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An Improvement on the Estimation of Pseudoresponse Spectral Velocity Using RVT Method

by L. Liu and S. Pezeshk

Abstract The basic assumption in the prediction of peak ground acceleration (PGA), peak ground velocity (PGV), and pseudoresponse spectral values by the random vibration theory (RVT) method is that the ground motion process is a bandlimited Gaussian random process (BGRP). However, for the estimation of pseudoresponse spectral values, the process is the output of a single-degree-of-freedom (SDOF) system subjected to the input of a BGRP. The output process is a narrow-band random process because a SDOF system acts as a narrow-bandpass filter. The property of a narrow-band process is significantly different from that of a bandlimited process. There is an obvious difference in the estimations of the pseudoresponse spectral values based on bandlimited or narrow-band process, especially in the lower frequency part. In this study, we propose an empirical method to improve the estimation of the pseudoresponse spectral values by the RVT based on the consideration of properties of a narrow-band Gaussian stationary process. Comparisons of our results with those of previous research studies and the time domain simulation (TDS) shows that our empirical approach improves the estimation of pseudoresponse spectral values in the long period range.

Introduction

The spectral response of ground motion, along with the peak ground acceleration (PGA) and peak ground velocity (PGV) are generally considered to be the characteristics of earthquake ground motion for the seismic design of engineering structures. Because of a lack of available strong ground motion recordings, Hanks (1979), McGuire and Hanks (1980), and Hanks and McGuire (1981) proposed a theoretical method to estimate these characteristics. They treated the ground motion as bandlimited finite-duration Gaussian white noise, with a source spectrum given by the omega-square model by Brune (1970, 1971) for far-field shear radiation. Hanks and McGuire (1981) used Parseval's theorem to predict the root mean square (rms) acceleration (a_{rms}) from the integral of the squared acceleration spectrum. Then, they used the random vibration theory (RVT) to relate the a_{rms} to the PGA. Boore (1983) and McGuire *et al.* (1984) extended the model to predict the PGV and the pseudorelative spectral velocity (PRSV). Boore (1983) estimated the PRSV using the general properties of bandlimited random process in the frequency domain and also from simulated accelerograms by filtering Gaussian white noise in the time domain. McGuire *et al.* (1984) obtained closed-form analytical solutions according to the RVT.

The prediction of pseudorelative spectral velocity in the lower frequency range using Boore's (1983) method is not as good as that of the PGA and the PGV. Boore and Joyner (1984) redefined the duration of ground motion time history on the basis that the ground shaking includes not only a stationary part but also a nonstationary part. Although their empirical relation improved the estimation of PRSV, the difference between the results of the RVT method and time domain simulations is obvious in some cases, especially in the long period part.

In this study, we proposed an improved relation to relate the duration correction with a property of narrow-band stationary Gaussian random processes. The results for various magnitudes, epicentral distances, and damping ratios show that this method improves the prediction of PRSV in most cases, particularly at lower frequencies.

Analysis

The Fourier amplitude spectra used to estimate the PGA or the PGV, by most authors, such as Aki and Richards (1980), McGuire and Hanks (1980), Herrmann and Kijko (1983), and Boore (1983), are based on the earthquake source spectrum given by the omega-square model by Brune (1970, 1971). Therefore, the power spectral density of ground motion is bandlimited with the corner frequency f_c of seismic source as the lower boundary and the high-cut frequency f_m as the upper boundary. However, when the pseudoresponse spectral values are estimated, the process is the output of a single degree-of-freedom (SDOF) system subjected to the input of a bandlimited random process. Hence, the Fourier amplitude spectra for the estimation of PRSV will be the Fourier amplitude spectra for PGA multiplied by the following function:

$$I(f, f_0) = \frac{f_0^p}{[(f^2 - f_0^2)^2 + (2\xi f f_0)^2]^{1/2}}, \tag{1}$$

where f_0 is the natural frequency of the SDOF system, ξ is the damping ratio of spectrum, the parameter p is a constant where $p = 2$ for pseudoresponse spectral acceleration (PRSA), and $p = 0$ for pseudoresponse spectral velocity. Because the damping ratio of most structures is less than 10%, there is a spike at f_0 of the function $I(f)$. The area of the power spectral density function of the response of a SDOF system will concentrate around f_0. Therefore, the response of the SDOF oscillator is a narrow-band random process with the natural frequency f_0 of the SDOF system as its central frequency of the power spectral density function. Figures 1 and 2 show the Fourier amplitude spectra for the estimation of PGA and PRSV at 2 Hz with damping ratio $\xi = 5\%$, magnitude $M = 7$, and epicentral distance $R = 10$ km. As it can be observed, the Fourier amplitude spectra is a narrow-band for the estimation of PRSV. Therefore, the prediction of PRSV should be based on the properties of a narrow-band stationary Gaussian random process.

To estimate PGA, PGV, or PRSV from the RVT method, we first calculate the rms value of the process according to Parseval's theorem (McGuire and Hanks, 1980). Then we determine the ratio of the maximum value and the rms. Hence, we can compute the maximum value from root mean square and the ratio. Boore (1983) estimated the ratio based on the number of the stationary random processes that upcross a certain level. However, for a narrow-band random process, the crossings tend to occur in clumps (Nigam,

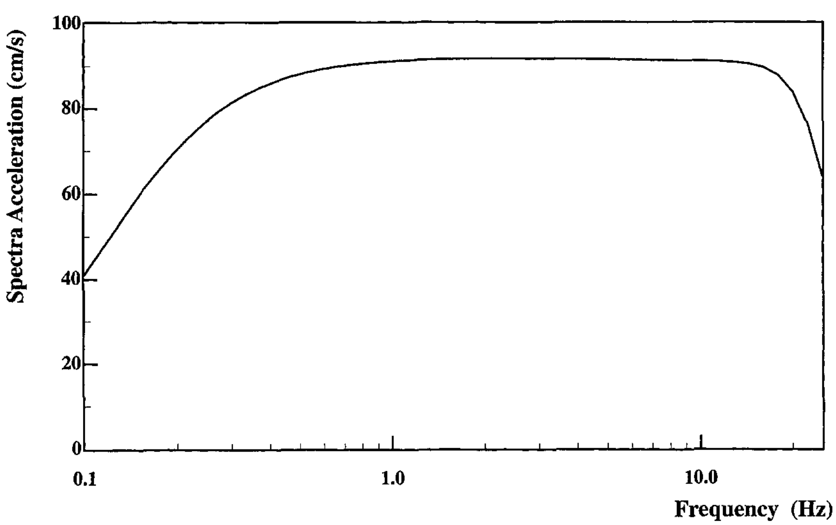

Figure 1. Fourier amplitude spectra of acceleration of bandlimited Gaussian random process.

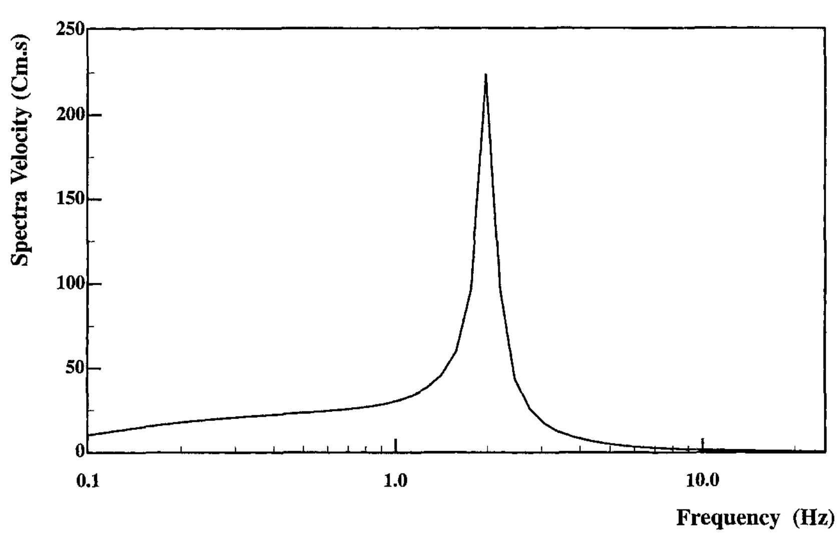

Figure 2. Fourier amplitude spectra of velocity response of a single degree-of-freedom structure subject to input of bandlimited Gaussian random process.

1983). The number of crossings are always overestimated when a process up-crossing number is used. This is reflected in the estimation of PRSV, particularly in the lower frequency range. Boore and Joyner (1984) attributed the overestimation to the definition of the duration of ground motion time history. Because the ground motion time history includes a stationary part and a nonstationary part, the real duration (which may affect the prediction of PRSV at long periods) of ground motion is usually longer than that of the stationary part. To consider the difference in duration, Boore and Joyner (1984) proposed the following relation to calculate the rms duration, D_{rms}, for the estimation of the rms value of the random process:

$$D_{rms} = D_s + D_0\left(\frac{\gamma^n}{\gamma^n + \alpha}\right), \tag{2}$$

where D_s is the duration of random process, and n and a are constants. They chose $n = 3$ and $a = 1/3$, respectively. The parameter γ is defined as

$$\gamma = D_s/T_0 \tag{3}$$

and

$$D_0 = T_0/2\pi\xi, \tag{4}$$

where T_0 is period of the SDOF system and ξ is the damping ratio of the spectra to be estimated.

To consider the property of narrow-band random process, instead of the constant α, we proposed to use the spectral density shape factor k as

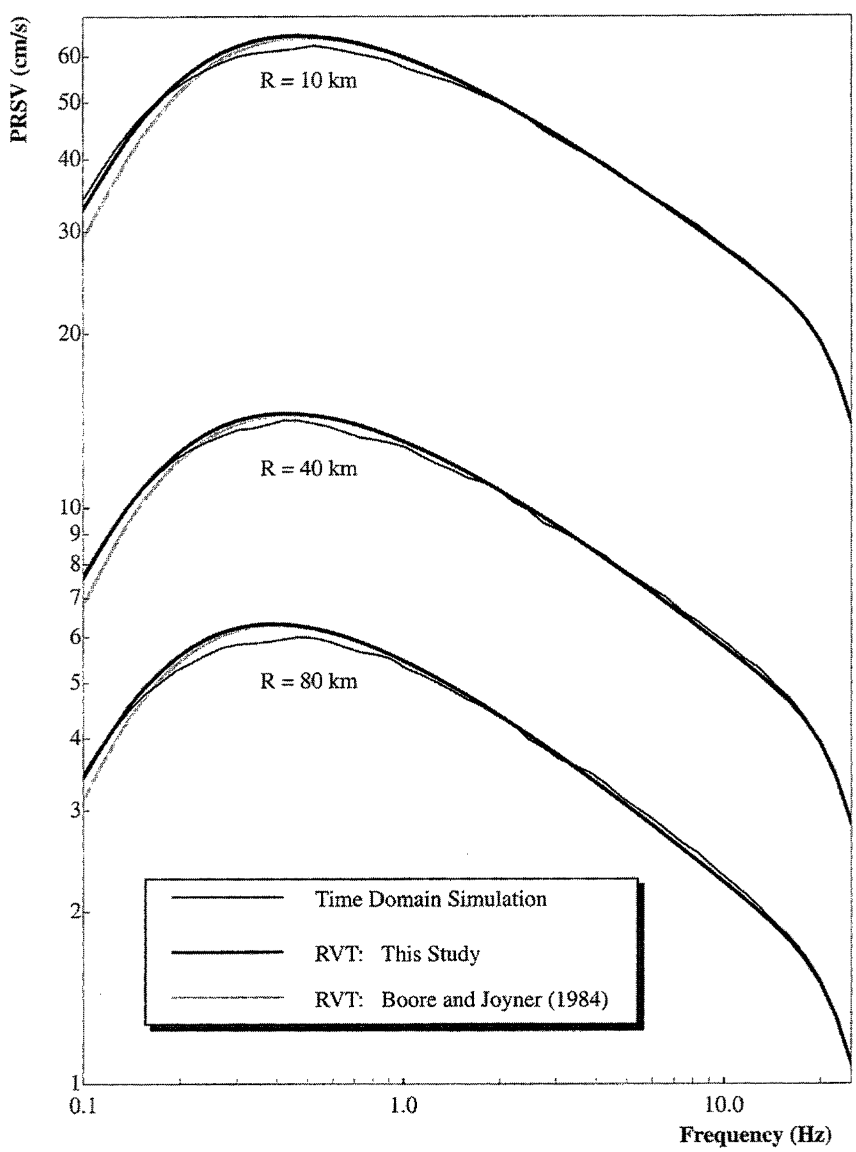

Figure 3. Comparison of pseudoresponse spectral velocity with $M = 7$ and $\xi = 0.05$.

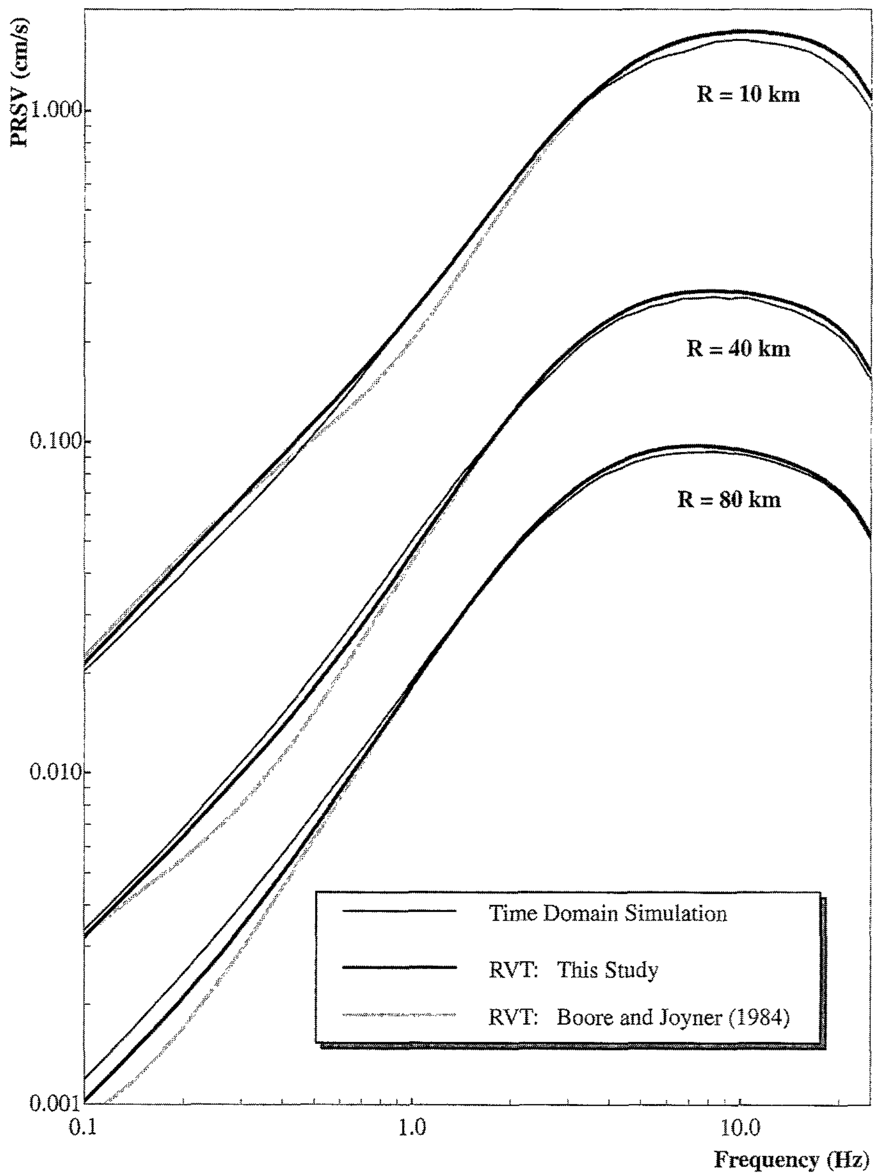

Figure 4. Comparison of pseudoresponse spectral velocity with $M = 4$ and $\xi = 0.05$.

$$k = \left[2\pi\left(1 - \frac{\lambda_1^2}{\lambda_0\lambda_2}\right)\right]^{1/2}, \tag{5}$$

where λ_i is the i-th spectral moment of the power spectral density function of random process, and

$$\lambda_i = \frac{1}{D_s}\int_0^\infty \omega^i\, G(\omega)\, d\omega, \tag{6}$$

where $G(\omega)$ is the one-sided power spectral density function determined by the Fourier amplitude spectra.

Therefore, the new equation to compute the rms duration can be written as

$$D_{rms} = D_s + D_0\left(\frac{\gamma^n}{\gamma^n + k}\right). \tag{7}$$

After comparing results for different n values, we propose to use $n = 2$. The larger the value of k is, the narrower the bandwidth of the random process will be. Thus, equation (7) includes the influence of bandwidth on the estimation of PRSV.

Comparison and Discussion

To compare our results with time domain simulations and Boore and Joyner's (1984) estimate, we calculated the pseudoresponse spectral velocity for magnitudes $M = 4$ and

7, epicentral distances $R = 10, 40$, and 80 km, and damping ratio $\xi = 0.05$ (Figures 3 and 4). In our calculations, there is no amplification due to the difference of rock materials between the earthquake focus and the site. The Brune's (1970, 1971) omega-square model is used as the source spectra. The duration is taken as (Herrmann, 1985),

$$D_s = 1/f_c + 0.05R, \tag{8}$$

where f_c is the source corner frequency. Other parameters used are: $\rho = 2.7$ g/cm^3, $\beta = 3.5$ km/sec scaling factor $c = 0.71 \times 0.55 \times 2.0$, stress parameter = 100 bars, high-cut frequency $f_m = 25$ Hz, geometric attenuation = $1/R$, and $Q(f) = 270 f^{0.87}$ (Dwyer *et al.*, 1984). For the time domain simulation (TDS), we used a total duration of 50 seconds, a time increment of $dt = 0.005$ sec, and the exponential amplitude envelope of Boore (1983). The program developed by Boore (1996) is modified to perform all the calculations with 700 runs for each magnitude, distance, and damping ratio.

The determination of source spectra is very important. This is especially true for the prediction of ground motion at close distances from a large earthquake due to irregularities in the distributions of slip, stress drop, and rupture velocity of seismic source. The omega square model by Brune (1970, 1971) is a simplified point source model without consideration for seismic source finiteness. Even though complicated seismic source models have been introduced by Joyner (1984) and Atkinson (1993), the Brune source spectral model is widely used because of its simplicity (EPRI, 1993; Boore *et al.,* 1996). At the same time, the RVT method is also used to predict the pseudospectral acceleration or velocity for a period greater than one second to meet the engineering needs (Atkinson and Boore, 1995; and Boore *et al.,* 1996).

The RVT method is the main method used to estimate the ground motion in areas with or without strong ground motion recordings because the RVT is a computationally fast method to estimate the characteristics from seismological models. On the other hand, we can separate the influences of seismic source, wave propagation, and local geological conditions in the determination of Fourier amplitude spectra of ground motion from a seismological model. Thus, we can easily take each influence into proper consideration in the model, and also we can improve the estimate with the increase of our knowledge about the seismic source, wave propagation, and local geological conditions. However, for the purpose of seismic resistant design of engineering structures, we need not only the PGA and PGV but the response spectral values at both high frequencies and low frequencies of ground motion, particularly for long span bridges and tall buildings. Therefore, in the future, attention must be focused on developing a more realistic seismic source model. More importantly, development of a new RVT method to improve estimating PRSV is also necessary because an accuracy problem always exists in the prediction of PRSV if one uses the bandlimited Gaussian random properties to predict the spectral values, no matter what source spectral model used.

Unfortunately, there is little improvement of the accuracy of the estimate of PRSV, so time domain simulation is still widely used by researchers (Boore *et al.,* 1996; EPRI, 1993) because there is no RVT method to satisfy the accuracy. In this study, we incorporated a parameter describing the property of filtered bandlimited random process in the determination of rms duration. Comparisons of the results of our method, time domain simulations, and results of Boore and Joyner (1984) show that our method improves the predication of PRSV in a wide range of magnitude, distance, and damping ratios.

In this study, we propose only a simple empirical method to consider the properties of narrow-band random process. Although there are some improvement in the estimation of PRSV, a theoretical method is needed based on the properties of narrow-band random process in the prediction of PRSV.

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Department of Civil Engineering
Campus Box 526570
The University of Memphis
Memphis, TN 38152
(L. L.)

Department of Civil Engineering
Campus Box 526570
The University of Memphis
Memphis, TN 38152
(S. P.)

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