

# Probabilistic Performance-Based Optimal Design of Steel Moment-Resisting Frames. I: Formulation

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**Abstract:** Significant progress has been made in the preceding two decades in the area of seismic engineering. Design codes are very quickly migrating from prescriptive procedures intended to preserve life safety to reliability-based design with less prescription intended to quantify risk associated with designs. Therefore, all stakeholders are given the opportunity to speak a common language (probability and risk) leading to structural designs that not only reliably preserve life safety after rare ground motions, but minimize damage after more frequent ground motions and thereby minimize life-cycle costs. Probabilistic performance-based design is in between traditional prescriptive design methods and full reliability-based design methodologies. The present paper provides an overview of a state-of-the-art model-code performance-based design methodology and casts this design procedure into multiple-objective optimization problems for single-story and multistory structural steel frameworks with fully and partially restrained connections. A methodology for applying an evolutionary (genetic) algorithm with radial fitness and balanced fitness functions is discussed in detail. A companion paper provides applications of the automated design algorithm to single-story frames and multistory frames with a variety of connection characteristics and beam-to-column moment capacity ratios.

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## Introduction

Performance evaluation of nonlinear dynamic systems is a complex process (Challa and Hall 1994; Aschheim and Black 2000; Deierlein and Mehanny 2000; Chopra 2001; Chopra and Goel 2001; Vamvatsikos and Cornell 2004). Although analytical methodologies for computing response histories have been available for decades through the pioneering work of Newmark (1959), and improvements in computational power since then have been significant, using second-order inelastic time-history analysis as a basis for design has yet to see widespread application in design practice for the following reasons:

- Understanding a system's response is cumbersome because a large amount of information is produced and its interpretation requires sound technical knowledge and extensive experience.
- Reliable ground motion input records have to be available with varying attributes, locations, and probabilities of exceedance.

- Current design procedures involve significant trial and error and, therefore, will be time consuming and computationally expensive if economy of design is pursued.

One might argue that headway in overcoming these challenges has been made through three significant developments:

- The generation of probabilistic performance-based seismic design model codes with subsequent verification using computational simulation and experimental research has evolved into practical model codes for performance-based design (SAC 2000).
- Researchers (Deierlein and Moehle 2004; Hamburger 2004) have begun detailed discussions and planning of future probabilistic performance-based design methodologies.
- A better understanding of the processes of earthquake ground motion generation and propagation, and seismic hazard analysis along with a growing number of instruments is making a larger number of ground motion time series available to structural engineers.
- Developments in soft computing applications in structural engineering has helped to automate the design process by intelligently performing the time consuming trial-and-error design cycles on the computer.

Performance-based design optimization is a combination of state-of-the-art performance-based seismic engineering and evolutionary computation into an automated design environment where design optimization is implicitly built into the process. Although there are significant discoveries left to be made, especially in the areas of structural behavior, it is now safe to say that the analytical techniques and computational power have facilitated a movement of design concern from life safety to minimization of economic and human loss during a structure's service life.

Structural design, by nature, is a trial-and-error process. The complexities of geometry necessitated by modern architectural

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needs and the stakeholder's desire to understand system behavior under different loading scenarios challenges a structural engineer to efficiently generalize the experience of previous designs into new projects. An adaptive methodology that can be automated is analogous to the engineer's experience-based trial-and-error procedures. Genetic algorithms (GAs) and evolutionary algorithms (EAs), which operate using computer simulation of natural evolution, have been applied successfully in recent years to structural design involving complex objectives and constraints with a large number of design variables (Camp et al. 1996; Cheng et al. 1999; Pezeshk et al. 2000; Foley 2001; Foley and Schinler 2001; Grierson and Khajepour 2001; Khajepour and Grierson 2001; Pezeshk and Camp 2001; Schinler 2001; Schinler and Foley 2001; Cheng 2002; Foley and Schinler 2003; Alimoradi 2004). The success of GAs and EAs is in no small part due to their power of adaptation and learning.

State-of-the-art research is developing practical, yet accurate, computational tools for simulating nonlinear dynamic response (OpenSees 2001). There also exists a methodology for reliability-based design and evaluation of real structures in test-bed programs (MAE 2005; PEER 2005). However, to date, there appears to be a gap in implementation of new analytical technologies into robust computational design tools to automate the process of performance-based design. This study's objective is to develop a practical and automated computational tool for implementation of probabilistic performance-based design of steel moment-resisting frame systems through the use of evolutionary computation and advanced computational methodologies. It should be noted that design objectives that involve societal consequences are very important considerations for performance-based seismic engineering. However, these objectives are outside the scope of the present effort.

## Probabilistic Performance-Based Design

Minimization of seismic risk for an individual structure can be achieved by optimizing performance under various seismic hazard scenarios. At the same time, efficient use of material and resources necessitates minimization of initial construction cost and expected repair costs over the structure's service life. For steel moment-resisting frames considered in this study, SAC (2000) presents analytical methods of quantifying structural performance at two levels of seismic hazard: Rare events with less than 2% probability of exceedance in 50 years (2/50 events); and more frequent seismic events with 50% probability of exceedance in 50 years (50/50 events). The present section will outline the important fundamentals of the probabilistic design methodology upon which the present effort is based. Further details can be found in SAC (2000).

### Performance Objectives

A performance objective consists of two components: A stated maximum level of expected damage (also called a performance level); and a level of seismic hazard. In a two level methodology (SAC 2000), performance objectives can be described as:

- 2% probability of performance inferior to collapse prevention in 50 years; and
- 50% probability of performance inferior to immediate occupancy in 50 years.

At this point, it should be emphasized that damage states only

consider structural components. Nonstructural damage (although extremely important) is not considered.

Collapse prevention (CP) is a postearthquake damage state in which significant degradation in stiffness and strength of the lateral force resisting system has occurred. The structure is on the verge of total or partial collapse. Immediate occupancy (IO) is defined as a postevent damage state for which repair is unnecessary. "The basic vertical and lateral force resisting systems of the building retain nearly all of their pre-earthquake strength and stiffness" (SAC 2000). CP level design principally controls the loss of life and the number of casualties, while IO level design is targeted to control the functionality of the constructed facility and the economic loss after an earthquake. This design method, at its core, is similar to procedures proposed to minimize "deaths, dollars, and downtime" (Deierlein and Moehle 2004; Hamburger 2004).

As a first step, association of a limit state with a level of seismic hazard in a stated exposure period (e.g., service life) is made. This is done by the designer in collaboration with the owner or other stakeholder considering minimum requirements in the design provisions. In the course of performance-based design, different levels of damage and functionality can be agreed upon in terms of the performance objectives. Within each performance objective, various performance levels are often associated with costs of construction and expected repair. Development of a methodology for creating decision-making curves to elucidate trade-offs between seismic performance and the cost of construction and expected repair is the focus of the present effort.

### Uncertainty in Meeting Performance

SAC (2000) presents a performance-based design procedure in a reliability format. The basic procedure consists of an evaluation of a set of demand-to-capacity ratios at given objective levels for various structural response parameters of interest. These response parameters could be quantities such as interstory drift angle or column compression force. Factors are introduced in the definition of the demand-to-capacity ratios to model uncertainty in response during seismic events, uncertainty in the analysis being able to reliably predict response, and uncertainty in the capacity. The demand-to-capacity ratios are used to evaluate confidence levels associated with the probability of the structure experiencing performance worse than the design performance objective during an exposure period.

Similar to the practiced method of load and resistance factor design (ACI 2005; AISC 2005), the procedure outlined in SAC (2000) is referred to as *demand and resistance factor design*. The theoretical basis and the assumptions made in the derivation of design equations are presented in the literature (SAC 2000; Cornell et al. 2002; Yun et al. 2002). The overall process is briefly outlined for completeness. The confidence parameter is used to quantify confidence in meeting performance objectives. This parameter is evaluated using computed demand and capacity for a variety of response parameters. The general expression for the confidence parameter is (SAC 2000)

$$\lambda = \frac{\gamma\gamma_a D}{\phi C} \quad (1)$$

where  $D$ =median demand for a defined response quantity (e.g., column compression force) from a structural analysis;  $C$ =median capacity for the same response quantity;  $\gamma$ =demand variability factor;  $\gamma_a$ =analysis uncertainty factor; and  $\phi$ =capacity reduction

**Table 1.** Parameters for Confidence-Level Determination (SAC 2000)

Fully restrained frames		Partially restrained frames	
Immediate occupancy performance	Collapse prevention performance	Immediate occupancy performance	Collapse prevention performance
$\beta_{UT,drift}=0.20^b$	$\beta_{UT,drift}=0.30$	$\beta_{UT,drift}=0.20$	$\beta_{UT,drift}=0.35$
$\beta_{UT,CCF}=\sqrt{0.0225+\beta^2}$	$\beta_{UT,CCF}=\sqrt{0.0225+\beta^2}$	$\beta_{UT,CCF}=\sqrt{0.0225+\beta^2}$	$\beta_{UT,CCF}=\sqrt{0.0225+\beta^2}$
$C_{drift}=0.02$	$C_{drift}=0.10$	$C_{drift}=0.01$	$C_{drift}=0.10$
$C_{CCF}$ from AISC (2005)	$C_{CCF}$ from AISC (2005)	$C_{CCF}$ from AISC (2005)	$C_{CCF}$ from AISC (2005)
$\phi=1.0^b$	$\phi=0.90$	$\phi=1.0$	$\phi=0.85$
$\gamma=1.5$	$\gamma=1.3$	$\gamma=1.4$	$\gamma=1.4$
$\gamma_a=1.02$	$\gamma_a=1.0$	$\gamma_a=1.02$	$\gamma_a=1.03$
$\gamma_a=e^{1.4\beta^2}$	$\gamma_a=e^{1.4\beta^2}$	$\gamma_a=e^{1.4\beta^2}$	$\gamma_a=e^{1.4\beta^2}$

<sup>a</sup> $\beta$ =coefficient of variation of the axial load values from the suite of nonlinear analyses. It should be noted that column compression force capacities are assigned using yield surfaces.

<sup>b</sup>Values given assume global interstory drift evaluation. Local interstory drift evaluation was omitted in the present study.

factor. Eq. (1) is evaluated for each structural response parameter considered at each performance level defined.

The confidence parameter is used to evaluate confidence levels in attaining the stated performance objectives. The confidence parameter is used in the following expression to determine confidence levels (SAC 2000)

$$\lambda = \exp \left\{ -b\beta_{UT} \left( K_x - \frac{1}{2}k\beta_{UT} \right) \right\} \quad (2)$$

where  $b$ =coefficient that relates incremental change in demand to an incremental change in ground motion ( $b=1.0$  implies a linear relationship);  $\beta_{UT}$ =uncertainty measure equal to the vector sum of the logarithmic standard deviation of the variation of demand and capacity from uncertainty;  $k$ =slope of the hazard curve in natural log-log coordinates; and  $K_x$ =standard Gaussian variate associated with the probability of  $x$  not being exceeded as a function of the number of standard deviations above or below the mean. The parameters used in the present study for the partially restrained (PR) and fully restrained (FR) frames considered are given in Table 1.

## Components of Design Procedure

The design procedure outlined in SAC (2000) is relatively new and a concise outline of how the procedure is employed in the present study is beneficial for the reader to fully appreciate the computational demands, the simplified design objectives, and the simplified constraints that can be formulated using the new methodology. The approach used in the present optimization formulations to address these issues is highlighted in the following.

### Evaluation of Demand

For each performance objective, evaluation of demand consists of selecting input earthquake characteristics (e.g., design spectrum, acceleration–displacement spectrum, or acceleration time histories) and appropriate structural analysis procedures that can be used to estimate the response parameters of interest (see the next section). In the present study, acceleration time histories and inelastic time-history analysis are used as the basis for demand evaluation.

## Response Parameters

SAC (2000) uses three response parameters for assessing performance of steel moment-resisting frames (MRFs): interstory drift angle; column axial compression force; and column (splice) tension force. Interstory drift has long been used as a measure of lateral stability of a structural framing system and for assessment of serviceability. In prescriptive design specifications or codes, drift limits are set in anticipation that these limits will minimize structural damage and preserve the stability of the structure during a ground motion event. These limits are also established to preserve structural stability for postevent recovery and/or rescue operations. Interstory drift is closely related to plastic rotation demand in the MRF girders, columns, connections, and  $P-\Delta$  instability. As a result, interstory drift is a reasonably good measure of local and global damage to the structure. Interstory drift is used as a response parameter in the present study.

Column axial compression is also a very useful response parameter for assessing the performance of MRFs. The axial compression in a column that resides within a MRF can be used to assess force-controlled behavior within the structural system. For example, force-controlled behavior may be inelastic buckling of a column and this may lead to inferior frame performance. The present study utilizes column compression force in the assessment of structural performance.

Column axial (splice) tension is also a highly important response measure. It is most critical in situations where column splices exist within the structure. These splices may not have suitable tension capacity and, therefore, they may limit structure performance during ground motion events. The present study considers one- and three-story steel frames and, therefore, it is assumed that splices do not exist in the frameworks considered. As a result, column tension forces are not considered as response parameters.

### Analysis Type

Four different types of structural analysis are allowed in SAC (2000) for the design of moment-resisting frames: Linear static procedure; linear dynamic procedure; nonlinear static procedure; and nonlinear dynamic procedure (NDP). They encompass varying degrees of computational cost and analytical complexity and naturally result in differing levels of accuracy.

Caution should be exercised in selecting the method of analy-

sis as some methods may not be applicable to specific structural systems (e.g., regular frames, torsion-sensitive buildings). However, if appropriate engineering judgment is exercised, NDP can be applied to all systems. It commonly results in more accurate response quantities than other methods, because all modes of vibration, geometric and material nonlinearity, and second-order effects can be captured in the analysis. The additional computational cost associated with carrying out this level of analysis is significant. Furthermore, selecting an appropriate suite of ground motions for carrying out the nonlinear dynamic analysis must be done with due diligence. The reason for this is that design spectra include a wide range of ground motion characteristics for a building site. A single acceleration time history (or limited suites of ground motions) is thought to be incapable of providing reliable designs for buildings. As a result, care must be exercised in selecting ground motion acceleration time histories for a given site and recurrence intervals.

In this study, nonlinear second-order time-history analysis is used as the analytical basis for the design automation. Today's computational advancements justify employment of such sophisticated analytical engines in the design optimization and/or automation to fully leverage state-of-the-art computation and state-of-the-art design methodologies.

### Ground Motions

The input ground motion can be represented in different ways, with the simplest form being a time history that is scaled to user-defined peak ground accelerations, and more precise descriptions being a design spectrum compatible strong motion time history. The intensity level associated with the ground motion used in structural analysis should be consistent with the performance objectives of design.

The most accurate estimate of response is normally attained through time-history analysis using large sets of input ground motion records whose median spectral acceleration falls as close as possible to a target design spectrum that described the hazard level associated with the design performance objective(s) at the site. In this study, a suite of seven ground motion acceleration time histories for IO performance and a second suite of seven histories for CP performance are utilized for determining response and evaluating performance.

Strong ground motion records representing 2 and 50% probabilities of exceedance in 50 years for the city of Los Angeles were chosen from the records developed in the SAC steel project (Somerville et al. 1997). A newly developed GA-based ground motion search and scaling program (Naeim et al. 2004) could also be used to perform the task of ground motion selection and optimal scaling.

The pool of ground motion records used to represent target design spectra of the National Earthquake Hazard Reduction Program Site Category D (ATC 1997a,b) with deaggregation of hazards of M 6.75–7.5 at closest distance of 2–20 km, and M 5–7 at 5–15 km, for 2/50 and 50/50, respectively. The horizontal components of the acceleration time histories are provided in strike-normal and strike-parallel components. The 2/50 earthquake records are chosen from the near-fault recordings or simulations with virtually no scaling, and the simulated time histories are for magnitude 7.1 events on the Elysian Park fault (a blind thrust fault with shallowest depth of 10 km). The 50/50 events are from crustal earthquakes on soil category D. The near-fault records cover a balance of faulting mechanisms such as strike slip, oblique, and dip slip (Somerville et al. 1997). These time histories

are used as input to the analytical model to compute the median response of the structure for the performance levels associated with the record probabilities. Fig. 1 illustrates the ground motion records used for the response simulation in the present study.

### Evaluation of Capacity

Capacity is the maximum force or displacement that a structural member, substructure, or structural system can support or undergo at a defined limit state. Quantifying uncertainty in meeting performance objectives through a mechanism such as Eq. (1) requires that capacities be defined. Each of the response parameters used in the performance evaluation have associated capacities that require definition.

FEMA-350 (SAC 2000) defines interstory drift angle capacities at two levels: Global and local. The global interstory drift capacity is thought to be a very good parameter for assessing a structure's ability to resist  $P-\Delta$  instability and collapse (SAC 2000). However, large interstory drifts can also place large demands on the connections within the structure, and interstory drift capacities defined with consideration of local connection response are also needed. The present study utilizes global interstory drift angle capacities. Local interstory drift angle capacities are omitted from consideration.

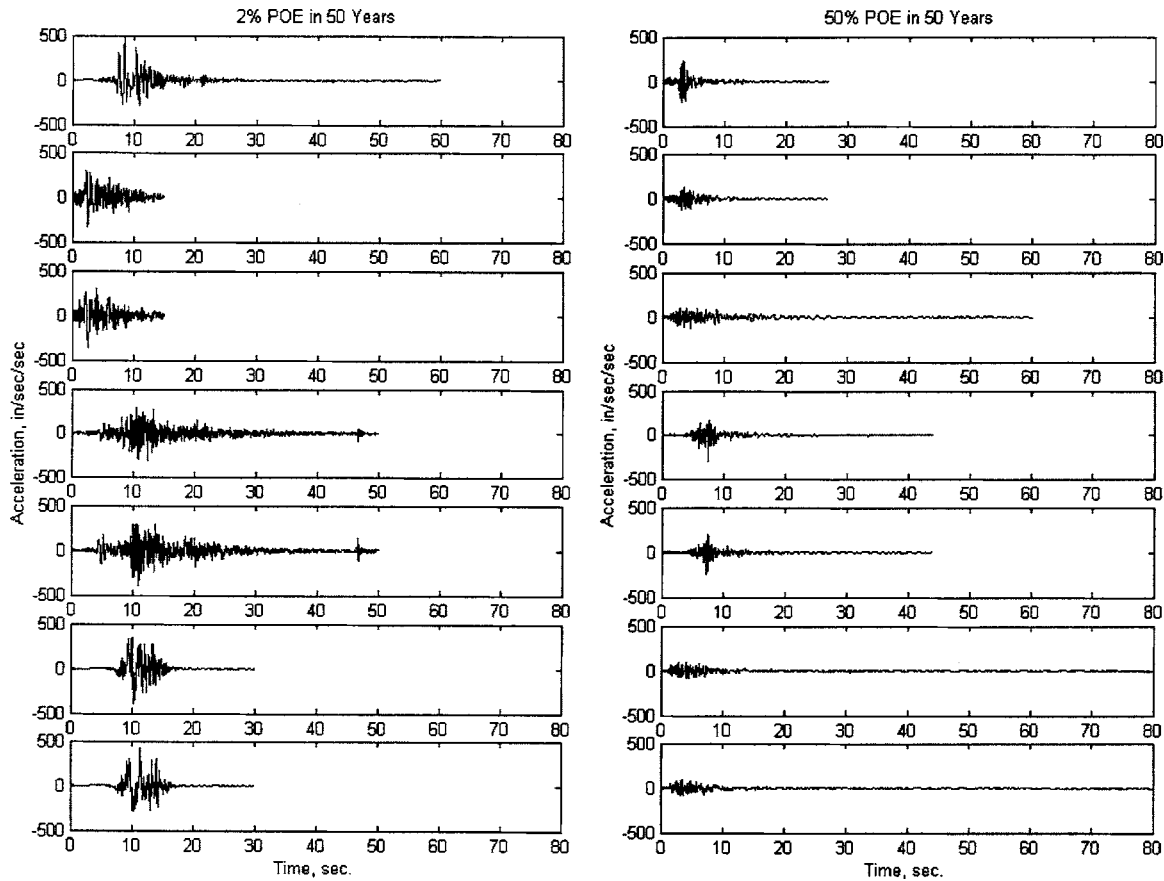
Column compression force (CCF) capacities are another very important contributor to the process of quantifying the confidence levels in meeting performance objectives. The present study uses yield surfaces with critical parameters that define the yield surface assigned using design specifications (AISC 2005). Fig. 2 illustrates the yield surfaces for beam elements and beam-column elements used in the present study.  $P_{yt}$ =tensile yield capacity of the member assuming the gross cross-sectional area.  $M_p^+$  and  $M_p^-$ =positive and negative plastic moment capacities for the bare steel member. Finally, the axial compression capacity in the absence of bending moment,  $P_{nc}^{minor}$ , is taken as the out of plane inelastic buckling capacity of the member. This point on the interaction diagram and the "kink" is connected conservatively with a straight line.

Column splice tensile capacities are also integral to the SAC (2000) methodology. As discussed earlier, the present study is limited to buildings that are three stories and less and the inclusion of column splices in these structures was omitted.

Collapse of a building structure is associated with the MRFs inability to support gravity loading during or after an event. This can also be defined as a system capacity consideration, although it is not considered directly in the SAC (2000) procedure. Collapse has been described as a ratcheting over of a framework resulting in progressive  $P-\Delta$  excursions (Gupta and Krawinkler 2000; Ibarra and Krawinkler 2004). Collapse can be described as a simple loss of lateral stiffness in combination with the frame being left in a significantly displaced configuration at the completion of the ground motion event. Deierlein and Mehanny (2000) introduced an objective measure called the stability index in order to assess the tendency for collapse. The incremental dynamic analysis technique (Vamvatsikos and Cornell 2004; Kunnath 2005) is another approach that will explicitly allow the designer to assess the tendency for collapse during or immediately following severe ground motion events.

Assessing collapse or near collapse via interstory drift is a relatively complex procedure and a standard methodology is lacking in this regard. Monitoring the numerical algorithm execution for appearance of negative definite or singular stiffness matrices was done, but it should be pointed out that the damping levels





**Fig. 1.** SAC ground motion acceleration time histories used in the present study (Somerville et al. 1997). It should be noted that 1 in./s<sup>2</sup>=0.0254 m/s<sup>2</sup>.

assumed in computing structure response (5% in the present study) may maintain a numerically stable structure during the ground motion event. A structure was deemed unstable if, during computed response, the numerical algorithm indicated an ill-conditioned system of equations. As methods for improving assessment of near collapse improve, these methodologies can be relatively easily included in the methodology proposed.

### Confidence in Meeting Performance

The essence of probabilistic performance-based design is to include uncertainty in a direct fashion. The SAC (2000) procedure incorporates uncertainty through definition of confidence levels on meeting performance objectives. Median structural demand parameters obtained from a set of response history analyses is used to find a ratio of probabilistic demand to capacity called the confidence parameter—Eq. (1). The ratio is calculated for all response parameters considered in the design. As outlined earlier, the present study considers column compression force and global interstory drift angle. Eqs. (1) and (2) are used to define confidence levels in meeting performance by considering global interstory drift and column compression force independently. The lower confidence level then becomes the controlling level for assessing a design's expected performance. Table 1 contains the parameters needed to utilize Eqs. (1) and (2).

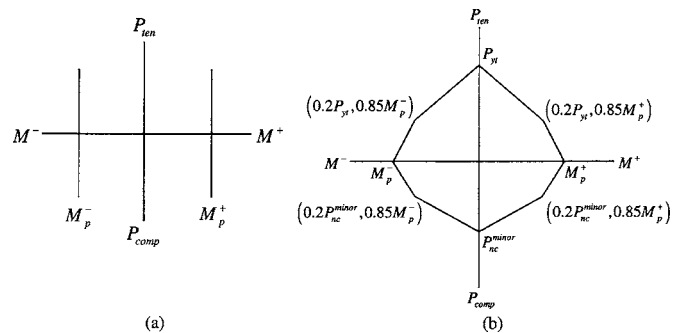
Confidence levels are computed by considering response parameters and then computing the corresponding confidence pa-

rameters. Assuming all SAC (2000) response parameters are considered, confidence parameters for each performance objective would be computed as

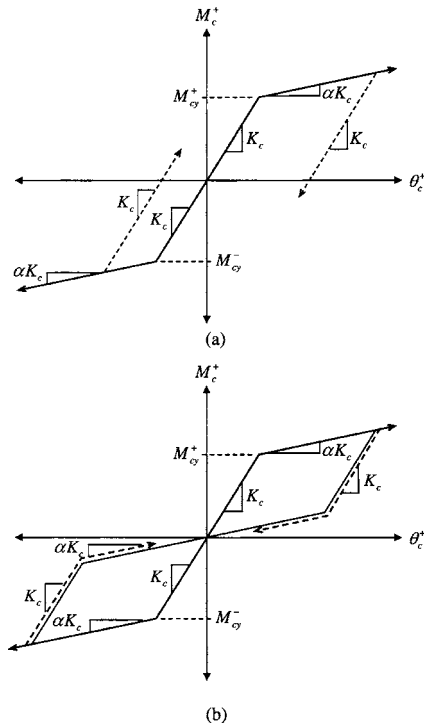
$$\lambda^{IO} = \max\{\lambda_{drift}^{Global\_IO}, \lambda_{drift}^{Local\_IO}, \lambda_{CCF}^{IO}, \lambda_{splice}^{IO}\} \quad (3)$$

$$\lambda^{CP} = \max\{\lambda_{drift}^{Global\_CP}, \lambda_{drift}^{Local\_CP}, \lambda_{CCF}^{CP}, \lambda_{splice}^{CP}\} \quad (4)$$

where  $\lambda$  is defined in Eq. (1). The subscript drift stands for inter-story drift angle and CCF represents the maximum median column compression force demand-to-capacity ratio obtained



**Fig. 2.** Yield surfaces for beam and beam-column members: (a) beam member with no axial force bending-moment interaction; (b) beam-column member with axial-force bending-moment interaction



**Fig. 3.** Connection models used in the present study: (a) nonlinear moment–rotation response without gap; (b) nonlinear moment–rotation response with gap

considering the entire response history for all column members in the framework. The resulting IO and CP confidence parameters are then used in conjunction with Eq. (2) to compute confidence levels associated with meeting each of these performance objectives. Based upon earlier discussions regarding response parameters, the confidence parameters for the present study are based upon the following controlling conditions:

$$\lambda^{IO} = \max\{\lambda_{drift}^{Global\_IO}, \lambda_{CCF}^{IO}\} \quad (5)$$

$$\lambda^{CP} = \max\{\lambda_{drift}^{Global\_CP}, \lambda_{CCF}^{CP}\} \quad (6)$$

It is worth noting that column compression force demand confidence parameter calculation appears to be inconsistent with use of interaction (yield) surfaces in inelastic analysis. The reason for this is that the interaction surface can replace use of design specification equations. This is often called advanced analysis (Clarke et al. 1992; White 1992, 1993; White and Nukala 1997; Bridge et al. 1998; Foley 2001; Foley and Schinler 2003). Design for CCF is based on the evaluation of Eq. (1). Here, demand is the largest median column compression force found for all members in the frame obtained from nonlinear second-order time-history analysis of the structure that is subject to a set of input ground motion time histories. Capacity is given in FEMA-350, which basically follows the AISC specifications. Uncertainty and capacity reduction factors are shown in Table 1.

### Optimized Design Problem Statements

One of the attractions to performance-based design optimization is that the design statements become very direct and greatly simplified from a stakeholder's perspective. For instance, a multi-objective reliability-based optimization problem can be relatively

simply stated as: Minimize the initial cost of construction; maximize the confidence level on the probability of damage not to exceed CP performance during the structure's service life (e.g., 50 years); and maximize the confidence that damage will not be worse than that associated with IO performance during the structure's service life. These types of design formulations can be more appealing to owners of buildings.

The present study considered both portal frame design and multistory frame design. It was decided to formulate two optimization problem statements. The portal frame analysis would likely not be as sensitive to the column compression force response parameter because the axial loading in the columns is likely to change very little during the ground motion events. However, in the case of multistory frames, the column compression forces during the events may change significantly and, therefore, the column compression force response parameter may become more important in assessing confidence in meeting performance. The present section outlines the optimization problem statements considered.

The first-optimization problem statement considered has been shown to be applicable to the portal frame considered in the present study (Alimoradi 2004). Therefore, the probabilistic performance-based design optimization problem statement for portal frames with partially or fully restrained connections can be stated as

*Minimize:*

$$Z_1 = C_{initial} \propto \sum_{i=1}^{N_{mem}} \rho_i A_i L_i \approx V_{columns} + V_{beams} \quad (7)$$

$$Z_2 = \frac{1}{q^{CP}} \quad (8)$$

$$Z_3 = \frac{1}{q^{IO}} \quad (9)$$

*Subject to:*

$$M_p^{col} \geq 1.2 \sum_{j=1}^{N_b} M_p^{bm} \quad \text{full-strength connections} \quad (10)$$

$$M_p^{col} \geq 1.2 \sum_{j=1}^{N_b} M_{cy}^{cn} \quad \text{partial-strength connections} \quad (11)$$

$$q^{IO} \geq q_{limit}^{IO} \quad (12)$$

$$q^{CP} \geq q_{limit}^{CP} \quad (13)$$

where  $C_{initial}$ =initial construction cost;  $\rho_i$ =material weight density for member  $i$ ;  $A_i$ =cross-sectional area of member  $i$ ;  $L_i$ =length of member  $i$ ;  $V_{columns}$ =volume of the columns in the frame;  $V_{beams}$ =material volume of the beams in the frame;  $q^{IO}$ =confidence level associated with meeting IO performance;  $q^{CP}$ =confidence level in meeting CP performance;  $M_p^{col}$ =plastic moment capacity of the column at a given beam-to-column joint;  $M_p^{bm}$ =plastic moment capacity of beam  $j$  at the same beam-to-column joint;  $M_{cy}^{cn}$ =yield moment capacity of connection  $j$  at the beam-to-column joint; and  $N_b$ =number of beams framing into the beam-to-column joint considered. Minimization of  $\lambda$  is equivalent to maximization of  $q$ .

Two nonlinear connection characteristics are considered in the present study. Both are shown in Fig. 3. The first is a bilinear connection with initial stiffness,  $K_c$ , hardening stiffness ratio,  $\alpha$ , and yield moment capacity,  $M_{cy}$ . In this connection, unloading is assumed to take place with the initial stiffness and there is no limit to the connection's rotation capacity. The second connection is also bilinear. However, hysteretic pinching is modeled in this connection. The hardening stiffness and the initial elastic stiffness are used to define the gap behavior associated with this connection.

Naturally, a given design that has a lower demand to capacity ratio provides higher level of confidence on the performance level for which the ratio of demand to capacity is obtained. Throughout this study, the minimization of  $\lambda$  is always analogous to maximization of the level of confidence. The choice of either variable is, however, up to the user. If communicating level of confidence with the stakeholder is preferred, the final minimal ratio of demand to capacity could be easily transformed to a level of confidence following Eq. (2).

The SAC (2000) procedure includes minimum confidence levels for assessing performance. If one assumes global behavior limited by interstory drift as the controlling response parameter, as done in the present study, the SAC (2000) methodology requires at least 50% confidence in attaining IO performance and 90% confidence in meeting CP performance objectives. The present optimization problem includes maximization of confidence for both performance levels and, therefore, a lower-bound limit was used as shown in Eqs. (12) and (13).

The second optimization problem formulation considered sought to ensure economy of designs by establishing a mechanism in which the confidence level based upon the global interstory drift response parameter is high *and* is as close as possible to the confidence level computed, using the column compression force response parameter (Alimoradi 2004). This formulation will be termed balanced design. An optimization problem formulated in this manner can be mathematically stated as

*Minimize:*

$$Z_1 = V_{\text{columns}} + V_{\text{beams}} \quad (14)$$

$$Z_2 = \frac{(q_{\text{CCF}}^{\text{CP}} - q_{\text{ISD}}^{\text{CP}})^2}{q^{\text{CP}}} \quad (15)$$

$$Z_3 = \frac{(q_{\text{CCF}}^{\text{IO}} - q_{\text{ISD}}^{\text{IO}})^2}{q^{\text{IO}}} \quad (16)$$

*Subject to:*

$$q_{\text{ISD}}^{\text{CP}} = q_o^{\text{CB}}, \quad q_{\text{ISD}}^{\text{IO}} = q_o^{\text{IO}} \quad (17)$$

in which  $q_o^{\text{CP}}$  and  $q_o^{\text{IO}}$  = target levels of confidence for design.

The relatively simplistic format of the optimization problems outlined above hides the complexity of the analysis needed to allow such simplistic statements. One should also note that there are two constraints that fall outside the realm of understanding for owners: The strong column weak beam (SCWB) criteria. The remaining expressions are posed in the language of owners. This is the beauty of probabilistic performance-based design.

It is recognized that minimizing initial construction cost on the basis of structure volume (weight) is overly simplistic for steel frame structures. However, the methodology being proposed is most certainly capable of easily including more complex initial

construction objectives and constraint functions as they develop. Other objectives and constraints for steel framed structures that include constructibility have been proposed and successfully implemented in optimized steel frame design problems (Liu et al. 2006).

The evaluation of confidence levels of the structure in attaining IO and CP seismic performance will require specific point-in-time loading at the time of the events. The following load combinations are used to evaluate seismic demands:

$$1.0D + 0.25L + 1.0E^{2/50} \quad (18)$$

$$1.0D + 0.25L + 1.0E^{50/50} \quad (19)$$

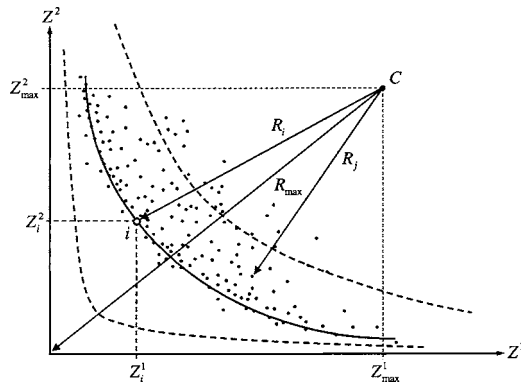
where  $E^{2/50}$  and  $E^{50/50}$  represent earthquake effects for 2/50 and 50/50 ground motions, respectively.

## Evolutionary (Genetic) Algorithm

The multiple objective optimization problem considered in the present study is highly complex when one considers that the analytical modeling for demand prediction requires inelastic time-history analysis to accurately quantify the stated objectives and constraint violations. Therefore, classical gradient-based optimization algorithms will be difficult (if not impossible) to utilize. However, the GA (a class of EA modeled on survival of the fittest theory) has been successfully applied in many areas of structural engineering as alluded to previously. The attraction to using a GA for solving problems like that considered is that gradients of objective functions and constraints need not be included in the algorithm for solution. This is especially important when inelastic time-history analysis is considered as well as uncertainty. In fact, it may not be possible to formulate a traditional gradient-based solution for the problem currently under consideration. EAs have been shown to easily provide optimized and automated design algorithms for complex inelastic-analysis-based design (Foley 2001; Foley and Schinler 2003).

A GA is a computational simulation of the natural evolutionary process to solve search and optimization problems. Early formulations of adaptable systems on machines go back to the early stages of computer software and hardware development (Levy 1992). It has taken a significant length of time for this subject to mature to the point where it is a practical tool. The pioneering work by Goldberg (1989), subsequent researchers, and the widespread availability of high-speed computers have paved the way for many applications of GAs in engineering. The fundamentals of the GA have been diligently and thoroughly described throughout the literature (Goldberg 1989; Mitchell 1997; Haupt and Haupt 1998; Michalewicz 1999; Coley 2001) and details will not be repeated here.

A GA driver (Carroll 2004) is used in the present study as a front end in the automated design algorithm. Various structural designs are represented in the traditional manner using binary string chromosomes that decode to the structural steel shapes used as design variables in the problems considered. The parameters used to control the GA and improve its performance are often problem dependent. Discussion of the two frames in the companion paper (Alimoradi et al. 2006) includes the GA parameters used to execute the automated design.



**Fig. 4.** Pareto front approximation in genetic algorithm for multiple-objective optimization

## Fitness Formulations

Application of the GA in the search for the optimal combination of design variables requires that the optimization problem statements be recast into unconstrained optimization problems. Since several optimization problem statements were considered in the present study, multiple fitness formulations were also considered.

Consider two competing objectives  $Z^1$  and  $Z^2$ , where it is desired to minimize both. If the two objectives compete with one another, the candidate designs can form decision-making curves in objective space. The group of designs that *dominate* all others is termed the Pareto front in objective space. If gradient-based optimization algorithms are used, this front can be approximated using weighting of individual objectives and then solving single-objective problems to find a set of single optimal solution points on the curve. Other approaches to solving multiple-objective optimization problems are available (Balling and Wilson 2001; Deb 2001). Two distinct fitness formulations are used in the GA formulations presented in this study. Each will be discussed in some detail in the following sections.

### Radial Fitness Formulation

The first formation of GA fitness is done within the context of multiple-objective optimization and Pareto optimal theory. Details of the formulation can be found in Alimoradi (2004). For the present discussion, we will limit fitness space to two dimensions as shown in Fig. 4. The cloud of circles in Fig. 4 represents candidate designs that have been generated during the execution of the GA. The hypothetical Pareto front is shown as the solid line. When more than two objectives are considered, these fronts are hyperquadrics in  $n$ -dimensional space. These fronts may be hyperspheres, hyperellipsoids, or hyperhyperboloids depending upon the convexity of the relationship among objectives. In all cases, the Pareto front (assuming minimization of all objectives) can be obtained by linking individual fitness to that individual's proximity to the origin in objective space. The physical analogy is very simplistic. The origin in objective space represents the best possible solution to which both objectives are at their minimums—zero.

Any definition of fitness employed in the GA should not hinder the development of enough points on the Pareto front to reasonably represent the relative significance of different objectives in decision space. In general, individual fitness should have the following characteristics:

- Individuals closer to the origin should have higher fitness; and

- Populations with higher fitness should have greater dispersion in objective space. In other words, higher fitness should be assigned to populations where greater separation between candidate designs along the Pareto front exists. This will aid designers with higher quality decision curves.

The aforementioned objectives may be translated into a polar coordinate system fitness function based upon the distance of solutions from the worst-case scenario design. In the two-dimensional objective space as shown in Fig. 4, this becomes minimization of  $1/R$ , where the radial distance  $R_i$  for any candidate design is

$$R_i = \sqrt{(Z^1_{\max} - Z^1_i)^2 + (Z^2_{\max} - Z^2_i)^2} \quad (20)$$

The terms in Eq. (20) are defined in Fig. 4. The radial distance is nothing more than the magnitude of the position vector connecting the worst design,  $C$ , to the design considered.

The radial fitness is defined in the present study using Eqs. (7)–(13) as its basis. When column compression force is omitted as a response parameter (appropriate for portal frames considered in this present study), the confidence levels in meeting performance objectives can be defined using global interstory drift limits. This correspondence is given below (SAC 2000)

$$q_{\text{limit}}^{\text{CP}} = 0.70, \quad d_{\text{limit}}^{\text{CP}} = 6.6\% \text{ (SMRF)}, \quad d_{\text{limit}}^{\text{CP}} = 5.9\% \text{ (OMRF)}$$

$$q_{\text{limit}}^{\text{IO}} = 0.70, \quad d_{\text{limit}}^{\text{IO}} = 1.5\% \text{ (SMRF)}, \quad d_{\text{limit}}^{\text{IO}} = 0.78\% \text{ (OMRF)}$$

SMRF denotes a special moment-resisting framework and OMRF denotes an ordinary moment-resisting frame. In general, SMRFs contain detailing and members that are thought to be capable of undergoing larger seismic demands more reliably than OMRFs.

SCWB behavior is also included. The fitness of the  $j$ th individual at the  $k$ th generation during the evolution can be written as (Alimoradi 2004)

$$F_{jk} = \left[ \frac{R_{jk}}{R_{\max}} \right] \left[ \frac{d_{\text{limit}}^{\text{CP}}}{d_{jk}^{\text{CP}}} \right] \left[ \frac{d_{\text{limit}}^{\text{IO}}}{d_{jk}^{\text{IO}}} \right] \left[ \frac{\sum M_p^{\text{col}}}{1.2 \sum M_p^{\text{bm}}} \right] \quad (21)$$

in which

$$R_{jk} = \sqrt{(d_{\text{limit}}^{\text{IO}} - d_{jk}^{\text{IO}})^2 + (d_{\text{limit}}^{\text{CP}} - d_{jk}^{\text{CP}})^2 + (W_{\max} - W_{jk})^2} \quad (22)$$

$$R_{\max} = \sqrt{(d_{\text{limit}}^{\text{IO}})^2 + (d_{\text{limit}}^{\text{CP}})^2 + (W_{\max})^2} \quad (23)$$

$d_{jk}^{\text{IO}}$  and  $d_{jk}^{\text{CP}}$  = mean interstory drift demands for individual  $j$  in generation  $k$  for IO and CP ground motion records, respectively. It should be noted that if partially restrained connections are included in the frame considered, Eq. (21) should have the plastic moment capacity of the beams changed to the corresponding end connection plastic moment capacity.

### Balanced Confidence Formulation

The second fitness formulation was developed to balance confidence levels for attaining performance objectives using predominant response parameters: global interstory drift and column compression force. This formulation stems from the design objectives posed in Eqs. (14)–(17). In a manner similar to that used for the radial fitness formulation, a second fitness for individual  $j$  at generation  $k$  in the evolution is written as (Alimoradi 2004)

$$F_{jk} = \left[ \frac{16,717}{V_{jk}} \right] \left[ \frac{0.01^2}{\delta \lambda^{\text{CP}} \delta \lambda^{\text{IO}}} \right] P^{\text{CP}} P^{\text{IO}} \quad (24)$$

where



$$\delta\lambda^{\text{CP}} = \begin{cases} 0.01 & \text{for } |\lambda_{\text{ISD}}^{\text{CP}} - \lambda_{\text{CCF}}^{\text{CP}}| \leq 1 \\ (\lambda_{\text{ISD}}^{\text{CP}} - \lambda_{\text{CCF}}^{\text{CP}})^2 & \text{otherwise} \end{cases} \quad (25)$$

$$\delta\lambda^{\text{IO}} = \begin{cases} 0.01 & \text{for } |\lambda_{\text{ISD}}^{\text{IO}} - \lambda_{\text{CCF}}^{\text{IO}}| \leq 1 \\ (\lambda_{\text{ISD}}^{\text{IO}} - \lambda_{\text{CCF}}^{\text{IO}})^2 & \text{otherwise} \end{cases} \quad (26)$$

$$P^{\text{CP}} = \frac{1}{1 + \left[ \frac{|\lambda_{\text{ISD}}^{\text{CP}} - \lambda_{\text{CCF}}^{\text{CP}}|}{0.05} \right]^2} \quad (27)$$

$$P^{\text{IO}} = \frac{1}{1 + \left[ \frac{|\lambda_{\text{ISD}}^{\text{IO}} - \lambda_{\text{CCF}}^{\text{IO}}|}{0.05} \right]^2} \quad (28)$$

in which  $\lambda_{\text{ISD}}^{\text{CP}}$  and  $\lambda_{\text{ISD}}^{\text{IO}}$  = confidence parameters for global interstory drift at CP and IO performance objectives, respectively;  $\lambda_{\text{CCF}}^{\text{CP}}$  = target confidence parameter for global interstory drift at the CP performance level based upon a user-defined level of confidence;  $\lambda_{\text{CCF}}^{\text{IO}}$  = target confidence parameter for global interstory drift at the IO performance level based upon a user-defined level of confidence; and  $V_{jk}$  = material volume of individual  $j$  at generation  $k$ . The value 16,717 is the largest material volume possible given the section sizes chosen for the set of all design variables.

## Concluding Remarks

A procedure for design of steel frames using a probabilistic (confidence-level)-based formulation consistent with the SAC (2000) methodology has been presented. This probabilistic methodology for design has been cast into several optimal design problem formats. Both optimal design formats contain multiple objectives. The first seeks to minimize the volume of structural members, while also seeking to maximize the confidence levels for attaining immediate occupancy and collapse prevention performance. The second optimization problem formulation makes an attempt at ensuring economy in design by establishing a mechanism in which the confidence level based upon the global interstory drift response parameter is high and is as close as possible to the confidence level computed using the column compression force response parameter. The formulations presented are capable of considering partial connection restraint and assessment of confidence levels for performance are based upon inelastic time-history analysis.

The optimization problems are intended to be solved using a GA and to this end, two fitness function definitions have been presented. The first utilizes a radial (polar coordinate) formulation that is applicable to multidimensional objective space. The second is a more traditional penalty function formulation.

The companion to the present paper (Alimoradi et al. 2007) proceeds with implementation of the previously described probabilistic optimal design problem algorithms in the automated design of a portal frame with partially and fully restrained connections, as well as a three-story multibay frame.

## References

- AISC. (2005) *Load and resistance factor design specifications for structural steel buildings*, American Institute of Steel Construction, Chicago.
- Alimoradi, A. (2004). "Probabilistic performance-based seismic design automation of nonlinear steel structures." Ph.D. dissertation, Univ. of Memphis, Memphis, Tenn.
- Alimoradi, A., Pezeshk, S., and Foley, C. M. (2007). "Probabilistic performance-based optimal design of steel moment-resisting frames. II: Application." *J. Struct. Eng.*, 133(6), 767–776.
- American Concrete Institute (ACI). (2005). "Building code requirements for reinforced concrete." *ACI 318-05*, Detroit.
- Aschheim, M., and Black, E. F. (2000). "Yield point spectra for seismic design and rehabilitation." *Earthquake Spectra*, 16(2), 317–335.
- ATC. (1997a). "NEHRP commentary on the guidelines for the seismic rehabilitation of buildings." *FEMA 274, ATC-33*, Building Seismic Safety Council, Washington, D.C.
- ATC. (1997b). "NEHRP guidelines for the seismic rehabilitation of buildings." *FEMA 273, ATC-33*, Building Seismic Safety Council, Washington, D.C.
- Balling, R. J., and Wilson, S. (2001). "The maxi-min fitness function for multi-objective evolutionary computation: Application to city planning." *Genetic and Evolutionary Computation Conf. (GECCO 2001)*, Morgan Kaufmann, San Francisco, 1079–1084.
- Bridge, R. Q., Clarke, M. J., Osterrieder, P., Pi, Y.-L., and Trahair, N. S. (1998). "Design by advanced analysis." *J. Constr. Steel Res.*, 46(103), 144.
- Camp, C., Pezeshk, S., and Cao, G. (1996). "Design of framed structures using a genetic algorithm." *1st U.S.-Japan Joint Seminar on Structural Optimization at Structures Congress XIV*, ASCE, Chicago, 19–30.
- Carroll, D. L. (2004). *FORTTRAN genetic algorithm driver*, CU Aerospace, Champaign, Ill., (<http://cu.aerospace.com/carroll/ga.html>).
- Challa, V. R. M., and Hall, J. F. (1994). "Earthquake collapse analysis of steel frames." *Earthquake Eng. Struct. Dyn.*, 23, 1199–1218.
- Cheng, F. Y. (2002). "Multiobjective optimum design of seismic-resistant structures." *Recent advances in optimal structural design*, S. A. Burns, ed., ASCE, Reston, Va., 241–256.
- Cheng, F. Y., Ang, A. H.-S., Lee, J. H., and Li, D. (1999). "Genetic algorithm for multi-objective optimization and life-cycle cost." *Structural engineering in the 21st century*, ASCE, New Orleans, 484–489.
- Chopra, A. K. (2001). *Dynamics of structures: Applications to earthquake engineering*, Prentice-Hall, Upper Saddle River, N.J.
- Chopra, A. K., and Goel, R. K. (2001). "A modal pushover analysis procedure to estimate seismic demands for buildings: Theory and preliminary implementation." *PEER Rep. 2001/03*, Pacific Earthquake Engineering Research Center, Berkeley, Calif.
- Clarke, M. J., Bridge, R. Q., Hancock, G. J., and Trahair, N. S. (1992). "Advanced analysis of steel building frames." *J. Constr. Steel Res.*, 23(1–3), 1–29.
- Coley, D. A. (2001). *An introduction to genetic algorithms for scientists and engineers*, World Scientific, River Edge, N.J.
- Cornell, C. A., Jalayer, F., Hamburger, R. O., and Foutch, D. A. (2002). "Probabilistic basis for 2000 SAC/Federal Emergency Management Agency steel moment frame guidelines." *J. Struct. Eng.*, 128(4), 526–533.
- Deb, K. (2001). *Multi-objective optimization using evolutionary algorithms*, Wiley, Chichester, U.K.
- Deierlein, G. G., and Mehanny, S. S. (2000). "On assessing near collapse in seismic performance-based design." *Advanced Technology in Structural Engineering—Structures Congress 2000* (CD-ROM), ASCE, Philadelphia.
- Deierlein, G. G., and Moehle, J. (2004). "A framework methodology for performance-based earthquake engineering." *13th World Conf. on Earthquake Engineering* (CD-ROM), Vancouver, B.C., Canada.
- Foley, C. M. (2001). "An object-oriented evolutionary algorithm for the advanced analysis-based design of partially- and fully-restrained steel

- frames." *Technical Rep. to the National Science Foundation Grant CMS 9813216*, Dept. of Civil and Environmental Engineering, Marquette Univ., Milwaukee, Wis.
- Foley, C. M., and Schinler, D. (2001). "Optimized design of partially and fully-restrained steel frames using distributed plasticity." *Structures Congress 2001* (CD-ROM), ASCE, Washington, D.C.
- Foley, C. M., and Schinler, D. (2003). "Automated design of steel frames using advanced analysis and object-oriented evolutionary computation." *J. Struct. Eng.*, 129(5), 648–660.
- Goldberg, D. E. (1989). *Genetic algorithms in search, optimization, and machine learning*, Addison-Wesley Longman, Boston.
- Grierson, D. E., and Khajehpour, S. (2001). "Conceptual design using adaptive computing." *GECCO 2001 Workshop on Optimal Structural Design Using Genetic and Evolutionary Computation*, Morgan Kaufmann, San Francisco, 62–67.
- Gupta, A., and Krawinkler, H. (2000). "Prediction of seismic demands for SMRFs with ductile connections and elements." *Rep. No. SAC/BD-99/06*, SAC Joint Venture.
- Hamburger, R. H. (2004). "Development of next-generation performance-based earthquake engineering design criteria for buildings." *Proc., 13th World Conf. on Earthquake Engineering (WCEE)* (CD-ROM), Vancouver, B.C., Canada.
- Haupt, R. L., and Haupt, S. E. (1998). *Practical genetic algorithms*, Wiley, New York.
- Ibarra, L. F., and Krawinkler, H. (2004). "Global collapse of deteriorating MDOF systems." *13th World Conf. on Earthquake Engineering (WCEE)* (CD-ROM), Vancouver, Canada.
- Khajehpour, S., and Grierson, D. E. (2001). "Conceptual design using adaptive search." *Structures Congress 2001* (CD-ROM), ASCE, Washington, D.C.
- Kunnath, S. K. (2005). "Incremental dynamic analysis." *2005 ASCE SEI Structures Congress* (CD-ROM), ASCE, New York.
- Levy, S. (1992). *Artificial life*, Vintage Books, New York.
- Liu, M., Burns, S. A., and Wen, Y. K. (2006). "Genetic algorithm based construction-conscious minimum weight design of seismic steel moment-resisting frames." *J. Struct. Eng.*, 132(1), 50–58.
- MAE. (2005). Mid-America Earthquake Center, [www.mae.ce.uiuc.edu](http://www.mae.ce.uiuc.edu).
- Michalewicz, Z. (1999). *Genetic algorithms+data structures—Evolution programs*, Springer-Verlag, Berlin.
- Mitchell, M. (1997). *An introduction to genetic algorithms*, Massachusetts Institute of Technology, Cambridge, Mass.
- Naeim, F., Alimoradi, A., and Pezeshk, S. (2004). "Selection and scaling of ground motion time histories for structural design using genetic algorithms." *Earthquake Spectra*, 20(2), 413–426.
- Newmark, N. M. (1959). "A method of computation for structural dynamics." *J. Engrg. Mech. Div.*, 85, 67–94.
- OpenSees. (2001). *Open system for earthquake engineering*, Pacific Earthquake Engineering Research Center, Berkeley, Calif., (<http://opensees.berkeley.edu/>).
- PEER. (2005). Pacific Earthquake Engineering Research Center, ([www.peer.berkeley.edu](http://www.peer.berkeley.edu)).
- Pezeshk, S., and Camp, C. (2001). "State of the on the use of genetic algorithm in design of steel structures." *Recent advances in structural optimization*, S. A. Burns, ed., ASCE, Reston, Va., 200–250.
- Pezeshk, S., Camp, C. V., and Chen, D. (2000). "Design of nonlinear framed structures using genetic optimization." *J. Struct. Eng.*, 126(3), 382–388.
- SAC. (2000). *FEMA-350: Recommended seismic design criteria for new steel moment-frame buildings*, Federal Emergency Management Agency, Washington, D.C.
- Schinler, D. (2001). "Design of partially and fully-restrained steel frames using an object-oriented evolutionary algorithm." MS thesis, Marquette Univ., Milwaukee, Wis.
- Schinler, D., and Foley, C. M. (2001). "An object-oriented evolutionary algorithm for advanced analysis based design." *GECCO 2001 Workshop on Optimal Structural Design Using Genetic and Evolutionary Computation*, Morgan Kaufmann, San Francisco, 73–76.
- Somerville, P., Smith, N. F., Punyamurthula, S., and Sun, J. (1997). "Development of ground motion time histories for Phase 2 of the FEMA/SAC steel project." *Rep. No. SAC/BD97-04*, SAC Joint Venture, Sacramento, Calif.
- Yamvatsikos, D., and Cornell, C. A. (2004). "Applied incremental dynamic analysis." *Earthquake Spectra*, 20(2), 523–553.
- White, D. W. (1992). "Advanced analysis/design of a typical moment frame." *10th Structures Congress*, ASCE, San Antonio, Tex., 330–333.
- White, D. W. (1993). "Plastic-hinge methods for advanced analysis of steel-frames." *J. Constr. Steel Res.*, 24(2), 121–152.
- White, D. W., and Nukala, P. K. V. N. (1997). "Recent advances in methods for inelastic frame analysis: Implications for design and a look toward the future." *Proc., National Steel Construction Conf.*, AISC, Chicago, Vol. 43, 1–24.
- Yun, S.-Y., Hamburger, R. O., Cornell, C. A., and Foutch, D. A. (2002). "Seismic performance evaluation for steel moment frames." *J. Constr. Steel Res.*, 128(4), 534–545.