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Improved Velocity and Displacement Time Histories in Frequency Domain Spectral-Matching Procedures

by Ali Shahbazian and Shahram Pezeshk

Abstract Existing spectral-matching techniques in the frequency domain distort the displacement time history of the ground motions. In the time domain spectral-matching procedures, scale functions (wavelets) are additive and, with an appropriate scale functional form, extra displacement will not be imposed to the record. However, matching in the frequency domain with a multiplicative scale function applied to the Fourier spectrum requires a special attention in the matching process to have control on the displacement time history. This study shows that the velocity value at the end of the record is not affected by the Fourier amplitude spectrum scaling, but the displacement may linearly increase (or decrease) boundlessly. Two numerical solutions are proposed to solve the displacement drift problem in the frequency domain. Following the proposed frequency domain baseline correction procedure, one does not need to perform baseline correction in the time domain after completing the spectral matching. As a result, acceleration, velocity, and displacement time histories will remain fully compatible, and the boundary conditions of the velocity and displacement time histories will be preserved. The proposed procedure is not limited to spectral-matching methods and can be used with any general filtering process to retain the final displacement of the record. The possibility of applying a zero-padding technique to the spectral-matching filter is also discussed. It is shown that applying an appropriate window along with the zero-padding technique can lead to a reasonable displacement time history. The proposed procedures can be easily added to the existing or new frequency domain spectral-matching algorithms without significantly disrupting the spectral-matching process.

Introduction

Design acceleration response spectra are generally used by engineers for linear structural analysis of typical regular structures. The design spectrum for a given site is typically obtained from a uniform hazard spectrum (UHS). While the starting point for seismic design may be a UHS produced by probabilistic seismic hazard analysis, many researchers and practitioners are now arguing that the UHS itself is not an appropriate basis for design. It is preferable to disaggregate the hazard and then construct a scenario response spectrum or even a conditional mean spectrum.

Time history analysis may be required when nonlinear performance of a structure needs to be addressed. Instances that require time history analysis include very tall or long structures, complex buildings with extreme mass and/or geometric irregularities, structures with base isolation or supplementary damping devices, structures designed for high ductility demand, and particularly critical structures for which any damage has potentially far-reaching consequences in terms of safety. One way to obtain the needed time histories is to generate artificial ground motions to match the target design spectrum. Alternative approach would be to select a suite of real ground motions from analogous past seismic events recorded in similar site conditions (McGuire *et al.*, 2001, Bommer and Acevedo, 2004). However, there is not much chance of finding real ground motions that have response spectra that coincide with a desired response spectrum.

A single-factor scaling of a record can bring the response spectrum of the ground motion in some proximity of the design spectrum, but a complete match is achieved only by reforming the response spectrum of the ground motion. When a mean structural response is required, a suite of spectrum-compatible time histories results in significantly less response dispersion than scaled ground motions. For instance, a study by Watson-Lamprey and Abrahamson (2006) shows that spectrum-compatible records reduce the standard deviation by about a factor of 2 compared to scaled accelerograms. With a limited number of existing ground motions, spectral-matching procedures allow prediction of the seismic response with a few time history analyses in a specific level of confidence (Bazzurro and Luco, 2006).

Historically, there are two spectral-matching methods that have been developed to match the response spectrum of a real ground motion to a predefined target spectrum. One method is the ground motion modification in the time domain (e.g., RSPMatch2005; Hancock *et al.*, 2006) and the other is in the frequency domain (e.g., RASCAL; Silva and Lee, 1987). While in the time domain, the matching is carried out by some specific additive functions (wavelets) (Kaul, 1978; Lilhanand and Tseng, 1987, 1988; Hancock *et al.*, 2006), in the frequency domain, the Fourier amplitude spectrum (FAS) is multiplicatively scaled in each frequency with no change in the phase spectrum of the record (Gasparini and Vanmarcke, 1976, Silva and Lee, 1987, Bolt and Gregor, 1993).

Both the velocity and displacement time histories are typically altered by manipulating the accelerogram in the spectral-matching process. As a result, the natural form of the displacement time history may be seriously distorted. The displacement time history is implicitly involved in structural analysis when the input motion is an accelerogram, so manipulating the accelerogram may lead to unreasonable structural displacements and drifts (Naeim and Lew, 1995). Because the additive form of the adjustment function applied to the ground motion in the time domain procedures, it is easy to restrain velocity and displacement values at the end of the record if a compensated wavelet functional form is appropriately selected (Hancock *et al.*, 2006). This is simply because the integration operation has linear functionality. However, matching in the frequency domain with a multiplicative scale function applied to the FAS requires a special attention in the matching process to have control on the displacement time history.

In this study, we present two procedures in the frequency domain that appropriately manipulate velocity and displacement time histories such that a time domain baseline correction is no longer required. The two proposed techniques have a minor effect on the matching procedure and at the same time keep the displacement time history realistic. It will be shown that the velocity value at the end of the record is not affected due to the frequency scaling of FAS. The displacement corruption resulting from a frequency domain spectral matching always has a constant pattern that helps to establish a procedure to preserve the final displacement value of the ground motion. Furthermore, it will be shown that a simple zero-padding in the time domain before matching in the frequency domain, as an instance of a filter, is not enough to have meaningful displacement time histories after the traditional matching process. Zero-padding of the acceleration time history before filtering is the current technique to control the relevant displacement time history in which the acceleration time history at the beginning and at the end of the duration is padded with a sufficient length of zeros before the filtering process.

It should be noted that the spectral matching in the frequency domain can be considered as an acausal filter. In record processing, a filter is called causal when the filtered signal at any time t depends only on the input signal preceding t. In contrast, a filter for which the output signal also depends on the future input signal is called acausal. Furthermore, spectral matching in the frequency domain is a zero-phase filter. A zero-phase filter is a filter having zero spectral phase in all frequencies in the frequency domain. It can be shown that zero-phase filters are acausal. The proposed methods in our present study for displacement control can be applied in any kind of the filtering process where zero-padding is unable to completely remove distortion from the displacement time history.

Velocity and Displacement in the Spectral-Matching Process

The proposed procedures are discussed through two examples. The example recorded ground motions were carefully selected to closely fall in the same magnitude–distance category (bin). Both ground motions were selected for a 6 to 7 magnitude and less than a 10-kilometer source-to-site distance for a given site class D soil condition. The first selected ground motion is the north–south component of the El Centro station recorded during the Imperial Valley earthquake of 1940 (magnitude 6.95 and distance 6.09 km), and the second selected ground motion is the east–west component of the Newhall station recorded during the Northridge earthquake of 1994 (magnitude 6.69 and distance 5.92 km). Figure 1 shows the acceleration, velocity, and displacement time histories of these ground motions. These two ground acceleration records are used as input motions to match a target spectrum in the frequency domain. Although in general practice for the spectral matching, more than two records are randomly selected from an appropriate bin, for this study only two ground motions are selected from the bin such that they exhaustively illustrate the possible displacement issues occurring in the frequency domain spectral matching.

The target spectrum used in this study is simply determined by the Building Seismic Safety Council (BSSC) Specification (2004) for a site class D in California with $1.25g$ and $0.46g$ of spectral accelerations for short period (0.2 s) and long period (1.0 s), respectively. It should be noted that in practice, a target spectrum is usually a result of a comprehensive site-specific hazard analysis. However, to keep the target spectrum well defined, we use a design code spectrum. We followed the matching approach of the computer code RASCAL in which the FAS of the accelerogram in each frequency is iteratively scaled by the ratio of the target to response spectrum at the natural frequency of the structure until a degree of agreement between the target and response spectra is reached. Figure 2 exhibits the target and response spectra before and after the matching is performed.

The resulting acceleration, velocity, and displacement time histories, assuming zero initial velocity and displacement, are presented in Figure 3. Velocity and displacement time histories are derived by single and double integration of

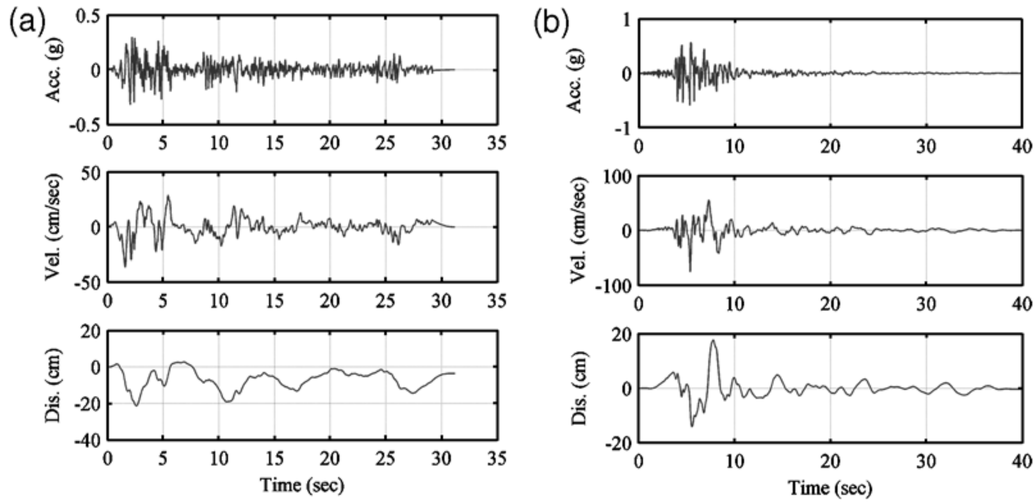

Figure 1. Acceleration, velocity, and displacement time histories prior to the spectral-matching process for (a) ground motion 1: Imperial Valley 1940, El Centro, north–south component; and (b) ground motion 2: Northridge 1994, Newhall, east–west component.

the accelerogram in the time domain, respectively. As is apparent in Figure 3, the displacement time history is distorted and goes toward infinity. This always happens regardless of what numerical integration method is used. Thus, a baseline correction in the time domain is necessary to have a physically meaningful displacement time history. Making a baseline correction in the time domain by fitting a smooth curve is not always easy, and using different functional forms may result in different time histories. In practice, deciding on the right functional form is critical, and some degree of uncertainty is always involved.

A final nonzero displacement value is not the same as having a permanent displacement due to a near source effect. Permanent displacement is a time-invariant step at the tail of the displacement time history and has a bounded value.

It is important to note that a baseline correction in the time domain will lead to a modification in the spectral phase. This is an issue if it is important to keep the phase spectrum natural in the frequency space. Moreover, by simply correcting the displacement time history and not correcting the velocity and acceleration, compatibility of the displacement with the acceleration and velocity motions will be lost (Malhotra, 2001; Boore and Bommer, 2005). It should be noted that for structural analysis purposes, the structural model is only exposed to the accelerogram as an input excitation motion; and, whenever a drift (structural relative displacement) in any point of the structure is required, it will be implicitly determined by double integration of the accelerogram. Consequently, the acceleration ground motion should carry the record's characteristics by itself and contain all the features of the record required for structural analysis. On the other hand, the velocity time history will lose boundary conditions (i.e., has nonzero values at the boundaries) if we apply time domain baseline correction by adding a constant baseline offset to the velocity to correct the linear trend of the displacement time history.

In the following, the origin of the error in the displacement time history imposed by the spectral matching in

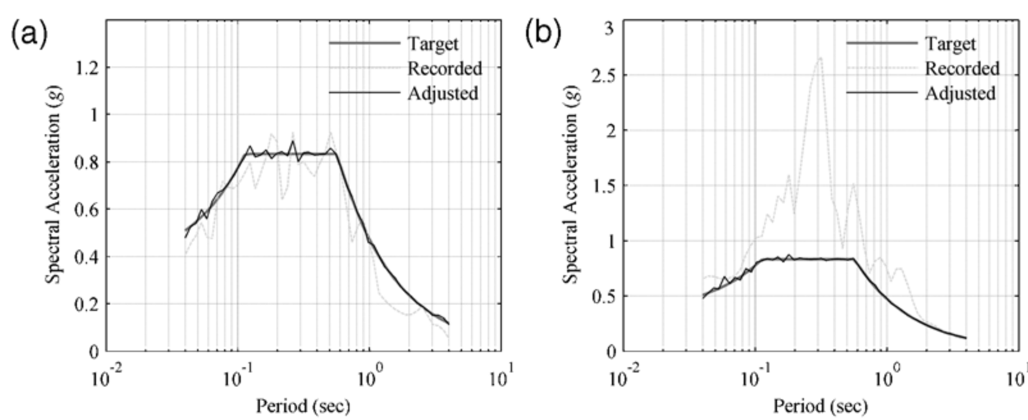

Figure 2. Spectral acceleration (5% damping) of the ground motion before (recorded) and after (adjusted) matching for (a) ground motion 1 and (b) ground motion 2. Target spectrum is chosen as a typical acceleration spectrum from BSSC (2004). Matching is conducted by the RASCAL spectral-matching algorithm (Silva and Lee 1987) with overall root mean square error of 3% (in logarithmic scale) over 0.04–4.0 s structural period range. Only the final result of the matching is illustrated.

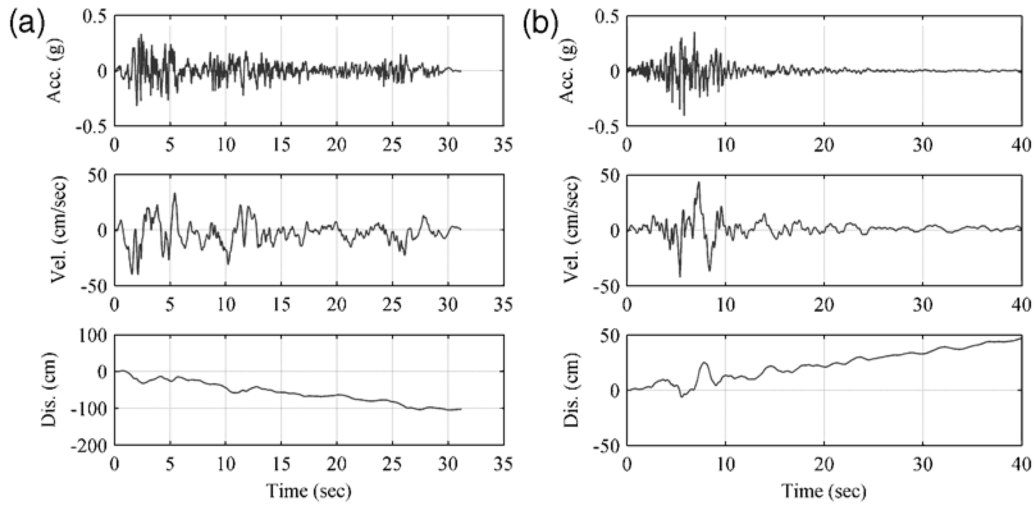

Figure 3. Acceleration, velocity, and displacement time histories after spectral matching for (a) ground motion 1 and (b) ground motion 2.

the frequency domain is quantitatively explored. Then, a frequency-domain baseline correction is proposed to control displacement and to maintain accuracy. It should be noted that spectral matching in the frequency domain is a simple application of zero-phase filtering of the record and exactly the same procedure can be followed for any other kind of zero-phase filters. Generally speaking, any function that multiplicatively changes a record in the frequency domain is a filter. When the filter includes no imaginary part in the frequency domain, it is called a zero-phase filter. For general filters, including filters affecting the spectral phase, the proposed procedure can be easily modified and extended for other applications.

Let us assume that the frequency domain counterpart of the recorded acceleration time history $a(t)$ is determined by

$$A(f) = \int_0^{T_D} a(t)e^{-i2\pi ft}dt, \tag{1}$$

where T_D is the duration of the accelerogram, f is the frequency, and $i = \sqrt{-1}$. Equation (1) transforms time series $a(t)$ to the frequency domain. Note that in the Fourier transform, if $a(t)$ has a finite length, then it is assumed that $a(t)$ is a T_D-periodic function, and, as a result, $A(f)$ will be a discrete function (i.e., it has values only at specific frequencies). Conversely, for a discrete time series $a(t)$, $A(f)$ will be a periodic function. However, it is more beneficial here to consider a continuous integration form of equation (1), rather than using the discrete form of summation, when deriving velocity and displacement from acceleration $a(t)$.

Scaling the FAS by an amplitude scale function, $S(f)$, the new acceleration record will be

$$a_{\text{new}}(t) = \int_{-f_{\text{nyq}}}^{f_{\text{nyq}}} S(f)A(f)e^{i2\pi ft}df, \tag{2}$$

where f_{nyq} is the Nyquist frequency.

The change in the acceleration, $\delta a(t) = a_{\text{new}}(t) - a(t)$, for such a scaling is

$$\delta a(t) = \int_{-f_{\text{nyq}}}^{f_{\text{nyq}}} [S(f) - 1]A(f)e^{i2\pi ft}df, \tag{3}$$

where $S(f)$ can be appropriately defined to make the original record's response spectrum match the target spectrum. One such scaling could be the ratio of the target to the response spectrum at the corresponding structural frequency (e.g., as in the RASCAL code), but generally it can be any scaling function as long as it satisfies the condition of taking only nonnegative real values throughout the entire frequency range. This is a general characteristic of zero-phase filters of any form. However, as will be discussed next, there is no need to place any assumption on $S(f)$, and the method can be easily extended to causal filters.

As a result, the velocity and displacement time histories will be adjusted as follows:

$$\delta v(t) = \int_{-f_{\text{nyq}}}^{f_{\text{nyq}}} \frac{S(f) - 1}{i2\pi f} A(f)[e^{i2\pi ft} - 1]df \tag{4}$$

and

$$\delta d(t) = \int_{-f_{\text{nyq}}}^{f_{\text{nyq}}} \frac{S(f) - 1}{-(2\pi f)^2} A(f)[e^{i2\pi ft} - 1]df - t\int_{-f_{\text{nyq}}}^{f_{\text{nyq}}} \frac{S(f) - 1}{i2\pi f} A(f)df, \tag{5}$$

where $\delta v(t)$ and $\delta d(t)$ are the velocity and displacement increments due to amplitude scaling in the frequency domain, respectively. Inspecting equation (4) and equation (5) reveals that the amount of the velocity and displacement adjustments at any time can be interpreted as the superimposition of all adjustment components for all frequencies such that

$$\delta v(t) = \sum_{f>0}^{f_{\text{nyq}}} v_{adj}(t, f) \tag{6}$$

and

$$\delta d(t) = \sum_{f>0}^{f_{\text{nyq}}} d_{adj}(t, f), \tag{7}$$

where $v_{adj}(t, f)$ and $d_{adj}(t, f)$ are quantities added to the velocity and displacement time histories at time t when the FAS is scaled by $S(f)$ in the frequency space. Note that in equation (6) and equation (7), frequency f varies from zero to f_{nyq} in the summation ($A(f)$ is a Hermitian function). Assuming that the velocity of the original record at the beginning and end of the motion is zero (which it is supposed to be), $A(f = 0)$ (which is equal to $\int_0^{T_D} a(t)dt$ from equation 1) is zero, and there is no concern for a velocity and displacement change enforced by the frequency zero.

Considering $\delta v(t)$ in equation (4), we have only a series of sinusoidal signals added to the velocity in the time domain due to the matching, whereas this is not the case for the displacement time history. The second term of equation (5) implies that, for each frequency, an extra linearly increasing (or decreasing) drift in displacement is added to the time history in addition to the sinusoidal signal. The result of such a linear offset in each frequency will be an overall linear trend in the time history that leads to a displacement time history distortion, as can be seen in Figure 3.

In particular, the velocity and displacement errors at the end of the record, T_D, can be determined. With a finite duration of T_D, $A(f)$ will be exclusively given at discrete frequencies $f_j = j/T_D$ where $j = 0, 1, 2, 3, \ldots$. Following equation (4) and equation (5), it is apparent that the sinusoidal parts of $\delta v(t)$ and $\delta d(t)$ diminish to zero at T_D. This means that the scaling of the FAS leads to no change in the final value of the velocity time history provided that the mean has been already removed from the original ground acceleration (if needed) before entering into the spectral matching. However, $\delta d(T_D)$ has a remainder that can be derived as follows:

$$\delta d(T_D) = \sum_{f>0}^{f_{\text{nyq}}} [S(f) - 1]p(f), \tag{8}$$

where

$$p(f) = -\frac{\text{Im}[A(f)]}{\pi f} = -2\text{Re}[V(f)] \tag{9}$$

in which $V(f)$ is the velocity in the frequency domain, and Re and Im indicate the real and imaginary parts of a complex function, respectively. Equation (8) explains that the total amount of displacement error at the end of the time history solely depends on the imaginary part of the acceleration ground motion. In other words, only the real part of the velocity ground motion in the frequency domain is responsible for the distortion in the displacement time history.

The variation of $p(f)$ for our example ground motions is illustrated in Figure 4, which illustrates that low frequencies are generally involved more in causing displacement drift. However, low frequencies in a very narrow band close to zero have less contribution. Overall, it follows the ground motion frequency variation in the frequency domain. Besides, depending on the amplitude scale function, $S(f)$, the resulting displacement error from each frequency may vary even more. Therefore, these imply that the complete removal of the extra displacement from an adjusted ground motion may not be achieved by a simple filtering of the time history after matching process is over. In the following, a procedure is proposed to solve the problem.

Frequency Domain Baseline Correction Procedure

Because displacement corruption is in the form of a straight line, to keep the displacement time history under control, it is only required to control drift at the end of the record. Comparing Figure 3 with Figure 4 illustrates that the displacement error imposed by just a few frequencies can create the same total displacement drift in the displacement time history resulting from all frequencies. For example, the drift of the

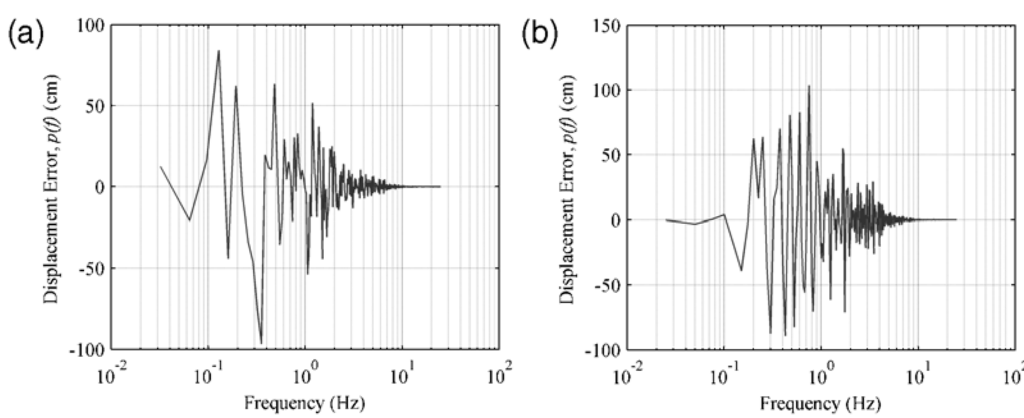

Figure 4. Variation of the final displacement quantity at each frequency for (a) ground motion 1 and (b) ground motion 2. Integration throughout all the frequencies will give the total final displacement of the record in the time domain. This also can be interpreted as the real part of the velocity in the frequency domain multiplied by a factor of −2. Note that $p|_{f=0} = -2d(T_D)$. $p|_{f=0}$ is not shown in this figure because of the logarithmic scale of the frequency axis.

final displacement in Figure 3b is approximately 50 cm, and just one frequency point in Figure 4b creates twice this displacement drift (100 cm) by itself. This means that the displacement time history is very sensitive to FAS scaling in the frequency domain, and any minor change can produce a significant error in the displacement. This sensitivity is much higher for low frequencies (except for a very narrow band close to frequency zero) than for high frequencies.

A procedure to solve this problem must be chosen carefully to have less interference with the main purpose of FAS filtering and at the same time remove undesired drift. Let us assume that the FAS scale function $S(f)$ has already been determined in the matching process without any concerns about the displacement drift. By setting $\delta d(T_D) = 0$ in equation (8), a modified FAS scale function, $S^*(f)$, must meet the condition

$$\sum_{f>0}^{f_{\text{nyq}}} S^*(f)p(f) = \sum_{f>0}^{f_{\text{nyq}}} p(f). \qquad (10)$$

The right side of equation (10) is equal to the final displacement of the record, which also could have been inferred by noticing that

$$\int_{-f_{\text{nyq}}}^{f_{\text{nyq}}} V(f)df = v|_{t=0} = 0 \qquad (11)$$

and

$$V|_{f=0} = \int_0^{T_D} v(t)dt = d(T_D). \qquad (12)$$

Unless the record has a permanent displacement, the right side of equation (10) will be zero. Many techniques can be found to define $S^*(f)$ in terms of $S(f)$. One simple and efficient definition for $S^*(f)$ is

$$S^*(f) = S(f) \times \begin{cases} q & p(f) \geq 0 \\ 1/q & p(f) < 0 \end{cases}, \qquad (13)$$

in which q is a positive scalar that corrects $S(f)$ to satisfy $\delta d(T_D) = 0$. Such a definition relies on the high sensitivity of the displacement time history to FAS filtering; therefore, one can make sure that nearly all features of the $S(f)$ filter will be preserved by using a q factor near unity. By applying the q factor, positive displacement drifts of the frequencies are canceled out by negative displacements of the other frequencies and a balance between them is created. All frequencies are involved in the correction model of equation (13), and those generating larger displacement deviations at T_D receive a proportionally larger compensation. Applying it to all frequencies ensures that only a minor modification of $S(f)$ is made and that it will not obstruct the principal purpose of the spectral-matching process (or the filtering process in general).

Substituting equation (13) into equation (10), the q modification factor can be conveniently obtained from the following quadratic equation (there is only one positive solution):

$$\left(\sum_{f>0|p(f)\geq 0}^{f_{\text{nyq}}} S(f)p(f)\right)q^2 - \left(\sum_{f>0}^{f_{\text{nyq}}} p(f)\right)q + \left(\sum_{f>0|p(f)<0}^{f_{\text{nyq}}} S(f)p(f)\right) = 0. \qquad (14)$$

The same concept that has been used here for zero-phase filters governs for causal filters as well. In this case, the general filter $S(f)$ (not a FAS scaling function anymore) is not a scalar value; and $S(f)p(f)$ in equations (10) and (14) should be substituted by $-\text{Im}[S(f)A(f)]/\pi f$ (or equivalently $-2\text{Re}[S(f)V(f)]$).

Numerical Results

Figure 5 displays the FAS scale function, $S(f)$, and its modified value $S^*(f)$ in the spectral-matching process. Using $S(f)$ results in uncorrected displacement error while $S^*(f)$ leads to a corrected displacement error at T_D, and no extra displacement is imposed during the matching.

As can be observed from Figure 5, there is only a minor difference between the scale functions, but it is enough to keep the original displacement time history at the end of the record unaltered. The q modification factor is applied at each iteration and is normally very close to unity. Note that each frequency will receive only either q or q^{-1} throughout the iteration process; therefore, the total modification for each individual frequency will be either $\prod q$ or $\prod q^{-1}$.

Variation of the q modification factor for two ground motions is shown in Figure 6. The two ground motions are matched in the frequency domain (Fig. 2) with overall root mean square error of 3%. Typical values of q may rarely go beyond the 0.99–1.01 range during the spectral-matching process by the RASCAL code, which means no noticeable modification in the FAS scale function is made.

Application of the proposed approach requires a careful elaboration of $\delta v(t)$ and $\delta d(t)$ in equation (4) and equation (5). Displacement offsets from individual frequencies determined by equation (5) are in the form of continuous integration and may slightly differ from the discrete result if the displacement is to be obtained from numerical double integration. The exact consistency is achieved when the same numerical technique is used for both the double integration of $a(t)$ to find $d(t)$ and the double integration of $\delta a(t)$ (given in equation 3) to find $\delta d(t)$ (given in equation 5).

Figure 7 shows acceleration, velocity, and displacement time histories of the matched ground motions in which the frequency domain baseline correction procedure is applied in spectral matching of the ground motions. The displacement time history looks realistic, and time domain baseline correction is no longer required. Comparing Figure 7 with Figure 3, no significant change in the time characteristics

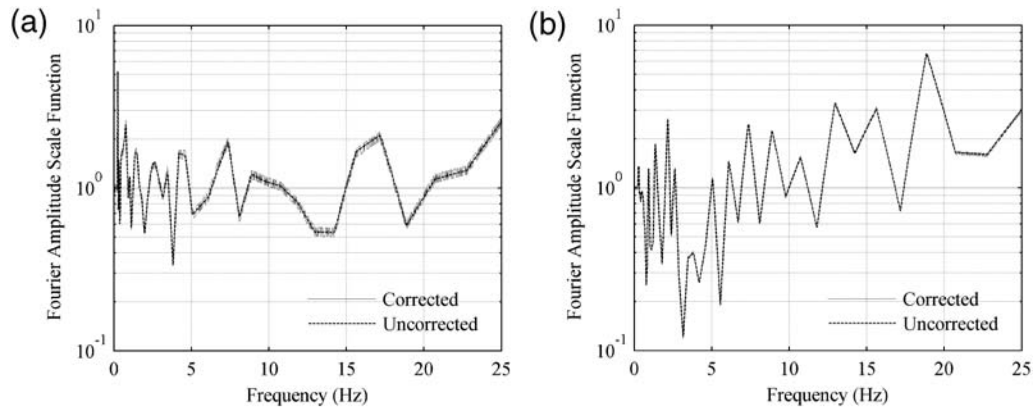

Figure 5. FAS scale functions $S(f)$ (uncorrected) and $S^*(f)$ (corrected) that make the ground-motion response spectrum match the target spectrum for (a) ground motion 1 and (b) ground motion 2. FAS scale function is corrected in each frequency according to equation (13) to remove the extra final displacement.

of the acceleration or velocity motions is detected due to applying the q modification factor.

Frequency Domain Baseline Correction versus Zero Padding

Depending on the application of the ground motions, various filters are widely used to reduce the ground motions' noise from different sources (e.g., environment, instrument, digitization process, and so on). Sources of the record contamination and methods to filter out the record noises are broadly discussed in the literature (e.g., Boore and Bommer 2005). Generally, a sufficient length of series of zero-points (zero-pads) is added before and after the accelerogram prior to filtering to prevent undesired distortion of the displacement time history (Boore 2005). Typically, the displacement time history of the filtered accelerogram without zero-padding

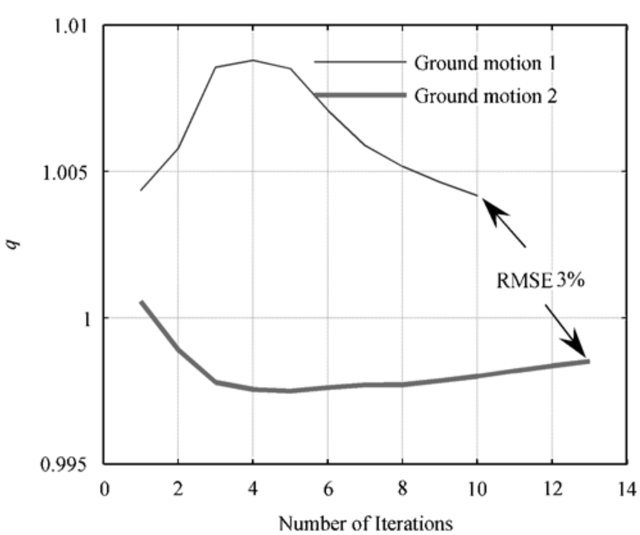

Figure 6. Variation of the q modification factor during the spectral-matching process for ground motions 1 and 2. For both ground motions, the spectral matching is performed until 3% root mean square error (RMSE) is met.

looks like the displacement record shown in Figure 3. A time domain baseline correction is necessary to have a meaningful displacement time history. Regardless of the type of the filter used, the time histories in Figure 3 illustrate the general feature of the filtering with no zero-padding.

For example, the SMSIM computer program (Boore 1996) uses the zero-padding technique in a stochastic model for ground-motion simulation. The zero-padding is needed because the ground motion is generated in the frequency domain as a product (a filtering process) of a randomly generated time history and a smoothed Fourier spectrum.

To better understand how zero-padding can help eliminate distortion of the displacement time history, the inverse transform of the FAS scale function (shown in Fig. 5) into the time domain is illustrated in Figure 8. This is, in fact, the impulse response of the spectral-matching filter that we use in the frequency domain.

Mathematically, the matching procedure in the frequency domain is the same as if one convolves the impulse response given in Figure 8 with the original accelerogram in the time domain. Because of its zero-phase property, the impulse response of the spectral-matching filter is symmetric and decays from both ends toward its midpoint. Generally, zero-phase filters take low values close to zero at the middle of the impulse response, as observed in Figure 8. Because the impulse response is originally calculated in the frequency domain at the same frequency points of the record, it has the same length as the record and has taken values at the same time points as the record.

Accelerograms end up with circular convolution when filtered in the frequency domain rather than regular convolution in the time domain and as a result produce a residual at the end of the record. Assuming the middle part of the impulse response is approximately zero, zero-padding of the accelerogram at the beginning and end of the record can remove the circular effect of the convolution, provided that the impulse response of the filter itself has grown in length accordingly as a result of zero-padding of the record.

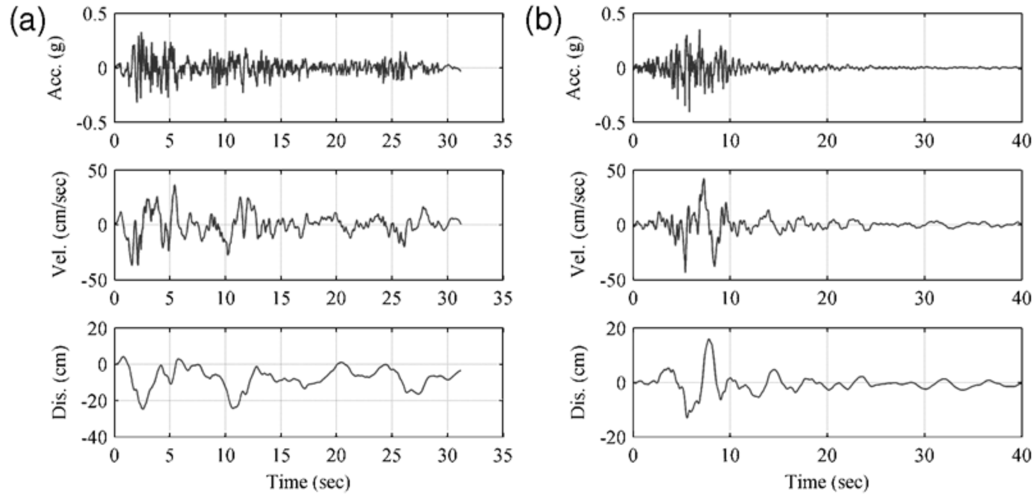

Figure 7. Acceleration, velocity, and displacement time histories after spectral matching when the q modification factor is applied during the matching process for (a) ground motion 1 and (b) ground motion 2.

As exhibited in Figure 9, zero-padding of the record mainly stretches the impulse response of the filter at the middle with no significant influence at both ends. Therefore, zero-padding of the record may significantly reduce the circular convolution residual.

Unfortunately, this is not the case for all kinds of filters. As can be seen from Figure 9, while we have small amplitudes in the central portion of the impulse response for the spectral matching of ground motion 2, the data points at the center of the spectral-matching impulse response of ground motion 1 are bounded within a finite interval that oscillates around zero. Thus, zero-padding of the record is not able to eliminate the residual, no matter what the length of the zero-padding is.

The effect of the zero-padding length of the accelerograms for the two example ground motions is compared in Figure 10. In each case, the ratios of the drift for the zero-padded record to the drift of the record with no zero-padding is presented. Drifts are determined at the end of the displacement time history after spectral matching is performed. As expected, ground motion 2 performs to some extent better than ground motion 1 with increasing length of zero-pad. However, neither of them can totally remove the drift. Increasing the length of the pad may even result in more displacement drift in this particular type of filter. Success of the zero-padding technique for drift removal depends on how small the amplitude of the impulse response of the zero-phase filter is in the central section.

The same approach is valid for causal filters. The impulse response of such filters only has significant values at the beginning and therefore a zero-pad will be needed at the end and not the beginning of the record.

It should be noted that the impulse response of filters widely used in record processing, such as zero-phase Butterworth filter, reaches to quite negligible values in the middle section (or shortly after the beginning of their impulse responses when they are used in causal form), and therefore zero-padding works well for them. The proposed frequency domain baseline correction can be a substitute where zero-padding is not able to totally correct the displacement drift. Zero-padding is not a general solution for some filters such as the traditional spectral-matching filter.

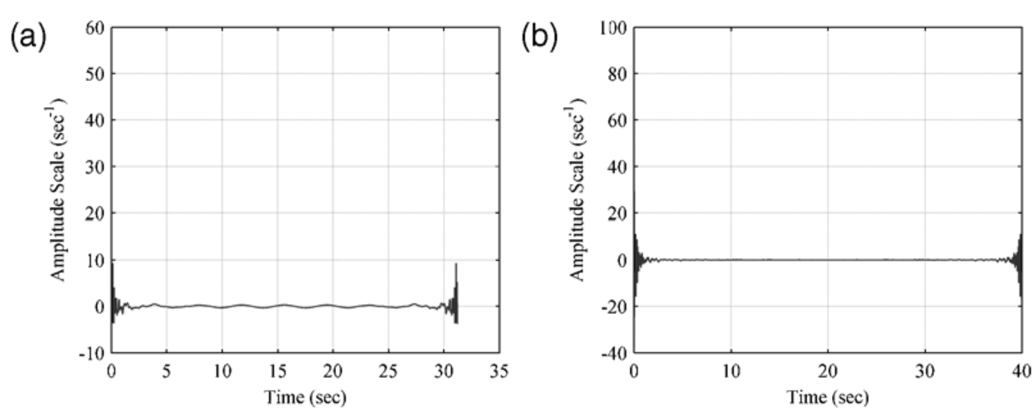

Figure 8. Impulse response of the FAS scale function in the spectral-matching process for (a) ground motion 1 and (b) ground motion 2.

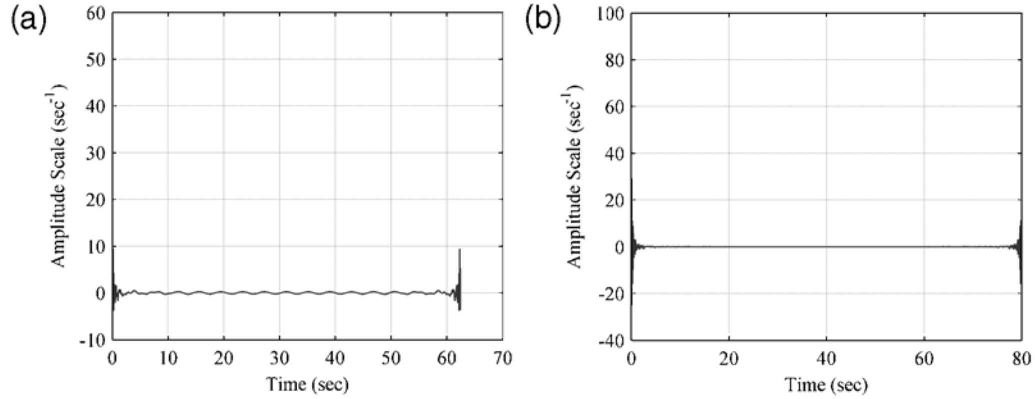

Figure 9. Impulse response of the FAS scale function in the spectral-matching process for (a) ground motion 1 and (b) ground motion 2. Accelerograms were zero-padded with half of the record length at the beginning and end of the record.

Windowing of the Spectral-Matching Impulse Response: An Alternative Procedure

To make the zero-padding technique applicable to spectral matching, the original time history is zero-padded at both ends with sufficient length of zero-padding. The impulse response of the matching filter as shown in Figure 9 should also have zero values at the middle in a length equal to the original time history record. Time windowing of the impulse response function can be used to ensure that we will have zero-value amplitude in the middle of the impulse response such that the matching process is not disrupted. Equivalently, time windowing can be performed in the frequency domain by convolution. In this procedure, a window is selected to smoothly taper to zero to reduce undesired frequency domain side effects on FAS scaling. Besides, the window must not add a phase angle to the matching filter, which has only positive scalar value.

The Hanning window is selected as a basis to construct an appropriate window. First, only the second half of the Hanning window with a length equal to the length of zero-padding in one end is selected and shifted to time zero; it is then extended with zeros to the length of the original acceleration ground motion plus another length of zero-padding. The phase spectrum for this window would be nonzero. Applying this window to the impulse response of the spectral-matching filter (a zero-phase function) will result in phase angle alteration of the record in the matching process. Furthermore, we only need to window the middle part of the impulse response of the matching filter and retain both ends. Hence, the new window is constructed by convolution with the similar but reversed window and then normalized to unity at time zero. This procedure will produce a zero-phase window (Oppenheim and Schafer 1999). The constructed window is illustrated in Figure 11.

As observed from Figure 11, the window has a smooth behavior in both the time and frequency domains with no ripple in the high frequencies. Because it obtains a nonzero value only at the ends with the same length of zero-padding, the circular effect of convolution will be totally removed. As a result, zero final displacement of the record will be preserved in the matching process. A sufficient length of the zero-padding will ensure that a significant part of the matching impulse response still remains after windowing and spectral matching converges to the target. Zero-padding length (window) must be large enough to allow the principal part of the impulse response of the matching filter to remain at the beginning and at the end. Otherwise, there will be no convergence to the desired level of accuracy with more iteration. The adequacy of the zero-padding length depends on the record, target spectrum, and degree of matching and typically varies from a few seconds to half of the original record's length.

The matched record without zero-padding sometimes has a noiselike oscillation at the end and exhibits behavior analogous to the beginning of the record due to the periodicity of the convolution. Another advantage of

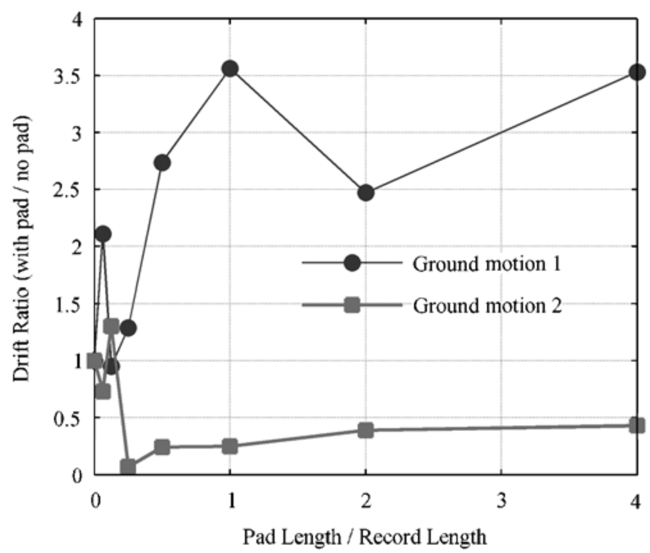

Figure 10. Variation of the displacement drift at the end of the time history with the different length of zero-padding in front and back of the record prior to the spectral-matching process. Pad lengths at both ends of the record are equal.

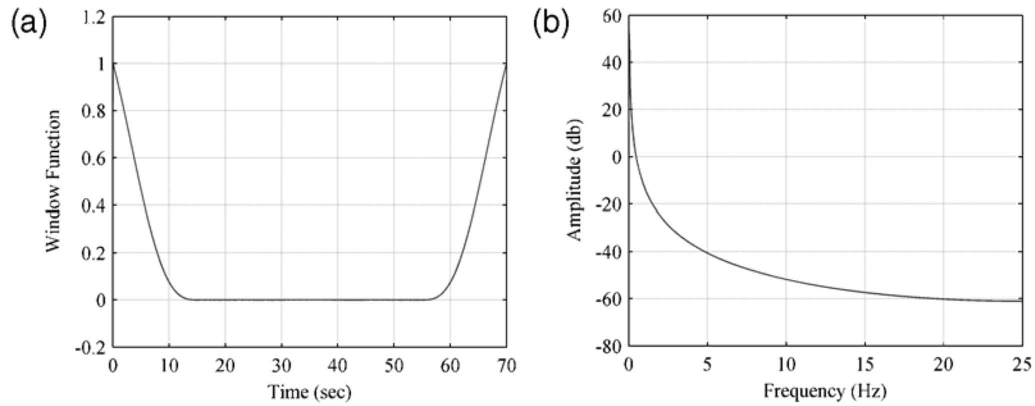

Figure 11. Zero-phase window function proposed for the impulse response of spectral-matching filter. (a) time variation and (b) Fourier amplitude. The window has the same length of zero-pad (15 s in this case) which is added to both ends of the record. The original record considered for this example has a length of 40 s.

the zero-padding is removal of this behavior and giving a realistic appearance to the matched record.

Figure 12 shows the acceleration, velocity, and displacement time histories of the matched ground motions in which the proposed Window-Pad technique is applied in spectral matching of the ground motions. Note that different zero-padding lengths were used in each ground motion case. Similar to the frequency domain baseline correction procedure, the displacement time history looks realistic, and the time domain baseline correction is no longer required. Furthermore, as observed from this figure, a slight undesired ripple seen at the end of the record in Figure 7 is removed by using the Window-Pad procedure. Despite the frequency domain baseline correction procedure, no numerical integration is involved; and it is insensitive to applied numerical integration.

Discussions and Conclusions

Two numerical techniques were proposed in this study that can be reliably used to resolve the displacement time history corruption problem arising during the spectral-matching process in the frequency domain. The displacement time history distortion due to the spectral-matching manipulation of the Fourier amplitude spectrum is eliminated directly in the frequency domain by the proposed procedures. Because the proposed procedures work in the same space where spectral matching is performed, the ground-motion spectral phase remains unchanged and the acceleration, velocity, and displacement time histories are compatible.

The proposed frequency domain baseline correction procedure to prevent the displacement distortion is in fact a filter. This filter takes advantage of the high sensitivity of the displacement time history to FAS scaling and modifies the spectral scaling filter to accurately remove displacement offset. The proposed method can work for any general multiplicative scaling of FAS (a general filter) regardless of the scaling function used or how it has been determined. If a permanent displacement occurs at the tail of the time history, it will be preserved after matching is performed. The Window-Pad technique, as an alternative procedure to

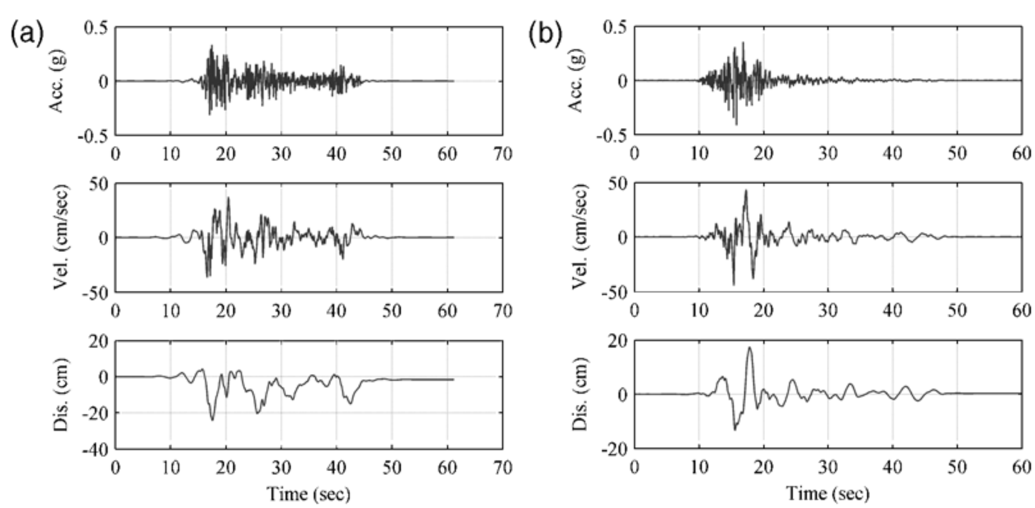

Figure 12. Acceleration, velocity, and displacement time histories after spectral matching when Window-Pad technique is applied during the matching process for (a) ground motion 1 with 15 s zero-padding and (b) ground motion 2 with 10 s zero-padding.

prevent displacement distortion in the frequency domain spectral matching, may alter the final nonzero displacement of the record, whereas the frequency domain baseline correction procedure can be a reliable method to solve the problem and retain the residual displacement. The proposed frequency domain baseline correction procedure can be easily added to the existing or any multiplicative frequency domain spectral-matching algorithms without significant disruption in the spectral-matching process.

Application of the proposed frequency domain baseline correction technique is not limited to spectral-matching methods. Any manipulation of the record may result in displacement corruption in the time domain. Such a distortion could be a result of filtering of the record for any reason in the frequency domain. The proposed procedure can be combined with both causal and acausal filters. Because the method involves all the frequencies in preserving the final displacement, it keeps the postfiltering shape of the displacement time history from time zero to the end of the record, except it makes the time history reach its original final displacement by a linear trend shift. Thus, it is more ideal for filters that are not intensely changing low frequencies to control the time characteristics of the displacement time history.

In the proposed Window-Pad procedure, the matched record will have a very smooth beginning and tail (this is an advantage over the frequency domain baseline correction technique). If no residual displacement originally occurred in the record, no final displacement will be left in the matched record. However, in the case of a record with permanent displacement, it may be modified slightly. The amount of this modification reduces with the higher sampling rate of time history. Applying this procedure creates a correlation among adjacent frequencies in the FAS scale function, which is another advantage of the Window-Pad procedure over the frequency domain baseline correction procedure.

Data and Resources

All data used in this paper came from the published sources listed in the references.

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