An Equivalent Point-Source Stochastic Simulation of the NGA-West2 Ground-Motion Prediction Equations

by Arash Zandieh, Shahram Pezeshk, and Kenneth W. Campbell

Abstract In this study, we use a genetic algorithm to invert horizontal ground-motion intensity measures (GMIMs) predicted from the empirical Next Generation Attenuation-West2 (NGA-West2) ground-motion prediction equations (GMPEs) to estimate a consistent and correlated set of seismological parameters to use with an equivalent point-source stochastic model. The GMIMs are peak ground acceleration and pseudospectral acceleration evaluated over a wide range of magnitudes, distances, and frequencies. The inversion is performed for $M_{3.5}^{8.0}$, $R_{RUP}=1$ to 300 km, $T=0.01$ to 10 s, and National Earthquake Hazard Reduction Program (NEHRP) B/C site conditions. Seismological parameters are obtained as a function of earthquake magnitude. The near-source geometric spreading was modeled as both magnitude- and frequency dependent to fit the empirical predictions. The agreement between the model and empirical predictions over all magnitudes and distances evaluated in this study is generally within 10%, with some local exceptions. The near-source geometric spreading is consistent with a distance decay of $R^{-0.8}$ to $R^{-1.3}$ at frequencies of $f \leq 1$ Hz for $M$ ranging from 3.5 to 8. Near 5 Hz, the distance decay is expressed as $R^{-1.17}$, on average at short distances. At larger frequencies, the near-source distance decay varies from $R^{-1.0}$ to $R^{-1.25}$. This stochastic model can be used for any application that requires a frequency-domain representation of the NGA-West2 GMPEs.

Electronic Supplement: Plots showing comparisons of predicted response spectra from the median Next Generation Attenuation-West2 (NGA-West2) ground-motion prediction equations (GMPEs) with the predicted response spectra for different magnitudes, distances, and frequencies, and residuals versus frequency between the predicted response spectra from the median NGA-West2 GMPEs and the predicted response spectra for different magnitudes, distances, and frequencies.

Introduction

The stochastic method, especially that based on a point-source model (Boore, 2003, 2005), is one of the methods used to simulate ground-motion intensity measures (GMIMs) in regions where recorded strong ground motions are sparse. Recent examples of such ground-motion models (GMMs) that were developed for central and eastern North America (CENA) as part of the Next Generation Attenuation-East (NGA-East) Project (Pacific Earthquake Engineering Research Center [PEER], 2015) are given in Boore (2015), Darragh et al. (2015), Hollenbeck et al. (2015), and Yenier and Atkinson (2015a,c). Previous point-source stochastic GMMs, as well as stochastic models developed using the finite-source stochastic method (Motazedian and Atkinson, 2005), used in the 2014 U.S. National Seismic Hazard Model (NSHM), are summarized in Petersen et al. (2014) and Rezaeian et al. (2014). The stochastic method is also used to simulate GMIMs in the hybrid empirical method (HEM) of Campbell (2003) that maps empirical estimates of GMIMs in a region with abundant strong ground motion data (the host region) to a region that lacks such data (the target region) using stochastic GMIM estimates in each region. A recent example of a ground-motion prediction equation (GMPE) developed using the HEM approach for the NGA-East Project is given by Pezeshk et al. (2015). An HEM approach that uses hybrid broadband simulations is given by Shahjouei and Pezeshk (2015). Previous HEM models used in the 2014 NSHM are summarized by Petersen et al. (2014) and Rezaeian et al. (2014). Campbell (2014) gives a comprehensive history and comparison of GMPEs that have been developed using the HEM approach.
If there is a comprehensive database of recordings in a given region, seismological parameters for the stochastic model can be estimated through an inversion process using the observed ground-motion data directly or using the empirical GMMs developed using the observed data. Scherbaum et al. (2006) used a genetic algorithm (GA) to determine seismological parameters for sets of stochastic models that matched empirically derived GMMs. These investigators observed that the overall quality of fit and the resulting model parameters strongly depend on the particular choice of the distance metric used for the stochastic model. They suggested the use of the hypocentral-distance metric with the stochastic simulations, which they found provided the lowest misfit for most of the empirical GMMs they used. However, an equivalent point-source metric has been found to do better for moderate-to-large earthquakes (Atkinson and Silva, 2000; Boore, 2009; Boore and Thompson, 2014, 2015; Yenier and Atkinson, 2014, 2015).

The NGA-West2 Project (Bozorgnia et al., 2014) made it possible to use a GA inversion approach to estimate stochastic seismological parameters from NGA-West2 GMPEs by providing a comprehensive database of peak and response-spectral GMIMs for small-to-moderate earthquakes in California and for large-magnitude global events (Ancheta et al., 2014) that was used to develop GMPEs appropriate for California, western North America (WNA), and other shallow-crustal active tectonic regions (Abrahamson et al., 2014; Boore et al., 2014; Campbell and Bozorgnia, 2014; Chiou and Youngs, 2014; Idriss, 2014). The purpose of this study is to estimate the seismological parameters to use with a stochastic point-source model appropriate for WNA and California from GMIMs predicted from the recently published set of empirical NGA-West2 GMPEs. We use a GA to perform the inversion of the GMPEs for moment magnitudes (M) ranging from 3.5 to 8.0 and closest distances to the fault-rupture surface (Rrup) ranging from 1 to 300 km. The inversion is performed for individual magnitudes ranging from 3.5 to 8.0 in increments of 0.5 magnitude units. The dependency of obtained seismological parameters on magnitude is investigated in this study.

We believe that calibrating an equivalent point-source model to GMPE predictions rather than from the recordings themselves provides a seismological model that is consistent with empirical models that are developed and calibrated using the expertise of ground-motion modelers.

The derived seismological parameters can be used to support point-source stochastic and HEM applications that use the NGA-West2 GMPEs to derive GMMs for CENA and other regions of the world where a sufficient number of strong-motion recordings from moderate-to-large earthquakes are not available, as was done by Pezeshk et al. (2015), who used a preliminary version of the inverted stochastic model developed in this study.

Methodology

Our goal is to find the set of seismological parameters to use with a simple point-source stochastic model that minimizes the misfit between the stochastically predicted GMIMs, which we will refer to as the predicted values, and the predicted GMIMs from the host GMPEs, which we will refer to as the observed values. Both predicted and observed values are compared for a specified reference-rock site condition, which we will describe later. For each magnitude, the misfit between the predicted and observed value is defined by the following residual:

\[ \text{res}_{ij} = \log(S_{ij}) - \log(G_{ij}), \]

in which \( S_{ij} \) is the observed value and \( G_{ij} \) is the predicted value of the GMIM for distance \( i \) and spectral period \( j \).

In this study, we use the GA inversion methodology of Holland (1975) and Goldberg (1989) to find the best combination of the seismological parameters for the stochastic model. This approach was first applied to GMPEs by Scherbaum et al. (2006). The GA focuses on a population of variables that are created randomly in the range defined by a set of constraints. The variables are grouped into sets, each of which is called a string, and are composed of a series of characters that defines a possible solution to the model. The performance of the variables, as described by an objective function defined later, and the set of constraints is represented by the fitness of each string. A mathematical expression, called a fitness function, calculates a value for a solution of the objective function. The best-fit solution gets the higher value, and the ones that violate the objective function and the constraints are penalized. Therefore, similar to what Darwin (1859) proposed happens in nature, the best-fit solutions will survive and get the chance to be a parent of the next generation of model solutions. In a crossover procedure, two selected parents reproduce the next generation. The crossover procedure first divides the selected parent strings into segments, and then some of the segments of a parent string are exchanged with corresponding segments of another parent string. This mutation operation guarantees diversity in the generated populations. This is done by switching 0–1 or vice versa in a randomly selected bit in the selected binary string to create a mutated string. Mutation prevents a fixed model of solutions from being transferred to the next generation (Holland, 1975; Goldberg, 1989).

The GA tries different combinations of the variables and finds the best solution that minimizes the sum of the squared residuals defined in equation (1) over all magnitudes, distances, and spectral periods (T) of interest. In this study, the inversion is performed separately for individual magnitudes. The performance of the inversion and the dependency of obtained seismological parameters on magnitude are investigated in this study. The inversion is performed for individual \( M \) ranging from 3.5 to 8.0 in 0.5 intervals (10 values of magnitude). The objective function that is minimized by the GA inversion is given by the following equation:
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Objective function = \sum_{i}^{N_i} \sum_{j}^{N_j} \text{res}_{ij}^2, \quad (2)

in which \(N_i\) is the number of \(R_{\text{RUP}}\) values, and \(N_j\) is the number of periods used in the inversion. For this study, the inversion is performed for 25 periods (\(N_i = 25\)) ranging from 0.01 to 10 s, uniformly distributed in log space, and for 30 values of \(R_{\text{RUP}}\) (\(N_j = 30\)) ranging from 1 to 300 km, uniformly distributed in log space.

Ground-Motion Prediction Equations

As discussed previously, we used the five GMPEs developed as part of the PEER NGA-West2 Project (Bozorgnia et al., 2014) to derive the empirical GMIM estimates in the WNA and California host region. These GMPEs are referred to as ASK14 (Abrahamson et al., 2014), BSSA14 (Boore et al., 2014), CB14 (Campbell and Bozorgnia, 2014), CY14 (Chiu and Youngs, 2014), and I14 (Idriss, 2014). These GMPEs used a vastly expanded NGA-West2 database (Ancheta et al., 2014) that included over 20,000 recordings from shallow-crustal earthquakes in California (\(M < 5.5\)) and in other similar active tectonic regions throughout the world (\(M \geq 5.5\)). We used the weighted geometric mean of the RotD50 (Boore, 2010) average horizontal GMIM predictions from the five GMPEs. We assigned the same weights to these models that were used to evaluate the NGA-West2 GMPEs for the 2014 NSHM (Petersen et al., 2014; Rezaeian et al., 2014). The weights were distributed evenly between four of the GMPEs, with I14 receiving one-half the weight of the other models. The geometric mean of the GMIMs was used, rather than individual estimates, to provide a more stable inversion and to be consistent with how the predictions would likely be used in HEM and other applications.

All of the NGA-West2 GMPEs that included regional site-response and anelastic attenuation terms were evaluated for the California region and for National Earthquake Hazard Reduction Program (NEHRP) B/C site conditions defined as a reference rock with \(V_{S30} = 760\) m/s, in which \(V_{S30}\) is the time-averaged shear-wave velocity in the top 30 m of a site profile (Building Seismic Safety Council [BSSC], 2009). To evaluate the NGA-West2 GMPEs, we used the strike-slip style of faulting on a vertical (90°-dipping) fault plane. We used a fault-rupture dip of 90° to evaluate those GMPEs that had dip as a predictor variable, which eliminated any hanging-wall effects from impacting the simple stochastic model. If hanging-wall effects are desired, we suggest that they are added to the final GMMs using, for example, the hanging-wall terms in the NGA-West2 GMPEs. ASK14 and CY14 include the depth to the top of the rupture surface \(Z_{\text{TOR}}\) as one of the predictor variables in their source-depth term. For each of these models, the default value of \(Z_{\text{TOR}}\) recommended by the developers for a future California earthquake was used. CB14 uses the hypocentral depth \(R_{\text{HYP}}\) to define source-depth effects. For this model, the default value of \(R_{\text{HYP}}\) recommended by CB14 based on the default value of \(Z_{\text{TOR}}\) from CY14 appropriate for California earthquakes was used. ASK14, BSSA14, and CY14 use \(Z_{L0}\), the depth to the 1.0 km/s shear-wave velocity (\(V_S\)) horizon beneath the site, to model sediment-depth and basin effects. CB14 uses \(Z_{2.5}\), the depth to the \(V_S = 2.5\) km/s horizon beneath the site, to model these effects. For these GMPEs, the default values of \(Z_{L0}\) and \(Z_{2.5}\) recommended by the developers for a California site with \(V_{S30} = 760\) m/s were used.

Except for BSSA14, which uses \(R_B\) (Joyner–Boore distance), the closest distance to the surface projection of the fault-rupture surface, as the distance metric the other GMPEs use \(R_{\text{RUP}}\) as the distance metric. Because the proposed model in this study is based on \(R_{\text{RUP}}\), we converted \(R_B\) to \(R_{\text{RUP}}\) for evaluating the BSSA14 model, using the relationships \(R_{\text{RUP}} = \sqrt{R_B^2 - Z_{\text{TOR}}^2}\), appropriate for a vertical fault. CY14 was evaluated with no directivity effects. If directivity effects are desired, we suggest that they be added to the final GMMs using, for example, the directivity term in CY14 or other terms developed as part of the NGA-West2 Project (Spudich et al., 2014).

Stochastic Ground-Motion Simulations

In the stochastic method, the ground-motion acceleration time series is modeled as filtered Gaussian white noise modulated by a deterministic envelope function defined by a specified set of seismological parameters (Boore, 2003, 2005). The filter parameters are determined by either matching the properties of an empirically defined spectrum of strong ground motion with theoretical spectral shapes or using reliable physical characteristics of the earthquake source and propagation media (Hanks and McGuire, 1981; Boore, 1983, 2003).

In the point-source model, the total Fourier amplitude spectrum (PAS) of a horizontal GMIM due to shear-wave propagation can be modeled by the following equation:

\[Y(M_0, R, f) = E(M_0, f)P(R, f)S(f)I(f)\]  \quad (3)

(after Boore, 2003), in which \(M_0\) is the seismic moment (dyn·cm), \(R\) is the hypocentral distance (km), \(f\) is the frequency (Hz), \(E(M_0, f)\) is the source term, \(P(R, f)\) is the path term, \(S(f)\) is the site term, and \(I(f)\) is the ground or instrumental response term. These terms are briefly described below. The reader is referred to Boore (2003, 2005) for a more comprehensive discussion of the point-source stochastic model.

Source Term

The source term \(E(M_0, f)\) in equation (3) is modeled with the Brune (1970, 1971) \(\omega^2\) point-source spectrum. Brune’s model is a single corner-frequency point-source spectrum in which the stress drop or stress parameter \(\Delta\sigma\) controls the spectral shape at high frequencies, and seismic moment \(M_0\) controls the spectral shape at low frequencies.
For this study, we use the shear-wave velocity and density of the crust in the vicinity of the earthquake source of $\beta_s = 3.5 \text{ km/s}$ and $\rho_s = 2.72 \text{ g/cc}$, respectively, based on the shear-wave velocity model derived in Boore (2016) for a generic NEHRP B/C site profile in WNA. As mentioned previously, the inversion is performed for individual magnitudes. Therefore, the stress parameter $\Delta \sigma$ is obtained as a function of magnitude in this study.

### Path Term

The path term $P(R, f)$ in equation (3) is traditionally separated into two components, commonly referred to as geometric attenuation (or spreading) and anelastic attenuation. The geometric attenuation term $Z(R)$ models the amplitude decay of the seismic wave due to the expanding surface area of the wavefront as it propagates away from the source. Anelastic attenuation, quantified by the quality factor $Q(f)$, models the amplitude decay of the seismic wave due to intrinsic (material) damping and scattering and is usually (but not always) found to be frequency dependent.

The quality factor is represented by a frequency-dependent relationship given by the following equation:

$$Q(f) = Q_0 f^n$$

(Boore, 2003), in which $Q(f)$ typically increases with frequency (i.e., $n > 0$). For this study, the functional form given in equation (4) is used for the inversion.

The geometrical attenuation or spreading term is defined by a piecewise function given by the following equation:

$$Z(R) = \begin{cases} R^{b_1} & R \leq R_1 \\ Z(R_1)(R/R_1)^{b_2} & R_1 < R \leq R_2 \\ \vdots & \vdots \\ Z(R_{n-1})(R/R_{n-1})^{b_n} & R > R_n \end{cases}$$

(after Boore, 2003), in which the values of $b_i$ represent the rates of decay (spreading coefficients) over the specified distance ranges. In this study, we used a trilinear functional form ($n = 3$) for modeling $Z(R)$. For the trilinear model, the slope parameters $b_1$, $b_2$, and $b_3$ are determined in the inversion, and the distance parameters $R_1$ and $R_2$ are set equal to the hinge points in the trilinear WNA path-duration model of Boore and Thompson (2014, 2015), resulting in transition distances of $R_1 = 45 \text{ km}$ and $R_2 = 125 \text{ km}$. The same transition distances are used for the geometric-attenuation and path-duration models because the rate of decay of seismic waves should correlate with the duration of these waves (D. Boore, personal comm., 2015), and it is important that the set of seismological parameters for a given region be internally consistent and not based on different studies with different assumptions. For the stable continental region of CENA, the value of $b_1$ is typically assumed to be associated with the decay of $Lg$ waves at regional distances and usually is set to a fixed value of $-0.5$ (e.g., Street et al., 1975). This is not necessarily the case for active crustal regions (ACRs) typical of the WNA. Therefore, in this study the coefficient $b_3$ is obtained by the inversion process consistent with coefficients $b_1$ and $b_2$ to capture the distance decay predicted by NGA-West2 GMPEs, to the extent that is possible by the inversion.

In this study, we investigated a frequency-dependent geometrical spreading term. The performance of the inversion improved considerably by considering the geometrical attenuation to be frequency dependent. For this purpose, the near-source geometrical spreading coefficient $b_1$ is allowed to vary by frequency in the inversion. However, coefficients $b_2$ and $b_3$ are not varied by frequency to reduce the trade-off between parameters.

To mimic the ground-motion saturation effects of source depth and finite faulting, we use the equivalent point-source distance metric $R_{PS}$ originally defined by Atkinson and Silva (2000) in place of hypocentral distance in the point-source stochastic simulations. This equivalent point-source distance is defined by the following equation:

$$R_{PS} = \sqrt{R_{RUP}^2 + h(M)^2},$$

in which $h(M)$ is the effective-depth term in kilometers, also referred to as the finite-fault factor by Boore and Thompson (2014). $h(M)$ is typically defined by the expression $\log h(M) = a + bM$, such as in Atkinson and Silva (2000) and Yenier and Atkinson (2014, 2015a,b,c). In this study, the effective-depth parameter $h$ is obtained directly from the inversion.

### Site Term

The site term $S(f)$ in equation (3) is defined as the product of a site amplification term $Amp(f)$ and a site-attenuation or diminution term $D(f)$.

In this study, the NGA-West2 GMPEs are evaluated for a reference NEHRP B/C site condition of $V_{530} = 760 \text{ m/s}$ and the default values of $Z_{1,0}$ and $Z_{2,5}$ associated with this value of $V_{530}$, to be consistent with their use in the 2014 NSHM (Petersen et al., 2014; Rezaeian et al., 2014) and with the recommendations of the NGA-West developers.

The site amplification factors implied by the NGA-West2 GMPEs are too complicated to be estimated in the inversion, due to their large number of degrees of freedom and complications caused by the trade-off of these site factors with other parameters in the seismological model. Instead, we use the generic NEHRP B/C shear-wave velocity profile in WNA and other ACRs that was developed by Boore (2016) as an update to the WNA generic-rock velocity profile of Boore and Joyner (1997) to represent the generic site amplification of a site with $V_{530} = 760 \text{ m/s}$. Campbell and Boore (2016) determined this approach to be the most appropriate of five proposed WNA NEHRP B/C crustal profiles used by various investigators to represent the site
profiles inherent in the NGA-West2 GMPEs. Site amplifications given in Boore (2016) were calculated from the square-root impedance method (Boore, 2013) using a generic NEHRP B/C $V_s$ profile and an updated relationship between $V_s$ and the density adopted by Boore and Thompson (2015). These site amplification factors are listed in Table 1. The amplification factors given in Table 1 do not include the shallow attenuation effects discussed below.

Anderson and Hough (1984) proposed a high-frequency FAS decay parameter $\kappa$ (kappa) that describes the observed high-frequency shape of the acceleration spectrum beyond the corner frequency that is independent of the stress parameter. After removing the effects of $Q(f)$, the remaining high-frequency decay is attributed to the effect of attenuation (kappa) in the seismic waves as they travel through the crustal profile beneath the site (e.g., Campbell, 2009). The corresponding site-attenuation term is given by the following equation:

$$D(f) = \exp(-\pi \kappa_0 f)$$  \hspace{1cm} (7)\hspace{1cm} (Boore, 2003), in which $\kappa_0$ (s) is the shallow attenuation parameter usually attributed to the crustal profile beneath the site. It includes the attenuation from both wave scattering and intrinsic (material) damping, or what Campbell (2009) refers to as the effective quality factor $Q_{ef}$. It can be calculated in a variety of ways, depending on the size of the earthquake and the available recordings (Ktenidou et al., 2014). We use $\kappa_0$ to define site attenuation because of its common use in engineering seismology (Campbell, 2009; Ktenidou et al., 2014).

For this study, we used the $\kappa_0$ model that was derived by Zandieh et al. (2016) from the high-frequency shape of the NGA-West2 GMPEs. They used the inverse random-vibration theory approach described by Al Atik et al. (2014) to calculate FAS from predicted values of response-spectral acceleration for all of the NGA-West2 GMPEs. They used these spectra to estimate $\kappa_0$, using the spectral-decay method of Anderson and Hough (1984). NGA-West2 GMPEs were evaluated for an NEHRP B/C site condition and for default estimates of depth to the top of rupture, hypocentral depth, and sediment (basin) depth consistent with this study. They derived estimates of $\kappa_0$ for magnitudes ranging from 3.5 to 8.0 and distances ranging from 5 to 20 km and used a mixed-effects model to derive equations for $\kappa_0$ as a function of magnitude. One of the main goals of the Zandieh et al. (2016) study was to develop a $\kappa_0$ model that could be used in inversions to develop stochastic models that are intended to mimic the predictions from the NGA-West2 GMPEs. Therefore, we used the $\kappa_0$ model developed by Zandieh et al. (2016) to avoid its trade-off with other high-frequency seismological parameters, such as $\text{Amp}(f)$ and $\Delta\sigma$, in the inversion. For this study, the $\kappa_0$ model derived by Zandieh et al. (2016) is adopted. Zandieh et al. (2016) is the model most representative of the average trend in the values of $\kappa_0$ with magnitude, if the weighted geometric mean of all five of the NGA-West2 GMPEs is used to define the GMIMs, with weights that are the same as used by Petersen et al. (2014). The $\kappa_0$ model 2 in Zandieh et al. (2016) is given by the following equation:

$$\kappa_0 = \begin{cases} 0.03367; & M \geq 4.4377 \\ 0.03367 + 0.00773(M-4.4377); & 4.4377 < M < 5.8794 \\ 0.04481; & M \geq 5.8794 \end{cases}$$  \hspace{1cm} (8)

**GMIM Response Term**

Up until now, the terms in the seismological model have been defined in terms of FAS, whereas the NGA-West2 GMPEs predict 5% damped pseudoabsolute acceleration response spectra (PSA). To invert the predicted values of PSA for a set of seismological parameters defined in terms of FAS, we must have a means of converting between the two types of GMIMs. This is done with the GMIM response term $I(f)$ in equation (3). This term can be evaluated either in the time domain using simulation or in the frequency domain using random-vibration theory (RVT). In this study, we use the RVT approach built into the point-source stochastic simulation program STRATA (Kottke and Rathje, 2008), which is consistent with the SMSIM program of Boore (2005).

The RVT method requires an estimate of the total response duration of the GMIM of interest (Boore, 2003). This

<table>
<thead>
<tr>
<th>Frequency $f$ (Hz)</th>
<th>Amplification $\text{Amp}(f)*$</th>
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<tbody>
<tr>
<td>0.010</td>
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<td>80.000</td>
<td>3.96</td>
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</table>

*Based on the generic NEHRP B/C $V_s$ site profile of Boore (2016).
is calculated as the sum of the source duration $T_S$ and the path duration $T_P$. The source duration for the Brune single corner-frequency source model is typically defined as the inverse of the corner frequency, or $T_S = 1/f_0$ (e.g., Boore, 2003). Path duration is more complex. Boore and Thompson (2014) used the NGA-West2 database to derive a new distance-dependent relationship for $T_P$ that they consider to be appropriate for ACRs such as WNA and California. We used their new path-duration term because of its consistency with the database used to develop the NGA-West2 GMPEs (Ancheta et al., 2014). Boore and Thompson (2014) defined the distance dependence of the path duration in terms of $R_{RUP}$, but Boore and Thompson (2015) noted that it should be defined in terms of $R_{PS}$, to be consistent with the use of this latter distance metric in the stochastic simulations. However, after reevaluation, Boore and Thompson (2015) suggested that no adjustment was needed to the value of duration given in Boore and Thompson (2014) if $R_{PS}$ instead of $R_{RUP}$ is used as the distance metric. Nevertheless, we decided to convert the value of $R_{RUP}$ to $R_{PS}$ using equation (6) for use in the stochastic simulation, to be consistent with the use of $R_{PS}$ as the equivalent point-source distance metric in the inversion. The resulting path durations are given in Table 2. They are constrained in the inversions because of their strong correlation with other seismological parameters in the model.

<table>
<thead>
<tr>
<th>$R_{RUP}$ (km)*</th>
<th>$T_P$ (s)</th>
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<tbody>
<tr>
<td>0.0</td>
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<tr>
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<td>2.4</td>
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<tr>
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<td>125.0</td>
<td>10.9</td>
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<tr>
<td>175.0</td>
<td>17.4</td>
</tr>
<tr>
<td>270.0</td>
<td>34.2</td>
</tr>
<tr>
<td>$T_P = 33.4 + 0.156(R_{RUP} - 270)$</td>
<td>$&gt; 34.2$</td>
</tr>
</tbody>
</table>

Use linear interpolation between nontabulated values.

*Convert $R_{RUP}$ to $R_{PS}$ using equation (6).

### Genetic Algorithmic Inversion

Table 3 provides a list of the seismological models and parameters that are needed to perform the point-source stochastic simulation. The last column indicates which ones are obtained from the literature and which are derived in the GA inversion. As indicated in the previous section, the constrained models and parameters are taken from recent published research because they are either too complex or too strongly correlated with other parameters to allow the inversion to converge. The remaining parameters are derived from the inversion. We used the GA inversion to determine the best combination of the remaining seismological parameters for the point-source stochastic model that matches the weighted median GMIMs predicted from the five NGA-West2 GMPEs.

As indicated in Table 3, the seismological parameters determined in the inversion are $\Delta\sigma$, $b_1$, $b_2$, $b_3$, $Q_0$, $\eta$, and $h$. Parameters are held constant in the inversion. The inversion is performed for individual $M$, ranging from 3.5 to 8.0 in 0.5 intervals (10 magnitudes), at 25 periods ranging from 0.01 to 10 s, uniformly distributed in log space, and at 30 values of $R_{RUP}$ ranging from 1 to 300 km, uniformly distributed in log space. Parameters are obtained for each magnitude. Parameter $b_1$ is obtained for each frequency used in the inversion.

Table 5 lists the final (lowest-misfit) model parameter values determined from the GA inversion. Separate values are given for each magnitude. Moreover, the near-source geometrical spreading decay parameter $b_1$ is provided as a function of frequency (at frequencies used in the inversion). The geometrical spreading coefficients $b_1$, $b_2$, and $b_3$ are shown in Figure 1. Figure 2 shows the effective-depth term. A functional form of $\log h(M) = a + bM$ is fitted to the depth values, giving $a = 0.1721$ and $b = 0.1275$, with an
### Table 5

Results of the GA Inversion

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<tr>
<th>$M$</th>
<th>$b_2$</th>
<th>$b_3$</th>
<th>$\Delta r$ (bars)</th>
<th>$Q_0$</th>
<th>$\eta$</th>
<th>$h$ (km)</th>
<th>$b_1$ (Hz)</th>
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</table>

### $h_1$ (Hz)

<table>
<thead>
<tr>
<th>$M$</th>
<th>$b_1$ (Hz)</th>
</tr>
</thead>
<tbody>
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<td>4</td>
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<td>7.5</td>
<td>1.189</td>
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<tr>
<td>8</td>
<td>1.181</td>
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</table>
The obtained effective-depth term fits the exponential model very well. The $\Delta\sigma$-values from the inversion are plotted against magnitude in Figure 3. The anelastic attenuation parameters $Q_0$ and $\eta$ obtained from the inversion along with the corresponding quality factor function $Q(f)$ are shown in Figure 4 for different magnitudes. The significance of this result is discussed later.

We recognize that there is a trade-off between seismological parameters that resulted from the inversion. In other words, parameters obtained in this study for the point-source stochastic model are correlated. Our goal was not to minimize the trade-off among the parameters. We intended to find a set of seismological parameters for a point-source model that fit the target NGA-West GMPEs. GA varies parameters independently and finds a combination that matches the target best. The GA does not guarantee that there will not be trade-offs among the model parameters, nor does it try to minimize them. However, we only allowed GA to vary each parameter in a physically meaningful range (see Table 4). Also, GA does not guarantee a unique solution. For each magnitude, GA finds a set of parameters that fit the target ground motions the best. Enough iterations were used to achieve convergence. GA provides a solution which is not necessarily the global or absolute best solution. By limiting each parameter, our GA runs typically result in similar parameters (this behavior was observed by applying GA a few times) and parameter trends with magnitude and frequency that are relatively smooth.

We emphasize the fact that the seismological model needs to be used in its entirety to obtain a consistent set of GMIM estimates between the empirical and simulated GMIMs. It is also important to mention that no calibration factors are needed to force the stochastic estimates of the GMIMs using the final parameter values to match the GMPE estimates (i.e., the observed data).

**Figure 1.** Geometrical spreading coefficients obtained from the genetic algorithm (GA) inversion of the Next Generation Attenuation-West2 (NGA-West2) ground-motion prediction equations (GMPEs): (a) coefficient $b_1$ as a function of magnitude and frequency, (b) coefficients $b_2$ and $b_3$ as a function of magnitude.

**Figure 2.** Effective-depth term $h(M)$ obtained from the GA inversion of the NGA-West2 GMPEs, along with a fitted functional form given by the equation $\log h(M) = a + bM$ ($a = 0.1721$, $b = 0.1275$, and $r^2$-value of 0.9971).

**Figure 3.** Stress parameter $\Delta\sigma$-values obtained from GA inversion of the NGA-West2 GMPEs as a function of magnitude.
In this study, results are shown for spectra from the inverted stochastic point-source model derived in Al Atik and Youngs (2014) for strike-slip faulting. The 95% confidence interval (CI) on the median NGA-West2 GMPEs are also plotted. Similar predictions from the GMPEs. In Figure 5, the 95% CI on uncertainty calculated as the standard deviation of median predictions of the NGA-West2 models: (a) parameters $Q_0$ and $\eta$ as a function of magnitude, (b) the quality factor function $Q(f) = Q_0 f^\eta$ for different magnitudes as a function of frequency.

**Figure 4.** Anelastic attenuation parameters and model obtained from the GA inversion of the NGA-West2 GMPEs: (a) parameters $Q_0$ and $\eta$ as a function of magnitude, (b) the quality factor function $Q(f) = Q_0 f^\eta$ for different magnitudes as a function of frequency.

**Discussion of Results**

Figure 5 compares the median predicted response spectra from the NGA-West2 GMPEs and the predicted response spectra from the inverted stochastic point-source model derived in this study. Results are shown for $M$ 3.5, 5.5, and 7.5 and $R_{\text{rupture}} = 1, 5.872, 28.321, 62.198, 112.21$, and 202.435 km. The 95% confidence interval (CI) on the median NGA-West2 GMPEs are calculated using the standard deviation $\sigma_{\text{GMPEs, Total}}$ defined as

$$\sigma_{\text{GMPEs, Total}} = \sqrt{\sigma_{\text{within-GMPEs}}^2 + \sigma_{\text{between-GMPEs}}^2}, \quad (9)$$

in which $\sigma_{\text{within-GMPEs}}$ is the average within-model uncertainty in median predictions of the NGA-West2 models derived in Al Atik and Youngs (2014) for strike-slip faulting, and $\sigma_{\text{between-GMPEs}}$ is the between-NGA-West2 GMPEs uncertainty calculated as the standard deviation of median predictions from the GMPEs. In Figure 5, the 95% CI on the median NGA-West2 GMPEs are also plotted. Similar plots for other magnitudes and distances are available in Figures S1–S40 (available in the electronic supplement to this article). Moreover, similar plots showing PSA values against distance for different magnitudes and distances are provided in Figures S41–S110. It should be noted that Al Atik and Youngs (2014) used magnitudes greater than $M$ 5 to derive the within-model uncertainty for the NGA-West2 GMPEs. However, their model is extrapolated down to $M$ 3.5 for purposes of Figure 5. Figure 5 shows generally good agreement between the median NGA-West2 GMPEs and the inverted stochastic model. The predicted PSA values are within the 95% CI of the median NGA-West2 GMPE predictions.

Figure 6 compares the magnitude scaling of PSA at 0.1, 1, 10, and 100 Hz for $R_{\text{rupture}} = 5.9, 10.6, 28.3, 62.2, 112.2$, and 246.4 km for the median NGA-West2 GMPEs and the inverted stochastic model. Figure 7 compares the distance scaling of PSA at 0.1, 1, 10, and 100 Hz for $M$ 3.5, 4.5, 5.5, 6.5, and 7.5 for the median NGA-West2 GMPEs and the inverted stochastic model. As can be seen in Figures 6 and 7, the magnitude and distance scaling predicted by the stochastic model obtained in this study is in good agreement with those from NGA-West2 GMPEs.

The misfit of the stochastic model with respect to the empirical GMPEs is clearly seen in the residuals plotted in Figure 8 for the same magnitudes and distances used in Figure 5. These residuals, calculated from equation (1), are defined as the difference between the logarithm base 10 of the predicted PSA of the empirical and stochastic models. In Figure 8, lines indicating the $\pm 10\%$ and $\pm 25\%$ difference between the PSA values from the inverted stochastic model with respect to the median NGA-West2 GMPEs are also plotted. Similar plots for other magnitudes and distances are available in Figures S1–S20. Moreover, similar plots showing residuals against distance for different magnitudes and frequencies are provided in Figures S1–S20.

As shown in Figure 8 and in Figures S1–S20, predictions from the stochastic models are within $25\%$ of the empirical model for many magnitudes, distances, and periods.

Figure 9 depicts magnitude, frequency, and distance combinations for which the PSA values predicted from the inverted seismological point-source model are different from the median NGA-West2 GMPE predictions by more than $25\%$. In general, differences larger than $25\%$ are not detected at $f > 24$ Hz for the entire distance and magnitude ranges.
used in the inversion. The maximum difference between the inverted model and empirical model is about 60%. These largest differences are observed at $M_{3.5}$ for $f < 15$ Hz, and distances less than 4 km are not considered to be of engineering interest.

Figure 10 shows magnitude, frequency, and distance combinations for which the ground-motion estimates from the inverted point-source model are within 10% of the median NGA-West2 GMPE predictions. From Figure 10, it can be observed that predictions from the inverted model are within 10% of the empirical model for the majority of magnitudes, distance, and frequencies used in the inversion. Beside the area with large deviations shown in Figure 10 there are areas with differences larger than 10% for $M \geq 5.5$ in the $\sim 8$–25 km distance range at different frequencies and the $\sim 40$–80 km distance range at $f > 0.2$ Hz. There are deviations greater than 10% at $f < 0.2$ Hz in the 20–200 km distance range for $M \leq 5.0$.

It is interesting to compare our results with those of Yenier and Atkinson (2015b; hereafter, YA15), who used average horizontal PSA from the NGA-West2 ground-motion database (Ancheta et al., 2014) for California earthquakes with $M \geq 3.0$, recorded by three or more stations within 400 km from the source, to derive an equivalent
They concluded that their equivalent point-source stochastic model can generally predict average PSA in California to within ±25% for $M \leq 7.5$, $R_{rup} < 400$ km and $f > 0.2$ Hz. In comparing this study with YA15, it is important to keep in mind three fundamental differences between these studies: (1) we invert on the GMIM predictions from the NGA-West2 GMPEs, whereas YA15 develop their model directly from the NGA-West2 data; (2) we performed our inversion at each magnitude individually, therefore all the seismological parameters are functions of magnitude, whereas YA15 assumes all parameters are independent of magnitude; and (3) we used a frequency-dependent near-source geometric attenuation term, whereas YA15 used a frequency-independent term. The following sections discuss differences between the two studies in terms of their near-source geometric attenuation, Brune stress drop, site amplification and attenuation, effective depth, and anelastic attenuation. After the discussion, predictions from the two models are compared. However, we do not intend to examine the differences between the predictions because the models are fundamentally different.

Near-Source Geometric Attenuation

YA15 conclude that their best-fit bilinear simulation model suggests that the attenuation in California can be
modeled with a distance decay of $R^{-1.3}$ within 50 km and $R^{-0.5}$ at further distances. They base this conclusion on their observation that the steeper near-source spreading does a better job at matching near-source attenuation trends than the traditional $R^{-1.0}$ model, particularly for events of $M < 5.5$. They also note that the $R^{-1.3}$ model requires an overall multiplicative calibration factor of $C_{\text{sim}}/0.0136$ to make the simulations match the observed response-spectral amplitudes for all magnitudes and distances, whereas the $R^{-1.0}$ model requires virtually no additional calibration ($C_{\text{sim}}/0.0136$), indicating that it provides the best overall fit to the data (Yenier and Atkinson, 2015b).

In this study, we used a magnitude- and frequency-dependent near-source geometrical spreading coefficient $b_1$ and magnitude-dependent coefficients $b_2$ and $b_3$. Geometrical spreading coefficients obtained from the GA inversion are shown in Figure 1. A systemic trend in $b_1$ with magnitude and frequency can be observed in Figure 1. At low frequencies, $b_1$ increases from about −1.3 up to about −0.8, with $M$ increasing from 3.5 to 8.0. Around 5 Hz, $b_1$-values from all magnitudes merge to an average of about −1.15. At $f > 5$ Hz, larger magnitudes of $M \geq 5.0$ show larger $b_1$-values. Around 100 Hz, $b_1$-values from all magnitudes merge to an average of about −1.1 (see Fig. 1). Coefficient
increases with magnitude from about $-1.2$ at $M_{3.5}$ to about $-0.4$ at $M_{8.0}$. Coefficient $b_3$ ranges from $-0.71$ to $0.47$ over this same magnitude range, as shown in Figure 1. The coefficient $b_3$ obtained in the study is generally smaller than the value of $-0.5$ typically attributed to the attenuation of $L_g$ waves (e.g., Street et al., 1975). It should be noted that there is a trade-off between coefficients $b_1$, $b_2$, and $b_3$ obtained from the inversion. It is important to recognize this correlation when using or interpreting these results.

Effective Depth

Yenier and Atkinson (2014) modeled the effective depth as $\log h(M) = -1.72 + 0.43M$ for moderate-to-large magnitude earthquakes. YA15 adopted the depth model of Atkinson and Silva (2000) expressed as $\log h(M) = -0.05 + 0.15M$ for $M < 6$ events and the relation by Yenier and Atkinson (2014) for $M > 6$ for a final model of $\log h(M) = \max(-0.05 + 0.15M, -1.72 + 0.43M)$. YA15 also introduced an alternative model expressed as $\log h(M) = -0.405 + 0.235M$ to prevent oversaturation at close distances for large events and high frequencies in forward modeling. This latter model implies a depth of about 3 km at $M_{3.5}$ and about 30 km at $M_{8.0}$.

The effective depth obtained in this study is shown in Figure 2. An exponential model of $\log h(M) = 0.17 + 0.13M$ is fitted to the depth values obtained from the inversion. The effective-depth values in this study increase from about 4 km at $M_{3.5}$ to about 15 km at $M_{8.0}$. They are similar to those by Atkinson and Silva (2000), which predict a depth of about 3 km at $M_{3.5}$ and about 14 km at $M_{8.0}$.

It is important to recognize that the results for near-source geometrical spreading coefficients $b_1$ might be correlated with the effective depth and our particular choice of the relationship used to convert the value of $R_{\text{rup}}$ used in the GMPEs to the equivalent point-source distance metric $R_{\text{PS}}$ used in the stochastic simulations.

Figure 6. Comparison of predicted response spectra from the median NGA-West2 GMPEs (dashed lines) with the predicted response spectra from the stochastic model derived from the GA inversion of the NGA-West2 GMPEs (solid lines), showing the magnitude scaling at $f = 0.1$, 1, 10, and 100 Hz.
Brune Stress Drop

YA15 found that the modeled Brune stress drop decreases with decreasing values of magnitude and hypocentral depth for $M < 6.0$ and $Z_{\text{HYP}} < 12$ km to match the NGA-West2 data. For events with $M 3.0$–$4.0$, the mean stress drop increases from $\sim 5$ bars at $Z_{\text{HYP}} < 5$ km to $\sim 120$ bars at $Z_{\text{HYP}} > 10$ km. For $M > 6.0$, stress drops increase from 50 to 160 bars over these same depths.

We obtained average stress drops that increase from 80 bars at $M 3.5$ to about 230 bars at $M 5.0$–$5.5$ and then decreases to 90 bars at $M 8.0$ for the NGA-West2 GMPEs, with an average hypocentral depth of around 9 km (Campbell and Bozorgnia, 2014). It is possible that the smaller values of $\Delta \sigma$ obtained by YA15 are enhanced by their adoption of a smaller value of $\kappa_0$ than we used (i.e., 0.025 s versus 0.034–0.045 s, depending on magnitude). It is also possible that the difference in the site amplification factors discussed in the next section contributes to these differences in stress drop.

Site Amplification and Attenuation

YA15 adopted a site-attenuation parameter of $\kappa_0 = 0.025$ s, based on the best-fitting equivalent point-source stochastic model of the vertical-component FAS of 11 $M \geq 6.0$ global earthquakes from active tectonic regions developed by YA14. YA15 used this value in conjunction with the site amplification model calculated from the CENA NEHRP B/C site profile proposed by Frankel et al. (1996), adjusted to a slightly different source velocity, to incorporate site response in their stochastic simulations.

In this study, we use the site amplification factors recommended by Campbell and Boore (2016), based on a generic site profile of an NEHRP B/C site in WNA developed by Boore (2016). The impact of using different site profiles is shown in Figure 11. This figure compares the site amplification factors, excluding and including site attenuation, from the site profiles used in YA15 and in this study. The site factors that include site-attenuation effects use

Figure 7. Comparison of predicted response spectra from the median NGA-West2 GMPEs (dashed lines) with the predicted response spectra from the stochastic model derived from the GA inversion of the NGA-West2 GMPEs (solid lines) showing the distance scaling at $f = 0.1, 1, 10,$ and $100$ Hz.
κ₀-values that are consistent with how the site factors have been used or are proposed to be used: κ₀ = 0.025 s for the site factors used by YA15 and κ₀ = 0.034–0.045 s adopted from Zandieh et al. (2016) and used along with the site factors used in this study. Including the effect of κ₀, the high-frequency amplification factors used in this study are smaller than those used in YA15. This figure also shows that the opposite is true at mid-to-low frequencies. These differences in site amplification will cause other spectral shape parameters, such as Brune stress drop, to compensate for these differences in the stochastic models. Past studies have typically used κ₀-values in the 0.035–0.040 s range for WNA site profiles (Campbell, 2011; Yenier and Atkinson, 2014, Pezeshk et al., 2015).

An Elastic Attenuation

The anelastic attenuation parameters Q₀ and η have values that are strongly dependent on the geometric spreading coefficient at large distances, characterized by the parameters b₂ and especially b₃ in this study. For this study, we did not use a far-source geometric spreading coefficient of −0.5

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**Figure 8.** Plots of residuals versus frequency between the predicted response spectra from the median NGA-West2 GMPEs and the predicted response spectra from the inverted stochastic model. (a) M 3.5, (b) M 5.5, and (c) M 7.5. Dotted and dashed lines indicate the ±10% and ±25% difference between the pseudoabsolute acceleration response spectra (PSA) values from the inverted stochastic model, with respect to the median NGA-West2 GMPE predictions.
consistent with the attenuation of $L_g$ waves (e.g., Street et al., 1975) but instead obtained $b_3$ directly from the inversion. The coefficient $b_3$ from the inversion is generally less than $-0.5$ (except at $M > 7.5$ where it is $-0.47$) with values as low as $-0.7$. The anelastic attenuation parameters $Q_0$ and $\eta$ obtained in this study are listed in Table 5. These parameter values along with the quality factor function $Q(f) = Q_0 f^{\eta}$ are shown in Figure 4. The values of $Q_0$ range from 226 to 276 and those of $\eta$ range from 0.55 to 0.63 for different magnitudes.

YA15 adopted the anelastic attenuation model of Raoof et al. (1999) for southern California for their bilinear model, which is given by the relationship $Q(f) = Q_0 f^{\eta}$ with $b_2 = -0.5$ and $\beta_S = 3.5$ km/s. YA15 slightly modified this model to have $Q_0 = 170.3$ to correspond to the value $\beta_S = 3.7$ km/s used in their study and to limit the value of $Q$ to a minimum value of 100. This latter limit only affects frequencies below 0.31 Hz ($T > 3.1$ s) for the bilinear model of YA15 and 0.26 Hz ($T > 3.9$ s) for the models used in this study. Because the minimum values of $Q$ for the models used in this study only go down to 55 (for $M = 6.5$) at $f = 0.1$ Hz ($T = 10$ s), its impact is negligible compared to using a minimum value of 100. For example, at magnitude $M = 6.5$ our value results in a 16% reduction in the value of FAS at 200 km, compared to using a minimum $Q$-value of 100.

Figure 12 compares PSA values at 0.1, 1, 10, and 100 Hz for $M = 3.5, 5.0, and 7.0$ for the stochastic models obtained in

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**Figure 8.** Continued.
this study and those from YA15. The YA15 model is evaluated for the Brune single-corner-frequency point-source model, coefficient $b_3$ of $-1.3$, and focal depth of 9 km. The larger differences between two models occur at lower magnitudes and close distances (< 10 km). We do not intend to explain the deviations between these two models due to fundamental differences between the two discussed above.

Summary and Conclusions

We used a GA to invert weighted geometric mean estimates of horizontal response-spectral acceleration from the empirical NGA-West2 GMPEs of Abrahamson et al. (2014), Boore et al. (2014), Campbell and Bozorgnia (2014), Chiou and Youngs (2014), and Idriss (2014) to estimate a consistent set of seismological parameters that can be used along with an equivalent point-source stochastic model to mimic the general scaling characteristics of these GMPEs. The inversion is performed for events of $M_{3.5–8.0}$, $R_{rup} = 1–300$ km, $T = 0.01–10$ s ($f = 0.1–100$ Hz), strike-slip fault mechanism, with a fault-rupture dip of 90° to avoid hanging-wall effects, NEHRP B/C site conditions ($V_{530} = 760$ m/s), default values of the sediment depths $Z_{1.0}$ and $Z_{2.5}$ recommended by the NGA-West2 developers for the specified value.

Figure 8. Continued.
One of the more interesting results of the inversion is the magnitude and frequency dependence of the near-source geometric spreading. The geometric spreading for $R_{RUP} \leq 45$ km is consistent with a distance decay of $R^{-0.8}$ to $R^{-1.3}$ at frequencies of $f \leq 1$ Hz for $M$ ranging from 3.5 to 8. Near 5 Hz, the distance decay is $R^{-1.17}$, on average. At larger frequencies, the near-source distance decay is represented by decays of $R^{-1.0}$ to $R^{-1.25}$. The stress parameter is a function of magnitude,

of $V_{S30}$, and the site-attenuation parameter $\kappa_0$ derived by Zandieh et al. (2016).

Figure 9. Frequency, magnitude, and distance combinations for which the PSA values predicted from the inverted seismological model are different from the median NGA-West2 GMPE predictions by more than 25%.

Figure 10. Frequency, magnitude, and distance combinations for which the PSA values predicted from the inverted seismological model are different from the median NGA-West2 GMPE predictions by less than 10%.
which increases from 80 bars at $M_{3.5}$ to 235 bars at $M_{5.0}$–5.5 and then decreases with increasing magnitude to about 90 bars at $M_{8.0}$.

The agreement over all magnitudes and distances evaluated in this study is generally within 10%, except for $M \geq 5.5$ in the $\sim$8–25 km distance range at different frequencies and the $\sim$40–80 km distance range at $f > 0.2$ Hz. There are deviations greater than 10% at $f < 0.2$ Hz in the $\sim$20–200 km distance range for $M \leq 5.0$.

We conclude that such agreement means that our point-source stochastic model can be used to provide a frequency-domain representation of the NGA-West2 GMPEs to use in any application that requires such a model.

Figure 11. Site amplification factors for shear-wave velocity profiles used in Yenier and Atkinson (2015b; hereafter, YA15) and in Boore (2016), the latter adopted for this study, excluding (left plot) and including (right plot) representative values of the site-attenuation parameter $\kappa_0$. The different spectral shapes of the site factors will cause other spectral-shape parameters in the stochastic model to compensate for these differences. The color version of this figure is available only in the electronic edition.

Figure 12. Comparison of predicted response spectra from the stochastic model derived in this study from the GA inversion of the NGA-West2 GMPEs (solid lines) with those from YA15 at $f = 0.1$, 1, 10, and 100 Hz.
Data and Resources

The evaluation of the Next Generation Attenuation-West2 (NGA-West2) ground-motion prediction equations (GMPEs) was done using the spreadsheet provided by the Pacific Earthquake Engineering Research Center (PEER) and available at http://peer.berkeley.edu/ngawest2/databases (last accessed June 2015). All other data used in this study are available in the references.

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References


