Multi-Criteria Based Approach to Identify Critical Links in a Transportation Network

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Abstract: A wide range of disruptive events occur on transportation networks which have intense impacts on network users' travel time. Moreover, limited funds is forcing national, regional and local governments to carefully prioritize their investments. Therefore, reliable quantitative tools are needed to help decision-makers in choosing their investments so that the allocation of available resources is optimized. In this research, the authors attempt to understand the relative importance of links in a road network and suggest a methodology to rank the links according to three importance factors while combining the network improvement investment decision and subsequent network user response in a feedback loop. The first factor is based on the link flows at equilibrium. The second factor is based on the importance of facilities served. The third factor is based on the number of origin-destination pairs served by a link. The proposed methodology is demonstrated with a small test network and with real scale transportation network. Sensitivity analysis is performed using various budget scenarios and it is found that with the increase in budget the ranking of critical links changes.

Author Keywords: Link ranking; Link criticality; Network design; Network Development; Link Importance factor.

1. Introduction

Past few years have seen a visible increase in research related to link criticality and the disruption of transportation networks (Sullivan et al. 2010) which has been largely motivated by major events like natural disasters, extreme weather (e.g. North and Central Georgia winter storm (NOAA 2014)), bridge collapses (Xie and Levinson 2011; Zhu et al. 2010), increased threat of terror attacks, constructions activities and major crashes. A wide range of severe and relatively short-term disruptive events can occur on transportation networks causing disturbances in traffic flows and forcing travelers to change routes. Some of these events have small but others can have intense impacts on travel time of network users. Partial flooding, visibility reductions, traction hazards due to extreme weather conditions, pavement deterioration, debris on the road, and a wide variety of traffic accidents are all examples of events that are likely to result in short-term, partial reduction of capacity on a affected link; while catastrophic events like the collapse of a bridge, a chemical spill, or a major accident are likely to have long-term effect and reduce the capacity of the affected link to zero (Sullivan et al. 2010). In addition, some links of a network can be more critical, and their failure can paralyze the day-to-day activities and emergency operations of the city or region. Therefore, impact of transportation network disruptions need to be thoroughly explored not only for natural calamities and evacuation planning but also for functions of day-to-day importance and emergency response.

Throughout the world, the road network system is undoubtedly considered as one of the most critical components of a country's infrastructure due to various roles like expediting economic growth, providing timely access for travelers and contributing to the nation's defense. Simply stated, the traffic volume or usage of a link is an important indicator of its criticality. For example, the United States has witnessed tremendous amount of growth in vehicle travel on the interstate highway system which is the heart of the nation's passenger and goods movement. These roadways also connect various origins and destinations and disruption of even one link can lead to significant changes in the travel pattern. On the other hand, there are some links inside city networks which are critical to maintain connectivity of emergency services like hospitals and fire stations. Certain parts of the network may be more important than others due to important destinations located at a place, or due to network topological factors, or the intensity of link usage. In other words, criticality of a certain link or group of links in the network involves both the intensity of the usage, disruption in the critical services due to component failing and the results of that failure for the system. The more critical the link, the more severe will be the damage to the system when that link is lost. If the rank of a link is high, disruptions in that link due to any reason may change the network flow and increase travel times by a large amount. All the above stated factors are important, and a single factor cannot decide the criticality of a link in isolation although different factors weigh differently for different planning agencies. Therefore, strengthening and maintaining the links of a transportation network must be based on a prioritization methodology that incorporates multiple factors given the dissimilarity in criticality of various links and budgetary limitations. Hence, there must be a system to determine the link criticality or rank order of the links in a road network based on multiple factors. Such a measure can be useful for multiple purposes such as to prioritize the maintenance funds, to decide the optimal location of link retrofitting, resource allocation for traffic surveillance and highway patrolling. In a world where resources are limited and where funds do not necessarily increase with the growing demand for infrastructure improvements, not to mention the increasingly costly maintenance of the age-old infrastructure, it is necessary to make well informed decisions when selecting specific links for retrofitting, repair and improvement.

This study develops a simplified framework for the determination of link criticality using multiple factors. The study identifies three important factors to decide the relative criticality or importance of links in each network: volume of network users served, connectivity to important facilities, and number of origins and destinations served, with the humble admission that these factors does not form the complete set of Importance Factors (I.F.s).

The paper proposes a framework for multi-criteria-based link ranking that not only incorporates aforementioned factors but also helps to understand how the ranking changes by the changes in the collective usages of network users due to road network improvement investments. The study result vindicates that link ranking changes post network improvement and also based on the level of improvement. As such, a framework that does not factor the changes in link importance due to proposed network improvements and is based on current state is likely to yield suboptimal decision making. The developed framework of link ranking methodology is easy to implement in practice and can be easily used by practitioners and decision-makers for prioritizing links of a road network for strategic decision making such as deciding the locations of security personnel, installation of traffic surveillance cameras, link strengthening (such as bridge retrofitting) and link improvement (such as resurfacing).

In this context, the objectives of the this research are to: (1) conduct a thorough literature review of different measure and approaches of link ranking; (2) design a methodological framework for link prioritization combining multiple importance factors while capturing the network users' path choice behavior in the form of user equilibrium, (3) compare the link rankings based on individual factor and the combined criteria, and (4) investigate the effect of budgetary allocation on the link raking and its spatial distribution. The study uses two networks to perform the numerical experiments; first a small test network to demonstrate the methodology and then a real-scale road network to test its validity for practice. The results of numerical experiment attest the validity of proposed methodology and help to understand the role of multiple factors in identifying the most critical links of a road network.

The rest of the paper is organized as follows. The next section summarizes literature on link ranking and related measures. Then the section after this is devoted to understanding the day-to-day functional aspects of road network. The section describes the three factors based on day-to-day functional aspects of network and the methodology of link ranking proposed in this study for finding the critical links based on these three factors. The three factors are appropriately defined, and the link ranking implementation framework is described in this section. Then the next section presents the results of the computation experiments along with some useful insights from the results. The last section concludes the paper and proposes future research directions.

2. Literature Review

Measures of link ranking have been studied for a long period of time with an emphasis on criticality of the network, network disruption and vulnerability. A summary of the link ranking measures found in the literature is presented in Table 1. The first measure mentioned in Table 1 is the link importance index. It measures the importance of a link based on average daily traffic and increase in travel cost due to a link's disruption. Most of the studies found in the literature measure the importance of the link due to disruptions in order to measure network vulnerability (Jenelius 2009; Jenelius 2010; Jenelius, Petersen, and Mattsson 2006; Jenelius and Mattsson 2012; Rupi et al. 2015). Similarly, there are various measures proposed in past that calculate the link criticality in a network (Li and Ozbay 2012; Luathep et al. 2011; Rodríguez-Núñez and García-Palomares 2014; Snelder, van Zuylen, and Immers 2012; Sullivan et al. 2010; Ukkusuri and Yushimito 2009). Some of the measures adopted in past studies have been found to compute the accessibility of a link in a network (D'este and Taylor 2003; Luathep et al. 2011; Sohn 2006; Taylor and Susilawati 2012). Some studies use the link efficiency in a network (Chen et al. 2012; Latora and Marchiori 2003; A. Nagurney and Qiang 2007; Anna Nagurney and Qiang 2007a; Anna Nagurney and Qiang 2007b; Qiang and Nagurney 2007) and others evaluate link interruptions and alternate routing (Berdica and Mattsson 2007; Snelder, van Zuylen, and Immers 2012). Some studies have also attempted to rank the links either based on combination of a different criteria or by using spatiotemporal patterns of alternative travel paths (Fang et al. 2012; Knoop et al. 2012).

2.1. Gaps in the Literature

Over the past decade, there has been a noticeable amount of research for analyzing the vulnerability of transportation network and prioritization of links motivated by major catastrophic events like accidents or natural calamities or evacuation planning. Even after the scrutiny of such a rich resource of findings in this domain, a distinct gap exists; the existing studies did not consider important factors based three characteristics simultaneously in the determination of link criticality, namely the (1) network characteristic, (2) flow characteristic and (3) location characteristics (e.g. location of important facilities). Some studies have attempted to compute combined criticality index; however, they did not consider the change in network flows resulting from network user response due to road capacity improvements. Moreover, most of the measures and indicators are complex and not ready to use for practitioners. It is important to have workable definitions of importance factors along with implementation simplicity for its applicability for practice. Given the potential for substantially different performance outcomes, selection based on importance factors to identify the most critical links on a network is imperative. For example, the arbitrary but common use of link ranking based on average daily traffic and travel time is not sound methodologically, nor is it necessarily realistic with respect to every day usage of link on transportation networks. This study endeavors to bridge these gaps in literature in this domain and proposes a methodological framework for link ranking using multiple criteria while capturing the network users' response to the network improvement investments.

3. Link Ranking Methodology

Common performance indicators used in past include the link-specific average annual daily traffic (AADT) collected from traffic counters, and the Volume-to-Capacity ratio (V/C) which is the output of common travel demand models (Margiotta, Eisele, and Short 2015). As transportation networks become more heavily used, the ranking approaches focusing on AADT and V/C may not be adequate because they are inherently localized and static in nature. The methodology proposed here attempts to rank the links of a network according to three importance factors based on three characteristics while combining the network improvement investment decision and subsequent network user response in a feedback loop. The three characteristics leading to three importance factors in this study are as mentioned below:

Flow Characteristics – the study uses link volume as a measure of importance of a link and it leads to importance factor 1 (ω_1).

Location characteristics – the study uses spatial locations of the important facilities to decide which links serve these facilities. In addition, study assigns differential importance to links based on facilities served. The differential importance is decided by Day-to-Day Criticality. It leads to importance factor 2 (ω_2).

Network Characteristics – the study uses number of used paths of various O-D pairs crossing through a link for assessing the importance of a link. It acts as the proxy for measuring the importance of link for connectivity in the network. This measure is likely to depend on the graph-theoretic property of a network in conjunction with user behavior. It leads to importance factor 3 (ω_3).

The method for the computation of three importance factors are explained next.

3.1. Computation of Importance Factor 1

For Computing Importance Factor 1, first, the link flows are determined by solving the traffic assignment problem while factoring the capacity enhancements due to network investment. Traffic assignment can be categorized as either static or dynamic traffic assignment. Static assignment assumes that traffic is in a steady state, and the time to traverse a link depends only on the number of vehicles on that link (Li and Ozbay 2012). Because of its simple mathematical formulation and solution procedure, static assignment is widely applied for evaluation of link criticality on the scale of a regional network. Typically, there are two types of static traffic assignment: user equilibrium (UE), which assumes that users reach equilibrium when they cannot improve their travel time unilaterally by switching routes, and system optimum (SO), which estimates link flows according to some system wide objective (e.g., minimization of total travel time). Although SO is desirable from planning perspective, the UE is more realistic from network user point of view. According to Sheffi (Sheffi 1985), the deterministic user equilibrium traffic assignment problem (UETAP) can be formulated as convex optimization problem. In the context of this study, the classical UETAP needs to be decomposed into two parts to incorporate changes in link cost functions due to investment decision. The resulting UETAP problem can be formulated as:

$$min \sum_{a\in A\setminus\bar{A}} \int_{0}^{x_a} t_a(x_a)dx + \sum_{a\in\bar{A}} \int_{0}^{x_a} \bar{t}_a(x_a)dx \quad (1a)$$

Subjected to:

$$\sum_{k} h_k^{rs} = q_{rs} \quad \forall\, r, s \tag{1b}$$

$$x_a = \sum_{r}\sum_{s}\sum_{a} h_k^{rs}\delta_{a,k}^{rs} \quad \forall\, a \tag{1c}$$

$$h_k^{rs} \geq 0 \quad \forall\, r, s \tag{1d}$$

where,

x_a = Flow on link a,

t_a = Cost of traveling on link a,

A = Set of links in the network,

$\bar{A}$ = Set of links considered for improvement, $\bar{A} \subset A$, decided by the investment decision

$\bar{t}_a$ = Cost of traveling on link a after capacity improvement,

h_k^{rs} = Origin-destination (O-D) flow on path k from r to s,

q_{rs} = O-D flow from origin r to destinations$s$, and

$\delta_{a,k}^{rs}$ = Binary value indicating that link a exists on path k between O-D pair r-s

The study uses the BPR function for the determination of link costs as shown below:

$$t_a(x_a) = \left[t_{0\text{a}}\left(1 + \beta\left(\frac{x_a}{C_a}\right)^{\alpha}\right)\right] \qquad \forall a \in A \backslash \bar{A} \tag{2a}$$

$$\bar{t}_a(x_a) = \left[\bar{t}_{0\text{a}}\left(1 + \beta\left(\frac{x_a}{C_a + \Delta C_a}\right)^{\alpha}\right)\right] \qquad \forall a \in \bar{A} \tag{2b}$$

Where,

C_a = Capacity of link a,

ΔC_a = Increase in capacity of link a after link improvement, decided by investment decision maker

t_{0a} = Free flow travel time of link a,

$\bar{t}_{0a}$ = Free flow travel time of link a after link improvement, (for simplicity this study assumes that $\bar{t}_{0a} = 0.95 \times t_{0a}$)

β, α = Link specific parameters.

The Slope-based Path Shift-propensity Algorithm (SPSA) developed by Kumar and Peeta (Kumar and Peeta 2014) has been used to obtain a precise solution of above stated UETAP formulation determining the link flows and set of used paths at UE in the network for different budget scenarios. Capability of SPSA to utilize the solution from previous iteration through warm start is especially useful for solving UETAP problem represented by Eqs. (1a) - (2b). Once the link flows are determined, the values are normalized to obtain importance factor 1 using Eqs. (3) and (4) as follows:

$$\omega_1 = \frac{x_a}{x_{max}} \quad \forall a \tag{3}$$

$$x_{max} = \max(X) \tag{4}$$

Where,

ω_1 = Importance factor 1,

x_a = Flow of link a,

x_{max} = Largest value of link flow at UE in the network,

X = Vector of link flows.

3.2. Computation of Importance Factor 2

Importance factor 2 is determined by identifying the important facilities or destinations served by the links in a network. In this study, five facilities are considered as important facilities for day-to-day use and emergency response: hospital, fire station, police service, school and grocery stores. Each facility has been given differential weights (θ^d) based on their day-to-day importance and emergency responsiveness. For this purpose, hospital has been given the highest weight (θ^1=5) followed by fire station (θ^2=4), police service (θ^3= 3), school (θ^4=2) and grocery stores (θ^5=1). Each link may serve from zero to all five important destinations. If a link serves multiple destination of same category, say two hospitals, then, each hospital is treated as an individual destination, that is, multiple hospitals are not accounted as one. If a link serves a destination of importance, it is given the weightage with respect to that destination and finally the weights are summed for each link. The basic premise is that links that serve most of the important destinations are used more often to serve the communities and hence those links are identified as the most important based on this criterion. Importance Factor 2 is calculated using the following equations:

$$\omega_2 = \frac{\sum_{d=1}^{n} \theta_a^d}{\sum_{d=1}^{n} \theta^d} \quad \forall a \tag{5}$$

$$\theta_a^d = \begin{cases} 5 & at\ d = 1\ \ if \text{ hospital is served by link } a, & 0\ otherwise \\ 4 & at\ d = 2\ \ if \text{ fire station is served by link } a, & 0\ otherwise \\ 3 & at\ d = 3\ \ if \text{ police service is served by link } a, & 0\ otherwise \\ 2 & at\ d = 4\ \ if \text{ school is served by link } a, & 0\ otherwise \\ 1 & at\ d = 5\ \ if \text{ grocery shop is served by link } a, & 0\ otherwise \end{cases} \tag{6}$$

Where,

ω_2 = Importance Factor 2

n = Number of important facilities/destinations considered (in this study $n = 5$)

θ^d = Destination weight for facility/destination type *d*

θ_a^d = Destination weight of link a.

The higher the value of ω_2, higher is the importance of the link. In this study, ArcGIS has been used to find the different destinations served by the road network. The OD pairs in the network and the set of used paths between those OD pairs are merged with the network data to find the important destinations served by the links.

3.3. Computation of Importance Factor 3

Importance factor 3 is based on the graph-theoretic property (GTP) in conjunction with network user behavior. GTP can help in better understanding of real-world network and aid in the ability to analyse them for link ranking. Intuitively, the most connected link should get the highest priority. Past studies have been dominated by centrality measures in the determination of link criticality, but centrality measure alone may not be sufficient without considering link usage intensity by spatially separated network users. This study proposes to use the number of used paths of various O-D pairs crossing through a link for assessing the importance of a link instead of centrality measure. A used path is defined as the path between an OD pair which has flow greater than zero. First, the link-path incidence matrix is obtained for the study area utilizing the information of used paths at UE for all O-D pairs. Then the number of paths served by links are determined using this incidence matrix. Finally, the total number of paths served by a link over all O-D pairs is normalized by the total number of O-D pairs in the network to determine Importance Factor 3. The following equation explains the calculation of Importance Factor 3:

$$\omega_3 = \frac{\sum_{p=1}^{n_{OD}} \psi_a^p}{n_{OD}} \quad \forall a \tag{7}$$

Where,

ω_3	=	Importance factor 3
ψ_a^p	=	Number of used paths between an O-D pair p that pass-through link a
n_{OD}	=	Total number of O-D pairs in the network.

Based on this criterion, links serving higher number of paths are given higher priority and vice versa. The Importance Factors are calculated in such a way that no value is greater than 1 or less than zero, that is, the factors are normalized and hence their range is $0 < \omega_1, \omega_2, \omega_3 < 1$. Once the three factors are determined for each link, a combined Importance Factor (*w*) is calculated by Eq. (8) as follows:

$$w = \beta_1\omega_1 + \beta_2\omega_2 + \beta_3\omega_3 \tag{8}$$

Where, β_1 and β_2 are positive weights given to ω_1 and ω_2 such that $\beta_1 + \beta_2 + \beta_3 = 1$.

The links are ranked based on combined Importance Factor (*w*) resulting into vector of ordered links R. Where, *i*th element (a_i) in vector R is arranged in decreasing order of their combined I.F., i.e.

$R = \{a_i | a_i \in A, w_i > w_{i+1} \forall i\}$, where, w_i is the combined I.F. of i^{th} element in R. The R acts as input for link improvement problem.

The decision maker investment problem or the link improvement problem can be formulated as below:

$$\min TSTT = \sum_{a \in R_m} x_a \bar{t}_a(x_a, \Delta C_a) \tag{9}$$

Subjected to:

$$\sum_{a \in R_m} g_a(\Delta C_a) \leq B \tag{10}$$

$$\Delta C_a \geq 0 \ \ \forall a \in R_m \tag{11}$$

Where,

$TSTT$	=	Total system travel time
R_m	=	First m elements of vector R
$g_a(.)$	=	Function for determining the cost of improving link a
B	=	Total budget.

The objective of link improvement problem represented by Eqs. (9) - (11) is to minimize the total system travel time by deciding the changes in link capacities $\{\Delta C_a\}_{a \in R_m}$ under budget constraint. In Eq. (9) $\bar{t}_a$ is computed based on Eq. (2b) and is a function of current flow on link a (x_a) and change in link capacity (ΔC_a). Eq. (10) ensures that the total improvement cost does not exceed the total given budget. Eq. (11) ensures that the added capacity ΔC_a for each candidate link are non-negative. The output of this problem is set of links ($\bar{A}$) and respective changes in link capacities $\{\Delta C_a\}_{a \in \bar{A}}$ which acts as input for the UETAP problem represented by Eqs. (1a) - (2b). The set of links considered for improvement is given as:

$$\bar{A} = \{a | a \in R_m, \Delta C_a > 0\} \tag{12}$$

Figure 1 shows the sequence of steps of proposed methodology using a flow chart. First UETAP is solved with the base condition as initialization. Then three importance factors and the combined I.F. are computed for each link in the network. The links are ranked in decreasing order of importance based on the value of combined I.F., such that the link having highest combined I.F. is ranked 1 followed by the second highest combined I.F. and so on. This results into vector R consisting of the set of links $\{x_a\}$ ranked in decreasing order of priority. The Importance Factors are used as a surrogate to reduce the feasible space and complexity of the problem at each iteration. The link improvement decision is obtained using both, the link ranking and the decision maker goal under budget constraint. Link ranking helps to identify set of potential links for improvements. In the simplest way, this study proposes to use the first m links as potential candidates for improvement (m is decided by expert judgment based on network size and budget). The set of links for improvements ($\bar{A}$) and level of capacity improvements (that includes no improvement) are decided by optimizing the decision maker objective (minimizing the total system travel time). Then the UETAP (represented by Eqs. (1a) - (2b)) is solved with new inputs ($\bar{A}$) and ΔC_a $\{a \in \bar{A}\}$. Then, the I.F.s (1 and 3) and the combined I.F. are updated for each link in the network. The process is continued in a feedback loop till R stops changing as shown in Figure 1. In summary, the link ranking procedure is the first step in the complete methodology and the subset of m ranked links is subjected to investment strategy process. However, whether that investment strategy is optimal will depend on the consistency of the decision maker's goal with the definitions of importance factors used to rank the links. The next section describes the solution approach undertaken to implement the above methodology.

4. Implementation Details

The flowchart of the proposed solution approach is presented in Figure 2. From the figure, it can be observed that the user equilibrium traffic assignment problem and the decision maker link improvement problem are solved in feedback loop alternatively till convergence. In this study the link improvement problem is solved by particle swam (PSwarm) optimization algorithm. Here it is important to mention that study uses PSwarm algorithm as it was able to deal with real-world size network with moderate computational time, but other heuristic algorithms can also be used for this purpose such as Memetic Algorithm (Pishvaee et al., 2010), Differential Evolution (Koh, 2007), Evolutionary Algorithms (Lau et al., 2009) and Hill climbing (Los and Lardinois, 1982).

The PSwarm algorithm is implemented in MATLAB to obtain a trial capacity expansion vector for the critical links. Then this vector is translated into new network capacities. The new network is then feedback to the UE solution algorithm. In this study the UE is solved using SPSA. The SPSA has been implemented through a C++ code. The SPSA yields a UE link flows which is used to calculate the first importance factor (ω_1) for the links. SPSA also provides the link-path incidence matrix which is used to compute the importance factor 3 for the links. Here, it is imperative to mention that Importance Factor 2 is based on network topology and spatial locations of facilities and needs to be computed only once and not in each iteration. Once, the importance factors are computed then the combined I.F. is found for the links. The links are then ranked based on their combined I.F. and then the critical links are sent to link improvement problem. This procedure is repeated until convergence. Convergence is measured by comparing the rank order vectors obtained between two consecutive iterations. Once the next set of ranks are obtained, the order is compared to the previous ranks. If they are the same, then the algorithm is converged, and we obtain the optimum ranks of the links.

The two important components of the proposed solution approach are: PSwarm optimization algorithm and the SPSA traffic assignment algorithm. A brief review of these two techniques are presented next.

4.1. The Particle Swarm Algorithm

The particle swarm (PSwarm) algorithm was proposed by Eberhart and Kennedy (Eberhart and Kennedy 1995) in an attempt to find the global optimizer of non-convex function without finding the derivative of the function. Two important benefit of using this method are: (i) no requirement of smoothness of objective function, and (ii) ability to find global optimum even under non-convexity of objective function and multiple domains of attraction. The PSwarm algorithm simulates the behavior of particles attempting to find the optimal position by self-exploring as well as exploiting the exploration of other particles. The population of particles is called swarm. Each particle is associated with position and velocity at any instant. At each iteration, the velocity vector of a particle is updated as the stochastic linear combination of (i) its own velocity in previous iteration (ii) direction to the particles best known position from the particle's current position, and (iii) direction to the swarm's best-known position from the particle's current position. In this sense, this method combines the local search (own experience) with the global search (population experience). At each iteration, particles update its positions based on its current position and updated velocity. The iteration is terminated when the norm of velocity vector of all particles is less than a predefined threshold value (ε) chosen based on the desired precision.

4.2. The Slope-based Path Shift-propensity Algorithm

The SPSA was proposed by Kumar and Peeta (Kumar and Peeta 2014) to devise a traffic assignment algorithm capable of generating a precise solution at moderate computational effort while maintaining the simplicity of execution for practice. It is an iterative algorithm and its convergence is theoretically proven. It uses the concepts of the path shift-propensity factor and the sensitivity of path costs with respect to path flows in the flow update process. The path shift-propensity factor is defined as the difference between the cost of a path and the cost of cheapest path for the related O-D pair. The slope of the path cost function is used as the measure of sensitivity of path costs with respect to path flow. The SPSA algorithm starts with all-or-nothing (AON) assignment or a warm start using previously known approximate solution as initialization. Then it checks for convergence criteria; if the initial solution does not satisfy the convergence criteria, then the SPSA flow update process is initiated. The SPSA equilibrates one O-D pair at a time in a sequential manner. The equilibration is the process of flow updates of paths aimed at decreasing the differences in cost of paths with non-zero flows between an O-D pair. For this purpose, it divides the set of paths between an O-D pair into two subsets: set of costlier paths and set of cheaper paths. Then flows are shifted from the set of costlier paths to the set of cheaper paths. It uses a line search to decide the optimal step size which decides the extent of flows shifts along the move direction. The move direction is determined by the vector of path shift-propensity factors and slopes of path cost function. The sequential approach helps in achieving faster convergence, but it may introduce the order bias leading to the solution noise. This issue is tackled partially by updating path sets simultaneously for all the O-D pairs before commencing the flow shifts for the O-D pairs at each iteration. In this sense SPSA combines merits of simultaneous and sequential approach. The simultaneous path set update also helps to decrease the computational cost especially for large scale networks. Once an O-D pair is equilibrated using SPSA flow update mechanism, then the next O-D pair in the sequence is brought into the equilibration process. Once all the O-D pairs are equilibrated, the convergence criterion is checked. If it is satisfied, the algorithm is terminated, else the next iteration is initiated. The convergence criterion adopted in this paper is the relative gap (Rgap) of 1.0E-6.

The pseudo code for the link ranking methodology is presented below:

Step 0: Initialization

Set counter $n = 1$. Set decision maker budget B.

Perform user equilibrium traffic assignment using SPSA. A new flow vector $\{x_a\}$ will be generated.

Step 1: Calculate importance factor ω_2 for all links using Eqs. (5) and (6) and destinations served data. The vector $\{\omega_2\}$ is generated which is preserved for all iterations.

Step 2: Calculate importance factor ω_1 for all links using Eqs. (3) and (4). $\{\omega_1\}$ will be generated as a vector of importance factors.

Step 3: Calculate importance factor ω_3 for all links using Eqs. (7) and link usage data. $\{\omega_3\}$ will be generated as a vector.

Step 4: Calculate combined I.F. (w) for all links using Eqs. (8). $\{w\}$ will be generated as a vector of combined I.F.

Step 5: Rank the links based on the descending order of the combined I.F. w in the vector $\{w\}$. The rank order of links (R^n) will be generated as a vector of link numbers $\{a\}$.

Step 6: if n>1 test convergence, if the convergence criterion is met, stop and accept the current solution R^n as the set of link ranks otherwise increase counter n by 1 and go to step 7. **Note:** The convergence is tested by comparing the rank order of links (R^n) with the previous rank order of links (R^{n-1}). If (R^n) is same as (R^{n-1}) the n convergence is achieved.

Step 7: Send vector R^n to the link improvement problem solved using PSwarm.

Update capacities of critical links and get new flows using SPSA and then go to Step 2.

Here it is imperative to mention an important limitation from implementation perspective arising due to non-uniqueness of UE path flows. UE path flows are theoretically non-unique. Different solution algorithms can result into different path flows, even multiple runs of same solution algorithm with significantly different initialization can result into new path flow solution. Changes in UE path flow solution can affect third importance factor. This issue can be handled by using a central solution in UE solution space which is considered as the representative of entire solution space, for example by using maximum entropy user equilibrium (MEUE) or entropy weighted user equilibrium (EWUE) solution for UETAP (Kumar and Peeta 2015). We have used SPSA for solving the UETAP for simplicity as the focus of paper is on demonstrating the proposed methodology. The issues arising due to non-uniqueness of path flow solution of UETAP can be resolved by post processing the SPSA solution (Kumar and Peeta 2015; Rossi, McNeil, and Hendrickson 1989) or by switching SPSA with other solution algorithm (e.g. TAPAS (Bar-Gera 2010), SOLA (Florian and Morosan 2014)) that provides central and most likely solution in the solution space. However, for simplicity, in the paper, this issue has been dealt partially by using SPSA with warm start. SPSA is initialized through warm start using path flow solution from previous iteration to improve consistency between solutions of two consecutive iterations.

5. Numerical Experiment

This section presents the results of the numerical experiments and discusses the link ranking results to validate the proposed method detailed in the previous section. First the implementation of the proposed methodology is demonstrated using a small network, then it is implemented for a real scale network of

Montgomery County. Sensitivity analysis is performed through multiple implementations of proposed framework for three different budgets for both small and real scale networks to determine the change in link ranking with the change in investment levels.

Small Test Network

To facilitate comprehensive analysis a small network consisting of 18 links (see Figure 3) was used to demonstrate the proposed methodology. In Figure 3, the number above the link represents the link number and the number inside the circle represents the node number. The nodes 1, 2, and 3 are the origins and the nodes 12, 13 and 14 are the destinations in this network.

As it is a small network all links was considered as the potential links for improvement (i.e. m was taken as 18 for network 1). Table 2 summarizes the numerical results for the small test network in the form of the link ranking for two budget scenarios after convergence (100 iterations). It can be observed from this table that with increase in budget, the ranking of the links changes significantly. Under zero budget allocation for improvement, link 9 was the most critical link. After having a budget allocation of 50 million, the ranking changes but link 9 remains the number one rank link. Link 8 becomes the second most critical after link 9 with increase in budget which was previously occupied by Link 14.

6. Numerical Results for Real Scale Network

Figure 4 highlights the ranking of links of the Montgomery County network (Test Network 2) based on the various importance factors. In this case m was taken as 20 (but it can be taken as any number less than the total number of links in a network). The network with RED color shows the first 10 links with the highest rank and the YELLOW marks the next 10 links. This identifies the most critical links used for serving different origins and destinations, carrying more flows and serving more important destinations. Figure 4(a) shows the top 20 links based on importance factor 1 which is based on the link flows (considering β_1=1 and β_2=0). This information also explains which links might be more prone to traffic congestion. Figure 4 (b) shows the top 20 links based on importance factor 2 (considering β_1=0 and β_2=1). This figure identifies the links which serve most of the important destinations like school, grocery shops, fire service, police station and hospital. The higher the number of important destinations served, higher is the rank of the link. This figure shows that the top links are concentrated at the bottom of the network which gives an idea about the distribution of the frequently used important locations. Figure 4(c) shows the top 20 links based on importance factor 3 based on the link usage for number of O-D pairs (considering β_1=0 and β_2=0). This is the most important ranking of all three rankings since it is based on number of O-D pairs served by a link at UE and signifies graph theoretical importance with respect to travel from all origins to all other destinations within the network. From the figure, it can be observed that the top ranked links appear in the center of the network which shows that these are the common links used to serve most origins and destinations. The comparison results (Figure 4(a)-4(c)) vindicates that link criticality changes significantly based on the importance factor used, and therefore a single measure of link criticality is not sufficient. Figure 4(d) shows the ranking of links based on the combined I.F. This ranking gives an idea of the overall ranking of links simultaneously using three important factors.

It is important to mention from the implementation perspective that in the computation of combined I.F. for link ranking, different weights are given to three importance factors. The sum of all three weights equals to one, thus require deciding only weights of two I.F.s. In this study, Importance Factor 1 and 2 are given a value of 0.3 (considering β_1=0.3 and β_2=0.3) whereas Importance Factor 3 assumes a value of 0.4. Relatively more weight is given to the third importance factor as it reflects topographic importance of a link. However, different weights than used in this study could be used based on preference of planning agency and it would be interesting to observe how the link ranking behaves when the weights are changed keeping all other parameters constant.

Figure 5 shows the change in link ranking due to different investment scenarios. This methodology is tested for 3 types of budgets scenarios. From Figure 5, it can be observed that with the increase in investment, the link ranking changes as expected and four sets of top 20 links of the different budget scenarios are not identical although some links are common. The spatial location of critical links can act as the guiding factor for strategic decision making such as where to place security cameras, potential locations of patrolling by security personnel and strengthening of links such as resurfacing or bridge strengthening.

Results show that transportation network link evaluation and ranking can be extremely non-intuitive both in terms of the effects the importance factors can have on the network and in terms of the individual impacts associated with the importance factors. Network-wide performance is difficult to predict by examining ranking outcomes on the individual road links comprising the network because even small changes on one part of the network have the potential to dramatically affect the network system. It is also counter-intuitive to consider the possibility that some capacity improvement projects worsen system-wide performance.

Careful prioritization and sequencing of link improvement projects are needed from a resource management perspective, as implementing improvement projects of certain combinations of links may reduce or even completely erase the benefits associated with individual link improvement projects. The benefits associated with the individual link improvements cannot simply be extrapolated across groups of links in an additive manner. Implementing a high-value critical individual link improvement is not necessarily beneficial to the roadway network as a whole and can result in a negligible or even adverse overall improvement in travel time. The outcome depends on the dynamics associated with the topology, location, and design specifications of the specific projects involved as well as with behavior of individual travelers. Future work is needed with respect to integrating other network performance measures into the prioritization and ranking process. A more detailed investigation of the non-linear dynamics associated with implementing link ranking procedures is also required as the ranking of a group of links instead of individual is a highly complex and sensitive work.

7. Conclusion

This research was undertaken with the motivation that day-to-day network uses are as important for the determining the criticality of links as the occurrence of disruptive events in transportation networks.

Moreover, due to resource limitation, planners and decision makers are not able to allocate required funds to all links for improvement. Link ranking thus helps identify the most critical links in the network thereby assisting the planners to make improvement decisions or make strategic decision such as identify links for resurfacing, potential locations to place security cameras, patrolling by security personnel and strengthening links such as retrofitting a bridge. Despite vast literature in the domain of link criticality and researchers have done very little to distinguish between various methodological approaches and combine the various ranking measures while incorporating impact of investment decisions and resulting network users' behavior. The methodology proposed in this research determines the ranking of links by considering three importance factors; (1) link flow: higher the flow more important is the link, (2) the importance of destinations served: higher the number of important destinations served, more important is the link, and (3) the graph theoretic property: weight based on how many paths of various O-D pairs are served by the link under UE. These are combined to determine the highest ranked links. Numerical experiments have been performed, first with a small 18-link test network to demonstrate the concept, and then using real scale Montgomery County network to test its usefulness for practice. Three Budget scenarios were considered for analyzing the link rankings for both networks. It was found that with the increase in budget, the ranking of link changes significantly. It was also observed that each importance factor has an individual effect on the ranking of links and with change in the importance factor or a combination of the three, the link ranking changes considerably.

The implementation of this methodology led to satisfactory and meaningful results for the test networks. The proposed methodology is simple to understand and implement for practice. The results obtained can be easily used by practitioners and decision-makers and can be relevant, for instance, for the allocation of limited resources for traffic surveillance, infrastructure maintenance and improvement. This methodology can also be used for project prioritization of larger networks.

The proposed methodology and results presented are based on some assumptions and limitations. First, the proposed methodology does not consider the effect of growth of population, demand uncertainty and changes in land-use pattern over time which may affect the link usage and hence the link ranking. Second, only five facility types were considered in the calculation of importance factor 2 and other facilities such as churches, recreational centers, community centers, special events and occasions can also be considered thereby increasing the accessibility level of users. Third, the methodology assumes stable financial environment or constant budget which may not always synchronize fully with the reality. Fourth, the proposed methodology did not account for capacity reduction from lack of maintenance or extreme events although it is a topic that needs to be investigated. Moreover, the social, political as well as environmental factors are not considered while ranking the links or prioritizing the improvement or maintenance projects which plays a crucial part in funding allocation for these network or roadway improvements projects. The limited funds available either allow a few really bad roads to be repaired while the rest of the system gets neglected, or the worst roads are left alone to allow maintenance of the rest of the system (Poston and Reyes 2016). Incorporating other objectives that capture the before mentioned factors is left as a future research topic that both academia and practice should consider investigating. Some of the improvement works may take very less time like fixing a small portion of roadway whereas some will take potentially longer period for improvement as well as a larger budget like that of a bridge repair. Moreover, some of these links may fall into the Metropolitan or City planning agencies whereas others will fall into the State jurisdiction. Hence, while selecting the network for analysis or project prioritization, these points need to be considered. These assumptions and limitations create potential for many worthy research directions. Future scope of research also includes: the exploration of the sensitivity due to the various weights to be used in the procedure, the examination of the impacts of smaller incremental changes in budget scenarios, and the multi-year link ranking tasks for long term planning. Link improvement problem can also be formulated in terms of discrete network design problem with multiple capacity levels for each link. Moreover, the problem can have multi-objectives such as consumer surplus, user cost, construction cost, reserve capacity, social surplus and others since the decision makers must consider several factors while making a critical decision. Another scope of future research can be to analyze the network for improvement in terms of prioritizing maintenance of an important link versus new lane construction. In another case, the budget constraint creates an interdependence among competing projects even if there are no network effects (e.g. link ranking effects in this case) and the optimal set of projects will shift for many investment scenarios even if just a simple incremental benefit cost methodology is used. This also appeals for a potential research for future. Finally, equity considerations under budget allocation can be considered for the ranking of links.

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Appendix

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Table 1. Literature on Link Ranking Measures in Transportation Networks

Author(s) Reference	Measure/Index	Definition
(Rupi et al. 2015)	Link Importance Index	Measure of the consequences of link Disruption
(Rodríguez-Núñez and García-Palomares 2014)	Station Exposure (Criticality)	Expected average travel time increase for trips when a randomly chosen link is disrupted
(Chen et al. 2012)	Impact area vulnerability index	Measure of relative change in network efficiency due to link closure
(Jenelius and Mattsson 2012)	Importance of a cell of a network	Total impact over all O-D relations in the network
(Taylor and Susilawati 2012)	Accessibility/Remoteness Index of Australia (ARIA)	Ratio of road network distance of a given intensity to the average distance of all locations
(Li and Ozbay 2012)	GIS-based multi-criteria cost estimation tool ASSIST-ME	Tool consisting of formulations used for the different types of costs
(Snelder et al. 2012)	Alternate route indicator	Proposed additional indicator for alternate routes
(Luathep et al. 2011)	Relative Accessibility Index (AI)	Measure for evaluating the socio-economic effects of link (or road segment) capacity degradation or closure
(Luathep et al. 2011; Sullivan et al. 2010)	Network Trip Robustness (NTR)	Sum all the individual NRI values for each link in the network divided by the total demand in the network
(Novak et al. 2012; Scott et al. 2006; Sullivan et al. 2010; Ukkusuri and Yushimito 2009)	Network Robustness Index (NRI)	Change in travel cost associated with rerouting all traffic in the system if that segment become unusable
(Jenelius 2010)	Flow-based redundancy importance measure	Weighted sum over closures on every other link
	Impact-based redundancy importance measure	Weighted sum of total impact on every other link
(Jenelius 2009)	Beta index	Ratio of number of undirected links in a region to the number of undirected nodes in that region
(Ukkusuri and Yushimito 2009)	Criticality of a network component	Change in the performance of the network after the removal or damage of one its components
(Latora and Marchiori 2003; Nagurney and Qiang 2007b; Qiang and Nagurney 2007)	N-Q measure	Defined in the context of network equilibrium. The measure captures demand and costs, and the underlying behavior of users of the network
(Berdica and Mattsson 2007)	Volume-delay functions	Travel time on each link as a function of traffic volume (vehicles per hour and lane), speed limit and link length (km)
(Jenelius et al. 2006)	Measuring Importance	Importance of a link with regard to the whole network
(Sohn 2006)	Accessibility Index	Significance score of a certain link based on the pre- and post-accessibility measures
(Latora and Marchiori 2003; Nagurney and Qiang 2007b) (Nagurney and Qiang 2007c)	L-M Measure	Network performance/efficiency measure for a given topology
(D'este and Taylor 2003)	Hansen Index and Black-Conroy Cumulative distribution index	Measures integral accessibility of a link/node

Table 2. Ranking of Links in the Test Network 1

Rank	Vector of links in decreasing order of rank based on combined weight at different levels of network improvement budget			
	0 (million $)	50 (million $)	100 (million $)	400 (million $)
1	8	9	3	10
2	9	8	15	13
3	11	13	1	4
4	15	14	11	17
5	3	5	9	9
6	1	2	8	15
7	18	6	2	8
8	4	15	13	16
9	6	3	10	1
10	5	10	16	7
11	17	4	14	11
12	14	18	5	3
13	2	1	4	14
14	13	11	17	12
15	16	16	7	6
16	7	7	6	2
17	10	12	18	5
18	12	17	12	18

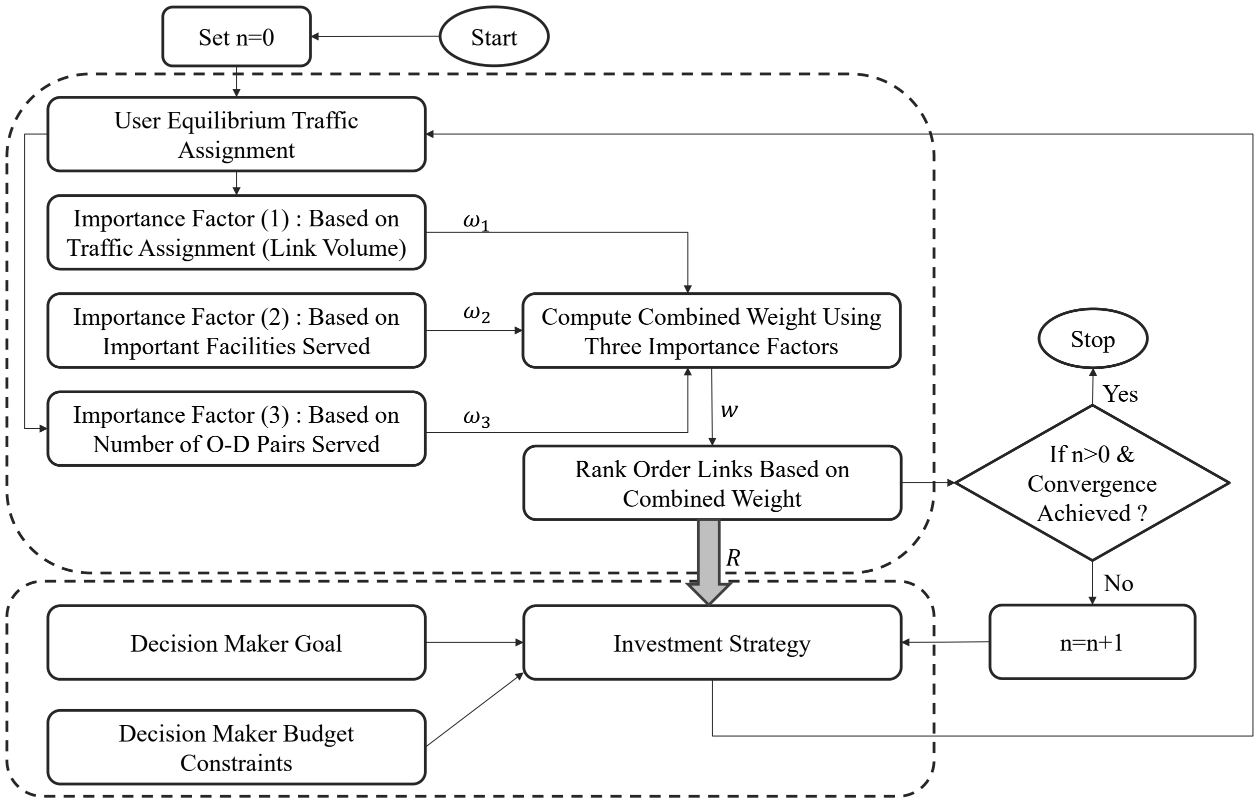


Figure 1- Link Ranking Methodology

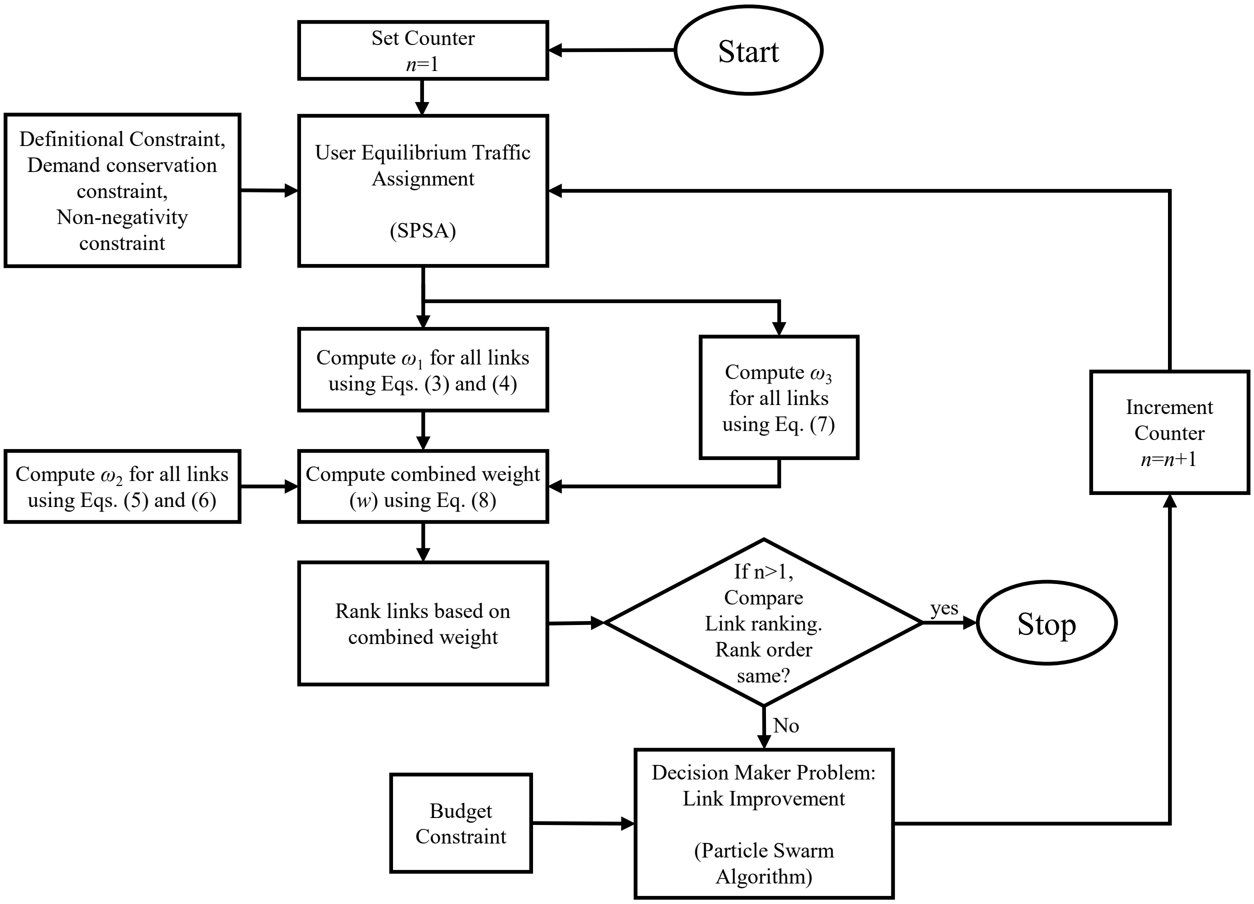


Figure 2- Implementation Flowchart of the Solution Approach

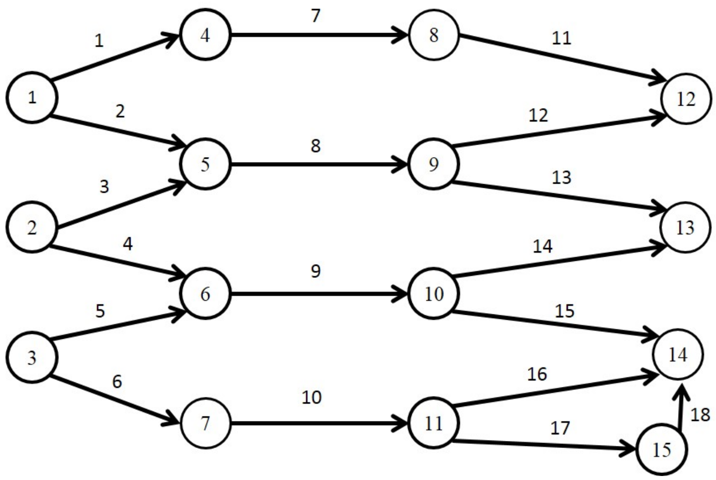


Figure 3- Test Network 1 Topology

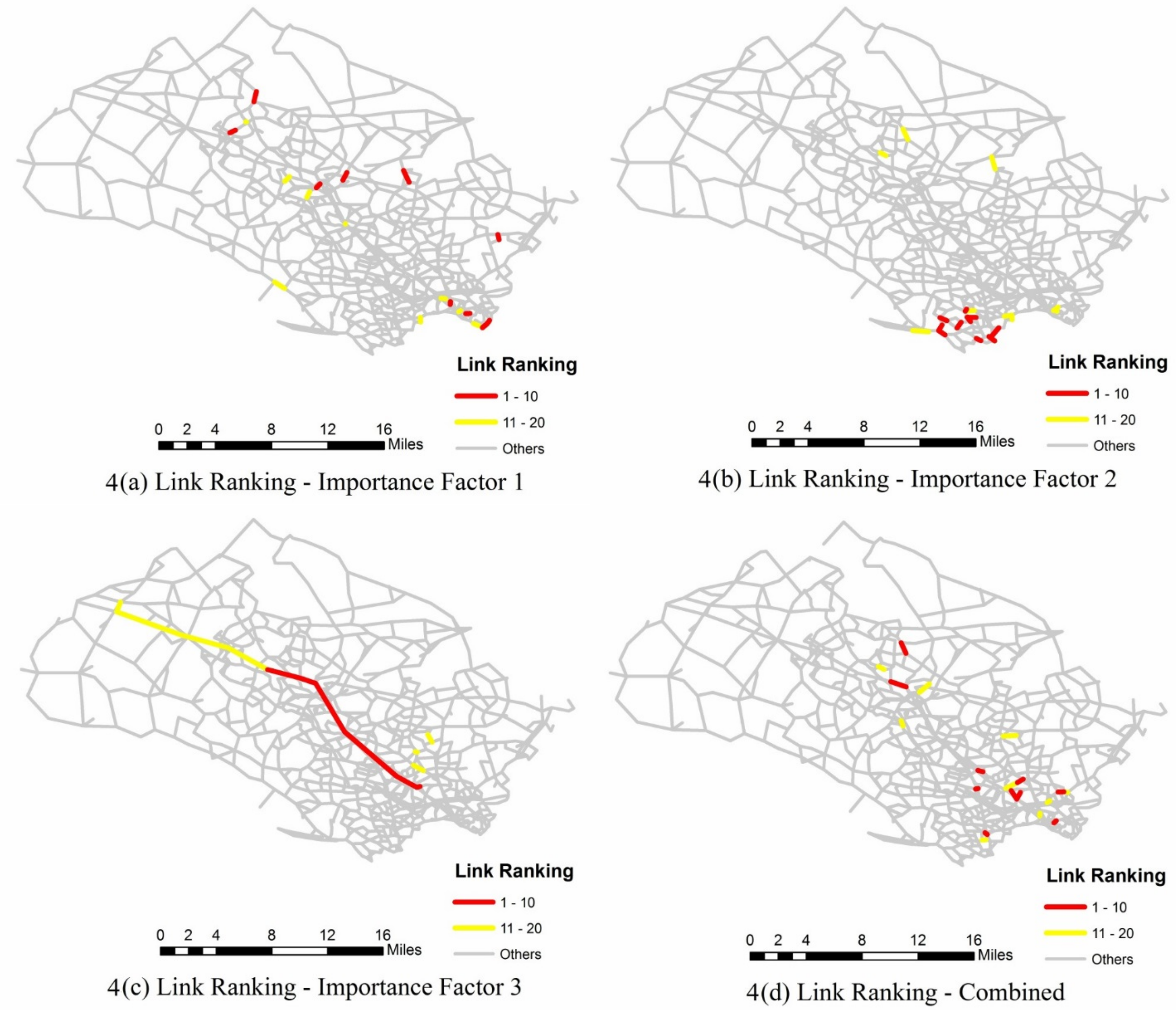
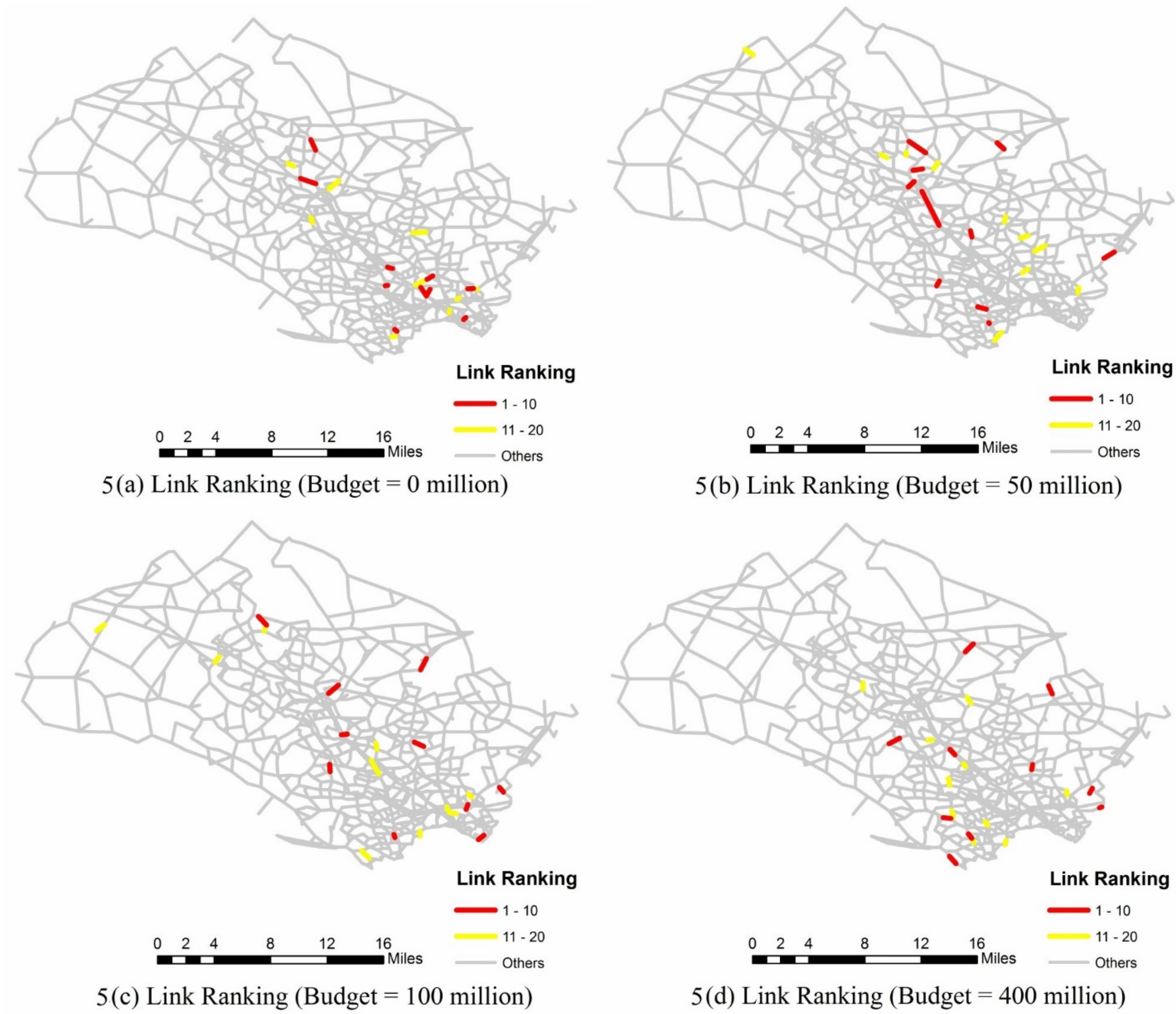


Figure 4- Link Ranking by Importance Factors for Test Network 2

(a) Link ranking – importance factor 1; (b) Link ranking – importance factor 2; (c) Link ranking – importance factor 3; (d) Link ranking – combined importance factor



5(a) Link Ranking (Budget = 0 million) 5(b) Link Ranking (Budget = 50 million)

5(c) Link Ranking (Budget = 100 million) 5(d) Link Ranking (Budget = 400 million)

Figure 5- Changes in Link Ranking due to Investment for Test Network 2

(a) Link ranking (budget = 0 million); (b) Link ranking (budget = 50 million); (c) Link ranking (budget = 100 million); (d) Link ranking (budget = 400 million)