Slab and Deck Weight

To simplify computation of the slab weight, we use the full depth of the slab, from bottom of deck to top of slab. Although this approach overestimates the volume of concrete, it is conservative. For the unit weight of reinforced concrete, we use the weight of plain concrete plus 5 pcf. Because slabs on formed metal deck are usually lightly reinforced (sometimes welded wire mesh, rather than reinforcing bars, is used), adding 5 pcf for reinforcement may seem excessive, but the deck itself can weigh between 2 and 3 psf.

An alternative approach is to use the thickness of the slab above the deck plus half the height of the rib as the thickness of concrete in computing the weight of the slab. In practice, the combined weight of the slab and deck can usually be found in tables furnished by the deck manufacturer.

Example 9.9 Floor beams are to be used with the formed steel deck shown in Figure 9.16 and a reinforced concrete slab whose total thickness is 4.75 inches. The deck ribs are perpendicular to the beams. The span length is 30 feet, and the beams are spaced at 10 feet center-to-center. The structural steel is A992, and the concrete strength is
120 $f_c' = 4$ ksi. The slab and deck combination weighs 50 psf. The live load is ~~40~~ psf, and there is a partition load of 10 psf. No shoring is used, and there is a construction load of 20 psf.

a. Select a W-shape.
b. Design the shear connectors. live-load
c. Compute ~~Check~~ deflections. The maximum permissible ~~total~~ deflection is 1/~~240~~ of the span length. 360

Solution Compute the loads (other than the weight of the steel shape). Before the concrete cures,

Slab wt. = 50(10) = 500 lb/ft

Construction load = 20(10) = 200 lb/ft

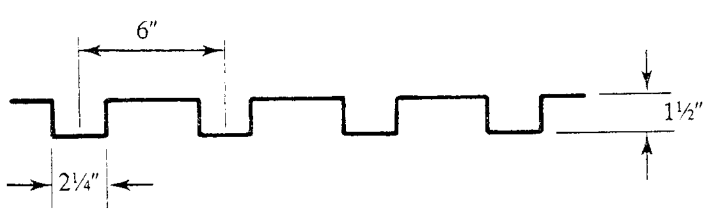
After the concrete cures,

Partition load = 10(10) = 100 lb/ft

Live load = ~~40~~(10) = ~~400~~ lb/ft

120 1200

FIGURE 9.16



LRFD Solution a. **Beam design**: Select a trial shape based on full composite behavior.

$$w_D = \text{slab wt.} = 500 \text{ lb/ft}$$

$$w_L = \text{live load} + \text{partition load} = \cancel{400}\ 1200 + 100 = \cancel{500}\ 1300 \text{ lb/ft}$$

1.300/2.68

$$w_u = 1.2w_D + 1.6w_L = 1.2(0.500) + 1.6(\cancel{0.500}) = \cancel{1.400} \text{ kips/ft}$$

$$M_u = \frac{1}{8}w_uL^2 = \frac{1}{8}(\cancel{1.400}\ 2.68)(30)^2 = \cancel{157.5}\ 301.5 \text{ ft-kips}$$

Assume that $d = 16$ in., $a/2 = 0.5$ in., and estimate the beam weight from Equation 9.4:

$$w = \frac{3.4M_u}{\phi_b F_y(d/2 + t - a/2)} = \frac{3.4(\cancel{157.5}\ 301.5 \times 12)}{0.90(50)(16/2 + 4.75 - 0.5)} = \cancel{11.7}\ 22.3 \text{ lb/ft}$$

Try a W16 × 26. Check the flexural strength before the concrete has cured.

$$w_u = 1.2w_D + 1.6w_L = 1.2(0.500 + 0.026) + 1.6(0.200) = 0.9512 \text{ kips/ft}$$

$$M_u = (1/8)(0.9512)(30)^2 = 107 \text{ ft-kips}$$

A W16 × 26 is compact for $F_y = 50$ ksi, and since the steel deck will provide adequate lateral support, the nominal strength, M_n, is equal to the plastic moment strength, M_p. From the Z_x table,

$$\phi_b M_p = 166 \text{ ft-kips} > 107 \text{ ft-kips} \qquad \text{(OK)}$$

After the concrete has cured, the total factored load to be resisted by the composite beam, adjusted for the weight of the steel shape, is

$$w_u = 1.2w_D + 1.6w_L = 1.2(0.500 + 0.026) + 1.6(\cancel{0.500}\ 1.300) = \cancel{1.431}\ 2.711 \text{ kips/ft}$$

$$M_u = \frac{1}{8}w_uL^2 = \frac{1}{8}(\cancel{1.431}\ 2.711)(30)^2 = \cancel{161}\ 305 \text{ ft-kips}$$

The effective slab width of the composite section will be the smaller of

$$\frac{\text{Span}}{4} = \frac{30(12)}{4} = 90 \text{ in.} \quad \text{or} \quad \text{Beam spacing} = 10(12) = 120 \text{ in.}$$

Use $b = 90$ in. For full composite action, the compressive force, C, in the concrete is the smaller of

$$A_sF_y = 7.68(50) = 384.0 \text{ kips}$$

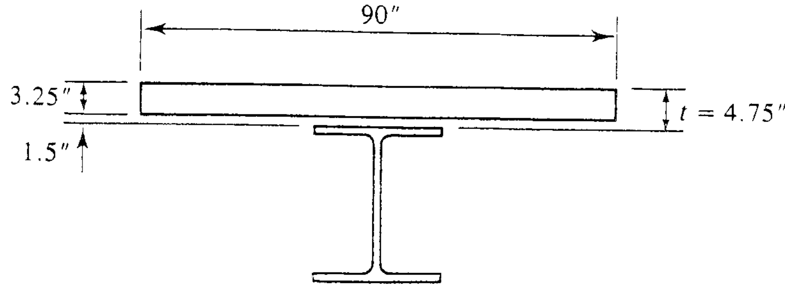
or

$$0.85f_c'A_c = 0.85(4)[90(4.75 - 1.5)] = 994.5 \text{ kips}$$

where only the concrete above the top of the deck has been accounted for in the second equation, as illustrated in Figure 9.17. With $C = 384.0$ kips, the depth of the compressive stress distribution in the concrete is

$$a = \frac{C}{0.85f_c'b} = \frac{384.0}{0.85(4)(90)} = 1.255 \text{ in.}$$

FIGURE 9.17



The moment arm of the internal resisting couple is

$$y = \frac{d}{2} + t - \frac{a}{2} = \frac{15.7}{2} + 4.75 - \frac{1.255}{2} = 11.97 \text{ in.}$$

and the design strength is

$$\phi_b M_n = \frac{0.90(384.0)(11.97)}{12}$$
$$= 345 \text{ ft-kips} > \cancel{161 \text{ ft}}\ 305\text{-kips} \qquad \text{(OK)}$$

Check the shear:

$$V_u = \frac{w_u L}{2} = \frac{\cancel{1.431}\ 2.711(30)}{2} = \cancel{21.5}\ 40.7 \text{ kips}$$

From the Z_x table,

$$\phi_v V_n = 106 \text{ kips} > \cancel{21.5}\ 40.7 \text{ kips} \qquad \text{(OK)}$$

Answer Use a W16 × 26.

b. **Shear connectors**: Because this beam has ~~a substantial~~ an excess of moment strength, it will benefit from partial composite behavior. We must first find the shear connector requirements for full composite behavior and then reduce the number of connectors. For the fully composite beam, $C = V' = 384.0$ kips.

Maximum stud diameter $= 2.5t_f = 2.5(0.345) = 0.8625$ in.

or $\frac{3}{4}$ in. (controls)

Try ¾-in. × 3-in. studs ($A_{sc} = 0.4418$ in.^{2}), one at each section.

For $f_c' = 4$ ksi, the modulus of elasticity of the concrete is

$$E_c = w_c^{1.5}\sqrt{f_c'} = 145^{1.5}\sqrt{4} = 3492 \text{ ksi}$$

From AISC Equation I3-3, the shear strength of one connector is

$$Q_n = 0.5A_{sc}\sqrt{f_c' E_c} \le R_g R_p A_{sc} F_u$$
$$= 0.5(0.4418)\sqrt{4(3492)} = 26.11 \text{ kips}$$
$$R_g R_p A_{sc} F_u = 1.0(0.6)(0.4418)(65)$$
$$= 17.23 \text{ kips} < 26.11 \text{ kips} \qquad \therefore \text{ use } Q_n = 17.23 \text{ kips}$$

The number of studs required between the end of the beam and midspan is

$$N_1 = \frac{V'}{Q_n} = \frac{384.0}{17.23} = 22.3$$

Use 23 for half the beam, or 46 total.

With one stud in each rib, the spacing is 6 inches, and the maximum number that can be accommodated is

$$\frac{30(12)}{6} = 60 > 46 \text{ total}$$

With one stud in every other rib, 30 will be furnished, which is fewer than what is required for full composite action. However, there is an excess of flexural strength, so partial composite action will probably be adequate.

Try 30 studs per beam, so that N_l provided = 30/2 = 15.

$$\Sigma Q_n = 15(17.23) = 258.5 \text{ kips} < 384.0 \text{ kips} \qquad \therefore C = V' = 258.5 \text{ kips}$$

Because C is smaller than A_sF_y, part of the steel section must be in compression, and the plastic neutral axis is in the steel section.

To analyze this case, we must first determine whether the PNA is in the top flange or in the web. This can be done as follows. If the PNA were at the bottom of the flange, the entire flange would be in compression, and the resultant compressive force, as illustrated in Figure 9.18, would be

$$P_{yf} = b_f t_f F_y = 5.50(0.345)(50) = 94.88 \text{ kips}$$

The net force to be transferred at the interface between the steel and the concrete would be

$$\begin{aligned} T - C_s = T - P_{yf} &= (A_sF_y - P_{yf}) - P_{yf} = 384.0 - 2(94.88) \\ &= 194.2 \text{ kips} \end{aligned}$$

This is less than the actual net tension force of 258.5 kips, so the top flange does not need to be in compression for its full thickness. This means that the PNA is in the flange.

FIGURE 9.18

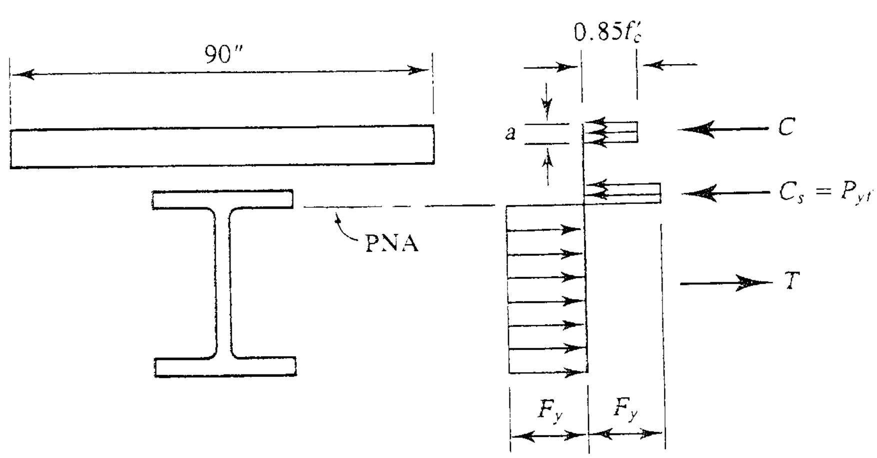
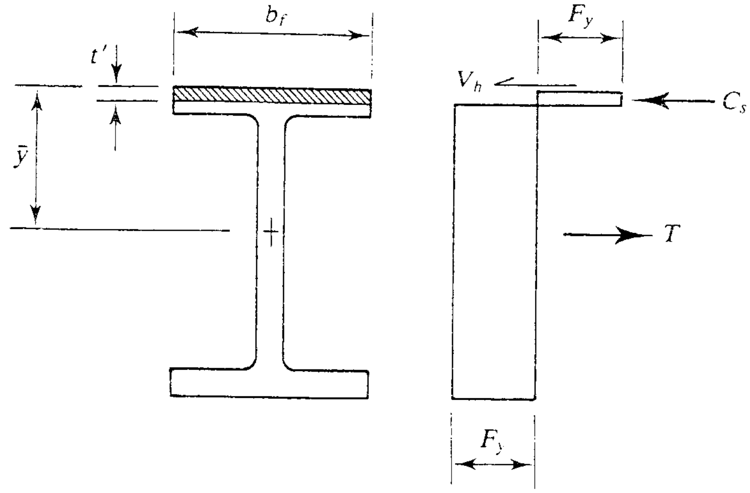


FIGURE 9.19



From Figure 9.19, the horizontal shear force to be transferred is

$$T - C_s = (A_sF_y - b_ft'F_y) - b_ft'F_y = V'$$

$$384.0 - 2[5.50t'(50)] = 258.5$$

Solving for the depth of compression in the flange, we obtain

$$t' = 0.2282 \text{ in.}$$

The tensile resultant force will act at the centroid of the area below the PNA. Before the moment strength can be computed, the location of this centroid must be determined. The calculations for $\bar{y}$, the distance from the top of the steel shape, are summarized in Table 9.5.

The depth of the compressive stress block in the concrete is

$$a = \frac{C}{0.85f_c'b} = \frac{258.5}{0.85(4)(90)} = 0.8448 \text{ in.}$$

Moment arm for the concrete compressive force is

$$\bar{y} + t - \frac{a}{2} = 9.362 + 4.75 - \frac{0.8448}{2} = 13.69 \text{ in.}$$

Moment arm for the compressive force in the steel is

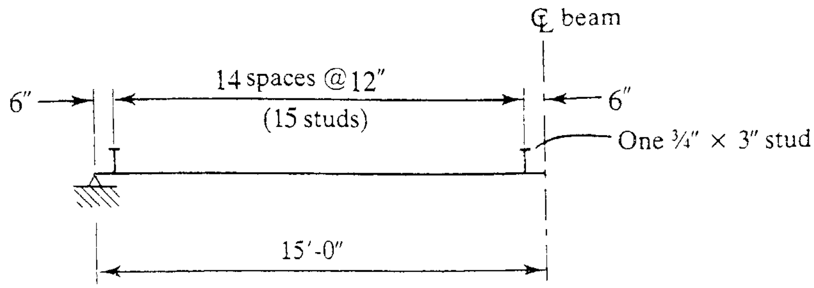
$$\bar{y} - \frac{t'}{2} = 9.362 - \frac{0.2282}{2} = 9.248 \text{ in.}$$

TABLE 9.5

Component	A	y	Ay
W16 × 26	7.68	15.7/2 = 7.85	60.29
Flange segment	−0.2282(5.50) = −1.255	0.2282/2 = 0.1141	−0.14
Sum	6.425		60.15

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = \frac{60.15}{6.425} = 9.362 \text{ in.}$$

FIGURE 9.20



Taking moments about the tensile force and using the notation of Figure 9.18, we obtain the nominal strength:

$$M_n = C(13.69) + C_s(9.248)$$
$$= 258.5(13.69) + [0.2282(5.50)(50)](9.248) = 4119 \text{ in.-kips} = 343.3 \text{ ft-kips}$$

The design strength is

305

$$\phi_b M_n = 0.90(343.3) = 309 \text{ ft-kips} > \cancel{161} \text{ ft-kips} \qquad \text{(OK)}$$

The deck will be attached to the beam flange at intervals of 12 inches, so no spot welds will be needed to resist uplift.

Answer Use the shear connectors shown in Figure 9.20.

c. **Deflections**: Before the concrete has cured,

$$w_D = w_{\text{slab}} + w_{\text{beam}} = 0.500 + 0.026 = 0.526 \text{ kips/ft}$$

$$\Delta_1 = \frac{5w_D L^4}{384EI_s} = \frac{5(0.526/12)(30 \times 12)^4}{384(29{,}000)(301)} = 1.098 \text{ in.}$$

The deflection caused by the construction load is

$$\Delta_2 = \frac{5w_{\text{const}} L^4}{384EI_s} = \frac{5(0.200/12)(30 \times 12)^4}{384(29{,}000)(301)} = 0.418 \text{ in.}$$

The total deflection before the concrete has cured is

$$\Delta_1 + \Delta_2 = 1.098 + 0.418 = 1.52 \text{ in.}$$

For deflections that occur after the concrete has cured, the moments of inertia of the transformed section will be needed. The modular ratio is

$$n = \frac{E_s}{E_c} = \frac{29{,}000}{3492} = 8.3 \quad \therefore \text{ use } n = 8$$

The effective width is

$$\frac{b}{n} = \frac{90}{8} = 11.25 \text{ in.}$$

Figure 9.21 shows the corresponding transformed section. The computations for the neutral axis location and the moment of inertia are summarized in Table 9.6.

FIGURE 9.21

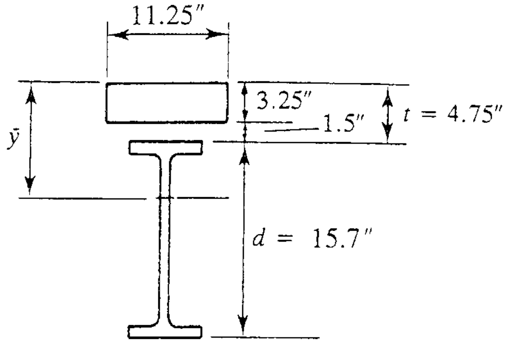


TABLE 9.6

Component	A	y	Ay	$\bar{I}$	d	$\bar{I} + Ad^2$
Concrete	36.56	1.625	59.41	32.18	1.906	165
W16 × 26	7.68	12.6	96.77	301	9.069	933
Sum	44.24		156.18			1098 in.^{4}

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = \frac{156.2}{44.24} = 3.531 \text{ in.}$$

Since partial composite action is being used, a reduced transformed moment of inertia must be used. From AISC Equation C-I3-3, this effective moment of inertia is

$$I_{\text{eff}} = I_s + \sqrt{\sum Q_n / C_f}\,(I_{tr} - I_s)$$
$$= 301 + \sqrt{258.5/384.0}\,(1098 - 301) = 954.9 \text{ in.}^4$$

The deflection caused by the live load is

insert →

$$\Delta_3 = \frac{5w_L L^4}{384EI_{\text{eff}}} = \frac{5(\overset{1.200}{\cancel{0.400}}/12)(30 \times 12)^4}{384(29{,}000)(954.9)} = \overset{0.7898}{\cancel{0.2633}} \text{ in.}$$

The deflection caused by the partition load is

$$\Delta_4 = \frac{5w_{\text{part}} L^4}{384EI_{\text{eff}}} = \frac{5(0.100/12)(30 \times 12)^4}{384(29{,}000)(954.9)} = 0.0658 \text{ in.}$$

The total deflection is

$$\Delta_1 + \Delta_3 + \Delta_4 = 1.098 + \overset{0.7898}{\cancel{0.2633}} + 0.0658 = \overset{1.95}{\cancel{1.43}} \text{ in.}$$

~~and~~

$$\cancel{\frac{L}{240} = \frac{30(12)}{240} = 1.50 \text{ in.} > 1.43 \text{ in.} \quad \text{(OK)}}$$

Answer The live-load deflection is satisfactory.

Insert for page 590:

The maximum permissible live-load deflection is

$$\frac{L}{360} = \frac{30(12)}{360} = 1 \text{ in.} > 0.7898 \text{ in.} \quad \text{(OK)}$$

~~Note that although the strength of this composite beam is far larger than what is needed, the deflection is very close to the limit.~~

ASD Solution a. **Beam design**: Select a trial shape based on full composite behavior.

1.300/1.800

$$w_D = \text{slab wt.} = 500 \text{ lb/ft}$$

1200 1300

$$w_L = \text{live load} + \text{partition load} = \cancel{400} + 100 = \cancel{500} \text{ lb/ft}$$

$$w_a = w_D + w_L = 0.500 + \cancel{0.500} = \cancel{1.000} \text{ kips/ft}$$

1.800 202.5

$$M_a = \frac{1}{8} w_a L^2 = \frac{1}{8}(\cancel{1.000})(30)^2 = \cancel{112.5} \text{ ft-kips}$$

Assume that d = 16 in., $a/2$ = 0.5 in., and estimate the beam weight from Equation 9.6:

202.5 22.53

$$w = \frac{3.4\Omega_b M_a}{F_y\left(\frac{d}{2} + t - \frac{a}{2}\right)} = \frac{3.4(1.67)(\cancel{112.5} \times 12)}{50\left(\frac{16}{2} + 4.75 - 0.5\right)} = \cancel{12.5} \text{ lb/ft}$$

Try a W16 × 26. Check the flexural strength before the concrete has cured.

$$w_a = w_{\text{slab}} + w_{\text{beam}} + w_{\text{const}} = 0.5000 + 0.026 + 0.200 = 0.7260 \text{ lb/ft}$$

$$M_a = \frac{1}{8} w_a L^2 = \frac{1}{8}(0.7260)(30)^2 = 81.7 \text{ ft-kips}$$

From the Z_x table,

$$\frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 110 \text{ ft-kips} > 81.7 \text{ ft-kips} \qquad \text{(OK)}$$

After the concrete cures and composite behavior has been achieved, the load and moment are

1.300 1.826

$$w_a = w_{\text{slab}} + w_{\text{beam}} + w_L = 0.500 + 0.026 + \cancel{0.500} = \cancel{1.026} \text{ kips/ft}$$

1.826 205

$$M_a = \frac{1}{8}(\cancel{1.026})(30)^2 = \cancel{115} \text{ ft-kips}$$

The effective slab width of the composite section is the smaller of

$$\frac{\text{Span}}{4} = \frac{30 \times 12}{4} = 90 \text{ in.} \qquad \text{or} \qquad \text{Beam spacing} = 10 \times 12 = 120 \text{ in.}$$

Use b = 90 in. For full composite behavior, the compressive force in the concrete at ultimate (equal to the horizontal shear at the interface between the concrete and steel) will be the smaller of

$$A_s F_y = 7.68(50) = 384.0 \text{ kips}$$

or

$$0.85 f'_c A_c = 0.85(4)[90(4.75 - 1.5)] = 994.5 \text{ kips}$$

Only the concrete above the top of the deck has been accounted for. With $C = 384.0$ kips, the depth of the compressive stress distribution in the concrete is

$$a = \frac{C}{0.85f'_c b} = \frac{384.0}{0.85(4)(90)} = 1.255 \text{ in.}$$

$$y = \frac{d}{2} + t - \frac{a}{2} = \frac{15.7}{2} + 4.75 - \frac{1.255}{2} = 11.97 \text{ in.}$$

and the allowable flexural strength is

$$\frac{M_n}{\Omega_b} = \frac{Cy}{\Omega_b} = \frac{384.0(11.97)}{1.67} = 2752 \text{ in.-kips} = 229 \text{ ft-kips} > \overset{205}{\cancel{115}} \text{ ft-kips} \quad \text{(OK)}$$

Check shear:

$$V_a = \frac{w_a L}{2} = \frac{\overset{1.826}{\cancel{1.026}}(30)}{2} = \overset{27.4}{\cancel{15.4}} \text{ kips}$$

From the Z_x table,

$$\frac{V_n}{\Omega_v} = 70.5 \text{ kips} > \overset{27.4}{\cancel{15.4}} \text{ kips} \qquad \text{(OK)}$$

Answer Use a W16 × 26.

b. **Shear connectors.** The design of shear connectors is the same for both LRFD and ASD. From the LRFD solution, for one ¾-inch × 3-inch stud every other rib,

$$M_n = 343.3 \text{ in.-kips}$$

The allowable moment strength is therefore

$$\frac{M_n}{\Omega_b} = \frac{343.3}{1.67} = 206 \text{ ft-kips} > \overset{205}{\cancel{115}} \text{ ft-kips} \qquad \text{(OK)}$$

Answer Use the shear connectors shown in Figure 9.20.

c. **Deflections**: The computation of deflections is the same for LRFD and ASD. See the LRFD solution. ■

9.8 TABLES FOR COMPOSITE BEAM ANALYSIS AND DESIGN

When the plastic neutral axis is within the steel section, computation of the flexural strength can be laborious. Formulas to expedite this computation have been developed (Hansell et al., 1978), but the tables presented in Part 3 of the *Manual* are more convenient. Three tables are presented: strengths of various combinations of shapes and slabs; tables of "lower-bound" moments of inertia; and a table of shear stud strength Q_n for various combinations of stud size, concrete strength, and deck geometry.