A JOINT TRAVEL DEMAND AND ENVIRONMENTAL MODEL TO INCORPORATE EMISSION PRICING FOR LARGE TRANSPORTATION NETWORKS

By

Sabyasachee Mishra, Ph.D., P.E.
Research Assistant Professor
National Center for Smart Growth Research and Education
University of Maryland
College Park, MD 20742
Phone: (301) 405-9424
Email: mishra@umd.edu

Timothy F. Welch, JD, LL.B
PhD Research Assistant
National Center for Smart Growth Research and Education
University of Maryland
College Park, MD 20742
Phone: (301) 405-4481
Email: twelch@umd.edu

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ABSTRACT

Emission reduction strategies are gaining greater attention to support the national objective for a sustainable and green transportation system. A large percent of emission contribution is from transportation modes, primarily from auto and truck travel. Reductions in highway travel require prudent planning strategies and modeling user’s response to planner’s policies. Modeling planning goals and user’s response is a challenging task. In this paper the authors present a joint travel demand and environmental model to incorporate vehicle emission pricing (VEP) as a strategy for emission reduction. First, the travel demand model determines the destination, mode and route choice of the user’s in response to the VEP strategy set by the planner. Second, the emission model provides NOx, VOC, and CO₂ estimates at a very detailed level. A Base-case and three models are proposed to incorporate VEP in a multimodal transportation network. The objective function of the Base-case is the minimization of Total System Travel Time (TST), and the models are designed with objective of minimizing of Total System Emission (TSE). User Equilibrium method is used in the demand model to model user responses and solved by Frank Wolfe algorithm. The Base-case represents “do-nothing” conditions and the three models address the interactions between planner’s perspectives and user responses to VEP strategies. The proposed model is applied to Montgomery County’s (located in the Washington DC-Baltimore region) multimodal transportation network. The case study results show that VEP can be used as a tool for emission reduction in transportation planning and policy.

Key Words: Emission pricing, travel demand, multimodal, user equilibrium, frank wolfe

INTRODUCTION

In the United States (U.S) autos and trucks generate 25 percent of CO₂ emissions and account for over 70 percent of its petroleum consumption (1). It is a surmountable task to achieve reductions in greenhouse gases and ozone creating compounds to sustainable levels. Federal and state agencies aim to minimize harmful effects of transportation on the environment to achieve greenhouse gas and ozone targets. In particular, an explicit consideration is given to reducing levels of NOx, VOC (ozone creating compounds) and CO₂ (greenhouse gas) through better operational level planning. Measuring and reducing emissions requires quantification in modeling activities and incorporation into planning methods. The choice of type and nature of emission functions are important for deriving accurate estimates in the planning of transportation activities. Currently, there are two types of feasible methods, one is a long-term solution to improve public transport such that the mode shift can cause a large emission reduction and the other is a short-term solution to influence traveler behavior by imposing vehicle emission pricing (VEP) such that there are minimal emissions produced in the system (2). Although the former is the ideal scenario, for a long-term sustainable solution, the later needs more careful analysis before determining additional cost to the user to achieve substantive emission reductions. In both
cases there is a need for the development of travel demand or behavioral models. Such models are capable of examining the travel behavior of road users and their responses to impedances such as VEP to achieve emission reductions. In this context, VEP can be used a tool to model traffic flows in a transportation system, by choosing an alternate route, being elastic to travel options or a combination of both.

In this paper, we propose a joint travel demand (called a demand model, hereafter) and emission model to achieve emission reduction for the major pollutants NOx, VOC, and CO₂. The demand model incorporates individual travel behavior in determining travel choice and route choice in response to emission pricing strategies set by the decision maker. The Environmental Protection Agency (EPA) developed MOtor Vehicle Emission Simulator (MOVES) is integrated with the demand model for estimation of pollutants. Developing such models is complex and involves two major hurdles. First, the difficulty lies in obtaining specific and accurate emission levels with which to model individual driver responses to examine how the targets are achieved. Second, the application of these models in real world case studies to assist federal, state and local agencies (or decision makers) to achieve emission reduction objectives.

The proposed methodology in the paper incorporates a VEP technique in a step by step manner. In the next section a literature review is presented, followed by model formulation and solution methodology. The case study section describes the overview of the region’s geography, and transportation network characteristics. Model application and scenario results are presented in the next section. Finally, the conclusion section summarizes the findings, depicts contributions of the paper and enumerates future scope of research.

23 LITERATURE REVIEW

Emission reduction strategies are well studied in the literature. In a decision making framework these strategies can be viewed as a Stackelberg’s game theory in bi-level modeling. The upper level represents the decision maker’s goal (or preference) based on a certain objective and the lower level represents user’s response (whether or not to follow) the upper level strategy. In transportation engineering, the upper level can be considered the minimization of emission through VEP and the lower level represents driver’s response to the policy at the upper level (3)(4). In some cases the upper level strategies are a priori, and exogenously defined and in others the strategies are concurrently optimized in conjunction with the lower level. The lower level is solved with the traffic assignment optimization problem, and the most common objectives are the minimization of cost and time (5). Although in reality the two objectives are nearly identical, as time is monetized in the assignment process, cost is considered a separate variable in order to achieve positive externalities like fundraising or market based trip decision making. Uncommon in the literature is the objective to minimize the environmental impacts of vehicular traffic through a pricing mechanism (or derivatives thereof).
A number of studies have developed emission factors before applying them in the assignment process. While factors may vary based on area type, speed, highway congestion, weather conditions and other factors, use of these factors require a large amount of effort in data collection. In addition, development of emissions factors requires a calibration process and may not lead to an accurate estimation of emissions. The EPA MOVES model has been extensively developed to measure emission factors for different regions in the U.S. While some researchers point out a few limitations in models like MOVES (6), where there is a specialized need for emissions modeling, for the purpose of this research design and other scenario-based research, MOVES provides an excellent tool to measure changes in emissions based on changes traffic demand, assignment and other network characteristics.

Several studies examine the relationship between network flow and system emissions (3)(7). Effective modeling efforts have focused on incorporating road pricing into the highway assignment algorithm via Waldrop’s User Equilibrium (UE) objective function and a Frank-Wolf (FW) solution approach (3). The results typically show that flow, as a function of link volume and capacity, has a substantial impact on the overall level of emissions. However this is not the only condition that potentially affects total emissions. Nagurney (7) points out several conditions which can produce increased network emissions which include (somewhat paradoxically) the addition of a link, a decrease in travel demand and improvements in link cost structure (i.e. travel on the link becomes cost effective). The results of the study point out that a reduction in emissions requires not only improvements to network flow, possibly through demand management (8)(9), but also with the addition of supply side management, most feasibly in the form of a VEP.

The most efficient and equitable method to achieve substantive reductions in vehicle emissions is through a pricing mechanism ((10), (11)). Previous studies show that pricing can have substantial impacts depending on the policy objective, particularly in terms of internalizing the negative effects of emissions and for the market effect of pricing on trip, mode and route choice (12). An effective emission pricing strategy requires a large enough network to provide alternative routes and links with low enough saturation to absorb additional volume in order to show significant emissions improvement (13). An effective pricing strategy is therefore one that either prices links in a given area (such a non-attainment area) or along a single corridor. On the other hand, in the absence of viable route and link alternatives a mix of pricing and demand elasticity, which is the assumption that at the right price some trips will not occur or travelers will switch modes, can be used to effectively reduce TSE ((4),(12), (14), (15)).

Therefore the application of VEP strategy should be incorporated for emission minimization to examine the interaction between decision maker’s policy and the user’s response. Application of VEP strategies for real world conditions is limited in the literature. This paper analyzes a Base-case depicting existing emission levels, and proposes a set of models to minimize emissions in a combined framework of user’s response to decision maker’s policies. The framework consists of a joint travel demand and emission model to incorporate VEP
strategies for emission minimization. In the remainder of the paper emission pricing and VEP is used interchangeably. The next section describes the model formulation. The notations for model formulation are presented below.

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<th>Explanation</th>
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<tr>
<td>( \alpha_a )</td>
<td>Constant, varying by facility type (BPR function)</td>
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</tbody>
</table>
\( \beta_a \): Constant, varying by facility type (BPR function)
\( \beta^k \): Weights for each term in the size variable \( (S_j) \)
\( \gamma^c \): Value of time (VOT) for user class \( c \)
\( \delta^r_{a,ij} \): Flow on link \( a \), a subset of path \( r \), connecting each Origin-Destination (O-D) pair \( (i-j) \)
\( \pi^p_m \): Mode specific constant for mode \( m \) and purpose \( p \)
\( \tau_a \): Toll value for link \( a \)
\( \phi_a \): Total emissions for link \( a \)
\( c \): User class
\( t_o \): Free flow time on link \( a \)
\( \beta \): Mode and attribute specific coefficient (mode choice model)
\( \phi \): Factor to convert emission price to travel time units
\( \omega \): A positive constant (exponential demand function)
\( n \): Assignment iteration number

2 MODEL FORMULATION

In this section the Base-case (do-nothing) and three different models incorporating optimal emission pricing are presented. Each section describes the model formulation and the appropriate need for decision making.

7 Base-Case

The base case represents “do-nothing” or current conditions without any VEP. The user behavior is studied in the trip assignment stage and is solved by classical user equilibrium method (5).

\[
\text{Minimize} \quad \sum_a \int_0^{x_a} \left( t_a(x_a) + \frac{\tau_a(x_a)}{\gamma^c} \right) dx_a \quad (1)
\]

subject to:

\[
\sum_r f^r_{ij} = q_{ij} \quad (2)
\]

\[
x_a = \sum_i \sum_j \sum_r f^r_{ij} \delta^r_{a,ij} \quad (3)
\]

\[
f^r_{ij}, q^r_{ij} \geq 0 \quad (4)
\]

The objective function shows minimization of TST of the network as per Wardrop’s first principle, which denotes that “no user can experience a lower travel time by unilaterally changing routes” (16). In simple terms the equilibrium is achieved when the travel cost on all
used paths is equal. The three terms in equation (1) represent the total travel cost. The first term, $t_a$, is the travel time for link $a$, which is a function of link flow $x_a$. The second term, $\tau_a$ is the toll value for link $a$. The toll value is converted to units of time by considering the value of time (VOT) for user classes. Realistically, VOT varies by user class. This term is included to incorporate the differential effect of toll roads (if a toll road exists in the network, zero otherwise), on travel costs for each class of users. The sum of these two terms in Equation (1) can be referred as user cost for link $a$ ($u_a = t_a(x_a) + \tau_a(x_a)\gamma c$). Equation (2) is a flow conservation constraint to ensure that flow on all paths $r$, connecting each Origin-Destination (O-D) pair $(i-j)$ is equal to the corresponding demand. In other words, all O-D trips must be assigned to the network. Equation (3) represents the definitional relationship of link flow from path flows. Equation (4) is a non-negativity constraint for flow and demand. The travel time function $t_a(.)$ is specific to a given link ‘$a$’ and the most widely used model is the Bureau of Public Roads (BPR) function given by

$$t_a(x_a) = t_o \left(1 + \alpha_a \left(\frac{x_a}{C_a}\right)^\beta_a\right) \tag{5}$$

where $t_o(.)$ is free flow time on link ‘$a$’, and $\alpha_a$ and $\beta_a$ are constants (and vary by facility type). $C_a$ is the capacity for link $a$. In the base model the objective is minimization of total system travel time. Emission is not a component of the base case. The emission parameters are derived from MOVES and utilized exogenously to the assignment method for the Base-case.

When the objective function is expanded to include the minimization of TSE, the emission model is moved into the assignment program. After the initial iteration of assignment, based on free-flow travel time, the resulting emissions are calculated using the MOVES model derived emissions factors. The factors used are based on link speed, volume, distance, time of day, facility type and area type. The pre-determined emissions cap is then subtracted from the newly calculated emissions which becomes the basis for the emissions charge at the end of each assignment iteration.

Model-1: Link Based Emission Pricing

When the decision maker has the objective to minimize total emission, link based emission pricing may be the first option. Based on a policy criteria such as links exceeding a threshold emission value need to be priced. The threshold value can be considered as an emission cap. The cap can be determined by calculating the link level emissions for the entire network in the base year by grams per mile for each link. The cap can be then set at the average of emissions per mile. The cap for each link can then be determined by multiplying the average grams of emissions per mile by the distance of each link. The emissions cap for each link is:
\[ \Phi_a = \left[ \frac{1}{N} \sum_{a=1}^{N} \left( \frac{\phi_a}{l_a} \right) \right] * l_a \]  

(6)

where \( \phi_a \) is the total emissions for link \( a \) calculated for each link in the base model and \( l_a \) is the link distance. Once the cap is determined, the emission price \( (e_a) \) can be incorporated into the travel demand model. The emission price can be converted to travel time units with an appropriate factor \( (\varphi) \) representing VOT in monetary terms. The revised user cost function for link based emission is

\[
u_a(x_a,e_a) = t_a(x_a) + \frac{\tau_a(x_a)}{\gamma^c} + \frac{\varphi e_a(x_a)}{\gamma^c}
\]  

(7)

where \( u_a(x_a,e_a) \) is the travel time function for Model-1, which incorporates emission pricing term \( e_a \). The objective function for Model-1 is similar to base case with the exception that the third term from equation (7) \( \frac{\varphi e_a(x_a)}{\gamma^c} \) is added to equation (1). The emissions price for each link is updated at the end of each assignment iteration based on the emissions produced as a result of that assignment over the predetermined cap, so that changes in the results of each assignment are reflected in the travel cost faced by each user which like travel time, will vary between assignment iterations. It should be noted that with the revised user cost function the objective of Model-1 is to minimize total system emission (TSE) as opposed to the Base-case where the objective was to minimize total system travel time (TST).

**Model-2: Corridor Based Emission Pricing**

Link based emission pricing provides overall estimates of the user behavior responses to a predefined threshold value. However, pragmatically it may be difficult to implement link based emission pricing. As a revised approach, emission pricing can be applied to corridors (set of contiguous links) instead of individual links. The difficult part is to determine which corridors need to be priced. The threshold value for corridor can be determined as follows.

\[ \Phi_b = \sum \left[ \frac{1}{N} \sum_{a=1}^{N} \left( \frac{\phi_a}{l_a} \right) \right] * l_a \]  

(8)

where \( \phi_b \) is the total emissions for corridor \( b \), calculated for each link in the base model and \( l_a \) is the link distance. Similar to equation (7) emission pricing for the corridor \( e_b \) can be incorporated into the user cost function. However implementation of corridor based emission pricing may be difficult in a demand model. Alternatively, the total pricing for a corridor can be
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proportionally attributed to each link of the corridor. The emission pricing for link in the corridor 
($e'_a$) can be obtained by the following equation.

$$e'_a = \left(\frac{e_b}{l_b}\right)l_a$$  \hspace{1cm} (9)

where $e_b$ is the total corridor emission price, $l_b$ is the length of the corridor, and $l_a$ is the 
length of link $a$. Similar to the pricing method in Model-1, the corridor emission price is 
determined at the end of each assignment iteration, based on the amount of emissions produced 
on the corridor over the cap for each iteration of the assignment model.

Model-3: Variable Demand Emission Pricing

Both link (Model-1) and corridor based emission pricing (Model-2) provide a good 
understanding of shifting routes and modes (discussed later in the paper) to minimize total 
system emission. These models will be very helpful to analyze the reduction of emissions as a 
result of VEP. However, both Model-1 and Model-2 do not consider the variability of demand 
because of changes in network conditions as a result of vehicle emission pricing. Alternatively, 
the users are not elastic to the emission pricing strategy. Demand elasticity can be incorporated 
to the base case by introducing inverse demand function in the objective function. Modeling 
variable demand completely changes the objective function. This can be represented as follows.

$$\text{Minimize} \sum_{a} \int_{0}^{x_a} \left(t_a(x_a, e_a) + \frac{\tau_a(x_a)}{y_c} + \frac{\phi e_a(x_a)}{y_c}\right) - \sum_{ij} \int_{0}^{q_{ij}} D_{ij}^{-1}(x_a, e_a)$$  \hspace{1cm} (10)

This formulation allows the decision maker to model the elasticity of the user behavior. 
The constraints for the variable demand model remain same as in the base case (see equations 2- 
4). The inverse demand function $D_{ij}^{-1}(.)$ is associated with O-D pair $i-j$. An exponential demand 
function is then used which is a function of potential demand and least user cost paths to 
determine the new demand $d_{ij}^{\varphi}(x_{ij})$ between O-D pairs is given by

$$d_{ij}^{\varphi}(x_{ij}) = \hat{d}_{ij}^{\varphi} \exp(-\omega \ast u_{ij}^c) \hspace{0.5cm} \forall \hspace{0.2cm} i, j, \tau$$  \hspace{1cm} (11)

where $\hat{d}_{ij}^{\varphi}$ is the potential demand between $i-j$, $u_{ij}^c$ is the least cost path between O-D 
pairs $i-j$ and $\omega$ is a positive constant. This new demand is then fed back into the highway 
assignment model. The users in the $i-j$ O-D pair are now elastic to the cost of travel ($u_{ij}^c$). 
Alternatively, as the cost increases the willingness to travel decreases. The emission cap for this 
case can be set at the discretion of the decision maker. Link based or corridor based emission pricing 
can be incorporated into the variable demand model (equation 6, and 8).
**Destination and Mode Choice**

The assignment procedure for the base case and three models are explained in the earlier section. The demand model should also be sensitive to the destination and mode choice as the network characteristics change. The following section describes steps on how destination choice and mode choice are implemented in the demand model. The utility $U_{ij}n$ of choosing a trip attraction destination $j$ for a trip $n$ produced in zone $i$ is given by:

$$U_{ijn} = S_j + \alpha L_{ij} + \sum \beta^k D^k_{ij} + \sum \beta^k D^k_{ij} N^k_n + \sum \beta^k Z^k_j + C_{jn}$$  \hspace{1cm} (12)

Where, $S_j$ is the size variable for destination zone $j$, $L_{ij}$ is the mode choice logsum between zone pair $ij$, $D^k_{ij}$ represents the various distance terms (linear, log, squared, cubed and square root), $N^k_n$ represent person, household or production zone characteristics for trip $n$ and is used for creating interaction variables with distance terms, $Z^k_j$ represents attraction zone characteristics (other than the size term), and $C_{jn}$ is a correction term to compensate for the sampling error in the model estimation (i.e., it represents the difference between the sampling probability and final estimated probability for each alternative). The size variable may consist of several different terms; up to four categories of employment in addition to households were used. Weights ($\beta^k$) for each term in the size variable were estimated along with all other model parameters as follows, where $E^k_j$ is employment of type $k$ in zone $j$:

$$S_j = \log \left( \sum \beta^k E^k_j \right)$$  \hspace{1cm} (13)

The destination choice model provides O-D demand for all trip purposes. A nested logit structure is formulated for mode choice, which is based on generalized utility functions for auto and transit travel. Separate utilities were developed to represent mode choice by trip purpose and time of day. The mode choice utility function is represented as follows:

The complete utility function for mode choice is as follows:

$$U^p_m = \pi^p_m + \beta^p_{1m} IVTT^p_m + \beta^p_{2m} TET^p_m + \beta^p_{3m} \times AOC^p_m + \beta^p_{4m} PC^p_m + \beta^p_{5m} \tau^p_m + \beta^p_{6m} WT^p_m + \beta^p_{7m} IWTa^p_m + \beta^p_{8m} IWTb^p_m + \beta^p_{9m} NOT^p_m + \beta^p_{10m} DA^p_m$$  \hspace{1cm} (14)

Where $\pi^p_m$ is a mode specific constant for mode $m$, and purpose $p$; $\beta$ in each term is the mode and attribute specific coefficient; $IVTT$ is the in-vehicle travel time, $TET$ is the terminal time, $AOC$ is the auto operating cost, $PC$ is the parking cost; $\tau$ is the toll value, $WT$ is the waiting time, $IWTa$ is the initial waiting time less than 7.5 minutes; $IWTb$ initial waiting time greater than 7.5 minutes; $NOT$ is the number of transfers, $TF$ is the transit fare; and $DA$ is the
drive access time. The mode choice model results in splitting O-D trip matrices into 11 travel modes (3 auto modes, and 8 transit modes). Three auto modes refer to Single Occupant Vehicles (SOV), High Occupant Vehicles with 2 occupants (HOV-2), and High Occupant Vehicles with three or more occupants (HOV-3+). Eight transit modes are walk and drive to bus, express bus, rail, and commuter rail.

**SOLUTION METHODOLOGY**

Figure 1 shows a flowchart of the solution algorithm for both the Base-case and the three emissions models. The algorithm relies first on inputs commonly found in demand models, which includes the characteristics of the transportation networks (highway and transit), socio-economic and other inputs needed for the trip generation, destination choice, mode choice and traffic assignment programs. The traffic assignment is solved with a FW algorithm. In the FW when the model has met the convergence criteria, the next model begins by running the first group of programs in the emissions model to setup the emissions inventory derived from the Base-case. The objective of the Base-case is minimization of TST. The algorithm then diverges to run Model-1, Model-2 or Model-3 (with objective of TSE). If the shortest emissions path is selected, the model performs the first iteration of the Model-1/ Model-2 modified assignment with the emissions minimization function. After the first iteration the model checks for convergence then recalculates emissions based on that iteration’s assignment results. The iterative procedure between assignment and emissions algorithms continues until the results meet the convergence criteria.

For Model-3, variable demand is calculated based on the least cost path. The variable demand model first skims the Base-case assigned network for congested travel times. The shortest path between all O-D pairs is calculated and used in the inverse demand function to calculate the new demand based on the shortest path. Once the new demand is calculated it is used in the assignment model. The assignment model then iterates between the emissions model and assignment until the convergence criteria for the assignment program is met. Once the assignment convergence is met the nearest shortest path from the equilibrium assignment, the demand for the next iteration is determined. The model iterates between emissions minimization assignment and variable demand until the model converges. The complete model structure is incorporated into the transportation planning software, Cube Voyager (17). The UE assignment algorithm is also modified in Cube Voyager to reflect VEP for all three models. A brief description of the step by step approach is shown in Appendix-A.
FIGURE 1 Solution Methodology
CASE STUDY

The proposed framework is applied to Montgomery County in Maryland. Montgomery is the most populous county in the state with a population close to one million, 400,000 households, and employment of 600,000. The county boundary with the transportation network is presented in Figure 2. The county contains parts of the heavily travelled roadways in the Washington DC-Baltimore region (Washington DC is referred to as Washington in the remainder of the paper). The county has an extensive highway network with the Capital Beltway (or Interstate-495), that surrounds Washington, passing through Montgomery County. Interstate-270 interstate forms one leg of an interstate triangle between Washington DC, Baltimore city and Frederick city. The county also contains a portion of route 29, one of the major state routes, which run between the Washington and Baltimore beltways. In terms of transit, the commuter rail MARC, which travels between Brunswick (North) to Union Station (South), travels completely through Montgomery County. A major metro line (called the Red Line) also travels through the county. In addition, Washington Metropolitan Transit Authority (WMTA) provides bus service and the county has extensive coverage of its own bus system called “RIDE-ON” with very high capacity and frequency.

FIGURE 2 Montgomery County Transportation Network
There are a total of 4,302 highway links in Montgomery County consisting of all facility types representing a total of 3,630 lane miles. In addition, the network consists of 73 bus lines. A number of transit options are available in the network. Montgomery county has an extensive continuous emission monitoring (CEM) program and the mission is to examine emission reduction strategies. This paper is geared towards this mission to propose a number of emission reduction strategies by using travel demand and emission models. However, the proposed methodology can be extended to other regions.

MODEL APPLICATION

The Montgomery county study area consists of 223 zones and 49,729 O-D pairs. The data is obtained from the Maryland Statewide Transportation Model developed by The Maryland State Highway Administration. The population, households (by size and five income categories), and employment (four industry categories) are used for trip generation purposes. The destination choice model adequately distributes the trips between O-D pairs as per the formulation presented in Equation (12). Further, the mode choice model splits between autos and transit. Additionally, the assignment process provides the shortest user cost paths to the O-D pair and represents user’s response. Details of trip generation, destination and mode choice are not presented in the paper for brevity and much emphasis is given to highway assignment. The model application section consists of results from the Base-case, and three models.

The summary of results is presented in Table 1. The first column shows the four cases analyzed. Further, for each case, the emission results from four facility types are presented. For Base-case and for each model the emissions in terms of NO\textsubscript{x}, VOC and CO\textsubscript{2} are shown in column (3), (4), and (5). In the Base-case, the objective is to minimize total system travel time (TST); the effect of the final assignment on total emissions was ignored by the assignment algorithm to satisfy the very purpose of the Base-case which is estimating emissions and not to apply VEP. Emissions are estimated after the traffic assignment stage. The freeways from Base-case resulted in 8.7 million grams of NO\textsubscript{x}, 1.1 million grams of VOC, and 1,877 million grams of CO\textsubscript{2}. Among all facilities in the base case, freeways produce the largest amount of emissions. A total of 1,887 million grams of emission resulted from the Base-case. A pictorial representation of link-based emission is shown in Figure 3(a). Interstates 495 and 270 are the highest contributors of total emissions. Other areas of concern are along route 29 (lower right corner going north/south) and Connecticut/Georgia avenues (orange links crossing I-495) which are all heavily travelled corridors.

For Model-1, a threshold value or emission cap is determined (please see equation (8)), however the VEP is based on the total amount of emissions that exceed the cap after each iteration of the assignment program, meaning the links to be priced and the amount of the VEP...
(\(e_a\)) changes with the iterative change in link flow (\(x_a\)). From the Base-case emission results, the 50th percentile of emissions in grams per mile is determined. The links over the emission cap are subjected to VEP in Model-1. The emission cap is estimated as $0.0006 per gram (or $60 per tonne) of emissions for travel on links that exceed a predefined emissions cap of 435 thousand grams per mile resulting from the emissions inventory from the Base-case. The assignment algorithm seeks to minimize TSE. Model-1 produces a substantial level of emission reduction resulting in total emissions of 1,433 million grams, and 6.4 million grams NO\(_x\), 0.9 million grams VOC, and 1,426 million grams of CO\(_2\) (Table 1). This amounts to a little over 24% reduction in all emissions. Among the facility types, freeways are still the major contributor (considering the dominate flow pattern over other facility types) but exhibit the largest reduction (44%) in emissions.

Figure 3(b) presents a map of link emissions in the study area. While Model-1 results in emission reductions, the difficulty in implementing enforcement policy, and the effect it has on non-charged areas outside Montgomery County and the impact on local roads that are able to absorb more emissions due to lower activity levels in the base model, would make this a difficult emissions pricing strategy. With new technologies such as GPS tracking and electronic pricing, it is likely that such a strategy could be implemented, but we suggest more a traditional pricing method, by selecting a single heavily emitting corridor as opposed to a single link.

<p>| TABLE 1 Emissions by Pollutant and Facility Type |</p>
<table>
<thead>
<tr>
<th>Model</th>
<th>Facility Type</th>
<th>NOX (gm)</th>
<th>VOC (gm)</th>
<th>CO2EQ (gm)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Freeway</td>
<td>4,619,807</td>
<td>401,952</td>
<td>851,912,900</td>
<td>856,934,658</td>
</tr>
<tr>
<td></td>
<td>Major Arterial</td>
<td>2,637,581</td>
<td>419,295</td>
<td>604,646,167</td>
<td>607,703,044</td>
</tr>
<tr>
<td></td>
<td>Minor Arterial</td>
<td>515,946</td>
<td>107,596</td>
<td>147,206,819</td>
<td>147,830,362</td>
</tr>
<tr>
<td></td>
<td>Other*</td>
<td>971,794</td>
<td>204,015</td>
<td>273,611,629</td>
<td>274,787,438</td>
</tr>
<tr>
<td>SUM</td>
<td></td>
<td>8,745,128</td>
<td>1,132,858</td>
<td>1,877,377,516</td>
<td>1,887,255,502</td>
</tr>
<tr>
<td></td>
<td>Freeway</td>
<td>2,591,969</td>
<td>226,547</td>
<td>479,541,402</td>
<td>482,359,917</td>
</tr>
<tr>
<td></td>
<td>Major Arterial</td>
<td>1,870,555</td>
<td>297,512</td>
<td>429,569,086</td>
<td>431,737,153</td>
</tr>
<tr>
<td></td>
<td>Minor Arterial</td>
<td>664,779</td>
<td>138,604</td>
<td>189,342,464</td>
<td>190,145,847</td>
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<tr>
<td></td>
<td>Other*</td>
<td>1,229,364</td>
<td>252,932</td>
<td>327,356,255</td>
<td>328,838,551</td>
</tr>
<tr>
<td>SUM</td>
<td></td>
<td>6,356,667</td>
<td>915,594</td>
<td>1,425,809,207</td>
<td>1,433,081,468</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Percent Change</th>
<th>Model-1</th>
<th>Model-2</th>
<th>Model-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>NOX (gm)</td>
<td>-43.89%</td>
<td>-30.34%</td>
<td>-8.45%</td>
</tr>
<tr>
<td>VOC (gm)</td>
<td>-43.64%</td>
<td>-29.53%</td>
<td>-7.51%</td>
</tr>
<tr>
<td>CO2EQ (gm)</td>
<td>-43.71%</td>
<td>-29.89%</td>
<td>-9.04%</td>
</tr>
<tr>
<td>Total</td>
<td>-43.71%</td>
<td>-29.89%</td>
<td>-9.04%</td>
</tr>
</tbody>
</table>

* "Other" represents collector and local facility types.
Model-2 represents the VEP for a single corridor. A corridor is selected where a set of contiguous links over 10 miles produce link based emissions higher than the cap (set at 50% of emissions per mile for link in the corridor in the Base-case). For the case study, an analysis of the highway network showed that Interstate-270 is the only corridor to meet the criteria. However, for a larger network, there can be multiple corridors, and the model can analyze such an effect. Interstate-270 is a heavily used corridor for commuting between its terminus in Frederick Maryland and Washington DC. Along this corridor are many densely developed areas in Montgomery County. Emissions from this corridor alone account for over 17% of total vehicle emissions in the Base-case. Model-2 imposes a charge of $0.0006 per gram of emissions for travel on links that exceed the predefined emissions cap of 1.2 million grams per mile resulting from the emissions inventory from the Base-case. The VEP is only applied to the selected I-270 corridor. Table 1 shows that total emissions for Model-2 are 1,789 million grams with 8.0 million grams NO$_x$, 1.1 million grams VOC, and 1,780 million grams of CO$_2$. As a result of the corridor pricing strategy, the largest contributor of emissions switches to major arterials. This is largely a result of users selecting alternative routes that are within Montgomery County but still result in lower travel time. While Model-2 leads to an overall reduction in emissions of over 5%, it results in increased emissions for all other facility types due to excess use which produces a higher volume of trips and a greater number of congested lane miles increasing emissions on major arterials by 18%, minor arterials 21% and other roads by 6.5%. Figure 3(c) shows the results of Model-2 on emissions in the study area. The results show there is a substantial improvement of generally over 50% reduction in emissions along the corridor. The figure also shows the impact of VEP on a single corridor. Neighboring links become saturated with traffic and emissions increase. However, these links are generally able to absorb enough extra volume and emissions that TSE is reduced.

Model-3 represents variable demand conditions, modeling the elastic nature of travelers’ behavior to the given network conditions. When the VEP strategy is employed in the variable demand model a number of users may first decide whether or not to make a trip subject to imposed user costs. The “willingness to travel” was not analyzed in the earlier two models. For the variable demand model, in equation (11) the value of $\omega$ is 0.01 representing the elasticity of users response to travel cost (9). The total emissions from Model-3 resulted in 1,717 million grams with 7.9 million grams NO$_x$, 1.0 million grams VOC, and 1,708 million grams of CO$_2$. For Model-3, a reduction in all pollutants for all facility types was achieved. The results show a reduction of 9% in total emissions, falling between the Model-1 and Model-2 results. Users’ response to levels of congestion and VEP on the 270 corridor helped to improve emissions by reducing the number of vehicle trips taken within the county.

Figure 3(d) shows the change in link level emissions produced by the combination of corridor VEP and variable demand. Much of the improvement in emission results from VEP and reductions in demand for travel in the county due to the added cost of travel along the 270 corridor. Travel costs also increase on arterials and local roads as the cost in time to divert
FIGURE 3(a) Total System Emissions Base-Case

FIGURE 3(b) Percent Change in Emissions Model-1

FIGURE 3(c) Percent Change in Emissions Model-2

FIGURE 3(d) Percent Change in Emissions Model-2
around the corridor and in fuel expense reduces demand for travel on many roads. Some local roads show an increase in emissions as the cost of travel is comparatively low on these roads and does not increase to a level high enough to reduce total demand for trips. Emissions on the VEP corridor show reductions ranging from 25% to 50% for all links.

SYNTHESIS OF MODEL RESULTS

The results for the Base-case and three models are presented in Table 3. Each model result is further broken down by the facility type. The first column of results in the table provide the total system travel time (TST), followed by the total system emissions (TSE) shown in columns (3) and (4). The table also shows the vehicle miles travelled (VMT), average speed, number of congested lane miles (lengths of road with a volume to capacity ratio of over 0.75), and the number of links that are charged (exceed the predetermined cap) that result from each model.

In the Base-case TST is 154,453 hours and total VMT is 4,199,854 hours, which results in an average speed of 27 miles per hour. Without any emissions reduction strategies total system emissions for the Base-case are at the highest level compared to the three models. This is in part due to amount of traffic volume on the highways which results in a total of 375 congested lane miles with the majority of those miles on freeways and major arterials.

Model-1 shows a 9.6% improvement in TST when each link that exceeds the predetermined emissions cap has VEP applied which is calculated between iterations of the assignment algorithm. The reductions in freeway travel time (57%) and increase in average speed (from 38 to 49 mph) is greatest for freeways, with a modest improvement in major arterials (a 32% decrease in TST and a one mph increase in speed). However there is an increase on minor arterials for TST (59%) and a reduction in average speed (from 23 to 18 mph) and for other roads (38% increase in TST and a four mph decrease in speed). This result is mirrored in the level of emissions (29% and 20% respectively) and number of congested lane miles (124% and 269%). Freeways and arterials show significant improvements in reducing both emissions and congestion while minor arterials and other roads results in an opposite trend. There are two phenomena acting in Model-1 to produce the results. First, the study area is only a small part of a larger transportation network that consists of links outside Montgomery County. As a result, users have a variety of alternative routes to select from when making a trip. When all links are potentially subjected to VEP, it is more cost effective for some users (with a low VOT) to select a route that is outside of the study area, effectively outside the potential VEP area. This produces an overall decrease in VMT and helps reduce system emissions and congestion. At the same time, by setting a target emissions level (cap) dependent on the 50th percentile of emissions per mile, some less travelled roads gain emissions capacity will other congested roads are significantly over capacity. Since the VEP is a measure of emissions over capacity, many of the lower facility types such as minor arterials, collectors and other roads become feasible non-
charged routes for users with a lower VOT; in other words, those that can spend the time to go around the charged links. As a result of this, emissions levels, travel time, VMT and congestion all significantly increase on these out of the way but cheaper routes.

Model-2 demonstrates that the corridor VEP model achieves a lower reduction in emissions (5% compared to 24% in Model-1) but only initiates a VEP strategy on 88 links compared to the 1,405 links required in Model-1. Model-2 results in higher average freeway speeds than the Base-case, providing a 39% reduction in freeway congested lane miles. The model also shows increases in TST (5%) and an increase over the Base-case total congested lane miles (20%). This occurs, much like in Model-1, because users find it cost effective to avoid travel on the VEP corridor and unlike in Model-1 there is no additional emissions charge for users to select a route that avoids the corridor. Because of the availability of cost effective routes within Montgomery County users chose major and minor arterials.

**TABLE 2 Summary of Results**

<table>
<thead>
<tr>
<th>Facility Type</th>
<th>Model-1</th>
<th>Model-2</th>
<th>Model-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freeway</td>
<td>TST</td>
<td>TSE</td>
<td>VMT</td>
</tr>
<tr>
<td></td>
<td>(hrs)</td>
<td>(gm)</td>
<td>(MPH)</td>
</tr>
<tr>
<td>Base-Case</td>
<td>41,319</td>
<td>856,934,658</td>
<td>1,550,307</td>
</tr>
<tr>
<td>Major Arterial</td>
<td>55,434</td>
<td>607,703,044</td>
<td>1,392,557</td>
</tr>
<tr>
<td>Minor Arterial</td>
<td>21,233</td>
<td>147,830,362</td>
<td>484,132</td>
</tr>
<tr>
<td>Other*</td>
<td>36,467</td>
<td>274,787,438</td>
<td>772,858</td>
</tr>
<tr>
<td>SUM</td>
<td>154,453</td>
<td>1,887,255,502</td>
<td>4,199,854</td>
</tr>
</tbody>
</table>

| Freeway        | TST     | TSE     | VMT     | Average Speed | Congested Lane Miles | Charged Links | Percent Change |
|                | (hrs)   | (gm)    | (MPH)   | (1) (2) (3) (4) (5) (6) (7) (8) | TST (9) TSE (10) VMT (11) Congested Lane Miles (12) |
| Model-1        | 41,319  | 856,934,658 | 1,550,307 | 38 | 197 | N/A | -56.70% | -43.71% | -42.98% | -93.34% |
| Major Arterial | 55,434  | 607,703,044 | 1,392,557 | 25 | 118 | N/A | -31.93% | -28.96% | -28.49% | -50.02% |
| Minor Arterial | 21,233  | 147,830,362 | 484,132  | 23 | 30  | N/A | -26.85% | -23.10% | -22.63% | -45.23% |
| Other*         | 36,467  | 274,787,438 | 772,858  | 21 | 30  | N/A | -19.94% | -18.02% | -17.54% | -36.06% |
| SUM            | 154,453 | 1,887,255,502 | 4,199,854 | 27 | 376 | N/A | -10.52% | -8.68%  | -8.21%  | -20.54% |

Model-3 measures the relationship between emission reduction goals and user response by implementing a variable demand model along with the corridor VEP strategy. The results of this combined model show improvement for all indicators and all facility types (except congested lane miles for minor arterials increases 7%). The improvements occur in part because of the corridor VEP implemented in this model but also because the variable demand component of model results in a reduction in the total number of vehicle trips both beginning and ending in O-D pairs within the study area. In total, there is a reduction in travel demand of over 63,000 vehicle trips for all O-D pairs in Montgomery County which is less than 16% of the 396,500 trips that would otherwise occur in the county. This reduction in vehicle trips leads to an overall...
average system speed of 28 mph, which is higher than the Base-case and Models 1 and 2. The results also show improvement in total congested lane miles which shows a 22% improvement over the Base-case. In a single instance, on minor arterials, congested lane miles increased. This model provides a good estimate on how users might react with knowledge about road conditions and travel cost is known.

**FIGURE 4 Convergence Results for All Models**

To test for convergence the gap (*see Appendix-A*) between assignment iterations in the case of the Base-case and models 1 through 2 and assignment iterations and loop iterations for model 3, was measured. Gap is a function of change in assigned link volumes and cost between iterations, so that as gap decreases the assignment gets closer to convergence which indicates that a change in travel cost for a link does not produce a significant change in the entire network. For this paper the convergence criteria was set at a gap of 0.002 for the iterative assignment and 0.005 for the variable demand iteration. Figure 4 shows the iteration results in terms of the gap for fifty model iterations. In all cases the models reached the convergence criteria in less than 26 iterations. The three models had computational times per iteration of 12 minutes for Model-1, 8 minutes for Model-2 40 minutes for Model-3. The assignment iteration time varies a result of the
emissions model feedback into the assignment algorithm. Model-1 has a longer processing time because more links require emissions calculations. Model-3’s computation time was longer because each variable demand iteration has a nested traffic assignment program that also had to iterate and converge. The Base-case and all three models were run in Cube Voyager using multi-core distributed processing in a Windows Server 2008. However the processing times may vary as the computer configuration is altered.

CONCLUSION

This paper presents VEP as a tool for emission reduction strategies. A joint travel demand and emission model incorporates the VEP strategies proposed in the paper. The demand model incorporates individual travel behavior in determining the choice of destination, mode and route in response to VEP strategies set by the decision maker. The emissions model, MOVES, developed by EPA is integrated with the demand model for estimation of link based pollutants. Developing such models is complex and involves two major hurdles. First, the difficulty lies in obtaining a certain emission level to model individual driver responses to examine how the targets are achieved. Second, the complexity involved in developing multiple strategies in real world case studies for assisting federal, state and local agencies (or decision makers) to achieve emission reduction objectives. The proposed model addresses both hurdles. First, a robust model formulation is presented incorporating differential VEP strategies. Second, the model is applied to a multimodal transportation network in Montgomery County in Maryland, where the roadway infrastructure connects to the Washington-Baltimore region.

A Base-case depicting existing conditions and three models are proposed. The existing conditions are depicted in the Base-case, and emission level is estimated after the traffic assignment. The Base-case resulted in high levels of emissions and led to the development of three models. Model-1 represents a link based VEP suggesting links exceeding a predefined emission level will be penalized. Model-2 demonstrates a practical way of VEP implementation at a corridor level (set of contiguous links) as opposed to link level. Model-3 p more advanced way of analyzing VEP through variable demand, incorporating the willingness to travel component as opposed to the deterministic nature of traffic assignment models. It should be noted that the emission cap is predetermined by the decision maker and the assignment algorithm defines the links in the network to be subjected to VEP in a joint travel demand and emission model. While the Base-case illustrates the existing emission levels, three proposed models provide insightful results in emission reduction.

The proposed model has several dimensions of significant impact and contribution to practice. First, development of a functional joint travel demand and emission model to incorporate VEP strategies. Second, procedural development of three models that incorporate decision maker’s strategies in emission minimization to examine user’s response. Third,
application of the proposed methodology successfully to a real world case study to present emission estimates for different policy levels.

This paper provides a theoretically robust framework for emissions reduction strategies and further examines user behavior using a joint travel demand and emission model. The results show that through the implementation of VEP, critical emission reduction targets are achievable. While some strategies such as charging all links that exceed a predetermined cap may not be feasible; others that implement VEP along a single corridor show improvements in emissions with fewer resource requirements. The tools developed in this paper offer planners and decision makers a method to test the implications of various emissions policy objectives which is particularly important in the current era of environmental impact awareness. This research can be extended to incorporate uncertainty in travel demand and consequences to emission modeling.

ACKNOWLEDGEMENT

The authors are thankful to the Maryland State Highway Administration (SHA) for their continued support in the development of Maryland Statewide Transportation Model (MSTM). The tremendous effort by the staff at Parsons Brinkerhoff in the data collection and model development is also greatly acknowledged. The evolution of MSTM is a result of continued research for the last four years at the National Center for Smart Growth Research and Education, at University of Maryland. The research on emission reduction strategies are aimed at development of a real world model that can be used by the state, county and local agencies.

REFERENCE

Appendix-A: Joint Travel Demand and Emissions Model Algorithm Summary

**Step 0: Initialization.**

Calculate initial demand \( q_{ij}^{c(0)} \) and feasible flow pattern \( f_{ij}^{c(0)} \), based on free-flow travel time. Set \( n = 0 \).

**Step 1a: Update.**

Set
\[
T_a(x_a) = t_a \left( 1 + \alpha_a \left( \frac{x_a}{c_a} \right) \right) \forall a;
\]  

**Step 1b: Update with variable demand.**

Update demand with inverse demand function
\[
d_i^\phi(\sigma_{ij}) = d_i^\tau \exp \left( -\omega \ast u_i^{c(m)} \right) \forall i,j,\tau
\]  

where \( u_i^{c(m)} \) is the least cost path between O-D pairs i and j and \( \omega \) is a positive constant.

**Step 2: Direction Finding.**

Find the shortest path
\[
u_{ij}^{c(n)} = t_a(x_a) + \frac{\tau_a(x_a)}{y^c}
\]  

Perform all-or-nothing assignment based on updated travel times and obtain auxiliary flows \( f_{ij}^{c(m)} \).

**Step 3: Move Size.**

Line search for optimal step size, solving for \( a \):

\[
\text{Minimize } \sum_a \int_0^{x_a} \left( T_a(x_a, e_a) + \frac{\tau_a(x_a)}{y^c} + \frac{\varphi e_a(x_a)}{y^c} \right)
\]

**Step 4: Flow Update with emissions charge.**

Find \( f_a^{ct(m)}, f_a^{ce(m)}, f_a^{cn(m)} \) with:

\[
u_{ij}^{c(n)}(x_a, e_a, \cdot) = t_a(x_a, e_a) + \frac{\tau_a(x_a)}{y^c} + \frac{\varphi e_a(x_a)}{y^c}
\]  

**Step 5: Assignment convergence criterion.**

\[
K \leq \frac{\text{Abs} \left[ \sum_{a \in A} \left( f_a^{(n)} * f_a^{c(n)} \right) - \sum_{a \in A} \left( f_a^{(n-1)} * \delta_a^{c(n-1)} \right) \right]}{\sum_{a \in A} \left( f_a^{(n-1)} * \delta_a^{c(n-1)} \right)}
\]

where \( K \) is a dimensionless convergence criterion.

For Model-1 and Model-2 step 1(b) is skipped. For Model-3, all the steps are executed. If inequality holds, terminate assignment and go to step 1a. Otherwise, set \( n = n + 1 \) and go to step 1.