Comparative study on parameter estimation methods for attenuation relationships

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Comparative study on parameter estimation methods for attenuation relationships

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Abstract

In this paper, the performance and advantages and disadvantages of various regression methods to derive coefficients of an attenuation relationship have been investigated. A database containing 350 records out of 85 earthquakes with moment magnitudes of 5–7.6 and Joyner–Boore distances up to 100 km in Europe and the Middle East has been considered. The functional form proposed by Ambraseys et al. (2005 Bull. Earthq. Eng. 3 1–53) is selected to compare chosen regression methods. Statistical tests reveal that although the estimated parameters are different for each method, the overall results are very similar. In essence, the weighted least squares method and one-stage maximum likelihood perform better than the other considered regression methods. Moreover, using a blind weighting matrix or a weighting matrix related to the number of records would not yield in improving the performance of the results. Further, to obtain the true standard deviation, the pure error analysis is necessary. Assuming that the correlation between different records of a specific earthquake exists, the one-stage maximum likelihood considering the true variance acquired by the pure error analysis is the most preferred method to compute the coefficients of a ground motion prediction equation.

Keywords: regression methods, attenuation relationship, Europe and the Middle East, pure error analysis, ground motion prediction equation

Introduction

Ground-motion prediction equations (GMPEs) are widely used to estimate ground motion intensity measures (GMIMs) such as peak ground acceleration (PGA) and spectral accelerations (SA) at different periods. Regression analysis is performed to detect a functional form between variables. The rudimentary formulation of GMPEs is given by

\[ Y = f(X_{es}, \theta) + \Delta, \]

where \( Y \) is often the natural logarithm or the base 10 logarithm of the ground motion observation, \( f(X_{es}, \theta) \) is the ground motion regression model, in which records information are included in \( X_{es} \) such as magnitude, distance, fault mechanism, and site condition, and \( \theta \) is the vector of unknown model coefficients. The total variability of a GMPE is demonstrated by \( \Delta \), which has a normal distribution with zero-mean and the standard deviation of \( \sigma \). This variability is composed of between-events (inter-event) variability, \( \Delta B \), and within-event (intra-event) variability, \( \Delta W \), which have normal distribution with zero-means and standard deviations of \( \tau \) and \( \phi \), respectively. Since the between and within event standard deviations are uncorrelated, the total standard deviation can be obtained form

\[ \sigma = \sqrt{\tau^2 + \phi^2}. \]  

It should be noted that the notations used in this paper, closely follow the notations used by Al Atik et al. (2010), since these notations are defined to be used in the development of the next generation attenuation (NGA) models.

The value of the standard deviation of a GMPE has a significant effect on the probabilistic seismic hazard assessment (PSHA) results (Bommer and Abrahamson 2006). In order to decrease the influence of the standard deviation on PSHA results, the amount of the standard deviation should...
Diverse methods have been introduced and developed to compute the unknown parameters of GMPEs. The oldest and most rudimentary method to compute coefficients of a functional form are the least squares estimation, and the weighted least squares estimation. Brilinger and Preisler (1984, 1985) employed the random-effect regression method. Later, McLaughlin (1991) and Joyner and Boore (1993) utilized maximum likelihood estimation. Joyner and Boore (1993) showed that their proposed weighting matrix for the maximum likelihood estimation leads to the same result from the Brilinger and Preisler (1984, 1985) method. Two-stage methods with different weighting matrices were developed by Fukushima and Tanika (1990), Masuda and Ohtake (1992), and Joyner and Boore (1993). Recently, methods based on genetic algorithm (GA) is getting widespread in which an optimization function is defined to determine the coefficients of GMPEs (Tavakoli and Pezeshk 2005, Sobhaninejad et al 2007, Tavakoli and Pezeshk 2007, Cabalar and Cevik 2009, Bagheri et al 2011, Yilmaz 2011).

The main goal of this study is to analyze and compare the above-mentioned methods to explore the differences between them as well as to understand advantages and disadvantages of each one. First, these methods are thoroughly described. Then, the considered attenuation relationship and the prepared database are reviewed. Afterward, the LH test introduced by Scherbaum et al (2004) in addition to various statistical techniques (Histogram of the standard normal distribution, Q–Q plot, and analysis of variance) and goodness-of-fit measures (t-test, Var-test, Chi-square test, Lillie-test, ks-test, Jarque–Bera test, Anderson–Darling test, and Shapiro–Wilk test) are applied. Finally, the strengths and limitations of each procedure will be compared and discussed.

**Parameter estimation methods in linear models**

In order to estimate unknown parameters in linear models, the following matrix form is used

$$Y = X\theta + \Delta,$$

where $Y$ is a $n \times 1$ vector of observations (outputs), $\theta$ is a $m \times 1$ vector of unknowns, $X$ is a $n \times m$ matrix of variables (inputs), and $\Delta$ is a $n \times 1$ vector of deviations. The parameter $n$ represents the number of observations and $m$ represents the number of unknown parameters. The elements of vector $\Delta$ have a normal distribution with the average of zero and the variance–covariance matrix which is defined as

$$\text{variance}(\Delta) = V = \sigma^2 W^{-1},$$

where $W$ is a known weighting matrix of observations, and $\sigma^2$ is an unknown positive coefficient which can be acquired from the expected value of the squared residuals. With respect to the type of variance matrix, $V$, different methods of the regression analysis can be employed to estimate unknown parameters.

**Unweighted least squares estimation**

The simplest case to solve equation (3), called ordinary least squares (OLS) estimation, occurs once the weighting matrix, $W$, is a diagonal one with identical elements. In this method, by using properties of matrices, we get

$$\hat{\theta} = (X^T X)^{-1} X^T Y,$$

in which $\hat{\theta}$ is the estimation of the unknown coefficients vector, $\theta$. If the matrix $X$ is a square matrix, it could be simply written as

$$\hat{\theta} = X^{-1} Y.$$

It must be emphasized that this solution is applicable once the inverse of input matrix exists. If the determinant of the matrix, $X$ or $X^T X$ approaches zero, an alternative algorithm called recursive least squares algorithm (Ljung 1999) can be used as follows

$$\begin{align*}
\theta_{i+1} &= \theta_i + S_i x_{i+1} (y_{i+1}^T - x_{i+1}^T \theta_i) \\
S_{i+1} &= S_i - \frac{S_i x_{i+1} x_{i+1}^T S_i}{1 + x_{i+1}^T S_i x_{i+1}}, \quad i = 0, 1, 2, \ldots, n - 1, \\
\theta_0 &= 0, \quad S_0 = \lambda I
\end{align*}$$

in which $I$ is an $m \times m$ identity matrix, $\lambda$ is a large arbitrary value, $x_i^T$ is the $i$th row of the matrix $X$ and $y_i^T$ is the $i$th element of the vector $Y$.

Once the unknown parameters are calculated, the estimation of the output vector is

$$\hat{Y} = X\hat{\theta},$$

and, therefore, the difference between the real amounts and predicted values gives the vector of residuals

$$\Delta = \hat{Y} - Y,$$

where $\hat{Y}$ is the predicted values of observations, $Y$ is the real amounts of observations and $\Delta$ is the vector of residuals.

One of the initial assumptions in the regression analysis is to have a constant variance (homoscedasticity) for the errors in the model to use OLS method. Thus, if this assumption is violated, the OLS method is not appropriate and the effect of non-constant variance (heteroscedasticity) should be considered by using a weighting matrix or transformation in the model. When the variance in a model is variable although the estimated coefficients are not biased, the total standard error is inaccurate and the variance of the estimators is not optimum. It is therefore necessary to account for the heteroscedasticity effects (e.g. biased standard errors) in the estimations of coefficients to derive best linear unbiased estimators (BLUEs). There are two strategies under the presence of heteroscedasticity indicating that the variance is non-constant. In the first strategy, efficient coefficients can be acquired by incorporating an appropriate weighting function in the regression analysis to take into account of heteroscedasticity. In the second strategy, the coefficients obtained by OLS are kept, but the variance would not be constant and variable variance should be used. The OLS method is considered as method 1 in this paper.
**Weighted least squares (WLS) estimation**

Once the weighting matrix is available but either it is off diagonal or the elements of the diagonal matrix are not identical because some observations are less reliable than the others, the OLS method does not result in obtaining BLUEs (Draper and Smith 1981). When the weighting matrix is diagonal, observations are independent but they have different variances, while when the weighting matrix has off diagonal elements, it means that observations are correlated to each other and they are dependent. In this case, the final solution is given by (Searle 1971)

$$\hat{\theta} = (X^T WX)^{-1}X^T W Y.$$  \hspace{1cm} (10)

For the WLS method, different kinds of transformation matrices are proposed. For instance, if the weighting matrix is diagonal, transformation matrix, $P$, can be estimated by

$$P = \sqrt{W}.$$  \hspace{1cm} (11)

If the weighting matrix is off diagonal and it is Hermitian positive-definite, it can be decomposed by using the Cholesky method (Gentle 1998)

$$P^T P = W,$$  \hspace{1cm} (12)

in which $P$ is an upper triangular transformation matrix with positive diagonal elements and $P^T$ is the transpose of $P$. Then, adjusted input and output matrices can be defined as

$$\tilde{X} = PX,$$  \hspace{1cm} (13)

and

$$\tilde{Y} = PY,$$  \hspace{1cm} (14)

in which $\tilde{X}$ and $\tilde{Y}$ are called the transformed input and output matrices. Therefore, using the weighting matrix acts in a way that the input and output matrices are transformed by the matrix $P$. Thus, the solution of the problem, can be rewritten as

$$\hat{\theta} = (\tilde{X}^T \tilde{X})^{-1} \tilde{X}^T \tilde{Y} = (X^T P^T PX)^{-1}X^T P^T PY.$$  \hspace{1cm} (15)

It should be noted that the $P \Delta$ matrix now has a constant variance covariance matrix. As it is mentioned in the OLS method, once the determinant of matrix is around zero, the recursive least squares method is suggested. Therefore, the recursive algorithm in equation (7) can be applied to estimate coefficients.

The weighting matrix for the WLS method can be determined by using either grouped data to estimate the variance for each group, absolute residuals, or squared residuals versus a specific variable. The WLS method is called method 2 in this paper.

**Unweighted one-stage maximum likelihood estimation**

For the one-stage maximum likelihood estimation, it is assumed that observations have a normal distribution, and accordingly, the probability density function of the observations $L$ (West et al 2007) is expressed as

$$L = (2\pi)^{n/2} |V|^{-1/2} \exp \left[ -\frac{1}{2} (Y - X\theta)^T V^{-1} (Y - X\theta) \right].$$  \hspace{1cm} (16)

where $|\cdot|$ is the determinant and $n$ is the number of observations. For an assumed $V$, maximizing equation (16) with respect to the vector $\theta$ leads to the same results obtained from OLS method.

**Weighted one-stage maximum likelihood estimation**

When the variance–covariance matrix, $V$, is either off diagonal or diagonal with unequal elements, and this matrix is unknown, the maximum likelihood method can be used.

By substituting $\sigma^2 W^{-1}$ instead of $V$ in equation (16) and then taking a natural logarithm, it can be expressed as

$$\ln(L) = -\frac{n}{2} \ln(2\pi) - \frac{n}{2} \ln \sigma^2 - \frac{1}{2} \ln|W^{-1}| - \frac{1}{2\sigma^2} [(Y - X\theta)^T W (Y - X\theta)].$$  \hspace{1cm} (17)

For a constant $W$, the vector $\theta$ can be estimated by using equation (10). As a result, $\ln(L)$ is a function of $\sigma^2$, and in order to maximize $\ln(L)$, its differentiation with respect to $\sigma^2$ has to be equal to zero.

To derive an overall unbiased estimation of $\sigma^2$, the following equation is proposed (Chatterjee and Hadi 2006)

$$\sigma^2 = \frac{1}{n - m} [(Y - X\theta)^T W (Y - X\theta)],$$  \hspace{1cm} (18)

where $m$ is the rank of matrix $X$ and $n - m$ displays the degrees of freedom.

For a specific $W$, values of $\hat{\theta}$, $\sigma^2$ and $\ln(L)$ values can be calculated. These values are calculated for different assumed $W$‘s, and then the final answer corresponds to the $W$ that generates the maximum value of $\ln(L)$.

Joyner and Boore (1993) proposed a block diagonal variance covariance matrix

$$V = \sigma^2 W^{-1} = (\tau^2 + \phi^2) \begin{bmatrix} v_1 & 0 & \ldots & 0 \\ 0 & v_2 & \ldots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & v_N \end{bmatrix},$$  \hspace{1cm} (19)

where $N_e$ is the number of events in the database, and $v_i$ is defined as

$$v_i = \begin{bmatrix} 1 & \gamma & \ldots & \gamma \\ \gamma & 1 & \ldots & \vdots \\ \vdots & \vdots & \ddots & \gamma \\ \gamma & \gamma & \ldots & 1 \end{bmatrix},$$  \hspace{1cm} (20)

where $\gamma$ is defined as the following equation

$$\gamma = \frac{\tau^2}{\tau^2 + \phi^2},$$  \hspace{1cm} (21)

in which $\tau^2$ and $\phi^2$ are the variances related to the between-events and within-event variabilities, respectively. The rank of the matrix $v_i$ is equal to $R_i$, the number of records for the event $i$ (Joyner and Boore 1993). The weighted maximum likelihood method is considered as method 3 in this paper.
**Two-stage methods**

The very first step in this method is to separate variables that are identical for a specific event such as magnitude and fault mechanism from the other variables which are different for various records of a specific event such as distance and soil condition. The linear equation can be written as

\[
Y = B_0 + \sum_{i=1}^{m} B_i f_i(x_i) + \sum_{j=1}^{n} A_j g_j(x_j),
\]

(22)

where \( f_i(x_i) \) and \( g_j(x_j) \) consist of identical variables and unequal variables for the same event, respectively. \( B \) and \( A \) are coefficients in this equation, \( B_0 \) is a constant in the equation, \( n \) is the number of unequal variables, and \( m \) is the number of identical variables. Also, the total number of records for all earthquakes and stations is \( N \), and the total number of different events is \( N_e \).

Now, function \( H \) can be defined as

\[
H = B_0 + \sum_{i=1}^{m} B_i f_i(x_i).
\]

(23)

In the first stage, unknown parameters related to the unequal variables for the same event, \( A_j \), are calculated by using the following equation

\[
Y_1 = X_1 \theta_1 + \Delta 1,
\]

(24)

where

\[
Y_1 = [Y_i],
\]

(25)

\[
\theta_1 = [A_1, A_2, \ldots, A_n, H_1, H_2, \ldots, H_{N_e}]^T,
\]

(26)

and

\[
X_1 = [g_1(x_{11}), g_2(x_{12}), \ldots, g_n(x_{1n}), E_{k1}, E_{k2}, \ldots, E_{kN_e}],
\]

(27)

for \( k = 1, 2, \ldots, N \) and \( \Delta 1 \) is a vector containing intra-event residuals, \( \Delta W \), with a normal distribution \( N(0, \phi^2) \). \( E_{ki} \)'s are dummy variables and \( E_{ki} = 1 \) if the record \( k \) comes from the event \( l \); otherwise, \( E_{ki} = 0 \). Then, equation (23) can be rearranged as

\[
\hat{H} = B_0 + \sum_{i=1}^{m} B_i f_i(x_i) + (\hat{H} - H) + \Delta B,
\]

(28)

where \( \Delta B \) is the intrinsic variability for inter-event residuals in the aforementioned equation. At this point, unequal variables, \( A_j \), are calculated. Now the identical variables, \( B_k \), can be estimated by the following equation

\[
Y_2 = X_2 \theta_2 + \Delta 2,
\]

(29)

where

\[
Y_2 = [\hat{H}_1, \hat{H}_2, \ldots, \hat{H}_{N_e}]^T,
\]

(30)

\[
\theta_2 = [B_0, B_1, B_2, \ldots, B_m]^T,
\]

(31)

and

\[
X_2 = [1, f_1(x_{11}), f_2(x_{12}), \ldots, f_m(x_{1m})],
\]

(32)

for \( l = 1, 2, \ldots, N_e \) and \( \Delta 2 \) is a vector containing errors containing \( \hat{H} - H \) and \( \Delta B \). Since the \( \hat{H} - H \) and \( \Delta B \) vectors are uncorrelated, the variance–covariance matrix of \( \Delta 2 \) can be written as

\[
V_2 = \text{Var}(\hat{H} - H) + \tau^2 I,
\]

(33)

where \( \tau^2 \) is the variance of the vector \( \Delta B \) and \( I \) is the identity matrix. \( \text{Var}(\hat{H} - H) \) is defined as

\[
[\text{Var}(\hat{H} - H)]_{l,k} = [\text{Var}(\hat{\theta})]_{l+n,k+n},
\]

(34)

and the right-hand side of the above mentioned equation can be estimated by

\[
\text{Var}(\hat{\theta}) = (X^T X)^{-1}.\phi^2,
\]

(35)

in which \( \phi^2 \) is the variance of the first stage. Finally, the likelihood of samples can be obtained from the following equation

\[
L_2 = \frac{1}{(2\pi)^{N_e/2}} |V_2|^{-\frac{1}{2}} \times \exp \left[ -\frac{1}{2} (Y_2 - X_2 \theta_2)^T V_2^{-1} (Y_2 - X_2 \theta_2) \right].
\]

(36)

Now the problem can be easily solved as a WLS problem if the weighting matrix \( V_2^{-1} \) is known. Since, \( \tau^2 \) is unknown, therefore \( V_2^{-1} \) is unknown. Different assumptions are suggested by Fukushima and Tanaka (1990), Masuda and Ohtake (1992), and Joyner and Boore (1993), for this weighting matrix. In this paper the Fukushima and Tanaka (1990), and Joyner and Boore (1993) weighting matrices are used to calculate the unknown parameters of the ground motion equation.

Joyner and Boore (1993) proposed a diagonal weighting matrix as

\[
w_i = \left( \frac{\phi^2}{R_i} + \tau^2 \right)^{-1},
\]

(37)

where \( R_i \) is the number of records for the event \( i \), and \( \phi^2 \) is the variance of the first stage. \( \tau^2 \) can be calculated by iterating starting from zero to obtain the maximum likelihood.

Fukushima and Tanaka (1990) suggested using a diagonal weighting matrix in which each earthquake’s weight is equal to the number of records for that event. Two-stage maximum likelihood method with no weighting matrix, Boore and Joyner weighting matrix, and Fukushima and Tanaka weighting matrix are called method 4, method 5, and method 6 in this paper.

**Genetic algorithm**

A GA is used to solve optimization problems based on methods that originate from the natural and biological evolution such as selection, mutation, and cross over (Holland 1975, Goldberg et al 1989). In a GA, the optimization problem is formulated by defining an objective function. Then, a set of initial solutions called an initial population to the objective function is generated. Each solution (called a chromosome) is a combination of all unknown parameters that need to be estimated to satisfy the optimization problem (Gen and Cheng 2000).
The next step is to find the values of the objective function for the population. The initial population can now be improved by using genetic operators including selection, cross over, and mutation. The fitter chromosomes from the initial population are selected to create the next population. Each step of generating a new population is called a generation. To generate the next population, the selected chromosomes can be mixed two by two (parent) to create a new chromosome (child) which has the properties of its parent. Another way to create a new chromosome for the next population is using chromosomes from the previous generation (Gen and Cheng 2000). In this situation, the parameters inside of the chromosome are modified in order to keep the genetic diversity and avoid being stuck in local minima.

This process repeats until either the maximum number of generations is reached or the value of the objective function is satisfactory, and then this iterative algorithm is terminated. Finally, the answer to the optimization problem is the solution from the last generation which matches better with the objective function. The GA method is considered as method 7 in this paper.

Database

The database for this paper is built based on the CD ROM of European and Middle Eastern strong motion (Ambraseys et al 2000). The database includes 462 triaxial strong motion records from 110 earthquakes within 261 stations in Europe and the Middle East for corrected acceleration, velocity, and displacement (Ambraseys et al 2004).

All published empirical GMPEs from 1964 are provided by Douglas (2003) and Douglas (2014a). Most of these GMPEs until 2000s were developed for PGA and 5% damped linear elastic pseudo-absolute response spectral acceleration (PSA) ordinates at different periods (Douglas 2014b); however, the estimation of other GMIMs such as PGV, PGD, Arias intensity, and duration in addition to PGA and PSA are nowadays getting popular (Douglas 2012). Of course, particular GMIMs such as PGA and PSA at 0.2 and 1 s are very important due to the usage of them in performing PSHA (Tavakoli and Pezeshk 2005, Rezaeian et al 2015). In this study, we use values of PGA and 5% damped linear elastic PSA at 0.2 and 1 s as representative GMIMs to assess the performance of described regression methods. In this study, we use the geometric mean of two horizontal components because it is the most widely used GMIM in GMPEs (Beyer and Bommer 2006, Douglas 2014a). Various schemes have been defined on how to treat two horizontal components in order to be employed in the GMPEs such as arithmetic mean, geometric mean, larger component, random component for each accelerogram. Boore et al (2006) proposed two new definitions for the GMIM that can be used in GMPEs called GMRotD50 (GM indicates geometric mean, Rot implies rotations, D stands for period-dependent rotations, and 50 means that the 50 percentile value is used for the measure) or GMRotI50 (GM indicates geometric mean, Rot implies rotations, I stands for period-independent rotations, and 50 means that the 50 percentile value is used for the measure). The advantage of these measures is sensor orientation independency that yields in removing the uncertainty caused by the sensor orientation in GMPEs. Later, Boore (2010) proposed another definition to be considered as a GMIM called RotD50 in which no geometric mean is used to derive the new measure. NGA-west attenuation relationships (Bozorgnia et al 2014) are based on RotD50 definition for the GMIMs. Beyer and Bommer (2006) demonstrated that decreasing the aleatory uncertainties of GMPEs due to adopting GMRotD50 or GMRotI50 as the reference GMIM is less than 2%. Here, we employ the same database for all regression methods; and thus, the usage of geometric mean will equivalently affect results obtained from all regression methods.

In this database, the moment magnitude, $M$, is considered for the magnitude scale. Records that do not have moment magnitudes have been removed. In addition, records that have moment magnitudes less than 5.0 have been omitted. Joyner and Boore distance, $R_{JB}$, is used for the distance scale. Records with Joyner and Boore distances less than or equal 100 km are considered in this database.

Records included in the database all have known fault mechanisms. The Frohlich and Apperson (1992) approach is applied to classify fault mechanisms. Therefore, four classifications, thrust, normal, strike-slip, and odd, are defined based on the $B$, $P$, and $T$ axes plunges for each record. Once the plunge of $P$ axis is more than 60°, the earthquake is considered as normal. When the plunge of $B$ axis is greater than 60°, the earthquake is classified as strike slip. Once the plunge of $T$ axis is more than 50°, the event is considered as thrust. Otherwise, the earthquake is considered as odd.

Records with unknown site conditions have been removed from the database. Boore et al (1993) method has been used to categorize site conditions based on the average shear-wave velocity in the upper 30 m of the site profile. Accordingly, site conditions are divided into 4 groups: very soft, soft, stiff, and rock. According to Boore et al (1993), once $V_{S30} < 180$ m s$^{-1}$, the soil is considered as very soft; once $180 \leq V_{S30} < 360$ m s$^{-1}$, the soil is considered as soft; once $360 \leq V_{S30} < 750$ m s$^{-1}$, the soil is classified as stiff; otherwise, the soil is categorized as rock.

A total of 350 records created by 85 earthquakes from the data of the CD ROM have been collected as the final database. This database has 27 singly recorded earthquakes out of 85. Table 1 and figure 1 present the distribution of the data with respect to the site condition and fault mechanism. Since there are only 7 records with the very soft soil site condition, the soft and very soft soil classifications are combined and considered as one group. Figure 2 displays the distribution of the database with respect to the magnitude and distance.

Functional form

To evaluate the aforementioned methods for the estimation of empirical attenuation relationships coefficients, Ambraseys et al (2005) equation is considered. The proposed functional form is
Table 1. The distribution of the database with respect to the site condition and the fault mechanism.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Soft</th>
<th>Stiff</th>
<th>Rock</th>
<th>Total</th>
<th>Percent (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>5</td>
<td>23</td>
<td>58</td>
<td>138</td>
<td>40</td>
</tr>
<tr>
<td>Strike-slip</td>
<td>1</td>
<td>15</td>
<td>24</td>
<td>49</td>
<td>15</td>
</tr>
<tr>
<td>Thrust</td>
<td>1</td>
<td>11</td>
<td>33</td>
<td>59</td>
<td>17</td>
</tr>
<tr>
<td>Odd</td>
<td>0</td>
<td>18</td>
<td>25</td>
<td>21</td>
<td>64</td>
</tr>
<tr>
<td>Total</td>
<td>7</td>
<td>67</td>
<td>140</td>
<td>350</td>
<td>100</td>
</tr>
</tbody>
</table>

\[
\log(y) = a_1 + a_2 M + (a_3 + a_4 M) \log \left( \sqrt{d^2 + a_5^2} \right) + a_6 S \theta + a_7 S \theta + a_4 F_1 + a_9 F_0,
\]

(38)

where \( a_1 \) through \( a_{10} \) are the unknown parameters in the equation. \( S \theta = 1 \) or \( S \theta = 1 \) if the soil conditions are soft or stiff, respectively; otherwise, they are equal to zero. \( F_1 = 1 \), \( F_1 = 1 \), or \( F_0 = 1 \) if the fault mechanisms are normal, thrust, or odd, respectively; otherwise, they are equal to zero. \( y \) is the GMIM of interest such as PGA and SA at different periods, and \( d \) is the Joyner and Boore distance.

Application of different methods

Equation (38) is nonlinear in coefficients due to the multiplication of \( a_1 \) and \( a_2 \) with \( a_5 \). The first step to apply regression methods is to linearize the equation using Taylor series. Therefore, matrices \( Y, \theta \), and \( X \) can be defined as follows

\[
Y = [\log y_1, \log y_2, \ldots, \log y_N]^T,
\]

(39)

where \( N \) is the number of records in the database which is 350

\[
\theta = [a_1, a_2, a_3, a_4, \Delta a_5, a_6, a_7, a_8, a_9, a_{10}]^T,
\]

(40)

and

\[
X = \left[ \begin{array}{cccccc}
1 & M_i & \log \left( \sqrt{d_i^2 + a_5^2} \right) & M_i \log \left( \sqrt{d_i^2 + a_5^2} \right) & M_i \log \left( \sqrt{d_i^2 + a_5^2} \right) & S_{N_i} S_{N_i} F_{N_i} F_{N_i} F_{O_i} \\
\partial & \partial a_5 & (a_i' + a_5 M_i) \log \left( \sqrt{d_i^2 + a_5^2} \right) & \left. \frac{\partial}{\partial a_5} \left( a_i' + a_5 M_i \right) \log \left( \sqrt{d_i^2 + a_5^2} \right) \right|_{a_5 = a_5} \\
\end{array} \right].
\]

(41)

in which \( i \) is the record number, and \( a_i', a_5', \) and \( a_5' \) are trial amounts for \( a_1, a_5, \) and \( a_6 \). \( \Delta a_5 \) is a perturbation of the starting point of \( a_5 \) coefficient that aim in improving the solution. Calculation of the coefficients is therefore an iterative process in which the parameter \( a_5 \) is updated by \( a_5 + \Delta a_5 \). Any positive number except zero can be used as a starting point for \( a_5' \) (Joyner and Boore 1993). This iteration is terminated once the amount of \( \Delta a_5 a_5' \) is less than a desired limit which is considered \( 10^{-6} \) in this paper because the coefficients are estimated with 5 decimal places. The initial assumption for \( a_5, a_6, a_7, \) and \( a_8 \) is 1.

Since there are many similar records, the determinant of the \( X^TX \) matrix approaches to zero for OLS method (method 1). Accordingly, the recursive algorithm defined by equation (7) is used to calculate coefficients of the equation. The result of this method is mentioned in tables 2-4 for PGA, PSA at 0.2, and PSA at 1 s, respectively. The coefficients obtained by Ambraseys et al (2005) are also reported for comparison. It is worth mentioning that the Ambraseys et al (2005) coefficients are derived using the larger horizontal component.

Douglas and Smit (2001) suggested that if their database is divided into 2 km by 0.2 M unit intervals, records gathered in each bin can be assumed as repeated records. Douglas (2004a, 2004b) showed that using the analysis of variance could be a useful tool to investigate the regional dependency of strong motion data. In order to perform analysis of variance, intervals of 5 km by 0.25M units were used by Douglas (2004a). Since the number of records in this study is limited, using those intervals leads to loss of many records in this investigation. Therefore, after exploring different unit intervals for the distance and magnitude, the database is divided into 10 km by 0.2 M unit intervals to have sufficient records in each bin. The equations from figures 3 and 4 display the dependency of the standard deviations estimated from unit intervals on magnitude and distance as well as distribution of them, respectively.

Gradients of the fitted lines depict that there is a decrease in the standard deviation, \( \sigma \), with increasing the magnitude and distance as Youngs et al (1995), Campbell (1997), and Ambraseys et al (2005) reported. Although the slope for the distance dependency is near to zero, the dependency of the error on the magnitude is considerable. In this regard, the dependency of the standard deviation on the magnitude cannot be rejected at the significance level of 5%. In fact, it can be said that earthquakes with higher magnitude are more informative and therefore have more weight. Hence, there are two approaches to treat heteroscedasticity. In the first approach, the coefficient derived from OLS method are used but the variance should be updated based on either the prior knowledge or performing pure error analysis. In the second approach, a WLS regression analysis (method 2) can be employed in which the weighting matrix can be derived.
from the dependency of the error on magnitude. Therefore, the weighting matrix which is determined by using grouped data is defined as

$$W = \begin{bmatrix} w_1 & 0 & \ldots & 0 \\ 0 & w_2 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & w_N \end{bmatrix},$$

(42)

where

$$w_i = \frac{1}{\sigma_i^2} = \frac{1}{(0.59163 - 0.06026M_i)^2},$$

(43)

in which $\sigma_i^2$’s are variances of the bins. To estimate the coefficients of the equation the recursive algorithm is used and these coefficients are tabulated in tables 2–4 for PGA, PSA at

**Figure 1.** The distribution of the database used in this study in the magnitude-distance database for different fault mechanisms. Rectangular signs represent soft soil, cross signs are for stiff soil, and circle signs represent rock.

**Figure 2.** The distribution of the database with respect to the magnitude and distance.
The coefficients corresponding to this method with \( \gamma = 0.15 \) are listed in tables 2. The coefficients might not be accurate because the weight is estimated from the given data.

The weighted one-stage maximum likelihood method (method 3) is an iterative process. In this method, the natural logarithm of likelihoods (equation (17)) of different values of \( \gamma \) are computed to find the one that is associated with the maximum likelihood of the pdf of observations. The value of the \( \gamma \) can vary from 0 to 1. After exploring this interval, it is found out that the \( \gamma \) related to the maximum likelihood is located between 0.1 and 0.2. Figure 5 shows that the maximum likelihood occurs when \( \gamma = 0.15 \). The coefficients corresponding to this method with \( \gamma = 0.15 \) are listed in tables 2–4 for PGA, PSA at 0.2, and PSA at 1 s, respectively.

In the first stage of the two-stage method, coefficients according to the terms that are not constant for records of a specific earthquake are determined. Then, the remaining coefficients, \( a_1, a_2, a_6, a_9, \) and \( a_{10} \) are calculated in the second stage. Equation (23) can be defined as

\[
H = a_1 + a_2 M + a_3 F_N + a_4 F_T + a_5 F_O,
\]

and the unknown coefficients vector for the stage 1 is

\[
\theta_1 = [a_1, a_4, \Delta a_6, a_9, a_7, H_1, H_2, \ldots, H_N]^T.
\]

After finding vector \( \theta_1 \), vector \( Y_2 \) and matrix \( X_2 \), then vector \( \theta_2 \), can be determined using the equation (29). These coefficients with considering different weighting matrices, no weighting matrix (method 4), Boore and Joyner (1993) weighting matrix (method 5), and Fukushima and Tanaka (1990) weighting matrix (method 6) are provided in tables 2–4 for PGA, PSA at 0.2, and PSA at 1 s, respectively. It should be noted that for these three methods, coefficients \( a_3, a_4, a_5, a_6, \) and \( a_7 \) are identical. The final

<table>
<thead>
<tr>
<th>Method 1</th>
<th>Method 2</th>
<th>Method 3</th>
<th>Method 4</th>
<th>Method 5</th>
<th>Method 6</th>
<th>Method 7</th>
<th>ADSS05</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_1 )</td>
<td>1.30548</td>
<td>2.00106</td>
<td>1.42397</td>
<td>1.19829</td>
<td>1.20243</td>
<td>1.22584</td>
<td>1.02689</td>
</tr>
<tr>
<td>( a_2 )</td>
<td>0.01357</td>
<td>−0.10512</td>
<td>0.00761</td>
<td>0.03463</td>
<td>0.03066</td>
<td>0.01864</td>
<td>0.05509</td>
</tr>
<tr>
<td>( a_3 )</td>
<td>−2.40643</td>
<td>−2.85904</td>
<td>−2.52433</td>
<td>−2.37750</td>
<td>−2.37750</td>
<td>−2.37750</td>
<td>−2.27262</td>
</tr>
<tr>
<td>( a_4 )</td>
<td>0.20708</td>
<td>0.28136</td>
<td>0.22046</td>
<td>0.20684</td>
<td>0.20684</td>
<td>0.20684</td>
<td>0.18687</td>
</tr>
<tr>
<td>( a_5 )</td>
<td>7.64047</td>
<td>7.27960</td>
<td>7.43578</td>
<td>5.79273</td>
<td>5.79273</td>
<td>5.79273</td>
<td>7.45975</td>
</tr>
<tr>
<td>( a_6 )</td>
<td>0.12160</td>
<td>0.15040</td>
<td>0.10268</td>
<td>0.05583</td>
<td>0.05583</td>
<td>0.05583</td>
<td>0.11770</td>
</tr>
<tr>
<td>( a_7 )</td>
<td>0.00834</td>
<td>0.01979</td>
<td>0.01468</td>
<td>0.00744</td>
<td>0.00744</td>
<td>0.00744</td>
<td>0.00224</td>
</tr>
<tr>
<td>( a_8 )</td>
<td>0.00446</td>
<td>0.00346</td>
<td>−0.05073</td>
<td>−0.12384</td>
<td>−0.08097</td>
<td>0.00109</td>
<td>0.01905</td>
</tr>
<tr>
<td>( a_9 )</td>
<td>0.08411</td>
<td>0.08977</td>
<td>0.06443</td>
<td>0.07511</td>
<td>0.06569</td>
<td>0.07829</td>
<td>0.10545</td>
</tr>
<tr>
<td>( a_{10} )</td>
<td>−0.05097</td>
<td>−0.02227</td>
<td>−0.05634</td>
<td>−0.06639</td>
<td>−0.05600</td>
<td>−0.04017</td>
<td>−0.03417</td>
</tr>
<tr>
<td>( \tau )</td>
<td>—</td>
<td>—</td>
<td>0.27410</td>
<td>0.27584</td>
<td>0.27584</td>
<td>0.27584</td>
<td>—</td>
</tr>
<tr>
<td>( \varphi )</td>
<td>—</td>
<td>—</td>
<td>0.11514</td>
<td>0.13037</td>
<td>0.12000</td>
<td>0.11373</td>
<td>—</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>0.29590</td>
<td>0.29713</td>
<td>0.29760</td>
<td>0.30510</td>
<td>0.30081</td>
<td>0.29837</td>
<td>0.29611</td>
</tr>
</tbody>
</table>

Note: \( \sigma \) is the overall unbiased standard deviation for each method. Also, note that the ADSS05 coefficients are derived using the larger horizontal component instead of the geometric mean of two horizontal components.

Table 3. Coefficients of the GMPEs obtained for PSA at 0.2 s.

<table>
<thead>
<tr>
<th>Method 1</th>
<th>Method 2</th>
<th>Method 3</th>
<th>Method 4</th>
<th>Method 5</th>
<th>Method 6</th>
<th>Method 7</th>
<th>ADSS05</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_1 )</td>
<td>1.00960</td>
<td>1.29580</td>
<td>1.02710</td>
<td>0.64180</td>
<td>0.60730</td>
<td>0.61030</td>
<td>1.42260</td>
</tr>
<tr>
<td>( a_2 )</td>
<td>0.09580</td>
<td>0.04960</td>
<td>0.10370</td>
<td>0.15680</td>
<td>0.15460</td>
<td>0.14670</td>
<td>0.03630</td>
</tr>
<tr>
<td>( a_3 )</td>
<td>−2.03750</td>
<td>−2.23270</td>
<td>−2.06750</td>
<td>−1.77320</td>
<td>−1.77320</td>
<td>−1.77320</td>
<td>−2.34290</td>
</tr>
<tr>
<td>( a_4 )</td>
<td>0.16190</td>
<td>0.19220</td>
<td>0.16260</td>
<td>0.12780</td>
<td>0.12780</td>
<td>0.12780</td>
<td>0.20640</td>
</tr>
<tr>
<td>( a_5 )</td>
<td>7.48150</td>
<td>7.62260</td>
<td>7.43230</td>
<td>5.24070</td>
<td>5.24070</td>
<td>5.24070</td>
<td>7.99480</td>
</tr>
<tr>
<td>( a_6 )</td>
<td>0.09380</td>
<td>0.09700</td>
<td>0.08150</td>
<td>0.03510</td>
<td>0.03510</td>
<td>0.03510</td>
<td>0.09590</td>
</tr>
<tr>
<td>( a_7 )</td>
<td>0.02960</td>
<td>0.03390</td>
<td>0.03100</td>
<td>0.02060</td>
<td>0.02060</td>
<td>0.02060</td>
<td>0.02780</td>
</tr>
<tr>
<td>( a_8 )</td>
<td>0.06680</td>
<td>0.07080</td>
<td>0.01910</td>
<td>−0.11360</td>
<td>−0.01040</td>
<td>0.06710</td>
<td>0.06940</td>
</tr>
<tr>
<td>( a_9 )</td>
<td>0.11410</td>
<td>0.11800</td>
<td>0.09340</td>
<td>0.06990</td>
<td>0.08980</td>
<td>0.11570</td>
<td>0.11530</td>
</tr>
<tr>
<td>( a_{10} )</td>
<td>−0.05640</td>
<td>−0.02960</td>
<td>−0.06880</td>
<td>−0.10870</td>
<td>−0.06910</td>
<td>−0.04410</td>
<td>−0.05500</td>
</tr>
<tr>
<td>( \tau )</td>
<td>—</td>
<td>—</td>
<td>0.30580</td>
<td>0.30840</td>
<td>0.30840</td>
<td>0.30840</td>
<td>—</td>
</tr>
<tr>
<td>( \varphi )</td>
<td>—</td>
<td>—</td>
<td>0.09020</td>
<td>0.11580</td>
<td>0.09140</td>
<td>0.08430</td>
<td>—</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>0.31800</td>
<td>0.31820</td>
<td>0.31880</td>
<td>0.32940</td>
<td>0.32160</td>
<td>0.31970</td>
<td>0.31820</td>
</tr>
</tbody>
</table>

Note: \( \sigma \) is the overall unbiased standard deviation for each method. Also, note that the ADSS05 coefficients are derived using the larger horizontal component instead of the geometric mean of two horizontal components.

ADSS05: Ambraseys et al. (2005).
coefficients for the Boore and Joyner method are determined when $\tau = 0.12$.

In order to apply the GA (method 7), the objective function which is an unbiased estimator of the variance is defined as

$$\text{Objective Function } = 10 \log \left( \frac{1}{N-10} \sum_{i=1}^{N} |\log(y_i) - \log(\hat{y}_i)|^2 \right),$$  \hspace{1cm} (46)

where $N$ is the number of records in the database, $N - 10$ is the number of degrees of freedom (10 is the number of unknown parameters), $y_i$ is the observed value, and $\hat{y}_i$ is the estimated value of the ground motion parameter for the record $i$.

For the selection of chromosomes, the roulette wheel definition is used. The cross over probability $p_c$ and mutation probability $p_m$ are defined 0.8 and 0.05, respectively. The probability of the mutation is a small number to avoid losing suitable chromosomes. The number of generation and the size of population are 1000 and 200, respectively (Tavakoli and Pezeshk 2005, Sobhaninejad et al 2007). It should be pointed out that repetition of the GA estimation does not generate identical coefficient. In this regard, we run this algorithm twenty times and the result for each coefficient is then computed by taking the mean of estimated values for that coefficient. The estimated coefficients for the geometric mean of PGA and PSA at 0.2 and 1 s values of horizontal components are provided in tables 2–4, respectively.
Comparison of different methods

As it is seen from tables 2–4, the estimated coefficients from different methods are not the same or even in similar ranges. The overall unbiased standard deviation for each method is also given in tables 2–4. The minimum standard deviation is for the method 1 (OLS) and the maximum standard deviation is derived for the method 4. Statistical comparison can be an appropriate way to compare regression methods with each other to see which method gives the best answer. In order to assess the accuracy of different methods the residual analysis is utilized to check whether the basic assumptions of errors are valid. Therefore, various statistical tests are chosen and applied to compare these methods. These tests can be applied on residuals defined as the difference between the observed and predicted values. The residual, \( R \), in models is obtained by

\[
R = \log(y_{observed}) - \log(y_{predicted}).
\]  

(47)

These residuals tend to have a correlation with the variables (West et al. 2007), since they are normalized by dividing them to the corresponding standard deviation. Therefore, normalized residuals, \( Z \), are defined as

\[
Z = \frac{P(\log(y_{observed}) - \log(y_{predicted}))}{\sigma}.
\]  

(48)

where \( P \) is the transformation matrix and \( \sigma \) is the overall unbiased standard deviation given by

\[
\sigma = \sqrt{\frac{\sum_{i=1}^{N}(\log(y_{observed}) - \log(y_{predicted}))^2}{N - 10}},
\]  

(49)

in which \( N \) is the number of data and \( N - 10 \) is the number of the degrees of freedom. 10 is the number of unknown parameters. Normalized residuals should follow the standard normal distribution in which the mean is zero and the standard deviation is one. Following tests are used to compare the described regression methods.

Graphical

The first graphical method is the Histogram which graphically summarizes characteristics of the distribution of a dataset (Montgomery and Runger 2003, Mendenhall and Sincich 2007). Histograms or bars in figure 6 illustrate the distribution of the residuals obtained by using the geometric mean of PGA values of horizontal components, so the spread of data and skewness can be graphically observed. The red lines display the probability distribution function (pdf) of the standard normal distribution. The number of bins equal to the square root of the number of elements in data. As it can be seen the distributions of the residuals are not perfect, but they approximately follow the standard normal distribution.

A Quantiles–Quantiles (Q–Q) plot shows the quantiles of a normal distribution versus the quantiles of the sample which is standard residuals here in order to compare the sample distribution with the standard normal distribution (Montgomery and Runger 2003, Mendenhall and Sincich 2007). Quantiles are the inverse of the cumulative distribution function (cdf) of a population at defined intervals. The ‘+’ signs will be linear, if the sample is drawn from a normal distribution. Figure 7 indicates that the quantiles of the normalized residuals obtained by using the geometric mean of PGA values of horizontal components for methods 2 and 3 are matched on the quantiles of the standard normal distribution. For the remaining methods, there is a slight difference especially at the upper bound of the normalized residuals.

Goodness-of-fit tests

The basic assumption for the normalized residuals is to follow the standard normal distribution. Hence several statistical tests have been proposed to understand the goodness of a fit for a model (Montgomery and Runger 2003, Mendenhall and Sincich 2007). We employ some of these statistical measurements to investigate the quality of fits by different regression methods for the residuals obtained by using the geometric mean of PGA values of horizontal components.

The first test is the \( t \)-test in which the normalized residuals are assessed to see if the mean is zero (null hypothesis). If the null hypothesis cannot be rejected, the test statistic has a Student’s \( t \) distribution. \( P \)-values of the \( t \)-test are tabulated in table 5. When a \( p \)-value which describes the probability of the null hypothesis is equal to or less than the considered significance level which is 0.05 in this paper, then the null hypothesis would be rejected at this significance level. Results state that all methods fail to reject the null hypothesis.

The second test is the Var-test which is a Chi-square measure and it evaluates whether the normalized residuals have a unit variance (null hypothesis). The \( p \)-values corresponding to this test from table 5 show that null hypothesis cannot be rejected in all described methods.

The next test is the Chi-square goodness-of-fit test that recognizes if the normalized residuals come out of the standard
normal distribution. Calculated $p$-values from table 5 display the null hypothesis cannot be rejected in all methods.

The next test is the Lilliefors test (Lillie-test) in which residuals are compared to observe if they follow a normal distribution. In this test the mean and variance are unknown. Lillie-test $p$-values are gathered in table 5. As it can be seen all methods fail to reject the null hypothesis at the desired significance level.

The fifth test is the ks-test (Kolmogorov–Smirnov test) that determines whether the cdf of the normalized residuals is matched with the cdf of the standard normal residual (null hypothesis). The $p$-values are listed in table 5. According to the estimated values null hypothesis cannot be rejected in all methods.

The next test is the Jarque–Bera test which evaluates if the skewness and kurtosis of the normalized residuals correspond to the standard normal distribution. The $p$-values are provided in table 5. These amounts show that all models fail to reject the null hypothesis.

The seventh test is the Anderson–Darling test which checks if the pdf of the normalized residuals matches the pdf of the

---

**Figure 6.** Histograms of the normalized residuals for different regression methods. Solid line shows the normal distribution with a mean of 0 and standard deviation of 1, $N(0, 1)$, and bars represent the distribution of the standard residuals.

**Figure 7.** $Q–Q$ plots of the normalized residuals for explained regression methods. Solid line shows the normal distribution with a mean of 0 and standard deviation of 1, and ‘+’ signs represent the quantiles of each regression method.
The standard normal distribution. The \( p \)-values provided in table 5 indicate that the null hypothesis cannot be rejected in all methods.

The last one, Shapiro–Wilk test is a test to determine whether the residuals are drawn from a normal distribution. The \( p \)-values corresponding to this test are given in table 5 and they show that all methods fail to reject the null hypothesis.

A larger \( p \)-value implies that the model is more confident (Scherbaum et al. 2004). Regarding to these results, method 2 has the largest \( p \)-value for the Var-test, Jarque–Bera test, and Shapiro–Wilk test. Method 3 has the largest \( p \)-value for the Chi-square test, Jarque–Bera test, and Anderson–Darling test. Also, it can be inferred that method 4 and method 5 have the weakest performance compared to the remaining regression methods since they have 4 and 3 smallest \( p \)-values respectively.

**LH test**

Most of the goodness-of-fit measures can only check one of the assumptions related to the standard normal distribution. Scherbaum et al. (2004) have introduced a likelihood-based method (LH test) in which all the assumptions would be assessed. In the LH test the likelihood of an observation that is a normalized residual for being equal to or greater than \( Z_0 \) with considering both tails of the distribution is given by

\[
LH(Z_0) = \frac{2}{\sqrt{2\pi}} \int_{Z_0}^{\infty} \exp\left(-\frac{Z^2}{2}\right) dZ.
\]

where \( Z_0 \) is a normalized residual. If the normalized residuals are completely matched with the standard normal distribution, the amounts of the LH test would be uniformly distributed between 0 and 1 and the median of the LH values should be 0.5.

The distributions of the LH values for the residuals obtained by using the geometric mean of PGA values of horizontal components are demonstrated in figure 8. The values of the LH are uniformly distributed for methods 2 and 3. The median of LH (MEDLH) is given in table 6. In addition, the mean, median, and standard deviation of the normalized residuals are tabulated in table 6 (MEANNR, MEDNR, and STDNR, respectively). Moreover, standard deviations corresponding to the median, mean, and standard deviation and estimated using the ‘delete-1’ jackknife resampling (Shao and Tu 1995) procedure (Scherbaum et al. 2004) are provided in table 6.

---

**Table 5.** The \( p \)-values of different goodness-of-fit tests.

<table>
<thead>
<tr>
<th>Method</th>
<th>( t )-test</th>
<th>Var-test</th>
<th>Chi-square</th>
<th>Lillie-test</th>
<th>ks-test</th>
<th>Jarque–Bera test</th>
<th>Anderson–Darling test</th>
<th>Shapiro–Wilk test</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.99987</td>
<td>0.75023</td>
<td>0.13911</td>
<td>0.26845</td>
<td>0.62833</td>
<td>0.35249</td>
<td>0.19389</td>
<td>0.10416</td>
</tr>
<tr>
<td>2</td>
<td>0.99585</td>
<td>0.75023</td>
<td>0.19397</td>
<td>0.39233</td>
<td>0.69754</td>
<td>0.50000</td>
<td>0.29371</td>
<td>0.53273</td>
</tr>
<tr>
<td>3</td>
<td>0.81209</td>
<td>0.74862</td>
<td>0.29785</td>
<td>0.21513</td>
<td>0.68280</td>
<td>0.50000</td>
<td>0.54339</td>
<td>0.37534</td>
</tr>
<tr>
<td>4</td>
<td>0.39566</td>
<td>0.72977</td>
<td>0.10788</td>
<td>0.34001</td>
<td>0.74324</td>
<td>0.27547</td>
<td>0.15202</td>
<td>0.08841</td>
</tr>
<tr>
<td>5</td>
<td>0.47915</td>
<td>0.68060</td>
<td>0.07991</td>
<td>0.48723</td>
<td>0.17364</td>
<td>0.28109</td>
<td>0.19114</td>
<td>0.08952</td>
</tr>
<tr>
<td>6</td>
<td>0.99990</td>
<td>0.75023</td>
<td>0.25812</td>
<td>0.38929</td>
<td>0.71991</td>
<td>0.32272</td>
<td>0.31259</td>
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<tr>
<td>7</td>
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<td>0.75023</td>
<td>0.14083</td>
<td>0.18410</td>
<td>0.55332</td>
<td>0.38726</td>
<td>0.17859</td>
<td>0.12252</td>
</tr>
</tbody>
</table>

Note: Method 1: OLS; method 2: WLS; method 3: one-stage maximum likelihood; method 4: two-stage maximum likelihood with no weighting matrix; method 5: two-stage maximum likelihood with Boore and Joyner weighting matrix; method 6: two-stage maximum likelihood with Fukushima and Tanaka weighting matrix; method 7: GA.

---

![Figure 8. The distributions of the LH values for described regression methods. Each histogram has 10 bins with length of 0.1.](image-url)
The results of the LH test.

<table>
<thead>
<tr>
<th>Method</th>
<th>MEDLH</th>
<th>(\sigma)</th>
<th>MEANNR</th>
<th>(\sigma)</th>
<th>MEDNR</th>
<th>(\sigma)</th>
<th>STDNR</th>
<th>(\sigma)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.52184</td>
<td>0.00036</td>
<td>0.00001</td>
<td>0.00283</td>
<td>0.02931</td>
<td>0.00217</td>
<td>0.98702</td>
<td>0.00198</td>
</tr>
<tr>
<td>2</td>
<td>0.51991</td>
<td>0.00037</td>
<td>0.00027</td>
<td>0.00283</td>
<td>-0.01188</td>
<td>0.00128</td>
<td>0.98702</td>
<td>0.00207</td>
</tr>
<tr>
<td>3</td>
<td>0.53896</td>
<td>0.00039</td>
<td>-0.01255</td>
<td>0.00283</td>
<td>-0.01741</td>
<td>0.00016</td>
<td>0.98694</td>
<td>0.00198</td>
</tr>
<tr>
<td>4</td>
<td>0.50833</td>
<td>0.00039</td>
<td>-0.04482</td>
<td>0.00283</td>
<td>-0.02729</td>
<td>0.00051</td>
<td>0.98690</td>
<td>0.00198</td>
</tr>
<tr>
<td>5</td>
<td>0.53931</td>
<td>0.00038</td>
<td>-0.03303</td>
<td>0.00250</td>
<td>-0.01213</td>
<td>0.00361</td>
<td>0.98216</td>
<td>0.00175</td>
</tr>
<tr>
<td>6</td>
<td>0.50926</td>
<td>0.00036</td>
<td>-0.00001</td>
<td>0.00283</td>
<td>0.03689</td>
<td>0.00100</td>
<td>0.98702</td>
<td>0.00195</td>
</tr>
<tr>
<td>7</td>
<td>0.52508</td>
<td>0.00036</td>
<td>-0.00012</td>
<td>0.00283</td>
<td>0.02089</td>
<td>0.00047</td>
<td>0.98702</td>
<td>0.00198</td>
</tr>
</tbody>
</table>

Note: MEDLH: median of LH; MEANNR: mean of normalized residual; MEDNR: median of normalized residuals; STDNR: standard deviation of normalized residuals; \(\sigma\): corresponding standard deviation.

ANOVA table.

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>Prob &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Columns</td>
<td>0.73</td>
<td>6</td>
<td>0.122</td>
<td>0.129</td>
<td>0.99267</td>
</tr>
<tr>
<td>Error</td>
<td>2304.71</td>
<td>2443</td>
<td>0.943</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>2305.45</td>
<td>2449</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: SS: The sum of squares for each source; df: The degrees of freedom for each source; MS: The mean squares for each source; F: F-statistic.

Scherbaum et al (2004) have used the values of the LH median, plus the mean, median and standard deviation of the normalized residuals to rank different ground motion models for seismic hazard analysis. Based on this scheme models fall into 4 groups in this ranking scheme. Models are ranked class C, as the lowest acceptable group once the LH median is greater than 0.4 and the absolute values of the median and mean of the normalized residuals with corresponding standard deviations are less than 0.75 and the standard deviation for the normalized residuals smaller than 1.5. If the LH median is more than 0.3 and the absolute values of the median and mean of the normalized residuals with corresponding standard deviations are less than 0.5 and the standard deviation for the normalized residuals smaller than 1.25, models is ranked class B. Models are ranked class A, as the most satisfactory group when the LH median is larger than 0.4 and the absolute values of the median and mean of the normalized residuals with corresponding standard deviations are less than 0.25 and the standard deviation for the normalized residuals smaller than 1.125. Eventually, models that could not satisfy these criteria fall into the unacceptable group, class D. As it can be seen from table 6, all methods are classified as class A.

Analysis of variance

Since table 6 clearly shows that the estimated means of normalized residuals obtained by using the geometric mean of PGA values of horizontal components from different regression methods are not equal, the technique called analysis of variance (ANOVA) is used to compare the means of normalized residuals obtained from these methods (Montgomery and Runger 2003, Mendenhall and Sincich 2007). In one-way ANOVA, the null hypothesis here is defined as if samples (normalized residuals) come from populations with equal mean which is zero. The results from one-way ANOVA are tabulated in table 7 and the box plot of these methods is demonstrated in figure 9. A box plot graphically shows the centrality and skewness of the distribution of a dataset. This plot also indicates the 25th, 50th (median), and 75th percentiles, in addition to minimum, maximum and outlier values (Montgomery and Runger 2003, Mendenhall and Sincich 2007). In accordance with the \(p\)-values from table 7, the null hypothesis cannot be rejected at the 5% significance level. Figure 9 also depicts that there is no evidence of significant discrepancy between the normalized residuals derived from various regression methods.

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Constancy of the variance

Constancy of the variance of residuals is one of the basic assumptions about the residuals and the violation of this assumption is considered as a deficiency in the model. To overcome this problem, it is suggested to use a transformation on the variables in the functional form or to consider an appropriate weighting matrix in the regression analysis (Draper and Smith 1981). Another treatment is to keep those estimated coefficients but to use the true variable variance for the prediction of various percentile of ground motion (Ambraseys et al. 2005).

One way to check constancy of the variance assumption is to use the pattern of residuals against variables. When this pattern is like a funnel it illustrates that the variance is variable and therefore using a transformation or WLS method is necessary to satisfy the basic assumptions of the regression analysis. In this paper, the pure error analysis (Draper and Smith 1981) is performed to estimate the dependency of the standard error on the magnitude and distance. These equations are tabulated in table 8.

No weighting matrix was used for methods 1, 4, and 7, but a weighting matrix is considered in the remaining methods. As it can be observed from the gradients of the reported equations in table 8, the dependency of the standard deviation on the distance is insignificant, while the dependency on the magnitude still exists and it is considerable even for methods in which the weighting matrix is applied except method 2. In this regard, the dependency of the standard deviation on the magnitude cannot be rejected at the 5% significance level and it should be accounted for in the regression analysis. Therefore, all methods, except the WLS that accounts for the weight which is obtained from the pure error analysis of residuals, do not yield in obtaining BLUEs. It means that methods in which a blind weighting matrix is supposed are not helpful, so there is a need to use the pure error analysis of residuals to estimate the accurate weight and then consider it in the regression analysis.

Variance of the estimators

The variance covariance matrices of the unknown parameters using the geometric mean of PGA values of the horizontal components for different methods are estimated and the diagonal elements are considered to calculate the standard deviation of each coefficient. All standard deviations for these methods are given in table 9. As it was expected the method 2 (WLS) has the minimum standard deviations for the most coefficients.

Therefore, using a weighting matrix derived from the pure error analysis leads to having the optimal standard deviations for the coefficients. Furthermore, table 9 implies that the coefficients $a_1$, $a_3$, and $a_5$ possess the highest standard deviations.

<table>
<thead>
<tr>
<th>Method</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>$a_4$</th>
<th>$a_5$</th>
<th>$a_6$</th>
<th>$a_7$</th>
<th>$a_8$</th>
<th>$a_9$</th>
<th>$a_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.72253</td>
<td>0.10549</td>
<td>0.49021</td>
<td>0.07175</td>
<td>0.87900</td>
<td>0.04393</td>
<td>0.03683</td>
<td>0.04370</td>
<td>0.05381</td>
<td>0.05046</td>
</tr>
<tr>
<td>2</td>
<td>0.70504</td>
<td>0.09657</td>
<td>0.47516</td>
<td>0.06542</td>
<td>0.84180</td>
<td>0.04187</td>
<td>0.03609</td>
<td>0.04274</td>
<td>0.05191</td>
<td>0.04560</td>
</tr>
<tr>
<td>3</td>
<td>0.74655</td>
<td>0.10994</td>
<td>0.50651</td>
<td>0.07474</td>
<td>0.81604</td>
<td>0.04436</td>
<td>0.03646</td>
<td>0.06004</td>
<td>0.07003</td>
<td>0.06695</td>
</tr>
<tr>
<td>4</td>
<td>0.65429</td>
<td>0.09850</td>
<td>0.50321</td>
<td>0.07547</td>
<td>0.72072</td>
<td>0.04652</td>
<td>0.03800</td>
<td>0.04505</td>
<td>0.05558</td>
<td>0.05204</td>
</tr>
<tr>
<td>5</td>
<td>0.73068</td>
<td>0.11000</td>
<td>0.50640</td>
<td>0.07519</td>
<td>0.80486</td>
<td>0.05730</td>
<td>0.04244</td>
<td>0.05031</td>
<td>0.06207</td>
<td>0.05811</td>
</tr>
<tr>
<td>6</td>
<td>0.63986</td>
<td>0.09632</td>
<td>0.44066</td>
<td>0.06609</td>
<td>0.70482</td>
<td>0.04429</td>
<td>0.03716</td>
<td>0.04406</td>
<td>0.05435</td>
<td>0.05089</td>
</tr>
<tr>
<td>7</td>
<td>0.74883</td>
<td>0.10859</td>
<td>0.50606</td>
<td>0.07368</td>
<td>0.89589</td>
<td>0.04414</td>
<td>0.03701</td>
<td>0.04392</td>
<td>0.05405</td>
<td>0.05070</td>
</tr>
</tbody>
</table>

Figure 10. The comparison between estimated accelerations from different regression methods and observations for a normal fault at a stiff site. Circle and rectangular represent the geometric mean and larger horizontal component, respectively.

<table>
<thead>
<tr>
<th>Method</th>
<th>M5 to M6</th>
<th>M6 to M7</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Method 1</td>
<td>Method 2</td>
</tr>
<tr>
<td>Method 2</td>
<td>Method 3</td>
<td>Method 4</td>
</tr>
<tr>
<td>Method 3</td>
<td>Method 5</td>
<td>Method 6</td>
</tr>
<tr>
<td>Method 4</td>
<td>Method 7</td>
<td>ADSS95</td>
</tr>
</tbody>
</table>

Table 9. Standard deviations of unknown parameters in each regression method.
and it can be confirmed by table 2 since these coefficients have the larger discrepancy for different applied methods.

Comparison with data

To visually explore the differences between the regression methods, we plot the decay rate (attenuation) of estimated accelerations with distance as well as observations (see figure 10). To this end, stiff site condition and normal fault mechanism are considered in obtaining accelerations using the functional form since the most data available are in this category. As can be seen, the difference between the estimated accelerations obtained from different regression method is insignificant particularly at distances more than 5 km. The small difference between decay rates at short distances less than 5 km can be attributed to the lack of data for this range of distance. On the other hand, all methods overestimate the acceleration at long distances. This can be related to the effect of ignoring the anelastic or intrinsic attenuation (Sedaghati and Pezeshk 2016) term in the functional form. In fact, this plot demonstrates that choosing an appropriate functional form is very important, whereas the influence of selecting a specific regression method to acquire the coefficients of that chosen functional form is insignificant. In figure 10, the GMPE proposed by Ambraseys et al (2005) for the larger component of PGAs is also plotted for comparison. As can be seen, this equation is above all other GMPEs derived in this study, since we used the geometric mean of horizontal components.

We also used geometric mean of SA values of two horizontal components at periods of 0.2 and 1 s to investigate the performance of different regression methods. All aforementioned tests are done for SAs at 0.2 and 1 s and results are similar to the results inferred using PGA.

Summary and conclusions

OLS estimation supposes that all events have an identical weight and reliability and it ignores the correlation between different records from a specific event. This estimation has a simple algorithm and the solution can be quickly found. WLS estimation assumes that some earthquakes are more informative and consequently have higher weight than the other earthquakes, but it neglects the correlation between records measured for an earthquake at different sites. One-stage maximum likelihood estimation by Joyner and Boore (1993) accounts for the correlation between different records and different earthquakes. Two-stage methods with various proposed weighting matrices separate the coefficients which are constant for records of a specific earthquake such as the magnitude and fault mechanism from the other coefficients like the distance and site condition. These methods are computation inefficient. The GA has a simple concept but it is computationally very slow. In addition the solution is not unique and each time this method runs various results can be obtained. We can summarize the results as follows:

- Pure error analysis should be performed as a part of developing GMPEs to gain insight about the true variance of the derived equations which can be applied to estimate different percentiles of ground motions to be used in PSHA.
- Various goodness-of-fit tests, graphical methods, and the LH test demonstrate that the performance of the WLS estimation and one-stage maximum likelihood method is better in comparison to the other considered regression methods. Of course, the WLS methods is not perfect because it neglects the correlation between records. Thus, the one-stage maximum likelihood method in which the true variance obtained from the pure error analysis is considered can be selected as the best regression method to derive coefficients of GMPEs.
- The ordinary GA is very slow and does not result in improving the standard deviation. Also, the statistical tests illustrate that the distribution of residuals of GA is very similar to the distribution of residuals of OLS. Therefore, the OLS estimation is preferred compared to the ordinary GA.
- Two-stage methods are considered as worst regression methods because not only are slow and complicated but also yield in higher standard deviations for the functional form. Further, the pure error analysis shows that the true variance is not constant even after considering a weighting matrix in the regression procedure.
- Choosing an appropriate functional form is very important to develop GMPEs, whereas the influence of selecting a specific regression method to acquire the coefficients of that chosen functional form is insignificant.

Acknowledgments

We are indebted to Professor Nicholas N Ambraseys who provided us the CD ROM of European and Middle Eastern strong motion data released by Ambraseys et al (2000). May his soul rest in peace. The authors would also like to thank anonymous reviewers for their thoughtful comments to improve this paper. This project was partially supported by the Tennessee Department of Transportation.

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