DNDP (Chang and Chang 1993; Farvaresh and Sepehri 2012; Haas and Bekhor 2016; Lou et al. 2009; Mishra et al. 2014a, 2015; Miandoabchi and Farahani 2011; Wang et al. 2015, 2013; Welch and Mishra 2014), or (iii) mixed (MNDP) network design problem (Yang and Bell 1998). NDP is typically formulated as a bi-level problem where: (i) the upper-level problem determines the network improvements decision, and (ii) the lower-level problem determines the travel pattern of the network users. This bi-level structure and computational complexity of the lower-level result in the NDP being one of the most challenging problems in transportation investment (Allende and Still 2012; Chen and Chen 2013; Colson et al. 2005; Fontaine and Minner 2014; Zhang and Gao 2009).

Formulating the NDP is easy but solving it is computationally challenging. As a result, public agencies seldom use it as a decision-making tool while at the same time limited literature has been published for cases of medium to large-scale transportation networks with single- or multi-period planning (Mathew and Sharma 2009; Wei and Schonfeld 1994). To address these issues, this research proposes an alternative approach and has the following contributions. First, we extend the single year NDP approach proposed by Wang et al. (2013) to a multi-period NDP (MPNDP) and present patterns of network investment (i.e., network size, planning period, budget, and demand) for several test and real-world networks. Second, we develop, calibrate, and evaluate a Multi-Period Econometric Network Investment Model (MENIM) that can be used as a surrogate to the NDP with comparable accuracy but slightly higher computational efficiency.

The remainder of the paper is organized as follows. A summary of the published literature on DNDP and MPNDP is presented in Section 2. Section 3 presents the methodology for MPNDP. Section 4 discusses the results from an application of MPNDP and their effects on policy analysis. The fifth section presents the econometric framework, calibration, and validation of the MENIM model. Policy implications, conclusions and future research are presented in the final section.

2. Literature Review

In this section, a brief review of the DNDP and MPNDP literature is presented to provide the foundation to understand the models and formulations proposed in this research. Since DNDP is considered in this research, the various formulations and solutions related to DNDP are thoroughly discussed.

2.1 Discrete Network Design problems

2.1.1 Bi-level formulations

One of the first efficient approaches to formulate the DNDP was proposed by LeBlanc et al. (1975) as a bi-level programming model in which the upper-level minimized the total system cost subject to investments on links, while the lower-level solved a User-Equilibrium (UE) problem with fixed demand. A branch-and-bound (B&B) algorithm was proposed to solve the resulting problem and a lower bound for the branch was derived by requiring the travelers' route choice behavior to follow the system optimal (SO) principle instead of UE principle (Wang et al. 2013). The algorithm was relatively inefficient because the lower bounds were relatively loose. A revised B&B algorithm was developed based on LeBlanc (1975) (Farvaresh and Sepehri 2012). The revised algorithm coped with explicit or implicit path enumeration, the link-node network representation with multicommodity flows and found a global optimal solution for DNDP while guarding against Braess' paradox.

Another discrete network design problem (DNDP) was proposed by (Chen and Chen 2013) where the variables were defined as a series of integers rather than binary 0 and 1. A solution algorithm was designed combining a B&B with the Hooke-Jeeves algorithm (Hooke and Jeeves, 1961). The algorithm was inefficient with respect to computational time and provided only local optimal solutions. (Chen et al. 2015) proposed a bilevel mixed NDP where the upper-level minimized the average travel time while the lower-level solved a dynamic user-optimal condition (formulated as a variational inequality problem). A surrogate-based optimization framework was proposed for solving the problem that produces computational time savings by exploring the input–output mapping surface in a more systematic and efficient way. (Gao et al. 2005) introduced a traditional bi-level programming model for the DNDP and then proposed a new solution algorithm by using the support function concept to express the relationship between improvement flows and the new additional links in the existing urban network.

2.1.2 Single-Level formulations

In another study, a single-level mixed integer linear programming (SL-MILP) formulation was presented for the bi-level DNDP by appropriately modifying the travel time function (Farvaresh and Sepehri 2011). The nonlinearity of the travel time function was removed by means of a convex-combination based linear approximation which takes advantage of a unimodular structure. The authors presented the Karush–Kuhn–Tucker (KKT) conditions in the lower-level problem for the optimality of link flows. This formulation was able to provide the optimal solution for small size problems, but it could not be applied to medium to large scale due to computational limitations. (Zhang et al. 2014) formulated NDP as a single-level problem with transit routes explicitly modeled by introducing a set of complementarity constraints (MPCC). An active-set algorithm was employed to solve the problem but was limited to small scale networks. (Fontaine and Minner. 2014) reformulated the NDP problem into a single-level by replacing the lower-level with its Karush-Kuhn Tucker conditions. The resulting non-linear model was linearized and solved by a Benders Decomposition algorithm to global optimality. The application was limited to small scale networks.

In another study, the CNDP and DNDP were formulated as a single-level optimization problem with equilibrium constraints (Wang and Lo, 2010). The authors transformed the equilibrium constraints into a set of mixed-integer constraints and linearize the travel time function. The integer variables correspond to the whether a link gets added capacity. (Luathep et al. 2011) formulated the mixed NDP as a single-level optimization problem with variational inequality constraints representing the UE condition.

The variational inequality constraints were needed to be satisfied for all feasible link flows. The MNDP is transformed into a mixed-integer linear programming (MILP) problem which was solved using a global optimization algorithm based on a cutting constraint method.

2.1.3 Other formulations

(Lou et al. 2009) proposed a DNDP formulation under demand uncertainty and a cutting plane scheme algorithm was proposed to solve the resulting problem. The algorithm could converge to the global optimum but only under certain condition that the relaxed robust discrete network design be solved globally using subset demand and worst-case demand problem be solved globally using capacity expansion. The proposed algorithm could not handle real life size problems. (Wang et al. 2015) proposed a novel DNDP formulation to determine optimal new link additions and optimal capacity simultaneously. A global optimization solution algorithm, called range reduction technique was proposed that includes linearization and outer approximations. In another study, a mixed NDP was formulated where the lower-level is a standard UE problem (Zhang and Gao 2009) and a locally convergent algorithm is proposed to solve the problem by applying penalty function method.

Other approaches for solving the DNDP in the literature include heuristic approach given by (Chang and Chang 1993; Haas and Bekhor 2016), genetic algorithms for solving the DNDP (Kim et al. 2007) and also particle swarm optimization for solving the DNDP of freight transportation (Yamada and Febri 2015).

2.2 Multi-period Network Design Problems

Research on the multi-period network investment decision making is limited. A multi-period NDP for the dynamic investment problem was developed (Wei and Schonfeld 1994) and a B&B algorithm was used to determine the optimal solution. In another study, a multi time period flexible network design problem was developed considering both demand uncertainty and demand elasticity (Ukkusuri and Patil 2009). This formulation relaxed the flexibility to make future network investment. The problem was formulated as a bilevel stochastic mathematical programming with complementarity constraints (STOCH-MPEC) in which the bi-level formulation is converted to a single-level using non-linear complementarity constraint conditions for the UE problem. Sun et al. (2011) explored the multi-period network design problem and proposed a bi-level programming model with uncertain demand where the upper-level maximized the consumer surplus with budget constraint. Moreover, there are various studies conducted in context of multi-period such as multi-period discrete facility location problem considering both transportation and location costs (Albareda-Sambola et al. 2012) and also in reverse logistics network design problem considering multi-period presenting a mixed integer programming formulation (Alumur et al. 2012). Finally, a multi-period multi-path refueling location model was developed to dynamically satisfy origin-destination trips (Li et al. 2016). It was formulated as mixed integer linear program and solved by heuristic based genetic algorithm.

2.3 Literature Gap

The review of the literature suggests that most published studies have focused on solving the single year DNDP or CNDP with little research focusing on multi-period network investments decision ((Lai and Shih (2013); Barmann et al. (2017); Kalinowski et al. (2015); Kumar and Mishra (2018); Mishra et al. (2016);

Ngo et al. (2020); Petersen and Taylor (2001); Hooghiemstra et al. (1999); Welch and Mishra (2014b); Sharma and Mishra (2013)). Several researchers reformulated the bi-level problem into a single-level using the lower-level's optimality conditions (e.g., KKT) but were not able to find the global optimal solution for medium to large scale problem instances. In this research, we propose a new modeling framework for the network design problem that combines the existing bilevel formulation with an econometric model. The proposed framework is applied to a multi-year network investment decision making problem. The proposed approach can be used as an alternative to the NDP for network improvement resource allocation as it identifies the relationship between network supply and demand with acceptable accuracy and reduced computational complexity. Using econometric modeling as opposed to hierarchical programming for network design problems significantly reduces the effort and expertise required by the transportation planner/engineer once the econometric model has been trained.

In this paper, we present the formulation of the bilevel DNDP proposed by Wang et al. (2013) which is extended to MPNDP using SO relaxation and it is tested on networks of various sizes. Then an alternative approach is developed using multinomial framework (MENIM) for evaluating allocation potentials of large-scale networks and identifying potential benefits and limitations of such an approach over MPNDP.

3. Problem Formulation (MPNDP)

In any given transportation network with a given set of link improvements, the MPNDP can be defined as the problem of finding the combination of link improvements during a predefined time-period that minimizes the total system travel time subject to several constraints. The MPNDP has three important features: a) link expansion or improvements can only be completed using the budget allocated for that time-period, and the remaining portion of the budget can be rolled over into succeeding periods, b) continuation of link improvements into successive periods is preserved, that is, the link improvements in one-period is carried over to the next period for improvements, and c) link improvement benefits continue in the subsequent time periods.

The MPNDP with multiple capacity levels is originally formulated as a bilevel programming model where at the upper-level problem the decision maker determines the link improvements that will minimize the Total System Travel Time (TSTT) within a prespecified budget. At the lower-level problem user behavior is replicated through a UE traffic assignment problem. In this problem formulation, it is assumed that demand and budget are constant for each period and the only link improvement available is a one or two-lane additions. The nomenclature used in this research was adopted from Wang et al. (2013) and is modified for the multi-period mathematical formulation of the MPNDP model and is presented next.

Sets

A	=	Set of links
N	=	Set of nodes
$L \subseteq A$	=	Set of candidate links with improvements
$I = \{0, 1, 2\}$	=	Set of number of lanes to be added
O_p	=	Set of Origin-Destination (O-D) pairs in period p
P	=	Set of years in the planning period ($P = \{1, 2, \dots .p\}$)

Parameters

B_p = Budget for improvements in period p;
d_a = \$1 million/mile, incremental cost of adding one lane, $a \in L$;
l_a = Length of link a, $a \in A$;
C_a = Initial capacity of link a, $a \in A$;
α_a, β_a = Parameters of travel time function on link a, $a \in A$;
ΔC_a = Additional capacity on link a, $a \in L$;
t_a = Free flow travel time on link a;
q_{op} = Trip rate between O-D pair o in period p;
μ = Parameter excluding first set of solution (= 2)

Decision Variables

x_{ap} = $flow\ on\ link\ a \in A\ at\ period\ p \in P$ (lower-level decision variable)
v_{ap}^{i} = 1 if link $a \in A$ is improved in period $p \in P$ with the addition of $i \in I$ lanes and zero otherwise (upper-level decision variable)

Auxiliary Variables

$t_{ap}(x_{ap})$ = Travel time on link a (Bureau of Public Roads (BPR) function) in period p
π_a = Improvements cost of link a,
U_p = Unspent budget in period p, $p \in P \cup \{0\}, U_0 = 0$;
C_{ap} = Capacity of link a in period p, $a \in A$;

Explicit Sets, Parameters and Variables

F_U = Objective function of MPNDP
F_L = User Equilibrium Objective function
f_{kp}^{o} = Flow on path k connecting O-D pair o in period p;
δ_{akp}^{o} = 1 if link a is on path k between O-D pair o in period p and 0 otherwise
K_{op} = Set of paths connecting O-D pair $o \in O_p$ in period p

3.1 Extended Multi-Period NDP formulation Using SO Relaxation Method

In general, it is very difficult to solve the bi-level NDP because the problem is non-convex and NP-complete (Farvaresh and Sepehri 2011) and the relationship between UE link flow and upper-level decision variable(s) (in this case, the optimum number of lanes added to each link) is not explicitly defined. According to (Wang et al. 2013), if the users' route choice decision follow the SO principle, then we would obtain a single optimization problem. Hence, the bilevel problem is relaxed and formulated into single-level using a global optimization method called SO-relaxation. It exploits the property that a road network design decision under SO principle can be considered as a good approximate solution under UE principle.

The solution approach consists of three main steps that involves solving the relaxed problem, solving the user equilibrium, and solving the relaxed problem excluding the generated solutions from the relaxed problem to get the second-best solution set. If the objective function of the relaxed problem (step 1) is less than the objective function obtained in *step 3*, then the solution generated in *step 1* is the optimal solution.

Step 1:

The relaxed multi-period problem (MPNDP) can be formulated as,

$$\min_{v,x} \sum_{p\in P} \sum_{a\in L} \sum_{i\in I} x_{ap}\, t_{ap}\left(x_{ap}\right) \quad (1)$$

Subject to

$$\sum_{i\in I} \sum_{a\in L} [(\pi_a)] \le B_p + U_{p-1} \;\; \forall\, p \in P \quad (2)$$

$$\sum_{k\in K} f_{kp}^{o} \; = \; q_{op} \;\; \forall\, p \in P, o \in O_p \quad (3)$$

$$x_{ap} \; = \sum_{o\in O} \sum_{k\in K} \delta_{akp}^{o} f_{kp}^{o} \quad \forall\, a \in A, p \in P, o \in O_p \quad (4)$$

$$\sum_{i\in I} v_{ap}^{i} \; = 1 \;\; \forall \;\; a \in L, p \in P \quad (5)$$

$$\sum_{p\in P} \sum_{i\in\{1,2\}} v_{ap}^{i} \; \le 1 \;\; \forall\, i \in I, a \in L \quad (6)$$

$$C_{ap} \; = C_a + i.\, v_{ap}^{i}\, \Delta C_a \;\; \forall\, i \in I, a \in L, p \in P \quad (7)$$

$$U_p = B_p + U_{p-1} - \sum_{i\in I} \sum_{a\in L} [(\pi_a)] \quad \forall\, p \in P \quad (8)$$

$$v_{ap}^{i} \; \in \{0, 1\} \;\; \forall\, i \in I, a \in L, p \in P \quad (9)$$

$$\pi_a = \sum_{i,p} i v_{ap}^{i}\, l_a d_a \;\; \forall\, a \in A, i \in I \quad (10)$$

$$f_{kp}^{o} \; \ge 0 \;\; \forall\, k \in K_{op}, p \in P, o \in O_p \quad (11)$$

$$U_p \ge 0 \quad (12)$$

Eq. (1) represents the objective function of the problem. The first component on the objective function is the total travel time for the set of links without improvement and the second component if the total travel time for the set of links with improvement. Eq. (2) estimates the total cost or the allocated resource for capacity expansion does not exceed the budget for the respective period. Eq. (3) ensures the demand is satisfied within the network which is formulated in terms of path flows. Eq. (4) is the definitional incidence relationship which defines the relation between link flow and path flow. Eq. (5) ensures that only one capacity level is chosen for each candidate link. Considering the mutually exclusive nature of link improvements, one link cannot be improved more than once in the planning period. Eq. (6) ensures overall mutual exclusiveness of link improvements within the planning period. Eq. (7) estimates the new link capacity at each time-period. Eq. (8) calculates the unused amount of budget for every period (which gets rolled over into the next budget allocation-see equation 2). Eq. (9) defines the decision variable as binary. The total cost for link improvements is calculated by Eq. (10) as the product of the link length and incremental cost. Finally, Eq. (11) ensures non-negative path flows and Eq. (12) ensures non-negative budget. It is assumed throughout the study that the link travel time function (BPR function) is strictly increasing and convex so that we can have a unique solution. While solving the multi-period user equilibrium problem, the free flow travel time of the corresponding links is assumed to be constant.

In this paper, it is assumed that the cost of a one-lane addition would equal the link length times the incremental cost and two-lane addition would cost twice the link length times the incremental cost. The link length, number of lanes addition and the construction cost assumed herein does not limit the generalization of the model and can be defined by any other.

The study uses the BPR function (Sheffi 1985) for the determination of link travel time as shown below:

$$t_{ap}\left(x_{ap}\right) = \left[t_{\mathrm{a}}\left(1+\beta_a\left(\frac{x_{ap}}{C_{ap}}\right)^{\alpha_a}\right)\right] \qquad \forall a \in A, p \in P \tag{13}$$

The parameters of BPR function used are $\beta_a = 0.15$ and $\alpha_a = 4$.

Step 2:

In this step, the user equilibrium problem is solved using the generated solutions from step 1 and assigning the updated capacities to the corresponding improved links. This is the traditional UE problem where it is assumed that the demand for travel is fixed and the users' route choice is characterized by the user equilibrium principle (Sheffi 1985). The total system travel time is also calculated using the user equilibrium link flows and link travel times.

Step 3:

This step is like *step 1* with the addition of another constraint (Eq. 14) that excludes the first set of solutions. This constraint is defined as follows:

$$\sum_{a\in L}\left[\sum_{all\ i\in I\ satisfying\ v_{ap}=0} v_{ap}^{i} + \sum_{all\ i\in I\ satisfying\ v_{ap}=1}(1-v_{ap}^{i})\right] \geq \mu \tag{14}$$

Eq. (14) excludes the first set of solutions and retains all other solutions.

Solution Algorithm

The solution algorithm utilized for this approach is adopted from (Wang et al 2013). The steps of the approach are given below briefly:

$Step\ 1: x_0 = argminF_L(x)$

Set i = 1

$Step\ 2: y_1 = argminF_U(y, x_0)$

$Step\ 3: x_i = argminF_L(y_i)$

$Step\ 4: set\ i = i + 1$

$Step\ 5: y_i = argminF_U(y, x_{i-1}): y \neq y_{i-1}$

$Step\ 6: if\ F_U(y_i, x_{i-1}) \geq F_U(y_{i-1}, x_{i-1})\ stop\ else\ go\ to\ step\ 3$

The SO-relaxation based method which is used to obtain the solution has a unique property, that is, constraint (14) eliminates previously generated solutions (y) which are not optimal. This property holds because each constraint in Eq. (14) excludes exactly one solution in non-optimal set. The value of objective function in step-1 is non-decreasing in each iteration of the method and with each excluded solution, one more constraint is added to the problem. Thus, the problem terminates in a finite number of iterations because the cardinality of set L is finite or when the objective function of step 1 is worse than the one in previous iteration.

4. Numerical Experiments

In this section, MPNDP was applied to a series of test and real-world networks and detailed analysis was conducted using various budget scenarios. In addition, the objective function, namely TSTT, is also analyzed under different budget scenarios. MPNDP is a mixed integer non-linear programming (MINLP) problem that can be solved by various optimization solvers. The algorithms are coded with MATLAB, calling the genetic algorithm (GA) solver to solve the MINLP model. A personal computer with Intel(R) Core (TM) i7 3.20 GHz CPU, 12.0 GB RAM, and Windows 7 Professional operating system is used for all tests. The relative optimality tolerance ε for approximating the upper-level objective function is set at 0.001. Note that the tolerance does not affect the quality of the final solution. However, the relative optimality tolerance for the objective function of UE (used in Step 2) is set at 1E-8, to ensure the quality of the UE link flows.

4.1 Case Study Networks

In this study, a total of eight transportation networks are used for testing MPNDP. Out of those eight, six are used in the MENIM model development and, two networks (Winnipeg and Montgomery) are used for MENIM validation. The networks in this study are selected so that a proportionate mixture of test and real-world networks with sizes varying from small-scale to large-scale is achieved. Scale is defined in terms of the number of nodes contained in each network. This mixture of networks is used for demonstrating the MPNDP as well as for practical application of the MENIM. Table 1 shows the case study networks used. Each of the test networks is analyzed for various budget scenarios. In this study the available budget was set equal to a percentage of the cost of adding 2 lanes to all the links in the network. That percentage ranged from 5 to 25 percent with an increment of 5 percent. In the remainder of this study we will refer to the total budget required to add 2 lanes to all the links in the network as optimal budget. In this study, a budget range between 5 and 25 percent was considered.

Table 1: Different types of case study networks

Network	Number of nodes	Number of Links	Number of non-zero OD pairs
Mahmassani Network	6	8	4
Sioux Falls	24	76	528
25-node	25	44	9
Anaheim	416	914	1416
Chicago Sketch	933	2950	93135
Winnipeg	1052	2836	4345
Atlanta	1102	2295	20736
Montgomery	1752	4420	50625

4.2 Econometric Framework Approach to NDP

To obtain link improvements solution for larger networks, we propose an alternative approach that can be used as a surrogate to the MPNDP for planning purposes. This approach uses an econometric framework to uncover the relationship between link improvements and network/demand properties without the need of a traffic assignment. The econometric framework proposed herein is a multinomial logit model (from now own referred to as MENIM). In this study, v_{ap}^{i} is the dependent variable where v_{ap}^{0} is taken as the base alternative.

Let i be the index for link improvement alternatives and S denote the universal choice set of the link improvements choice S = {0 = no improvements, 1 = one lane addition, 2 = two lane addition}. It is very likely that decision maker q only considers a subset S_q (of S), while making the actual choice. In MENIM, the utility associated with alternative i can be estimated as follows:

$$U_q^i = V_q^i + \varepsilon_q^i = \boldsymbol{\beta}_i \boldsymbol{X}_q^i + \varepsilon_q^i \tag{15}$$

Where U_q^i is the vector of dependent variables obtained from the MPNDP model, $V_q^i = \boldsymbol{\beta}_i \boldsymbol{X}_q^i$ is the observed part of the utility, $\boldsymbol{X}_q^i$ is the vector of explanatory variables, and $\boldsymbol{\beta}_i$ is the corresponding column vector of coefficients, and ε_q^i is standard error or random variable that captures all unobserved factors that is independent and identically distributed across alternatives and decision makers. So, the probability of a decision maker q choosing an alternative i from a set of alternatives S_q is given by:

$$P_q(i|S_q) = \frac{exp^{V_q^i}}{\sum_{i \in C_q} exp^{V_q^i}} \tag{16}$$

Estimation of the MENIM is straightforward. The loglikelihood function can be written as:

$$\sum_{i=1}^{n} \left(\sum_{i=0}^{I} I(i|S_q) \boldsymbol{\beta}_i \boldsymbol{X}_q^i - \log\left(\sum_{i=0}^{I} exp(\boldsymbol{\beta}_i \boldsymbol{X}_q^i) \right) \right) \tag{17}$$

where $I(.)$ is the indicator function. Thus, each observation contributes two terms to the loglikelihood function. The parameters of the model are estimated using Maximum Likelihood Estimation (MLE). MLE attempts to find the parameter values or vector of coefficients, $\boldsymbol{\beta}_i$ that maximize the above loglikelihood function.

4.3 Dataset and Model Results

The dataset consists of link improvements from all the links of six networks (Mahmassani, Sioux Falls, 25-node, Anaheim, Chicago Sketch and Atlanta) that are identified from MPNDP. It is combined with the corresponding link properties that are used as explanatory variables such as natural log of capacity of the link, link flows, volume to capacity ratio (V/C ratio) of link, link length, total number of paths served between an O-D pair, the year when the link gets improvement, allocated budget for improvement and the functional classification of the link. The first five variables are continuous variables and the rest are categorical. Each row of the data consists of a link with improvements level and its properties such as capacity, link flow, year of improvements, V/C ratio, budget, functional classification, and total number of paths served in the network. A link improvement with zero lane capacity addition was chosen as the reference alternative.

Table 2 presents the results of the MENIM model. From the results in Table 2, it can be observed that links with higher capacities are less likely to get improved than links with lower capacities. It is also observed that links with higher V/C ratio are more likely to get a 2-lane addition than links with lower V/C ratio. On the other hand, it is interesting to note that longer links are less likely to get improved compared to shorter links. This can be explained by the fact that longer links incur higher construction costs to get improved. It is also interesting to note that links serving a higher number of paths between OD pairs are more likely to get improved since these are important links that are frequently used in the network. Also, if the link is an interstate or major arterial, it is more likely to get improved. Table 2 also shows model statistical fit parameters such as log-likelihood value, Akaike information criterion (AIC), Bayesian information criterion (BIC) and rho-square. The cutoff value for the logit model used is 0.7 on the AUC (Area Under the Curve) ROC (Receiver Operating Characteristics) curve. AUC - ROC curve is a performance measurement for classification problem at various thresholds settings. AUC-ROC value of 0.7 means there is 70% chance that the MENIM model will be able to distinguish between 1-lane improvement and 2-lane improvements.

These results are intuitive and expected. There were also other factors such as the year in the planning period in which the link gets improved. However, these factors were found to be insignificant in the model. These results provide ample information for the planners and decision makers to consider while selecting the links for expansion in the network.

Table 2: MENIM results

Variables Description	**One lane addition**		**Two lane addition**	
(Base Alternative: Zero lane addition)	*Coeff.*	*Std.err*	*Coeff.*	*Std. err*
Constant	-19.258	0.073***	-19.196	0.073***
Log(capacity of link)	-0.113	0.017***	-0.099	0.017***
Volume to capacity ratio of link	0.181	0.078**	0.252	0.078***
Length of link	-0.070	0.008***	-0.059	0.008***
Total number of paths served	0.090	0.042**	0.082	0.042*
Interstate (Yes=1, No=0)	1.329	0.035***	1.345	0.035***
Arterial (Yes=1, No=0)	1.101	0.031***	1.122	0.031***
Year 1 improvements (yes=1, no=0) (base: no improvement)	43.236	0.024***	42.324	0.024***
Year 2 improvements (yes=1, no=0)	50.227	0.024***	49.197	0.024***
Year 3 improvements (yes=1, no=0)	48.995	0.025***	48.011	0.025***
Year 4 improvements (yes=1, no=0)	42.302	0.027***	41.304	0.027***
Year 5 improvements (yes=1, no=0)	51.559	0.028***	50.569	0.028***
Budget = 10% (yes=1, no=0) (base: Budget = 5%)	2.834	0.044***	2.624	0.044***
Budget = 15% (yes=1, no=0)	1.554	0.043***	1.511	0.043***
Budget = 20% (yes=1, no=0)	1.105	0.042***	0.955	0.042***
Budget = 25% (yes=1, no=0)	0.469	0.041***	0.299	0.041***
Number of Observations				31,435
Number of Parameters Estimated				16
Loglikelihood				-53,219
Loglikelihood at convergence				-1.693
Rho-square				0.952
AIC				106,471
BIC				106,605
** 90%* ***95%* ****99%* *cut-off value =0.7*				

4.4 Model Validation

To evaluate the predictive accuracy of the MENIM, a validation exercise was undertaken in which the link improvement results (Observed) at the two capacity levels obtained from the MPNDP model were compared to the number of link improvements from the MENIM model (Predicted). The validation data set consists of the network properties and the link improvements from the MPNDP of the Winnipeg and Montgomery County, MD networks. The MENIM model is then utilized to calculate the predicted probabilities for each link of the validation dataset. Next, these predicted probabilities are translated to link improvement choices, that is, 0, 1 or 2 values. The probabilities are compared for each link and the link improvement choice corresponding to the highest probability is selected for that link. The following example demonstrates the translation from predicted probability to link improvement choice:

Link Improvement Choice	Predicted Probability	Selected Choice
0	0.232	
1	0.452	✓
2	0.316	

Once the predicted number of link improvements were calculated for each category, the average (Total AVG) between the matched percent of zero, one lane and two-lane addition categories were calculated. The results of the MENIM model validation are presented in Table 3. These results showcase the favorable performance of the MENIM model as the percent of link improvements matched closely with that from the MPNDP model.

Table 3: Model validation

Budget	Planning Period	Number of Links	Link Improvements							
			Montgomery				Winnipeg			
			0 lane addition	1 lane addition	2 lane additions	Total AVG (%)	0 lane addition	1 lane addition	2 lane additions	Total AVG (%)
5%	Year 1	Observed	4100	219	101		2392	301	143	
		Matched	3501	195	86		1962	221	99	
		Matched (%)	85%	89%	86%	87%	82%	73%	69%	75%
	Year 2	Observed	4121	202	97		2389	292	155	
		Matched	3317	164	82		1819	196	101	
		Matched (%)	80%	81%	85%	82%	76%	67%	65%	70%
	Year 3	Observed	4123	201	96		2454	255	127	
		Matched	3441	142	64		2192	221	106	
		Matched (%)	83%	71%	67%	74%	89%	87%	84%	87%
	Year 4	Observed	4124	211	85		2446	268	122	
		Matched	3393	156	54		2125	240	110	
		Matched (%)	82%	74%	64%	73%	87%	89%	90%	89%
	Year 5	Observed	4134	191	95		2448	262	126	
		Matched	3564	129	60		2191	212	104	
		Matched (%)	86%	67%	64%	72%	90%	81%	82%	84%
10%	Year 1	Observed	3815	404	201		2125	483	228	
		Matched	**3166**	**305**	**137**		**1896**	**407**	**179**	

Table 3 (Continued)

		Matched (%)	83%	76%	68%	76%	89%	84%	78%	84%
	Year 2	**Observed**	3862	351	207		2168	436	232	
		Matched	3332	258	151		1952	361	191	
		Matched (%)	86%	74%	73%	78%	90%	83%	82%	85%
10%	**Year 3**	**Observed**	3831	359	230		2178	462	196	
		Matched	3425	330	210		1856	381	151	
		Matched (%)	89%	92%	91%	91%	85%	82%	77%	82%
	Year 4	**Observed**	3860	370	190		2108	484	244	
		Matched	3473	306	145		1990	440	218	
		Matched (%)	90%	83%	76%	83%	94%	91%	89%	92%
	Year 5	**Observed**	3794	414	212		2418	282	136	
		Matched	3215	342	176		1949	204	88	
		Matched (%)	85%	83%	83%	83%	81%	72%	65%	73%
15%	**Year 1**	**Observed**	3597	559	264		1793	690	353	
		Matched	3152	479	227		1483	554	276	
		Matched (%)	88%	86%	86%	86%	83%	80%	78%	80%
	Year 2	**Observed**	3500	608	312		1862	630	344	
		Matched	2795	463	230		1570	514	281	
		Matched (%)	80%	76%	74%	77%	84%	82%	82%	83%
	Year 3	**Observed**	3546	579	295		2013	526	297	
		Matched	3124	479	238		1693	400	220	
		Matched (%)	88%	83%	81%	84%	84%	76%	74%	78%
	Year 4	**Observed**	3538	575	307		2589	174	73	
		Matched	3150	462	196		2410	162	59	
		Matched (%)	89%	80%	64%	78%	93%	93%	81%	89%
	Year 5	**Observed**	3989	253	178		2671	105	60	
		Matched	3623	225	143		2320	69	34	
		Matched (%)	91%	89%	80%	87%	87%	66%	57%	67%
20%	**Year 1**	**Observed**	3210	824	386		1501	884	451	
		Matched	2568	627	269		1260	706	341	
		Matched (%)	80%	76%	70%	75%	84%	80%	76%	80%
	Year 2	**Observed**	3236	795	389		1639	798	399	
		Matched	3012	653	278		1613	696	306	

Table 3 (Continued)

20%		**Matched (%)**	93%	82%	72%	82%	98%	87%	77%	87%
	Year 3	**Observed**	3268	760	392		2443	262	131	
		Matched	2666	602	300		2150	229	119	
		Matched (%)	82%	79%	77%	79%	88%	87%	91%	89%
	Year 4	**Observed**	3989	285	146		2640	133	63	
		Matched	3484	254	126		2200	97	37	
		Matched (%)	87%	89%	86%	88%	83%	73%	59%	72%
	Year 5	**Observed**	4322	63	35		2691	96	49	
		Matched	3935	58	24		2323	66	27	
		Matched (%)	91%	92%	69%	84%	86%	69%	55%	70%
25%	**Year 1**	**Observed**	2997	922	501		1381	951	504	
		Matched	2790	793	425		1310	839	438	
		Matched (%)	93%	86%	85%	88%	95%	88%	87%	90%
	Year 2	**Observed**	2972	946	502		1876	621	339	
		Matched	2711	882	395		1758	591	274	
		Matched (%)	91%	93%	79%	88%	94%	95%	81%	90%
	Year 3	**Observed**	3397	683	340		2651	113	72	
		Matched	3063	575	264		2424	98	59	
		Matched (%)	90%	84%	78%	84%	91%	87%	82%	87%
	Year 4	**Observed**	4138	183	99		2744	55	37	
		Matched	3616	146	67		2457	49	30	
		Matched (%)	87%	80%	68%	78%	90%	90%	81%	87%
	Year 5	**Observed**	4392	19	9		2797	22	17	
		Matched	3961	12	6		2564	20	14	
		Matched (%)	90%	64%	69%	75%	92%	90%	81%	88%

To test the predictive strength of MENIM for future scenarios, another validation exercise is performed. Using the two networks of Montgomery County, MD and Winnipeg new validation data is obtained using the MPNDP, for future demand. In planning, a 20-year horizon is typically used when evaluating transportation needs and solutions. Different traffic growth pattern types can be present in a study area. In this study, and for simplicity, a linear growth is assumed, that is, traffic demand increases linearly over time. Therefore, growth factor = 1 + (G x N) = 1 + (0.05*20) = 2. Here, G is the linear annual growth rate and N is the number of years for the future demand. In this case, a linear annual growth rate of

5% is assumed. The future demand is then obtained by multiplying the base year demand with the growth factor. Once the data is obtained for the validation, table 4 is created similarly to table 3 which shows the average percentage of matched links for each budget scenario. Like table 3, table 4 also shows that percent of link improvements from MENIM model matched closely with the percent from MPNDP model.

Table 4: Model validation - future demand scenarios

Budget	Planning Period	Number of Links	Link Improvements							
			Montgomery				Winnipeg			
			0 lane addition	1 lane addition	2 lane additions	Total AVG (%)	0 lane addition	1 lane addition	2 lane additions	Total AVG (%)
5%	Year 1	Observed	4132	197	91		2434	275	127	
		Matched	3417	136	47		2076	228	99	
		Matched (%)	83%	69%	52%	68%	85%	83%	78%	82%
	Year 2	Observed	4126	189	105		2421	254	161	
		Matched	3549	175	75		1947	189	84	
		Matched (%)	86%	93%	71%	83%	80%	74%	52%	69%
	Year 3	Observed	4107	202	111		2461	244	131	
		Matched	3566	179	97		2007	174	87	
		Matched (%)	87%	89%	88%	88%	82%	71%	66%	73%
	Year 4	Observed	4163	169	88		2483	233	120	
		Matched	3362	117	54		2136	205	97	
		Matched (%)	81%	69%	61%	70%	86%	88%	81%	85%
	Year 5	Observed	4139	191	90		2463	237	136	
		Matched	3681	151	66		2263	216	118	
		Matched (%)	89%	79%	74%	81%	92%	91%	87%	90%
10%	Year 1	Observed	3842	404	174		2153	463	220	
		Matched	3578	349	142		1878	362	155	
		Matched (%)	93%	86%	82%	87%	87%	78%	70%	79%
	Year 2	Observed	3819	403	198		2087	504	245	
		Matched	3650	349	153		1865	399	158	
		Matched (%)	96%	87%	77%	87%	89%	79%	64%	78%
	Year 3	Observed	3861	381	178		2195	437	204	
		Matched	3392	285	116		2006	360	162	

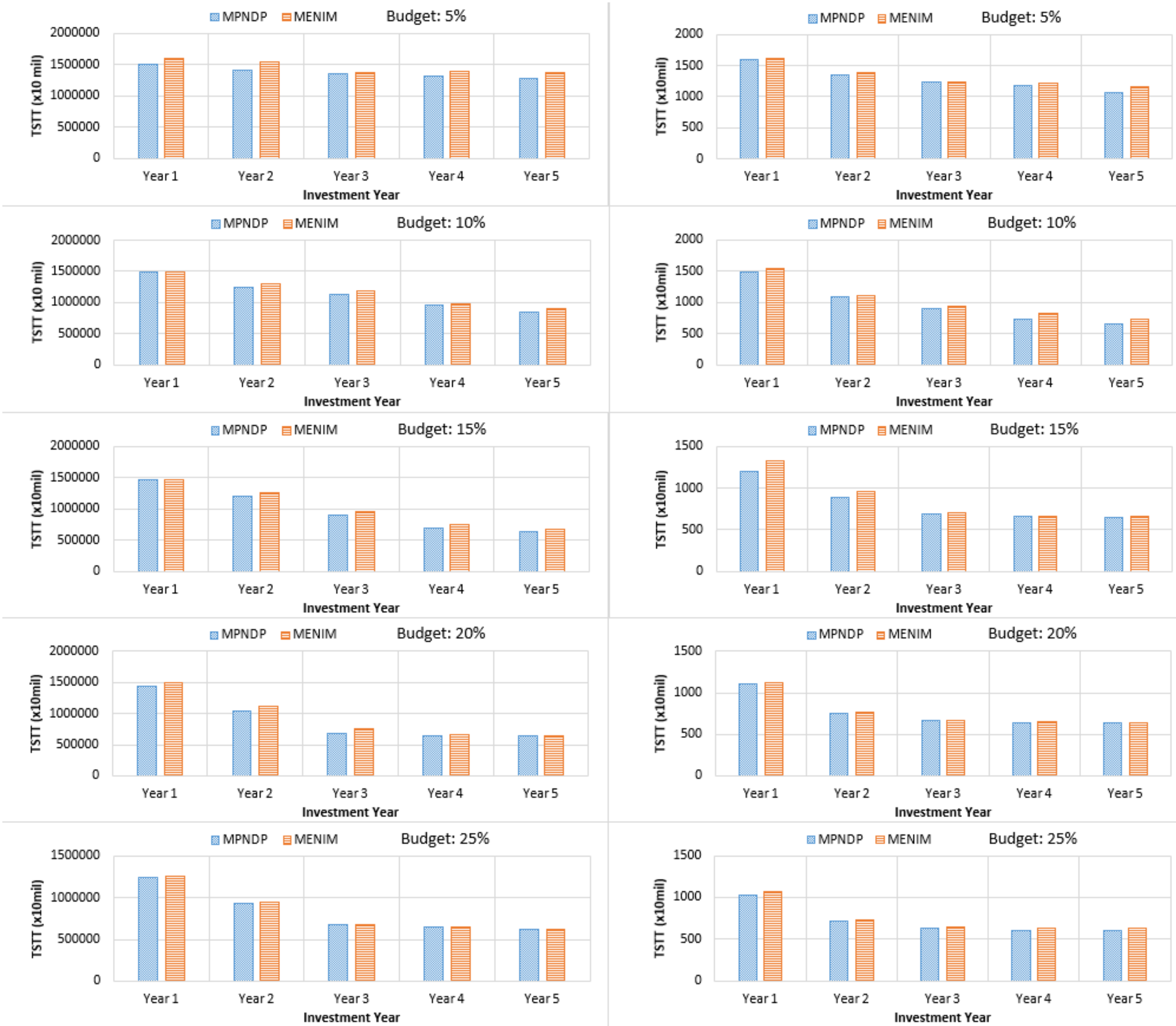
Table 4 (Continued)

10%		Matched (%)	88%	75%	65%	76%	91%	82%	80%	84%
	Year 4	Observed	3837	384	199		2120	517	199	
		Matched	3121	298	110		1855	444	134	
		Matched (%)	81%	78%	55%	71%	87%	86%	67%	80%
	Year 5	Observed	3824	384	212		2496	222	118	
		Matched	3665	345	125		2269	174	62	
		Matched (%)	96%	90%	59%	82%	91%	78%	53%	74%
15%	Year 1	Observed	3488	642	290		1824	701	311	
		Matched	3090	464	206		1685	544	245	
		Matched (%)	89%	72%	71%	77%	92%	78%	79%	83%
	Year 2	Observed	3524	593	303		1860	641	335	
		Matched	3188	516	231		1749	591	285	
		Matched (%)	90%	87%	76%	85%	94%	92%	85%	90%
	Year 3	Observed	3613	532	275		1990	582	264	
		Matched	3102	446	187		1580	451	159	
		Matched (%)	86%	84%	68%	79%	79%	77%	60%	72%
	Year 4	Observed	3563	552	305		2578	189	69	
		Matched	2853	424	204		2135	165	59	
		Matched (%)	80%	77%	67%	75%	83%	87%	85%	85%
	Year 5	Observed	3842	398	180		2692	99	45	
		Matched	3379	313	106		2460	90	31	
		Matched (%)	88%	79%	59%	75%	91%	91%	70%	84%
20%	Year 1	Observed	3220	808	392		1562	855	419	
		Matched	2760	682	284		1422	755	331	
		Matched (%)	86%	84%	73%	81%	91%	88%	79%	86%
	Year 2	Observed	3305	737	378		1560	872	404	
		Matched	2808	576	267		1200	642	261	
		Matched (%)	85%	78%	71%	78%	77%	74%	65%	72%
	Year 3	Observed	3266	750	404		2417	282	137	
		Matched	2748	613	298		2114	251	115	
		Matched (%)	84%	82%	74%	80%	87%	89%	84%	87%
	Year 4	Observed	3846	381	193		2673	121	42	
		Matched	2876	270	139		2117	111	35	

Table 4 (Continued)

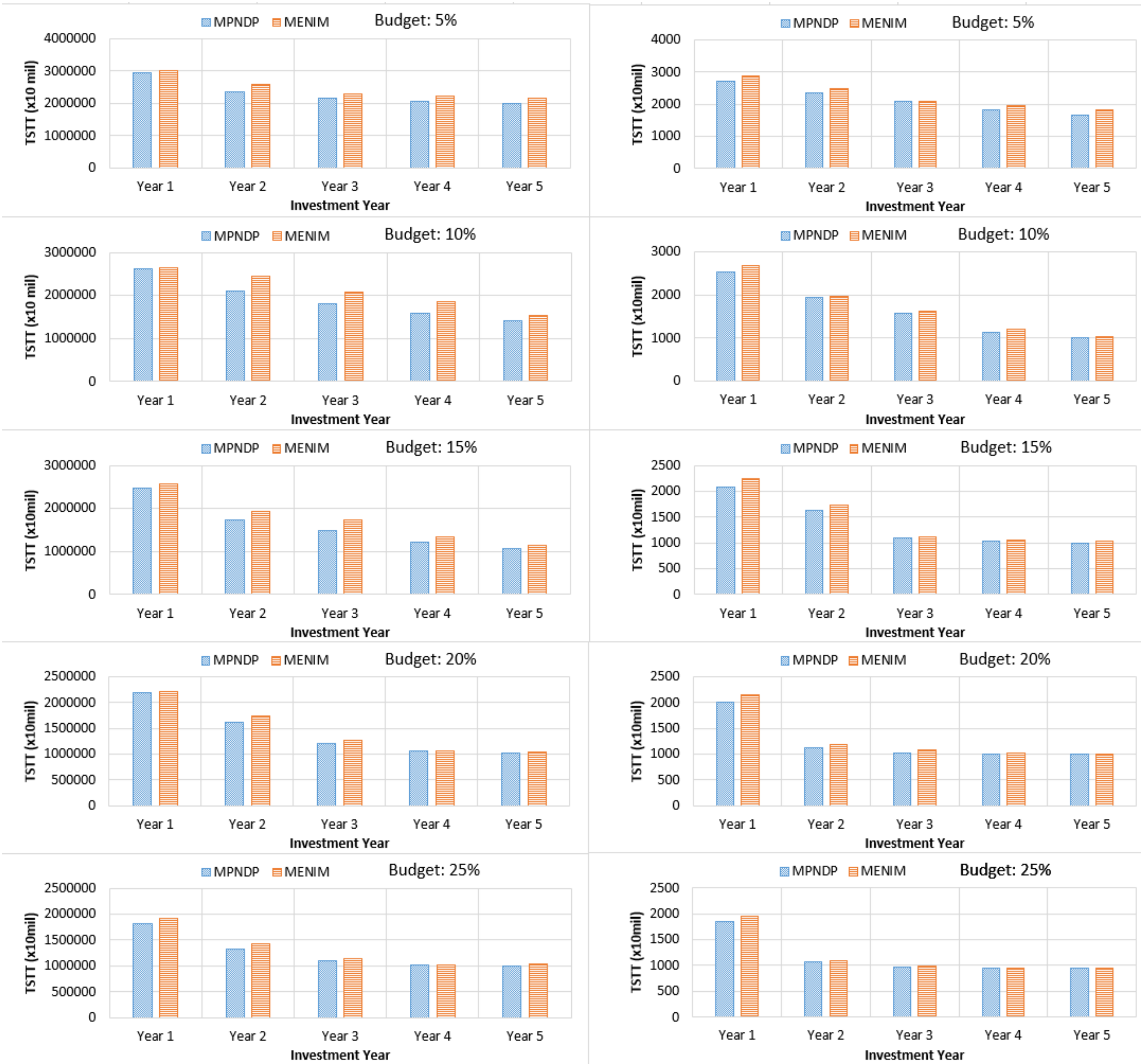
20%		**Matched (%)**	75%	71%	72%	73%	79%	92%	84%	85%
	Year 5	**Observed**	4295	90	35		2686	98	52	
		Matched	3639	76	23		2159	63	30	
		Matched (%)	85%	84%	67%	78%	80%	65%	57%	67%
25%	**Year 1**	**Observed**	3031	902	487		1294	1014	528	
		Matched	2729	793	402		1238	923	462	
		Matched (%)	90%	88%	82%	87%	96%	91%	88%	91%
	Year 2	**Observed**	2955	959	506		1752	742	342	
		Matched	2585	802	360		1602	647	268	
		Matched (%)	87%	84%	71%	81%	91%	87%	78%	86%
	Year 3	**Observed**	3482	620	318		2532	209	95	
		Matched	3307	554	276		2330	165	65	
		Matched (%)	95%	89%	87%	90%	92%	79%	69%	80%
	Year 4	**Observed**	4085	228	107		2601	159	76	
		Matched	3475	171	65		2284	145	63	
		Matched (%)	85%	75%	61%	74%	88%	91%	83%	87%
	Year 5	**Observed**	4310	67	43		2716	78	42	
		Matched	4006	56	28		2608	62	27	
		Matched (%)	93%	84%	65%	81%	96%	79%	64%	80%

Further validation was performed by comparing the TSTT of the two networks, Winnipeg and Montgomery County, MD, calculated using the MPNDP and the MENIM model. Figure 1 (a) shows the TSTT using MPNDP and TSTT using MENIM by budget and planning period for Montgomery, MD network and figure 1 (b) shows the TSTT for Winnipeg network. The TSTT calculated by MENIM is comparable with the TSTT from MPNDP across the planning period for all budget scenarios for the two networks. Similarly, figure 2 shows the TSTT using MPNDP and TSTT using MENIM by budget and planning period for the future demand scenarios for Montgomery, MD network and figure 1 (b) shows the TSTT for Winnipeg network which are also close and comparable. In addition, table 5 shows the relative difference in TSTT between MENIM and MPNDP (calculated as $\frac{TSTT(MPNDP)-TSTT(MENIM)}{TSTT(MPNDP)}$) across various periods and budget. The lower value of the relative difference across all conditions shows how the MENIM approach is comparable to the MPNDP approach.



(a) Montgomery, MD Network **(b) Winnipeg Network**

Fig. 1. TSTT comparison.



(a) Montgomery, MD Network **(b) Winnipeg Network**

Fig. 2. TSTT comparison - future demand scenarios.

Table 5: Model validation – Relative Difference in TSTT between MENIM and MPNDP

Budget	Planning Period	Relative Difference	
		Montgomery	Winnipeg
5%	Year 1	-1.8%	-6.2%
	Year 2	-10.8%	-5.9%
	Year 3	-6.4%	-0.1%
	Year 4	-7.8%	-6.0%
	Year 5	-8.0%	-8.2%
10%	Year 1	-0.3%	-6.0%
	Year 2	-16.4%	-0.6%
	Year 3	-14.1%	-2.7%
	Year 4	-16.4%	-5.7%
	Year 5	-7.2%	-1.4%
15%	Year 1	-4.4%	-8.4%
	Year 2	-10.6%	-5.7%
	Year 3	-15.6%	-0.6%
	Year 4	-9.9%	-3.1%
	Year 5	-7.0%	-5.0%
20%	Year 1	-0.2%	-7.8%
	Year 2	-7.1%	-5.9%
	Year 3	-5.4%	-6.9%
	Year 4	-0.3%	-1.7%
	Year 5	-1.0%	-0.7%
25%	Year 1	-6.2%	-5.6%
	Year 2	-7.3%	-2.5%
	Year 3	-4.3%	-1.2%
	Year 4	-0.7%	-0.6%
	Year 5	-2.9%	-1.5%

4.5 Policy and Planning Analysis Discussion

Roadways in transportation networks need to be improved and/or expanded regularly to meet the needs of growth in travel demand and to ease traffic congestion. However, planning for transportation network improvements over time is one of the challenges encountered by public agencies due to various challenges such as user behavior, budget limitations and policy constraints. Prioritizing roadway network infrastructure investment decision especially for long range planning is a major problem faced by the decision makers or planning agencies. Limited funds are forcing national, regional, and local governments to carefully prioritize their investments. These investments are usually long lasting, practically irreversible and costly. Hence the planners and decision makers need to resort to an efficient and innovative prioritization technique to ensure that the projects undertaken are significant and that the most effective utilization of resources takes place.

The multi-period planning model proposed in this research shows efficient budget allocation for network improvement to the links in such a way that the total system travel time of the network is minimized. Six networks are used as case studies for MENIM development and two large networks are used for the MENIM validation. The larger networks (Atlanta and Montgomery) have higher minimized total system travel times compared to the small and medium networks. The multi-period observations suggest that the multinomial logit model presented in this research displays results comparable to the optimization model and can be used as an effective surrogate tool of resource allocation for network improvement by decision makers or transportation planning agencies. Thus, this model can be used effectively in transportation asset management process for managing transportation infrastructure with the objective of improved decision making for resource allocation for multiple periods, that is, which programs/projects should the decision makers spend/invest their funding for the best long-term benefit. To adopt this procedure in real-world networks, the decision makers need to obtain data on network variables of their respective networks as training samples to calibrate the MENIM model. Most of the data variables can be obtained from the base-year network and the number of link installations can be obtained from past completed projects and identifying on which links the improvements were made. The calibrated model can then be used like any other statistical model to identify the links in the network needed for improvement in the future long-range transportation plans. This model can be further enhanced to aid in making informed decision about managing the network over the whole lifecycle considering network performance, economics, and engineering. Resource allocation for sub-elements of the network such as pavements, bridges, congestion, safety, etc., are necessary for sound information to support long-term investment decision-making.

5. Conclusion

This research proposed two different models in the context of multi-period network investments. First, the single year network design problem proposed by Wang et al. (2013) was extended to a multi-period framework (MPNDP) considering budget, policy, and other constraints. Numerical experiments involving different budget allocations were conducted for the MPNDP based on several case study networks to analyze the multi-period formulation. Second, the optimal results from the MPNDP were used to develop a MENIM model to obtain reasonable multi-period network investments. MPNDP is applied to six medium- to large-scale networks and the resulting patterns were analyzed. The patterns provide insights that can be used by the public agencies to obtain the corresponding level of cost and benefits (in terms of TSTT) associated with various network investments. It was found that with an increase in budget as well as network size, the total system travel time decreases. The methodology and policy measures presented in this research enable a decision maker to allocate resources efficiently within the planning horizon. The validation results also show that the MENIM can be used as an effective surrogate for the MPNDP for capacity expansion decision of larger networks.

NDP is considered as an NP-hard problem and therefore the computational time and cost becomes higher with larger network data. Moreover, since the problem is formulated for multiple years, the number of constraints and decision variables becomes significantly larger thus increasing the computational requirements even for example networks. The single-period problem already consists of many integer variables and thus multi-period problems will have an even higher number of integer variables. Because of this, the computational power of the mixed integer non-linear problem (MINLP) solvers becomes inefficient. The MENIM is proposed as an alternative approach to MPNDP to attempt to simulate results of reasonable accuracy and comparable efficiency for the analysis of larger networks. The authors would also like to note that for medium to large networks, and with the existing current computational capabilities and computer memory restrictions, the MPNDP may only be implemented with a cluster of computers and even then, the computational time is restrictive. The advantage of the MENIM is that it can be trained for a subset of the network at hand, where the MPNDP can be implemented, and then applied to the full network.

There are several limitations to this research that needs to be mentioned here. In order to train the MENIM model, one needs data such as how many lanes were installed in an existing network. Such data may not be readily available and need to be acquired from past completed projects in the network which may be time-consuming. In addition, it cannot be verified if those completed projects correspond to optimal decisions which may reduce the accuracy of the MENIM model. Lastly, the demand in the network is considered constant and the uncertainty is neglected.

For future research, multiple objectives can be considered in the MPNDP since planners must consider various factors while making network design decision. The TSTT can be combined with consumer surplus, construction cost or social surplus for example. The MPNDP can also be formulated in terms of safety issues. The research can also be extended to handle the interdependencies among projects and demand/budget changes in each period. Extension of this approach to other applications such as bridge project investment criteria is also worth considering.

Another future avenue is the development of efficient methodological techniques to solve multiyear transportation problems and bounding techniques to obtain approximate solutions. A more realistic, time-dependent probability assessment of multiple periods would be essential for determining the best possible solution sets. The results will offer rich insights on public policies, such as the budget allocation to maximize benefits or consumer surplus or minimize emissions. The major issue would stem from the stochasticity of demand. How to incorporate this stochasticity into modeling of roadway network infrastructure investment decision making is a challenge in both modeling and solution methodology.

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