Inelastic Displacement Spectra for Bridges Using the Substitute-Structure Method

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Abstract: The design of bridge structures for seismic loading by displacement-based procedures has become preferred over force-based procedures in modern specifications. One key step in displacement-based design (DBD) is the estimation of inelastic displacement. Current AASHTO specifications rely on a linear response spectrum analysis with an amplification factor, $R_d$, for conventionally designed bridges and on the substitute structure method (SSM) for analysis of isolated bridges to estimate inelastic displacements. Both methods are used in this study for a selected site and target acceleration response spectrum. The procedures for both are outlined in detail. Displacement spectra from nonlinear response history analyses (NLRHA) are generated for seven ground-motion sets and compared to results from the two simplified methods. The SSM is shown to produce inelastic displacement estimates that more closely match NLRHA results than do results from the AASHTO $R_d$ method over a wide range of periods for the specific conditions of (1) a large modal magnitude earthquake, (2) a class D subsurface profile, (3) accelerations characteristic of the selected site, and (4) reduced initial damping to mimic tangent-stiffness-damping. DOI: 10.1061/(ASCE)SC.1943-5576.0000279. © 2015 American Society of Civil Engineers.

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Introduction

Current AASHTO (2011) estimates of inelastic displacement for bridges of conventional design (nonisolated) rely on the equal displacement rule (EDR). EDR states that the inelastic displacement of a structure is equal to the displacement that the structure would have experienced if all elements remained in the elastic range, and is complemented in AASHTO with a short-period amplification factor, $R_d$. For isolated bridges, AASHTO (2010) adopts the substitute structure method (SSM) of analysis. SSM uses the secant stiffness to determine an effective period. Hysteretic behavior is treated as added viscous damping.

Other methods have been proposed for estimating the inelastic displacement without resorting to nonlinear response history analysis (NLRHA). Among these are methods that propose rules for a displacement amplification factor, $C_m$, to be applied to the elastic displacement as an estimate for inelastic displacement. More recently, Bozorgnia et al. (2010) proposed a ground-motion prediction model to estimate the inelastic displacement. Parameters $C_m$ and $R_d$ are directly analogous, simply different terminology.

In a recent study, Khose and Singh (2014) reported that the equivalent linearization methods for inelastic displacement, in the displacement-controlled spectral range, may underestimate response results. Further research will be helpful in developing tools to estimate inelastic response through simplified analytical methods in the design office.

The purposes of this study are to (1) demonstrate that the AASHOT $R_d$ method may underestimate inelastic response over a wide range of periods for large magnitude ground motions on Site Class D profiles, and (2) propose alternative methods for estimating inelastic displacement spectra.

To accomplish these objectives, a detailed discussion of the AASHTO $R_d$ method is provided. The SSM is fully described and other methods proposed in the literature are discussed. Important issues related to damping levels for both SSM and NLRHA are highlighted.

Seven ground-motion sets were scaled to a target response spectrum for a hypothetical bridge site on an AASHTO Class D subsurface profile. NLRHA analyses were performed on each set to develop the mean inelastic displacement spectra. The $R_d$- and SSM-based inelastic spectra were generated from the same target spectrum and compared to the results from NLRHA.

AASHTO $R_d$ Method for Conventional Design

Given the complexity of performing NLRHA and the potential difficulty in interpreting the results, it is certainly desirable to have a simplified, yet accurate, method of estimating inelastic displacement. Response spectrum techniques are the method of choice in most engineering offices. This is not likely to change soon.

Nonlinear behavior results in energy dissipation, which historically has been treated as added equivalent viscous damping. The stiffness is not constant during loading for nonlinear systems. The issue then becomes determining which values for effective damping and effective stiffness, when used in a response spectrum analysis, will produce results similar to those obtained in NLRHA.

One option for engineers is to use the initial stiffness and the elastic, initial-stiffness-based viscous damping in a linear response spectrum analysis to estimate nonlinear response. In this case, the assumption implies that for a given initial stiffness and a given ground motion, a yielding structure will experience the same displacement as a nonyielding structure. This equal displacement rule (EDR) assumption has been used in practice extensively and has
been shown to be valid, though not across the entire range of periods (Bozorgnia et al. 2010). The AASHTO $R_d$ method is based on the added assumption that the EDR is applicable only at periods longer than a characteristic site period, $T^*$. Therefore, an amplification factor, $R_d$, is applied to displacements at periods shorter than $T^*$. The characteristic site period, $T^*$, is given by Eq. (1).

$$T^* = 1.25T_S = 1.25 \frac{S_{D1}}{S_{D2}}$$ (1)

where $T_S$ = period defining the onset of the constant spectral velocity region for the design spectrum; $S_{D1}$ = spectral acceleration at a period of 1 s; and $S_{D2}$ = short-period spectral acceleration.

The amplification factor, $R_d$, according to AASHTO (2011), is determined by Eq. (2).

$$R_d = \left(1 - \frac{1}{\mu}\right) \frac{T^*}{T} + \frac{1}{\mu}$$ (2)

where $\mu$ = displacement ductility, defined as the ratio of maximum displacement to yield displacement.

Given the elastic response spectrum acceleration, $(SA_{EL})_T$, the inelastic displacement demand, $(SD_{INEL})_T$, follows as shown in Eq. (3).

$$(SD_{INEL})_T = R_d(SA_{EL})_T = R_d \left(\frac{T}{2\pi}\right)^2 g \cdot (SA_{EL})_T$$ (3)

$(SA_{EL})_T$ is the elastic spectral acceleration for 5% damping at the initial period, $T$. The process may be repeated at many different periods to obtain the inelastic displacement spectrum as generated using the AASHTO $R_d$ method.

**Substitute-Structure Method for Isolated Bridges**

The SSM has been incorporated into Guide Specifications for Seismic Isolation Design (AASHTO 2010). The method was first proposed by Gulkän and Sozen (1974) and further developed by Priestley et al. (2007) and others. For a single-degree-of-freedom (SDOF) oscillator, the secant stiffness, $K_{EFF}$, at maximum displacement with effective viscous damping, is used to establish inelastic displacement demands (see Fig. 1). Damping includes an initial elastic component (historically taken as 0.05 times the critical value) plus a hysteretic component. Various hysteretic rules are available for inelastic response (Priestley et al. 2007). For this study, the bilinear model was selected. According to Priestley and Grant (2005), the initial elastic component of damping ($\xi_o$) for the substitute structure analysis should be reduced from the typical 5% ($\xi_o$) by applying factors $\lambda_1$ and $\lambda_2$ to mimic a tangent-stiffness-based damping solution, as opposed to an initial-stiffness-based damping solution, as presented in Eqs. (4)–(6).

$$\xi_o = \xi_o \cdot \lambda_1 \lambda_2$$ (4)

$$\lambda_1 = \sqrt{\frac{\mu}{1 + \alpha \mu - \alpha}}$$ (5)

$$\lambda_2 = \alpha + \left(1 - \frac{\alpha}{\pi}\right) \left[\cos^{-1}\left(\frac{\mu - 2}{\mu}\right) - \frac{2(\mu - 2) \sqrt{\mu - 1}}{\mu^2}\right]$$ (6)

The parameter $\alpha$ is the ratio of postyield ($K_o$) to initial ($K_i$) stiffness values for a bilinear system, as seen in Fig. 1.

The total system damping ($\xi_{EFF}$) for SSM analysis is taken as the reduced elastic component, $\xi_o$, added to the hysteretic component ($\xi_{hys}$).

$$\xi_{EFF} = \xi_o + \xi_{hys}$$ (7)

The reduction applied to the elastic component is related to the belief that solutions for which the damping formulation is tangent-stiffness-based predict the true inelastic response of structures more accurately than do solutions for which the damping formulation is initial-stiffness-based (Priestley et al. 2007). Many computer solutions for inelastic response update the stiffness at each time step, but not the damping coefficient. It seems logical that the damping should be updated at each time step as well.

For the hysteretic component in Eq. (7), two options are considered here. Dwairi et al. (2007) proposed the model given by Eqs. (8a) and (8b) for the hysteretic damping component.

![](Fig. 1. Effective stiffness parameters)
\[ C_{EP} = \begin{cases} 85 + 60(1 - T_{EFF}), & T_{EFF} < 1 \\ 85, & T_{EFF} \geq 1 \end{cases} \]

where \( T_{EFF} \) is the secant-stiffness-based effective period of a bilinear oscillator [see Eq. (15), presented later].

Dwairi’s model is based on elastic-perfectly-plastic hysteresis combined with the analysis of a large set of ground-motion records. The model presented by Priestley et al. (2007) is the second model used in this study for the hysteretic component.

\[ \xi_{hys} = \frac{C_{EP}}{100} \left( \frac{\mu - 1}{\pi \mu} \right) \]  

(8a)

\[ C_{EP} = \frac{85}{\left( \frac{1}{T_{EFF}} \right)} \]  

(8b)

where \( T_{EFF} \) is the secant-stiffness-based effective period of a bilinear oscillator [see Eq. (15), presented later]. Typically, the Dwairi model results in a lower damping value than the bilinear model. Table 1 lists the \( \mu \) and \( \alpha \) combinations used in this study, along with the calculated damping values for the SSM analysis. Other parameters in Table 1 will be discussed later.

Once the effective damping has been established, a model for response modification due to the increased damping is needed. AASHTO has its own models and several others have been proposed (Priestley et al. 2007). Three of the models are given here, along with the Dwairi model for conventionally designed bridges, such as those relying on plastic hinging in columns. Eq. (12) is the EC8 model for far-field conditions and Eq. (13) has been proposed for sites where pulse-type ground motions are expected. Eq. (14) is an early model based on work by two of the pioneers in the field of earthquake engineering, Professors Nathan M. Newmark and William J. Hall.

\[ B_L = (\xi_{EFF} / 0.05)^{0.30} \leq 1.70, \text{ AASHTO (2010)} \]  

(10)

\[ R_\xi = (\xi_{EFF} / 0.05)^{0.40}, \text{ AASHTO (2011)} \]  

(11)

\[ R_\xi = (0.10 / (0.05 + \xi_{EFF}))^{0.50}, \text{ Eurocode 8} \]  

(12)

\[ R_\xi = (0.07 / (0.02 + \xi_{EFF}))^{0.25}, \text{ Pulse-type} \]  

(13)

\[ R_\xi = 1.31 - 0.191 \ln(100 \xi_{EFF}), \text{ Newmark–Hall} \]  

(14)

Some codes specify a factor, \( B_L \), by which the response is divided, while other models adopt a factor, \( R_\xi \), by which the response is multiplied. For isolated structures, AASHTO limits the reduction to \( B_L = 1.70 \), equivalent to \( R_\xi = 0.59 \). This is equivalent to limiting the total effective damping to 30% of critical and is adopted for the present study as well. Even at a small ductility, for example, \( \mu = 2 \), the hysteretic damping for an elastic-perfectly-plastic oscillator is 31.8%; thus, the 30% limit is recommended for use with the SSM analysis. Table 1 lists the theoretical effective damping values (\( \xi_{EFF} \)), but in computing \( R_\xi \), \( \xi_{EFF} \) was limited to 0.30 in this study.

Once the response modifier, \( B_L \) or \( R_\xi \), has been determined, the inelastic displacement computation is straightforward and given by Eqs. (15) and (16).

\[ T_{EFF} = 2\pi \sqrt{\frac{m}{K_{EFF}}} = T \sqrt{\frac{\mu}{1 + \alpha \mu - \alpha}} = \lambda_1 T \]  

(15)

\[ (SD_{INEL})_T = \frac{(SA_{EL})_{TEFF}}{B_L} \cdot g \cdot \left( \frac{T_{EFF}}{2\pi} \right)^2 = R_\xi \cdot (SA_{EL})_{TEFF} \cdot g \cdot \left( \frac{T_{EFF}}{2\pi} \right)^2 \]  

(16)

where \((SA_{EL})_{TEFF} = \text{elastic spectral acceleration at period } T_{EFF} \). Note that for \( T_{EFF} \), Eq. (15) is valid only in the case of a bilinear oscillator.

For either the AASHTO \( R_\xi \) method or the SSM, an inelastic displacement spectrum may be generated given an elastic acceleration spectrum. This elastic acceleration spectrum could be the mean of a suite of records or the code-based design response spectrum for a given site.

A curious aspect of the SSM is the predicted solution at low ductility values. The method predicts inelastic displacements smaller than elastic displacements over a wide range of periods when the ductility is reduced to a value dependent on the postyield stiffness ratio, \( \alpha \). When the effective period is within the constant velocity region of the target spectrum (i.e., when \( SA_{EL} \) is proportional to \( 1/T \)), the inelastic response displacement can be obtained as given by Eq. (17).

\[ (SD_{INEL})_T = \frac{S_{D1}}{\lambda_1 T} \cdot g \cdot \left( \frac{\lambda_1 T}{2\pi} \right)^2 = \frac{S_{D1}}{T} \cdot g \cdot \left( \frac{T}{2\pi} \right)^2 \cdot \lambda_1 \cdot R_\xi \]  

(17)

Eq. (17) can be simplified to Eq. (18).

\[(SD_{INEL})_T = (SD_{EL})_T \cdot \lambda_1 \cdot R_\xi \]  

(18)

where \((SD_{EL})_T = \text{elastic response spectral displacement at } 5\% \text{ viscous damping and at the initial period, } T \). Therefore, whenever the product of \( \lambda_1 \) and \( R_\xi \) is 1.00 and the effective period is within the constant velocity region of the target spectrum, the SSM predicts inelastic displacements exactly equal to the elastic displacement (i.e., the EDR is exactly satisfied). Using

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<th>( \xi_{eff}(%) )</th>
<th>( \mu_{\text{LIMIT}} )</th>
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</table>
the 30% cap on effective damping along with the AASHTO method [Eq. (10)] for computing \( R_f \), the ductility values (\( \mu_{\text{LIMIT}} \)) that make the product of \( \lambda_1 \) and \( R_f \) equal to 1.00 are given in Table 1. So, in a sense, the SSM confirms the EDR within certain period ranges. However, as previously noted, the damping equations [Eqs. (4)–(9)] are based on maximum ductility, while response to ground shaking is at levels smaller than the maximum for much of the load duration (with the effect more pronounced the lower the ductility). It seems logical that the theoretical effective damping may need to be reduced to obtain accurate results with SSM at low levels of ductility.

Before proceeding to the nonlinear response history analysis and comparison among methods, two other alternatives for inelastic response spectra generation are discussed briefly.

### Other Methods for Inelastic Displacement Spectra

Two additional methods for estimating inelastic displacements are summarized here. First, several studies have proposed displacement amplification factors to be applied to elastic displacements as an estimate for inelastic displacement. Among these is the model proposed by Watanabe and Kawashima (2014) in Eqs. (19) and (20).

\[
SD_{\text{INEL}} = C_{\mu} \cdot SD_{\text{EL}}
\]

where \( C_{\mu} = 1 - (c - 1) \left( \frac{T - a}{a e^{bT}} \right) \)

and \( a, b, c \) are values of \( C_{\mu} \) when \( T = 0 \); \( a \) is period at which \( C_{\mu} \) becomes 1 (i.e., the EDR is exactly satisfied); and the quantity \( (a + 1/b) \), as defined by Watanabe and Kawashima (2014), represents the period at which \( C_{\mu} \) takes on its minimum value. Values for the constants \((a, b, c)\) for ductility values of 2, 4, 6, and 8 were developed by Watanabe and Kawashima (2014). The study also provided estimation of the period at which the displacement amplification factor follows, not the EDR, but the EER (equal energy rule), i.e., the period at which:

\[
C_{\mu} = \frac{\mu}{\sqrt{2\mu - 1}}
\]

Another detailed analysis of proposed \( C_{\mu} \) factors is presented in an Earthquake Engineering Research Center report (Chopra and Chintanapakdee 2003).

As an alternative means of developing inelastic spectra for shallow crustal earthquakes on active tectonic regions, Bozorgnia et al. (2010) developed a ground-motion prediction equation for inelastic displacement of a bilinear, elastic-perfectly-plastic (EPP; \( \alpha = 0 \)) system using a logarithmic model given by Eq. (22).

\[
\ln \left( \frac{F_y}{W} \right) = f_{\text{mag}} + f_{\text{dis}} + f_{\text{fit}} + f_{\text{mag}} + f_{\text{site}} + f_{\text{sed}}
\]

where \( f_{\text{mag}} \) is a magnitude term, \( f_{\text{dis}} \) is a distance term, \( f_{\text{fit}} \) is a style-of-faulting term, \( f_{\text{mag}} \) is a hanging-wall term, \( f_{\text{site}} \) is a shallow site response term, and \( f_{\text{sed}} \) is a sediment depth term. The dependent variable was taken as \( F_y/W \) for the ground motion prediction equation (GMPE) development. This may be converted to a constant ductility inelastic displacement spectrum. Recognizing that \( F_y/K_i = D_f = D_{\text{MAX}}/\mu \), Eq. (23) may be derived.

\[
T = 2\pi \sqrt{\frac{W}{gK_i}} = 2\pi \sqrt{\left( \frac{F_y}{K_i} \right) \left( \frac{W}{F_y} \right) \left( \frac{1}{g} \right)}
\]

\[
= 2\pi \sqrt{\left( \frac{D_{\text{MAX}}}{\mu} \right) \left( \frac{W}{F_y} \right) \left( \frac{1}{g} \right)}
\]

\[
\therefore SD_{\text{INEL}} = D_{\text{MAX}} = \left( \frac{T}{2\pi} \right)^2 (\mu g) \left( F_y/W \right)
\]

The model requires definition of several seismological parameters including moment magnitude \( M_w \), rupture distance \( R_b \), Joyner-Boore distance \( R_y \), depth to top of rupture \( Z_{\text{TOP}} \), depth to attain a shear wave velocity of 2.5 km/s \( Z_{2.5} \), fault dip and rake, peak ground acceleration (PGA), and average shear wave velocity in the upper 30 m \( V_{30} \).

Bozorgnia et al. (2010) discuss the range of periods over which inelastic displacements are similar to elastic displacements, confirming the EDR, but for restricted period ranges.

### NLRHA

The inelastic spectra generation algorithm used for this study is an initial-stiffness-based damping algorithm (Hachem 2000). Hence, for NLRHA generation of spectra, the elastic damping is reduced in accordance with the approximate rule given by Priestley et al. (2007) and reproduced here as Eq. (24).

\[
\xi_{\text{NLRHA}} = \xi_{\text{EDR}} \left[ 1 - 0.1(\mu - 1)(1 - \alpha) \right] / \lambda_1
\]

By definition, \( \xi_{\text{NLRHA}} \) is the elastic component of damping, in an initial-stiffness-damping-based NLRHA, which will approximate tangent-stiffness-damping-based results. To be clear, the elastic damping in initial-stiffness-based damping NLRHA is taken as that given by Eq. (24). The effective damping in a simplified SSM analysis solution is taken as that given by Eq. (7). For NLRHA using an algorithm incorporating tangent-stiffness-damping, no reduction of the damping percentage would be required in the NLRHA. For the \( \mu \) and \( \alpha \) combinations used in this study, Table 1 lists the damping values used for NLRHA.

### Ground Motion Records for Nonlinear Analysis

For developing a target elastic acceleration response spectrum, a hypothetical bridge site is selected, characterized by a modal earthquake, \( M_w = 7.7 \), Site Class D \( (V_{30} \text{ between } 180–360 \text{ m/s}) \) subsurface conditions, and with a target spectrum based on three control points: \( \lambda_s = 0.445 \), \( S_{\text{DI}} = 0.906 \), and \( S_{\text{DI}} = 0.392 \). These values form the basis for the target spectrum shown in Figs. 2 and 3. The purpose of this study is not to suggest a record set appropriate for the hypothetical site, but to evaluate differences among the AASHTO \( R_f \) method, NLRHA, and the SSM in estimating nonlinear response.

Seven sets of 10 ground-motion pairs were used for this study. All records were obtained from PEER (Pacific Earthquake Engineering Research Center 2011). The records are described in Table 2.
While the purpose of this paper is not to synthesize a set of design ground motions for the hypothetical site, either of the last two sets might arguably be interpreted as such, given that match to spectral shape was identified in a recent study (NEHRP Consultants Joint Venture 2011) as the single most important factor in ground-motion selection at far-field sites. Other important factors include moment magnitude ($M_W$), source-to-site distance ($R$), tectonic setting (active tectonic versus stable continent), and site conditions ($V_{S30}$, profile depth, etc.). The NEHRP report is an excellent reference for engineers who need to select and scale ground motions for design. Further assistance in obtaining information for a site may be found with (1) hazard deaggregations (USGS 2015) to establish $M_W$, $R$, and $SA_{EL}$ data; and (2) the OpenSHA platform at OpenSHA.org (Field et al. 2003) for inferred $V_{S30}$ values.

For each selected record pair, scaling was performed on the geometric mean of as-recorded, horizontal components to minimize the mean-square-error (MSE) at periods between 0.09 and 10.0 s. This is a much wider range than would normally be used in practical ground-motion selection and scaling. With 10 record

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**Fig. 2.** Single event 10-record-pair mean spectra

**Fig. 3.** Mixed event 10-record-pair mean spectra
pairs, this results in 20 accelerograms per set. Each component for a given record received the same scale factor. The former online ground-motion database at PEER (Pacific Earthquake Engineering Research Center 2011) was used for the scaling and selection process. An updated version of the PEER ground-motion database is now available and contains many more records and scaling options (Pacific Earthquake Engineering Research Center 2014). The mixed-event set without restrictions (Set 6) includes the records with best fit to the target spectral shape without regard to earthquake magnitude, site conditions, source-to-site distance, or pulse characteristics. For the mixed-event set with restrictions (Set 7) the records with best fit to target spectral shape, with moment magnitude between 7.3 and 7.9, with $V_{530}$ between 0 and 760 m/s, with source-to-site distance less than 90 km, and with no pulse-type characteristics were selected. The choice of 760 m/s as the upper limit on $V_{530}$ is a liberty taken to more closely match spectral shape, given that the upper limit for Class D sites is 360 m/s.

The data for each of the record sets is presented in Tables 3–9, and the mean elastic acceleration spectra for the sets are shown over the target spectrum in Figs. 2 and 3. Included in the tables are the scale factors (SF) used for each record.

For the target spectrum chosen, note that $T^* = 1.25(S_D/S_{30}) = 0.54$ s. So, according to the AASHTO $R_d$ method, for periods greater than 0.54 s, the estimate of inelastic displacement is equal to the elastic displacement (i.e., the EDR applies).

Application of the SSM requires a determination of the long-period transition period, $T_T$. This is the period at which the displacement spectrum becomes a maximum. While AASHTO does not provide a means of determining $T_T$, FEMA includes maps for $T_T$ (FEMA 2009). For Imperial Valley, Loma Prieta, and Landers, $T_T = 8–12$ s. The value of $T_T$ selected for SSM analysis is critical, as this is the effective period at which the inelastic displacement spectrum reaches a plateau that will be apparent in the plots subsequently presented in this study. For this study, a value of $T_T = 12$ s
was adopted. Inelastic spectra in this study have been based on initial period (i.e., when the initial period is $T_f$, find the inelastic displacement, $SD_{INEL}$). Initial period, rather than effective period, was selected because the analysis used for bridge structures in AASHTO currently leads the engineer to estimate initial periods of the structure, not effective periods. Hence, when the effective period is $T_L$, the initial period is given by Eq. (25).

$$T = T_f \left(1 + \frac{\alpha \mu - \alpha}{\mu} \right)$$

The corresponding inelastic displacement—the maximum over all periods—is given by Eq. (26).

$$SD_{INEL,MAX} = \frac{SD_{H}}{T_f} \cdot g \cdot \left(\frac{T_f}{2\pi} \right)^2 \cdot R_{\xi}$$

The periods at which the maximum inelastic displacement occurs, and the maximum inelastic displacements, are presented in Table 10. The inelastic displacement is the same for all cases because the effective damping for SSM analysis was limited to 30% ($\xi_{EFF} = 0.30$) for each case.

### Table 8. Best Fit Record Pairs 1 (No Restrictions on $M_W$)

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<th>$R_{rup}$ (km)</th>
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<td>274.5</td>
<td>1.2269</td>
</tr>
</tbody>
</table>

### Table 9. Best Fit Record Pairs 2 (Restrictions on $M_W$)

<table>
<thead>
<tr>
<th>NGA number</th>
<th>EQ Name</th>
<th>Station name</th>
<th>$R_{gb}$ (km)</th>
<th>$R_{rup}$ (km)</th>
<th>$V_{S30}$ (m/s)</th>
<th>SF</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>Kern Co</td>
<td>Taft Lincoln</td>
<td>38.42</td>
<td>38.89</td>
<td>385.4</td>
<td>2.5013</td>
</tr>
<tr>
<td>1149</td>
<td>Kocaeli</td>
<td>Atakoy</td>
<td>56.49</td>
<td>58.28</td>
<td>310.0</td>
<td>3.0919</td>
</tr>
<tr>
<td>1158</td>
<td>Kocaeli</td>
<td>Duzce</td>
<td>13.6</td>
<td>15.37</td>
<td>281.9</td>
<td>0.9681</td>
</tr>
<tr>
<td>1177</td>
<td>Kocaeli</td>
<td>Zeytincarnu</td>
<td>59.18</td>
<td>59.88</td>
<td>341.6</td>
<td>3.2370</td>
</tr>
<tr>
<td>1211</td>
<td>Chi-Chi</td>
<td>CHY052</td>
<td>38.7</td>
<td>39.02</td>
<td>573.0</td>
<td>3.4082</td>
</tr>
<tr>
<td>1236</td>
<td>Chi-Chi</td>
<td>CHY088</td>
<td>37.48</td>
<td>37.48</td>
<td>318.5</td>
<td>1.9942</td>
</tr>
<tr>
<td>1495</td>
<td>Chi-Chi</td>
<td>TCU055</td>
<td>6.34</td>
<td>6.34</td>
<td>359.1</td>
<td>1.2668</td>
</tr>
<tr>
<td>1521</td>
<td>Chi-Chi</td>
<td>TCU089</td>
<td>0</td>
<td>9</td>
<td>671.5</td>
<td>1.3418</td>
</tr>
<tr>
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<td>Chi-Chi</td>
<td>TCU122</td>
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<td>475.5</td>
<td>1.2615</td>
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<td>Chi-Chi</td>
<td>TCU129</td>
<td>1.83</td>
<td>1.83</td>
<td>511.2</td>
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</table>

### Analysis of Selected Ground Motions

*SeismoSpect 1.3.0* was used to generate ground-motion parameters for the records used in the analysis. Table 11 summarizes the mean ground-motion parameters for each record set. The most glaring differences among the various record sets are the values for peak ground displacement (PGD) and for the ratio of PGD to peak ground velocity (PGV), each of which is smaller for the Loma Prieta, Imperial Valley, and Landers record sets. The Arias Intensity and the cumulative absolute velocity are larger for the Landers record set than for any other. Finally, the significant duration is significantly lower for Loma Prieta than for any of the other record sets. PGD, PGD/PGV, Arias Intensity, and significant duration have each been proposed as measures of damage potential from strong ground shaking.

### Inelastic Displacement Spectra Estimation

Inelastic displacement spectra were initially computed for two post-yield stiffness values, $\alpha = 0.00$ (EPP) and $\alpha = 0.05$, and for two displacement ductility values, $m = 6$ and $m = 12$. Subsequent to the initial parameter sets, EPP analyses were run at $m = 2$ and $m = 4$.

Three methods were used to compute the inelastic displacement spectra:
1. NLRHA using *Bispec 1.1.2* (Hachem 2000). An updated version of *Bispec*, with many more features than the version used here, is now available
2. The AASHTO $R_p$ method
3. SSM

Ductility values chosen for this work include the maximum permitted in AASHTO for conventionally designed bridges ($\mu = 6$) and a much higher value ($\mu = 12$), taken to be representative of a
bridge on very stiff substructures isolated with lead-rubber-bearings (LRB).

The $C_m$ method was not considered further in this study as the intent was to bypass the need to compute an amplification factor to be applied to elastic displacements, and instead to directly generate inelastic displacement spectra. The PEER GMPE was not pursued further in this study, since the goal was to generate inelastic spectra directly from a target elastic acceleration spectrum without detailed knowledge of the seismological parameters needed in the GMPE.

For ductility values of 6 and 12, no reduction in the idealized hysteretic damping given by Eq. (9) was incorporated. For a ductility value of 4, the Dwairi hysteretic damping model of Eq. (8) was adopted. For a ductility value of 2, the idealized model of Eq. (9) was initially used, and a reduced value for effective damping equal to 20% of that indicated by the idealized value was used.

For a large range of periods, underestimated response results, relative to NLRHA results, were generated by the AASHTO $R_d$ method. Figs. 4–7 illustrate the results for ductility values of 6 and 12, in terms of inelastic spectral displacement versus initial period. Recall that the AASHTO $R_d$ method uses the EDR for periods larger than $T^*$, which is equal to 0.54 s in the present study. Hence, for periods beyond 0.54 s in each of the plots, the AASHTO $R_d$ method and the elastic spectrum coincide. Clearly, for each of the record sets adopted in this study, the SSM provides more severe and more accurate results (assuming that NLRHA provides the correct answer) at these high ductility values of 6 and 12. Displacement amplification at periods much longer than 0.54 s is indicated in the plots for both NLRHA and SSM results.

Fig. 8 depicts the results for the case in which $m = 2$, $a = 0$, and no reduction in idealized effective damping is taken. Using the rationale developed in an earlier section (“The Substitute-Structure Method for Isolated Bridges”), the limit ductility for this condition is $\mu_{limit} = 2.93$. As indicated in Fig. 8, the SSM, in fact, predicts inelastic displacements below the elastic displacements (since $\mu = 2 < 2.93$) when the full theoretical effective damping is used. It is also evident in Fig. 8 that the NLRHA results do not support the decrease in inelastic displacement below elastic displacement. Fig. 9 was obtained by reducing the effective damping to 20% of the calculated value and provides one means of estimating inelastic displacements for low ductility values. Fig. 10 represents results for the case of $\alpha = 0.00$, $m = 4$, and the Dwairi damping rule, which produced results more closely matching NLRHA for this case.
The prediction of lower inelastic displacements compared to elastic displacements for low ductility values is an issue which requires more study.

Summary and Conclusions

Seven sets of ground-motion record pairs (10 record pairs per set) were selected and scaled to the target response spectrum for a site. NLRHA on bilinear single-degree-of-freedom systems of varying postyield to initial stiffness ratios ($\alpha$ values) were carried out for each of the seven record sets and inelastic displacement spectra were generated.

Two simplified analysis methods—SSM and the AASHTO $R_d$ method—were used to generate inelastic displacement spectra from the target acceleration spectrum for comparison to NLRHA results.

Methods from the literature were proposed by which appropriate effective viscous damping levels may be computed for both the SSM ($\xi_{Eff}$) and for NLRHA ($\xi_{NLRHA}$).

No near-field effects were assessed in this study. For near-field sites, a response modifier for increased effective damping that incorporates pulse behavior, such as that from previous research by...
others [Eq. (13)], could be adopted. For nonisolated structures, the use of a modified rule for effective damping could be employed with all other aspects of the procedure identical to that presented here. For example, Priestley et al. (2007) include a model given by Eq. (27) for thin Takeda hysteresis (with other models included as well). Thin Takeda hysteresis has been proposed as applicable to bridge columns experiencing plastic hinging. The cited reference would prove most helpful in developing inelastic displacement spectra for hysteretic rules other than the bilinear rule used in this study.

\[
\xi_{\text{EFF}} = \mu^{-0.378} \xi_{\text{el}} + 0.215 \left( 1 - \frac{1}{\mu^{0.642}} \right) \times \left( 1 + \frac{1}{(T_{\text{EFF}} + 0.824)^{0.444}} \right)
\]

(27)

The substitute-structure method for estimating inelastic displacement in bilinear oscillators provided more accurate results compared to the AASHTO \( R_d \) method for the record sets used in this study.

**Fig. 7.** Inelastic displacement spectra; \( \alpha = 0.05, \mu = 12 \)

**Fig. 8.** Inelastic displacement spectra; \( \alpha = 0.00, \mu = 2 \)
this study. For this reason, the substitute structure method is proposed as a potential alternative for estimating inelastic displacements in bridge structures, isolated or otherwise. Specifically, the following conclusions are made:

- For ductility values of 6 and 12, and for postyield stiffness values of 0.00 and 0.05, SSM inelastic displacement results computed by the guidelines presented in this study closely matched NLRHA results over a wide range of periods.
- SSM-based inelastic displacement spectra generally followed the NLRHA-based spectra more closely than did the $R_d$-based spectra for the specific conditions of (1) a large modal magnitude earthquake, (2) a Class D subsurface profile, (3) accelerations characteristic of the selected site, and (4) reduced initial damping to mimic a tangent-stiffness-based damping solution.
- AASHTO $R_d$ method analytical results underestimated NLRHA inelastic displacements over a significant period range for the ground-motion sets used in this study. The $R_d$ method is currently used in AASHTO for ductility levels of $\mu = 6$ and less.
- The SSM does, in fact, confirm the EDR, though over a limited period range. The range over which the EDR applies,
according to SSM theory, is a function of $\mu$, $\alpha$, $\xi_{EFF}$, $R_{\xi}$, and shape of the elastic acceleration spectrum.

- For low levels of ductility, damping from idealized, theoretical expressions may need to be reduced in order to obtain accurate estimates of inelastic displacement using SSM.
- The critical factors that determine the spectral shape for inelastic displacement are $S_{DI}$, $S_{DS}$, $T_i$, and $R_{\xi}$. The selection of a value for $T_i$ determines the maximum inelastic displacement and the period at which that displacement occurs.

Three alternatives to the AASHTO $R_T$ method are proposed for consideration in estimating inelastic displacements in bridge structures, as follows:

1. Inelastic displacement spectra derived from the SSM with secant stiffness, $K_{EFF}$; appropriate effective damping, $\xi_{EFF}$; and appropriate response modification, $R_{\xi}$.
2. Inelastic displacement spectra derived from NLRHA on appropriately selected and scaled ground-motion records using either (1) an initial-stiffness-damping-based algorithm with reduced initial elastic damping intended to mimic a tangent-stiffness-damping-based solution, or (2) a tangent-stiffness-damping-based algorithm; and
3. The GMPE from the literature (Bozorgnia et al. 2010) for shallow crustal earthquakes on active tectonic regions when it is possible to define the required seismological parameters ($M_w$, $R_{\text{JK}}$, $Z_{\text{TOR}}$, etc.).

Both SSM and NLRHA analyses for this study incorporated damping values intended to mimic a solution corresponding to tangent-stiffness-based damping as opposed to a solution corresponding to initial-stiffness-based damping.

**Suggestions for Practical Application**

The SSM may prove useful to engineers needing inelastic displacement spectra for the design of bridge structures. The method apparently provides reasonable estimates of inelastic displacement spectra when appropriate damping rules, dependent upon the type of hysteretic response expected during strong ground shaking, are applied. To apply the SSM in developing inelastic displacement spectra in the design office, the engineer needs each of the following:

1. A design basis elastic acceleration response spectrum for 5% damping, which provides the starting point. This data is typically available in the design specification (AASHTO). In some cases, the engineer may need design response spectra for different return periods. In such cases, one of the online USGS deaggregation tools (USGS 2015) will prove useful. The 2008 deaggregation tool provides data (spectral accelerations, magnitude, and distance) based on rotated geometric mean spectra, while the 2002 deaggregation tool provides as-recorded geometric mean spectra. Current bridge design practice is apparently based on as-recorded geometric mean spectra, while the use of the 2002 data may be preferred over the 2008 data, though the differences between the two may be small. For further discussion on these issues, consult the literature (Watson-Lamprey and Boore 2007; Baker and Cornell 2006). Note, as well, that the nature of the ground motion in terms of geometric-mean versus maximum-direction spectra is important and varies among design codes (Ghosh 2014; Stewart et al. 2011). Current AASHTO design ground motions for bridges are geometric-mean based while current ASCE7 ground motions for buildings are maximum-direction based.
2. For the type of hysteretic behavior expected in the structure, a rule for effective viscous damping, $\xi_{EFF}$, is required. Priestley et al. (2007) or Priestley and Grant (2005) are both excellent references for estimation of this parameter for various hysteretic rules.
3. A model for response modification, $R_{\xi}$, due to damping in excess of 5% of critical is required. Several of these models were discussed in the paper [Eqs. (10)–(14)]. Another model (Stafford et al. 2008), which includes duration effects, may prove useful when the duration of strong ground shaking is an explicitly defined or known value.
4. A method for computing the effective period, $T_{\text{EFF}}$, is needed. It would seem that in most cases, the expression used here [Eq. (15)] is adequate. For situations in which this is judged inadequate, alternative expressions may be found in the literature (Blandon 2004).
5. The long-period transition period, $T_L$, is needed to determine where the design spectrum transitions from constant velocity to constant displacement and, hence, the period at which the displacement spectrum is capped. Values for $T_L$ may be determined from FEMA maps (FEMA 2009). With these five tools in hand, the engineer may construct an inelastic displacement spectrum using the SSM outlined in this paper.

When properly selected and scaled ground motions (NEHRP Consultants Joint Venture 2011) are available, inelastic spectra may be generated using NLRHA. One of the more important criteria for ground-motion selection is match to spectral shape. Eq. (28) (Katsanos et al. 2010) is useful in assessing the shape match between record and target spectra.

$$D_{\text{RMS}} = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{(SA_{\text{GM}})_i}{PGA_{\text{GM}}} - \frac{(SA_{\text{TAR}})_i}{PGA_{\text{TAR}}} \right)^2$$

The smaller $D_{\text{RMS}}$, the more closely the record spectrum shape matches the target spectrum shape. $N$ is the number of periods defining the period range over which match to spectral shape is to be evaluated.

**Notation**

The following symbols are used in this paper:

- $B_L$ = factor by which displacement response is divided to estimate inelastic effects due to increased effective damping;
- $C_\mu$ = factor by which elastic displacement is multiplied to estimate inelastic displacement for a given ductility;
- $D_{\text{MAX}}$ = maximum inelastic displacement;
- $D_{\text{RMS}}$ = measure of spectral shape match;
- $D_s$ = yield displacement;
- $K_d$ = postyield stiffness of a bilinear system;
- $K_i$ = initial stiffness of a bilinear system;
- $N$ = number of periods defining a range for spectrum match evaluation;
- $PGA_{\text{GM}}$ = PGA for a ground motion record;
- $PGA_{\text{TAR}}$ = PGA for a target response spectrum;
- $R_{\xi}$ = factor by which displacement response is multiplied to estimate inelastic effects due to increased effective damping;
- $SA_{\text{EL}}$ = elastic response spectrum acceleration at initial period, $T$.

(SA_{EL})_{TEFF} = \text{elastic response spectrum acceleration (g) at effective period, } T_{EFF};
(SA_{GM})_{TI} = \text{ground motion spectral acceleration (g) at period } T_i;
(SA_{TAR})_{TI} = \text{target spectral acceleration (g) at period } T_i;
SD = \text{spectral displacement};
(SD_{EL})_{TI} = \text{elastic response spectrum displacement at initial period, } T_i;
(SD_{INEL})_{TI} = \text{inelastic response spectrum displacement at initial period, } T_i;
S_{DS} = \text{design spectral acceleration at a period of 0.2 s for a site};
S_{DI} = \text{design spectral acceleration at a period of 1 s for a site};
SF = \text{scale factor by which a ground motion record is multiplied};
T = \text{initial period of a bilinear oscillator};
T_{EFF} = \text{secant-stiffness-based, effective period of a bilinear oscillator};
T_L = \text{period at which spectral displacement reaches a maximum};
V_{S30} = \text{average shear wave velocity in the upper 30 m of a soil profile};
Z_{TOR} = \text{depth to top of rupture};
Z_{2.5} = \text{depth to } V_{S30} = 2.5 \text{ km/s};
\alpha = \text{ratio of postyield to initial stiffness values for a bilinear system};
\lambda_1 = \text{ratio of effective period to initial period};
\lambda_2 = \text{factor used in determining the appropriate value for initial viscous damping};
\mu = \text{displacement ductility} = D_{MAX}/D_0;
\mu_{LIMIT} = \text{displacement ductility at which SSM predicts inelastic displacements exactly equal to elastic displacements in the constant velocity region of the target spectrum};
\xi_{el} = \text{initial, elastic damping; historically taken as 0.05};
\xi_o = \text{initial, elastic component of viscous damping used in the SSM to mimic a tangent-stiffness-based solution};
\xi_{sys} = \text{added viscous damping from hysteretic behavior};
\xi_{EFF} = \text{total, effective, equivalent viscous damping used in the SSM};
\xi_{NLRHA} = \text{initial, elastic component of viscous damping used in an initial-stiffness-based NLRHA algorithm to mimic a tangent-stiffness-based solution}.

References

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