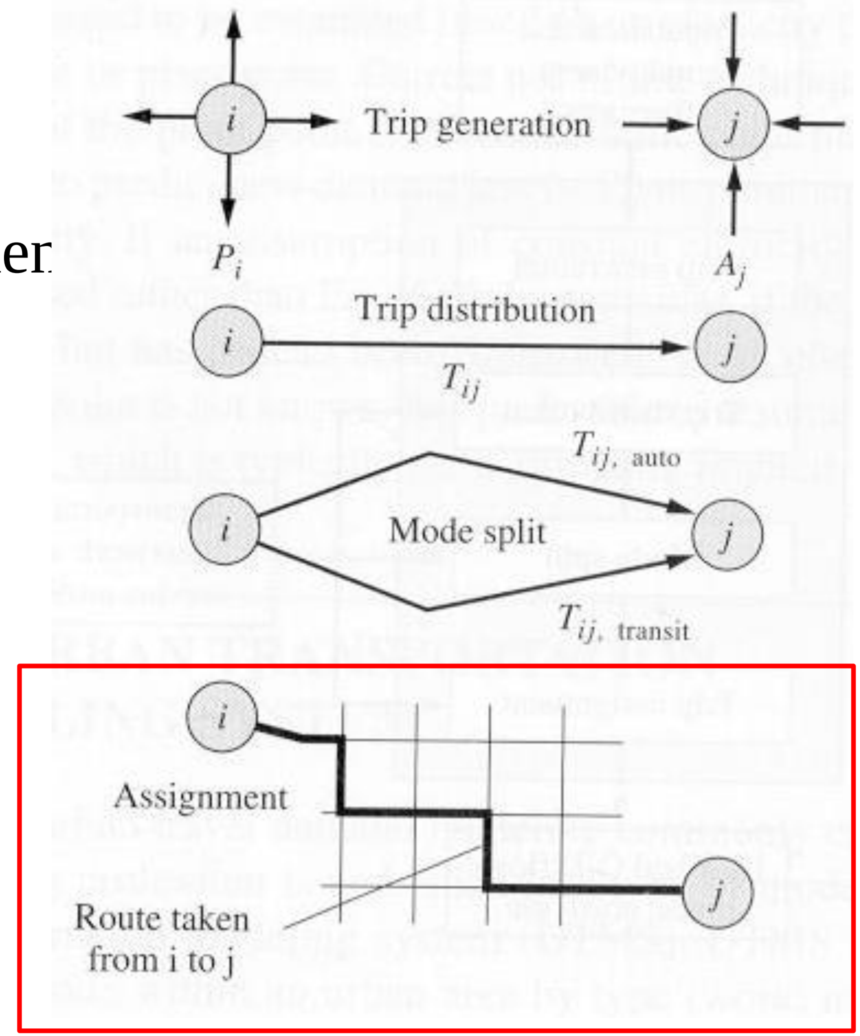
TRAFFIC ASSIGNMENT

Outline

1. Introduction
2. Assignment types and modeling methods
3. User equilibrium (UE) assignment
4. Solution methods and issues
5. Validation

Overview

- The procedure used to obtain expected traffic volume on the network is known as trip assignmen



Traffic Assignment

- Final step: Traffic assignment
  - What: When a person decides where to go and which mode to use, he/she has to decide on the route to take
  - Alternatives: The set of O-D routes in the route choice set for the person
  - Why: Traffic assignment estimates the route/link flows and travel times in the network

Traffic Assignment

- Traffic assignment
  - How: By distributing the total O-D demand between various routes for that O-D pair
  - Network loading mechanism: The process of loading O-D trip table to the network links
  - Difficulty: Individuals are not homogenous
    - Behavior: Different individuals behave differently based on their socio-economic characteristics and past experience
    - Choice set: Different individuals may have different route choice sets
    - Objective: Routing objective may differ across individuals

Assignment Definition

- Given
  - A graph representation of the urban transportation network
  - The associated link performance functions, and
  - An origin-destination matrix
- Find
  - the flow and the associated travel time on each of the network links.
- This problem is known as that of traffic assignment as the objective is to assign the O-D matrix onto the network.

Significance

- Significance of traffic assignment
  - Represents the “basic” level of what we mean by “traffic conditions”
  - Essential to make planning, operational, renewal, and policy decisions
  - Provides “feedback” to trip distribution and mode split steps of the 4-step model
  - Provides input to assess and influence energy and environmental impacts
  - Aids transportation operators in making “supply” decisions
  - Key methodological engine for intelligent transportation systems (ITS) applications

Cost Function

- Conventional Economics: Demand (D) and Supply (S)
- Equilibrium between D&S defines the price
- Equilibrium point
  - Marginal cost=Marginal revenue f
- In TA Cost is a function of a number of attributes
  - Distance
  - Free Flow Speed
  - Capacity
  - Speed-Flow Relationship
  - Fares, Fuel, ...
- Demand=Origin-Destination (OD) matrix
- Supply=Network Capacity

Dimensions

- Some dimensions of traffic assignment
  - Mode: Non-scheduled, scheduled (transit)
  - Time: Static, dynamic
  - Randomness/uncertainty: Deterministic, stochastic (demand and/or supply)
  - Objective: User equilibrium (UE), system optimal (SO), boundedly-rational (BR)
  - Behavior: Several factors (such as familiarity, risk willingness, etc.)
  - Function: Descriptive, prescriptive

Dimension-Mode

- (Road) Traffic assignment
  - Assign non-scheduled O-D trip demand for each O-D pair
- Transit assignment
  - Assign passengers who use the routes on a transit network using the transit O-D demand
  - Increasing future importance as transit (bus, rail, etc.) becomes a preferred solution
- Intermodal assignment
  - Assign intermodal trips (such as park-and-ride, ferry and bus)
  - Increasingly important in the future
  - Future research needs

Dimension-Time

- Static traffic assignment
  - Time is not a consideration
  - O-D trip rate is constant
  - Link travel times are constant
  - Appropriate for analysis of "off-peak" and/or homogenous conditions
  - Useful for long-term planning purposes

- Dynamic traffic assignment (DTA)
  - Time-dependency of traffic conditions is explicitly considered
  - O-D trip demand and travel times (flows) are time-dependent
  - Appropriate for "peak-period" analyses and to capture time-dependency when it is a significant aspect of the analysis
  - Useful for real-time operations and management, including for assessing various ITS strategies
  - In recent years, used for planning applications

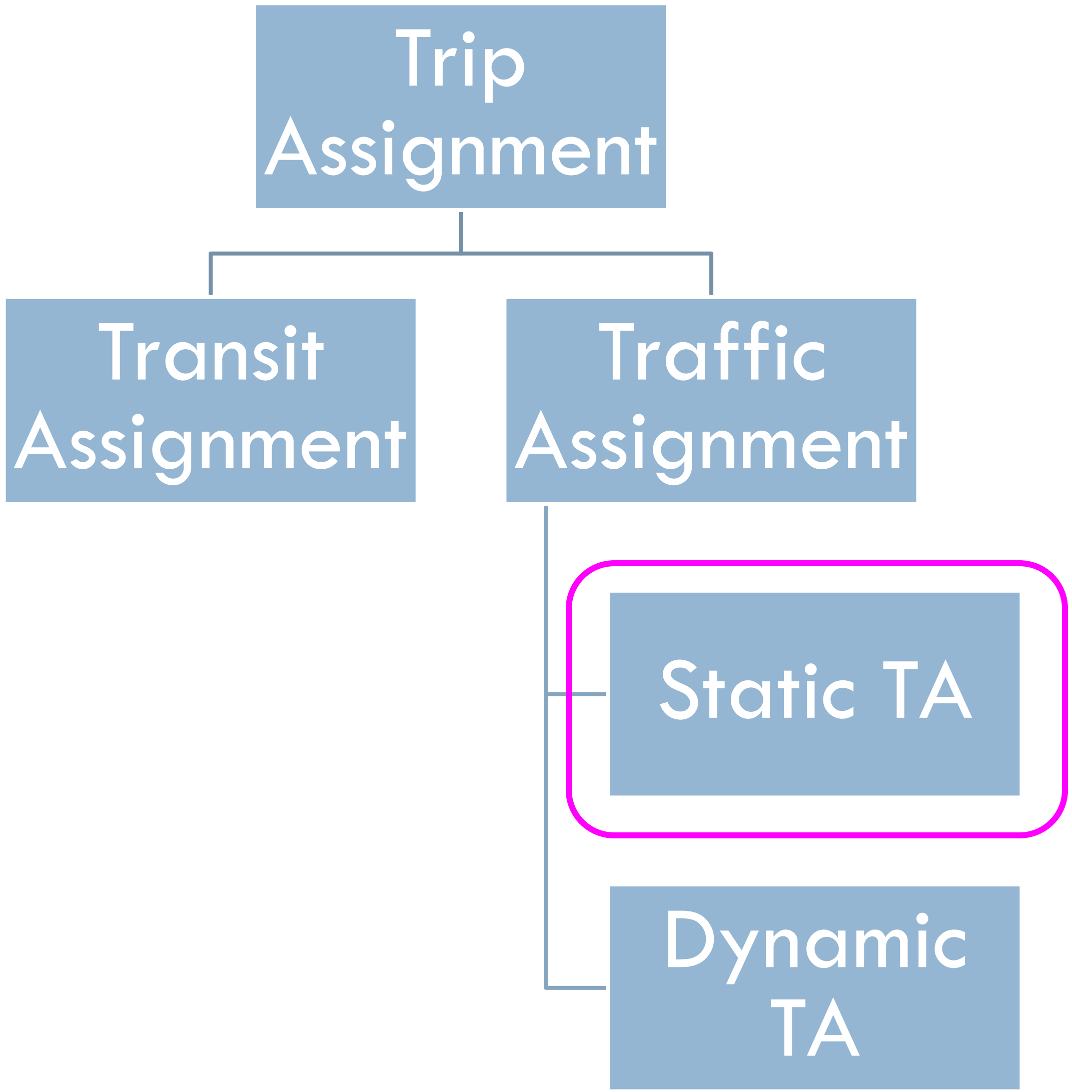
Dimension-Objective

- User equilibrium assignment
  - Will be discussed in detail later
  - User behavior is “selfish”
  - Reasonable estimate of actual driver behavior
  - Equilibrium
  - Adequate for long-term planning
- System optimal assignment
  - “Socially optimal”
  - Seeks best system performance
  - Behaviorally untenable; not an equilibrium
  - Requires coordination and/or collaboration
  - Provides a benchmark for comparing various traffic management strategies
  - Useful for developing prescriptive traffic strategies
  - Useful for many ITS applications

Dimension- Objective

- Deterministic assignment
  - Demand, supply, performance aspects are known *a priori*
  - Focus is on randomness, not time-dependency
- Stochastic assignment
  - One or more of demand, supply, and performance characteristics have randomness
  - Useful for modeling heterogeneity in individuals
  - For example, stochastic user equilibrium (SUE) assumes that individuals perceive link/route travel times differently (based on their behavioral tendencies)

Assignment Types

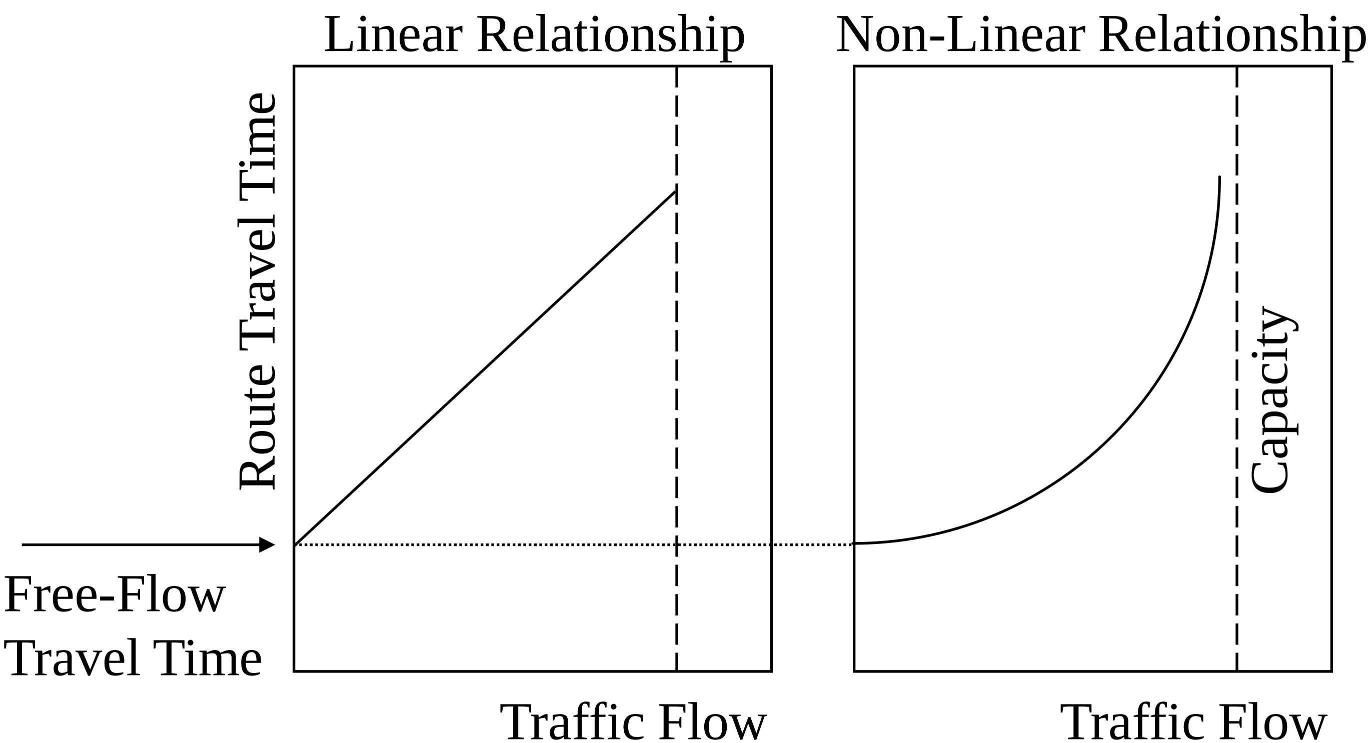


Static Traffic Assignment-Assumptions

- Standard assumptions
  - O-D demand is constant (does not vary with time)
  - Link travel times are time-invariant
  - A route flow exists on all the links comprising that route simultaneously
- Performance function: Travel time on a link depends only on the flow on that link and does not depend on flows on other links (though this assumption is not necessary)

Link Performance Functions

- Mathematical Relationship Between Traffic Flow and Travel Time

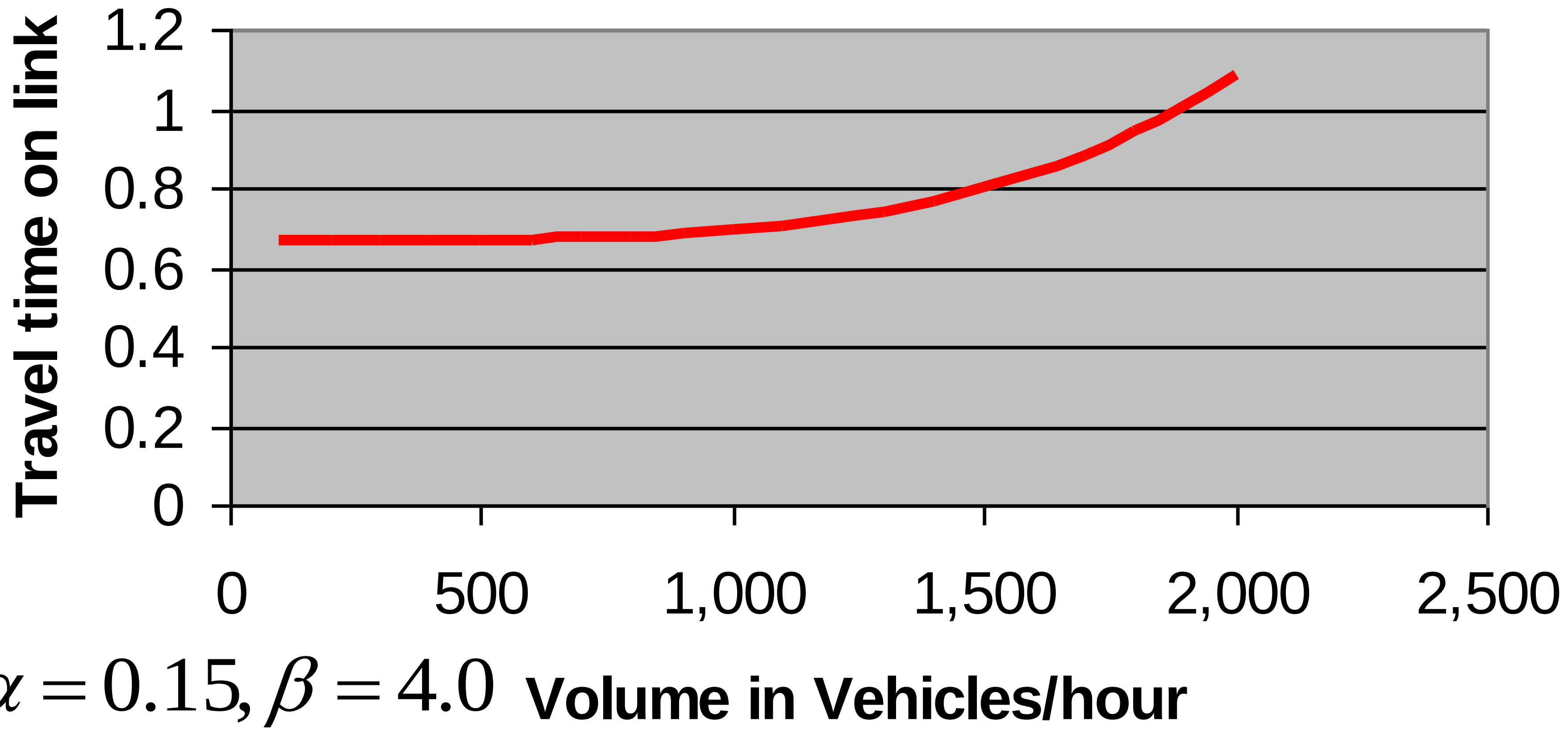


Link Performance Functions

- A steady-state link performance function is a positive, increasing, and convex curve.

- Typical link performance functions do not consider queued vehicles in the traffic stream

Link Performance Functions

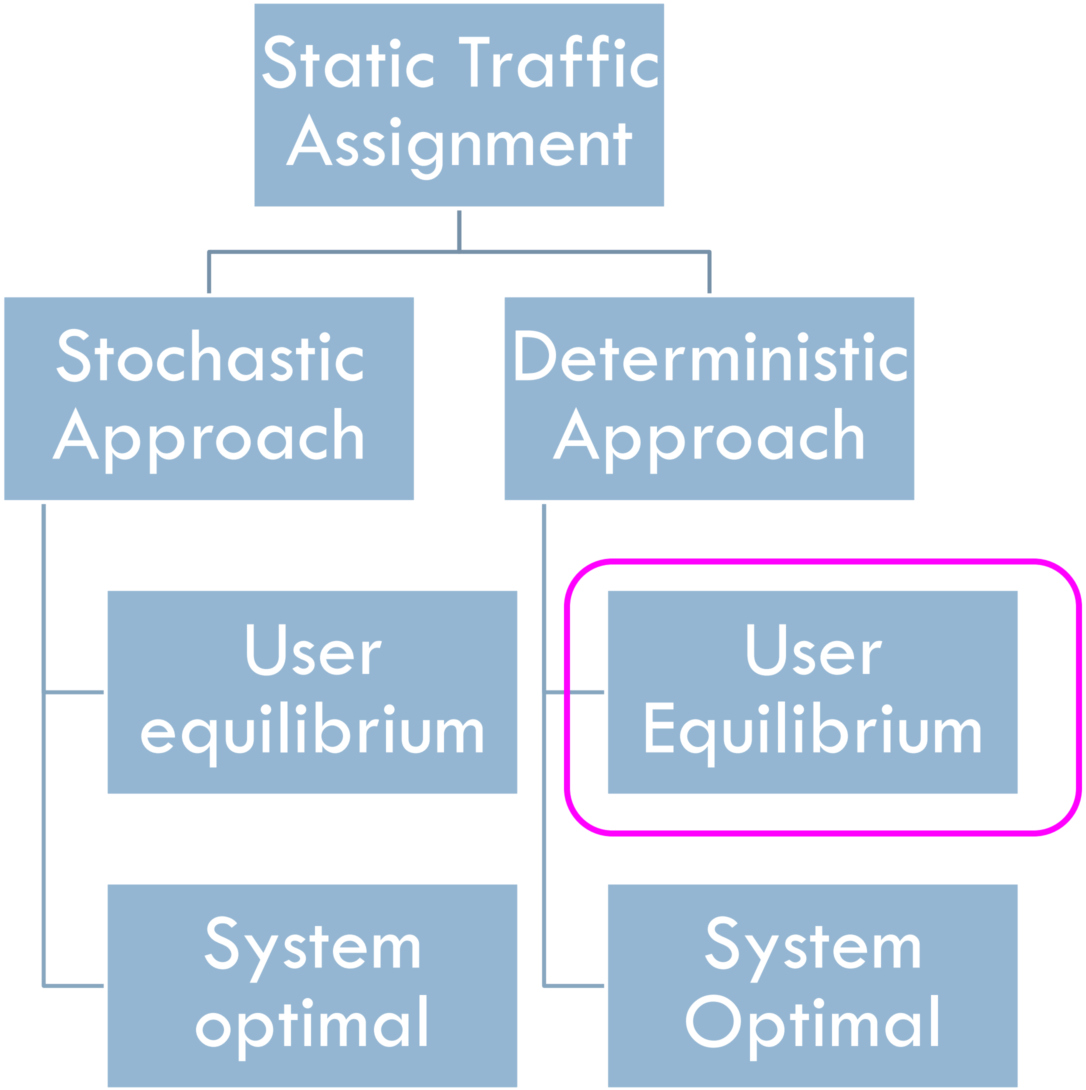


$\alpha = 0.15, \beta = 4.0$

V=volume, C=capacity, t_0=free flow travel time

$$t = t_0 \left[1 + \alpha \left(\frac{V}{C} \right)^{\beta} \right]$$

Static UE Assignment



Definition of Equilibria

- To solve the traffic assignment problem, it is required that the rule by which motorists choose a route be specified.
- It is reasonable to assume that every motorist will try to minimize his or her own travel time when traveling form origin to destination.
- A stable condition is reached only when no traveler can improve his/her travel time by unilaterally changing routes.

Equilibrium

- UE definition implies that
  - motorists have full information (choice set and travel times),
  - motorists consistently make the correct route choice decision
  - all motorists are identical in their behavior
- These assumptions can be partially relaxed in the context of route choice under information provision.
  - distinction between the travel time that individuals perceive and the actual travel time
  - This definition characterizes the stochastic-user-equilibrium (SUE) condition.

Wardrop's Equillibrium

- Flow allocation rules
  - Wardrop's first principle

    "For each O-D pair, the journey times on all used routes are equal, and less than or equal to those on any unused route"
    - Defines User Equilibrium (UE) flow
  - Wardrop's Second principle

    "The total system travel time is minimum (that is, the average journey time is minimum)"
    - Defines System Optimal (SO) flow

UE Characteristics

- Characteristics
  - At user equilibrium, the travel time on all used routes are equal and less than or equal to those on any unused route
  - At user equilibrium, no user can improve his/her travel time by unilaterally switching routes

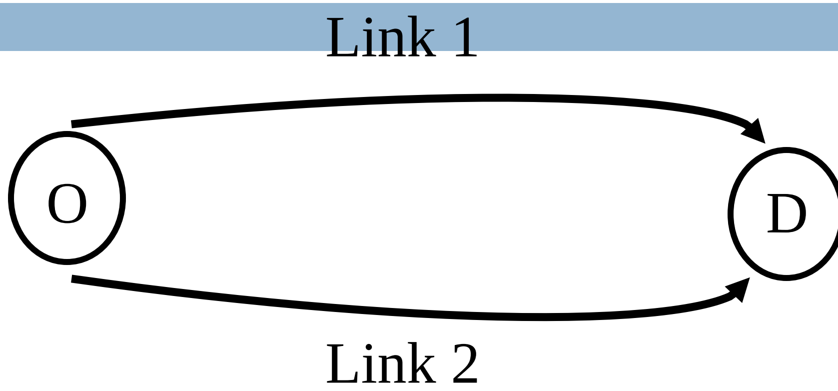
UE-Assumptions

- Assumptions
  - Individuals have full knowledge of travel times on all possible routes
  - All individuals are identical in their behavior (for example, perceive travel time identically)
  - Travel time is the only factor in the decision-making (all individuals unilaterally seek to decrease their travel times)

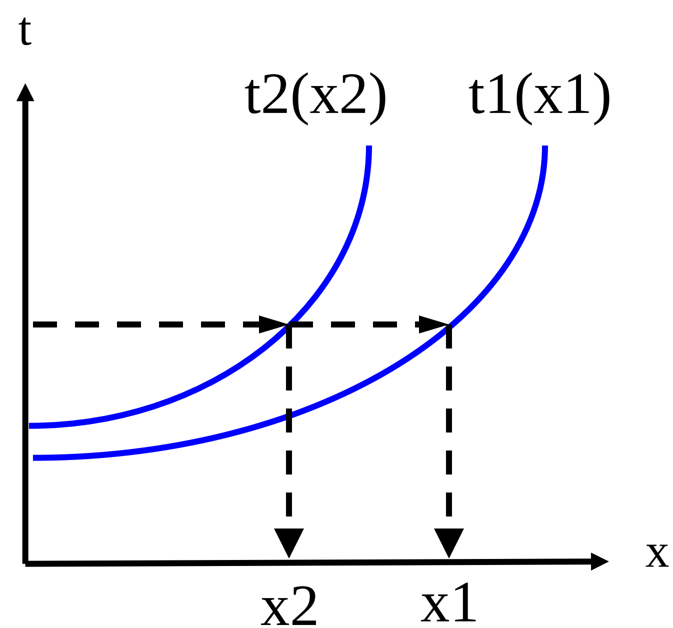
UE-Concept

- Example
  - Given
    - Network with 1 O-D pair and 2 routes (each route has just one link)
    - O-D demand = 10
    - Link performance functions (shown on next slide)
  - Determine UE:
    - Route flows
    - Travel times

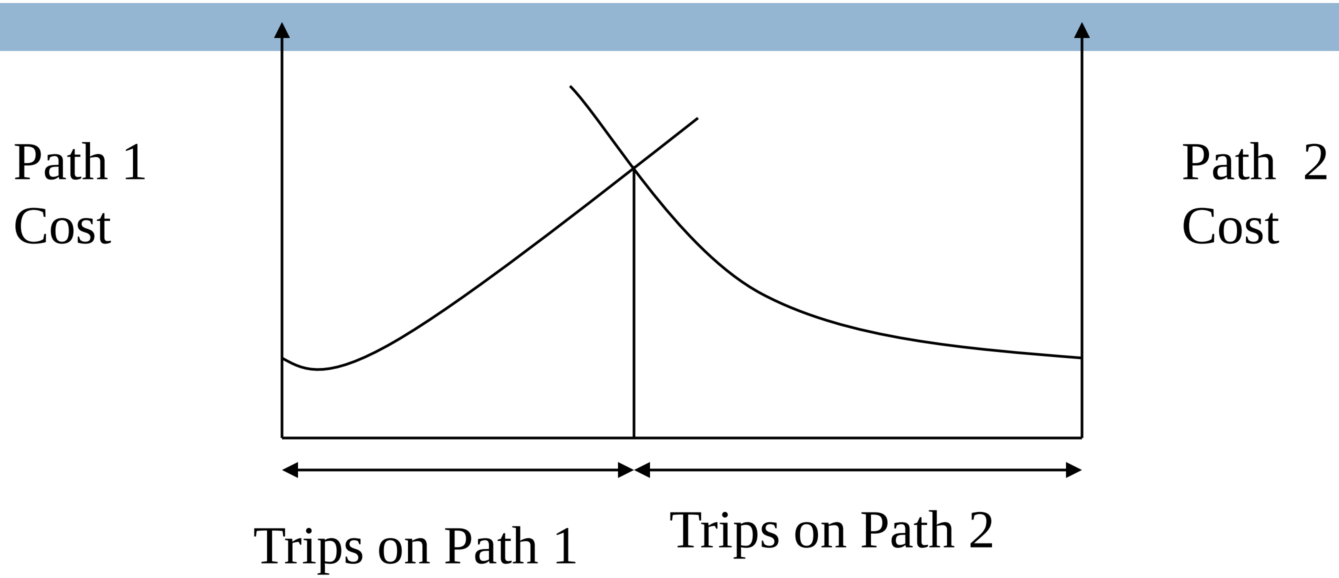
A simple example of UE



$q^{OD} = x1 + x2$



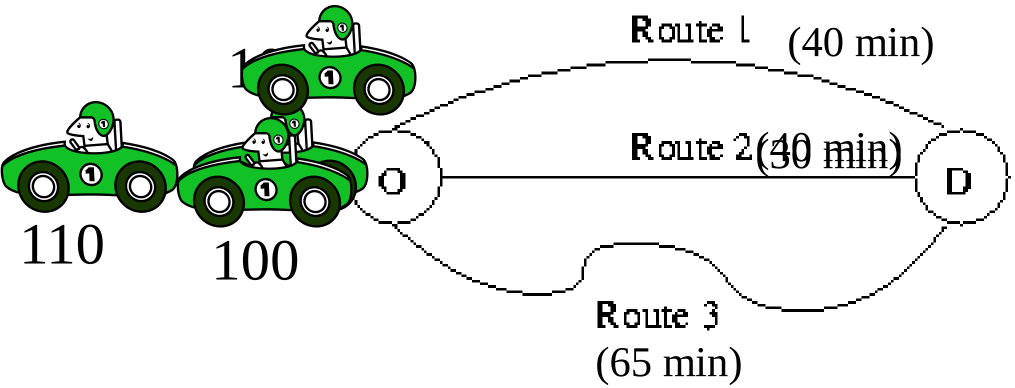
Operational UE

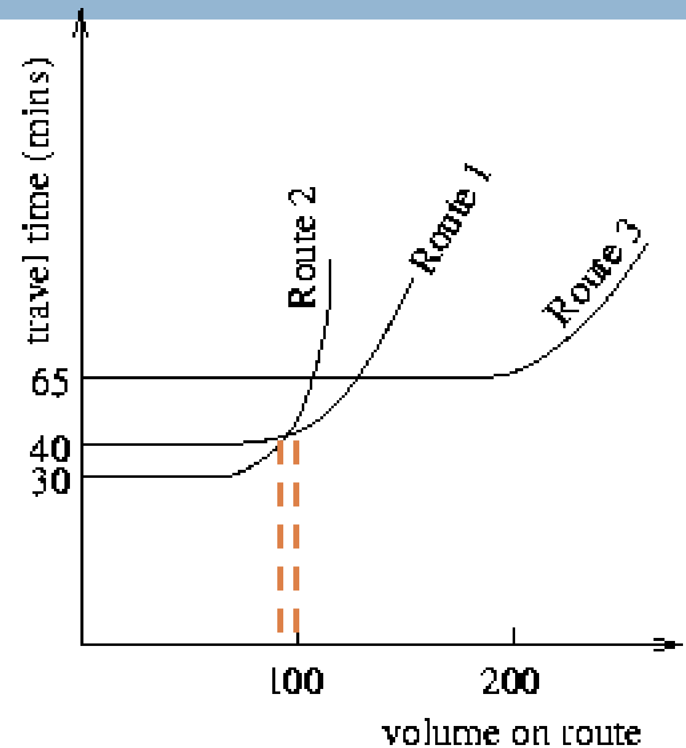


Operational definition of UE:

For each O-D pair, at user equilibrium, the travel time on all used paths is equal, and (also) less than or equal to the travel time that would be experienced by a single vehicle on any unused path.

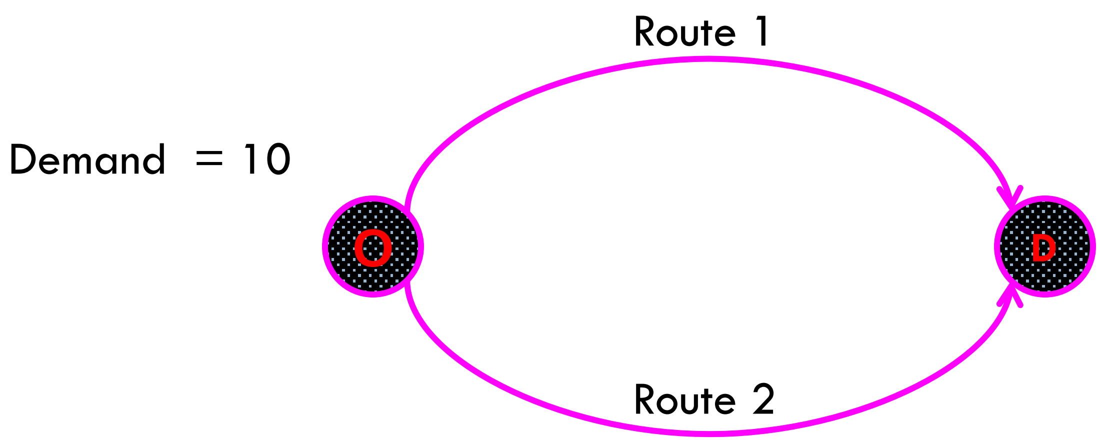
Example UE





User Equilibrium is reached when no traveler can improve his travel time by unilaterally changing routes.

UE-Example



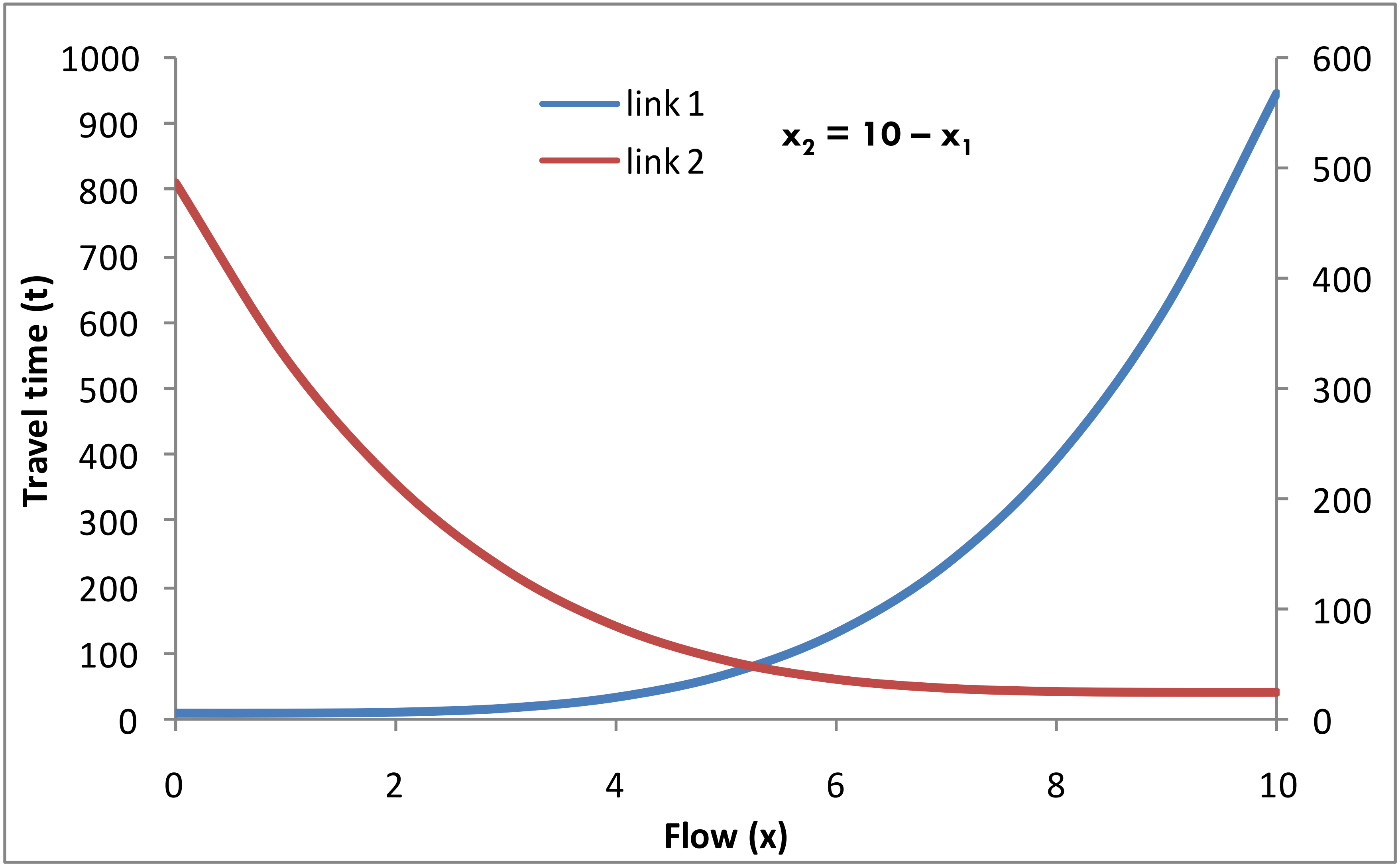
$$t_1 = 10\left[1 + 0.15\left(\frac{x_1}{2}\right)^4\right]$$

$$t_2 = 25\left[1 + 0.15\left(\frac{x_2}{3}\right)^4\right]$$

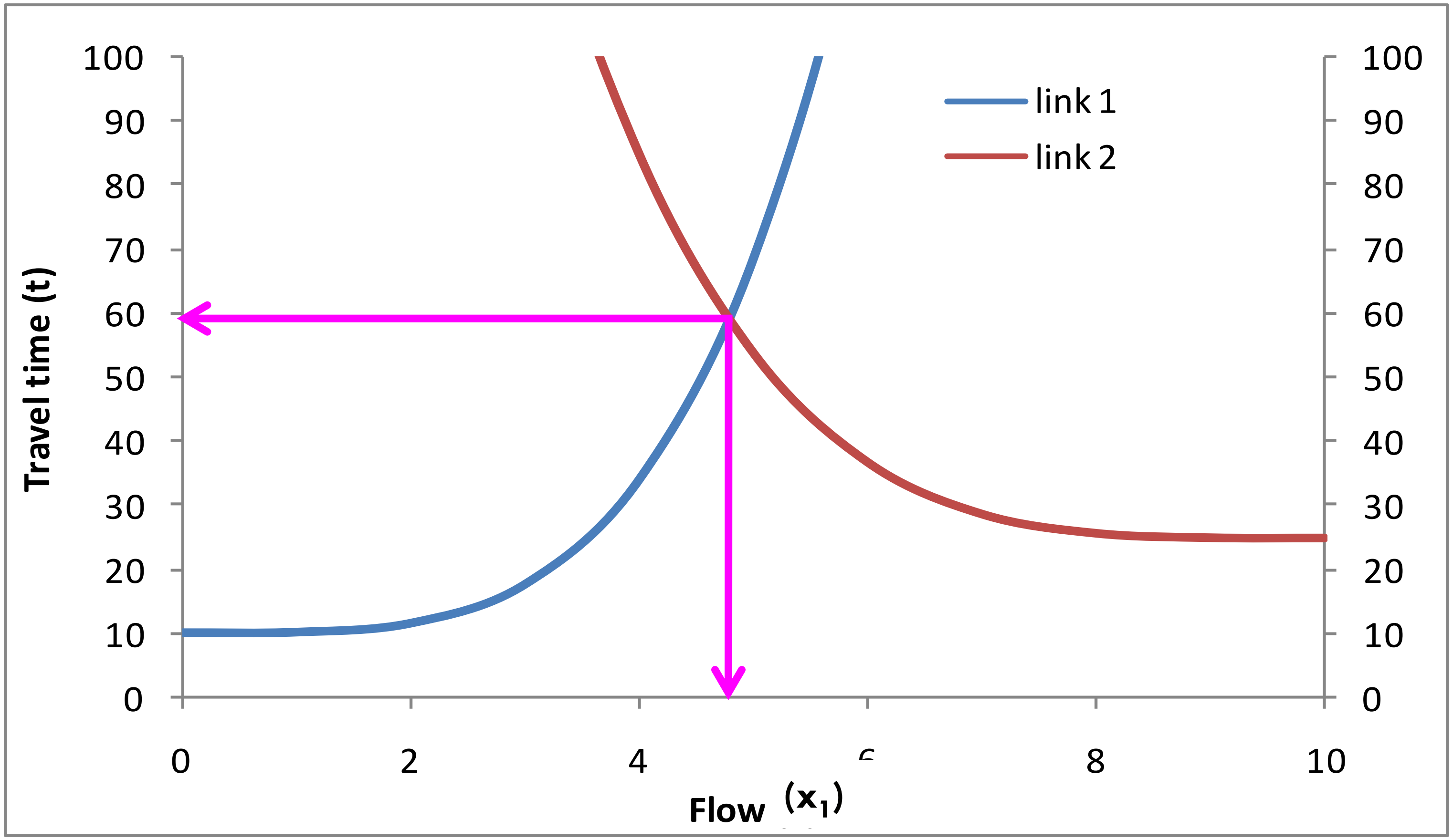
t denotes time x denotes flow

Ref: Sheffi (1984)

UE-Example Concept

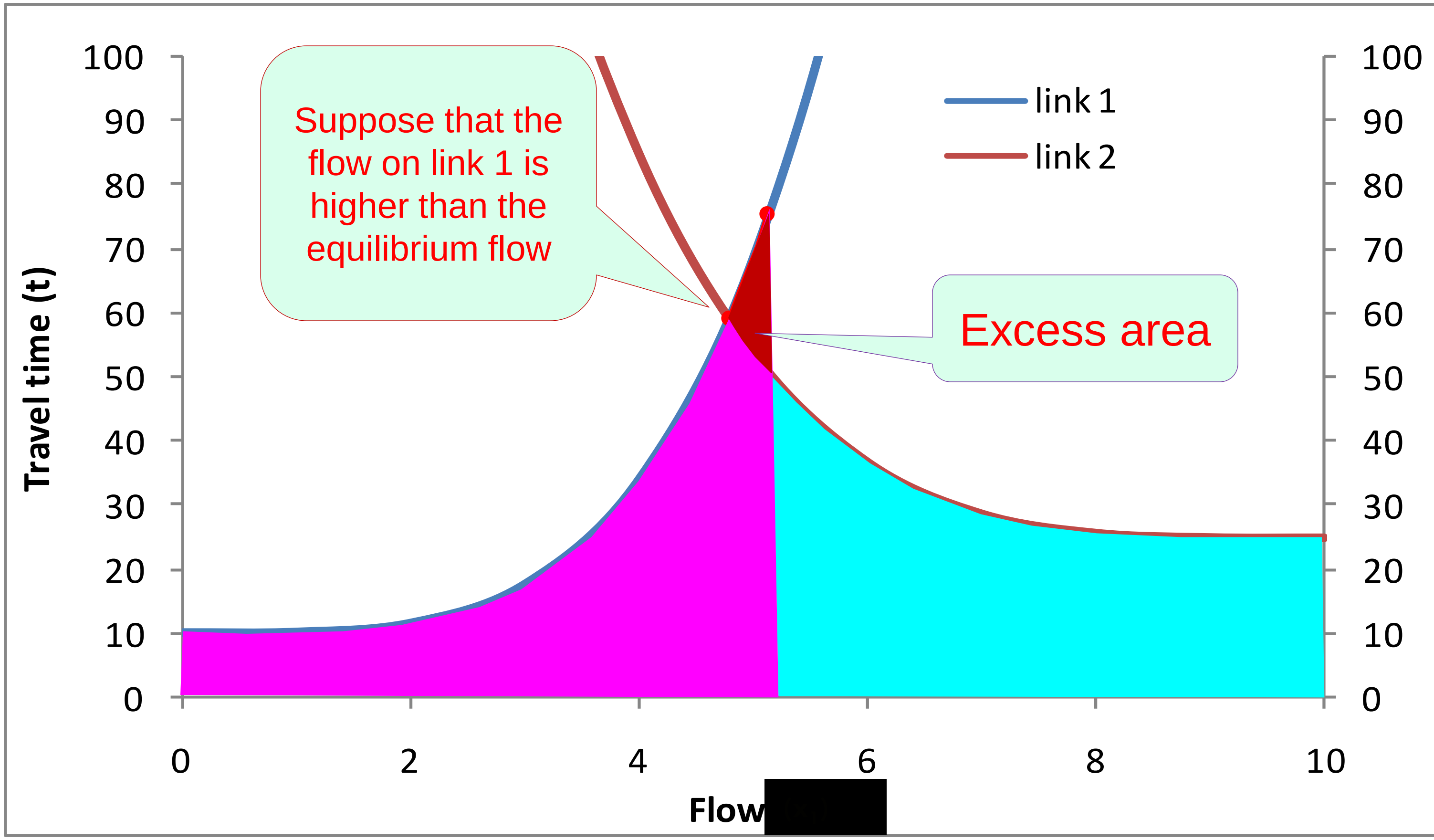


UE-Example Concept



- Flow on link 2 (x_2) = $10-x_1$
- $x_1=4.8$, $x_2=5.2$; $t_1=t_2=59$ (approx)

UE-Example Concept



- Sum of area under the curves is minimum at equilibrium point

Graphical Approach

- Problem with graphical approach
  - Cannot be used when an O-D pair has more than two routes
  - Cannot be used when routes have more than one link, and when some links are common to routes on the same or different O-D pair

UE Approaches

- Heuristic methods <u>(we will not talk in this presentation)</u>
  - *All-or-nothing (AON)*
  - *Capacity restraint*
  - *Incremental assignment*
- Analytical methods (Feasible direction methods)
  - Frank-Wolfe (F-W) algorithm
  - Link-based methods
    - Modified F-W methods
  - Origin-based methods
  - Path-based methods

Analytical Approach

- Feasible direction methods
  - Mathematical formulation for UE problem as an optimization problem
  - Consists of objective function and constraints
  - Equivalency of optimization problem and UE conditions

Formulating the Assignment Problem

- NOTATIONS
  - Network G (*N,A*)
  - *N* is set of consecutively numbered nodes
  - *A* is a set of consecutively numbered arcs (links)
  - *R* denote the set of origin centroids (which are the nodes at which flows are generated)
  - *S* denote the set of destination centroids (which are the nodes at which flows terminate)
  - q_{rs} is the trip rate between origin "r" and destination "s" during the period of analysis
  - x_a and t_a, represent the flow and travel time, respectively, on link a

Formulating the Assignment Problem

NOTATIONS

$t_a = t_a(x_a)$: where $t_a(.)$ represents the relationship between flow and travel time for link a

f_k^{rs} : represents flow on path k connecting origin r and destination s such that path k $\in K_{rs}$

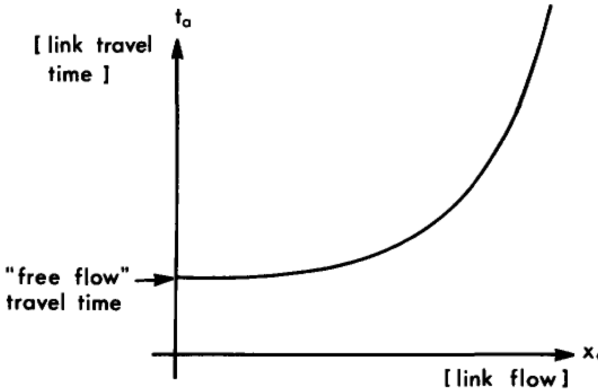
c_k^{rs} : is travel time on path k is the sum of the travel time on the links comprising this path.

$$c_k^{rs} = \sum_a t_a \delta_k^{rs} \quad \forall k \in K_{rs}, \forall r \in R, \forall s \in S$$

where

$\delta_k^{rs} = 1$ if link is part of path k connecting O-D pair $r-s$

$= 0$ otherwise



Using the same indicator variable, the link flow can be expressed as a function of the path flow, that is

$$x_a = \sum_a \sum_r \sum_s f_k^{rs} \delta_{a,k}^{rs}$$

User Equilibrium Formulation

$$\min\ z(x) = \sum_a \int_0^{x_a} t_a(\omega)\, d\omega$$

subject to

$$\sum_k f_k^{rs} = q_{rs} \qquad \forall\ r, s$$

flow conservation constraint

$$f_k^{rs} \geq 0 \qquad \forall\ k, r, s$$

Non negativity constraint

$$x_a = \sum_a \sum_r \sum_s f_k^{rs} \delta_{a,k}^{rs} \qquad \forall a$$

$$\delta_{a,k}^{rs} = \begin{cases} 1\ if\ \ link\ a\ is\ on\ path\ k\ between\ o-d\ pair\ rs \\ \quad 0\ \ \ otherwise \end{cases}$$

definitional constraints

Significance of User Equilibrium

- Significance:
  - Reasonable assumption for representation of human behavior
  - In order to asses the network performance for given demands UE conditions are assumed
- Limitations:
  - Assumption that each user minimizes travel time implies each user has perfect information on all conditions and routes
  - Individuals are assumed to behave identically

UE-Solution Method

- Developments
  - Beckmann et al. (1956) proved the equivalency between their transformation and UE problem
  - They also proved that their formulation has unique solution in terms of link flows
  - Frank-Wolfe (F-W) algorithm (1956) was used to solve Beckmann's UE formulation (most commonly used)
  - Since then many researchers have contributed to this field and many algorithms have developed

F-W Formulation

- In Frank-Wolfe algorithm, the updated route flows in each iteration are obtained by combining the current set of route flows with current all-or-nothing assignment flows:

$$f_{kij}^{(n+1)} = \begin{cases} (1-\alpha) f_{kij}^{n} & kij \neq kij^{*} \\ (1-\alpha) f_{kij}^{n} + \alpha f_{ij} & kij = kij^{*} \end{cases}$$

- In each iteration, the flow on minimum cost route increases and that on other routes decreases for an O-D pair
- The shift of flows from expensive routes to the cheapest route is proportional to current flow and step size
- The route flows are used to generate the link flows
- In terms of the link flows:

$$\boxed{x_a^{n+1} = x_a^n + \alpha_n (y_a^n - x_a^n),}$$

F-W Merits

- Merits
  - Easy to implement
  - Converges very fast in early iterations
  - Solution is much better than heuristic techniques such as incremental assignment
  - Requires less memory (RAM)

F-W Limitations

- Limitations
  - Tails badly into creep and is very slow in reaching objective function minimum (less efficient)
  - Provides only link flows, but for many planning applications, we need route flows

UE Note

- Demand for travel depends on the activity pattern, and hence not uniform over time and space.
- However transportation planners analyze networks only for certain periods of the day – morning peaks, evening peaks etc. depending on objective of analysis
- => O-D flows are considered constant for such analysis (steady-state) -> static assignment
- Flow is present simultaneously on all links of a path (static conditions)

System Optimal Assignment

$$\min \ z(x) = \sum_a x_a \, t_a(x_a)$$

Total system travel time

subject to

$$\sum_k f_k^{rs} = q_{rs} \qquad \forall \, r, s$$

flow conservation constraint

$$f_k^{rs} \geq 0 \qquad \forall \, k, r, s$$

Non negativity constraint

$$x_a = \sum_a \sum_r \sum_s f_k^{rs} \delta_{a,k}^{rs} \qquad \forall a$$

Definitional constraint

$$\delta_{a,k}^{rs} = \begin{cases} 1 \ if \ \ link \ a \ is \ on \ path \ k \ between \ o - d \ pair \ rs \\ \quad 0 \quad otherwise \end{cases}$$

SO Properties

- The SO formulation is subject to the same set of constraints as the UE problem and differs only in its objective function

- The SO flow pattern does not generally represent an equilibrium solution in congested networks

- Consequently, the SO flow pattern is not an appropriate descriptive model of actual user behavior

Significance of System Optimal

1. In many transportation system analysis problem it is useful to know the best performance possible for the network and OD demand

2. This is useful for control action (pricing, tolling) as well as to compare alternative solution strategies

3. Solution procedures for SO are virtually identical to those for UE

Solving UE

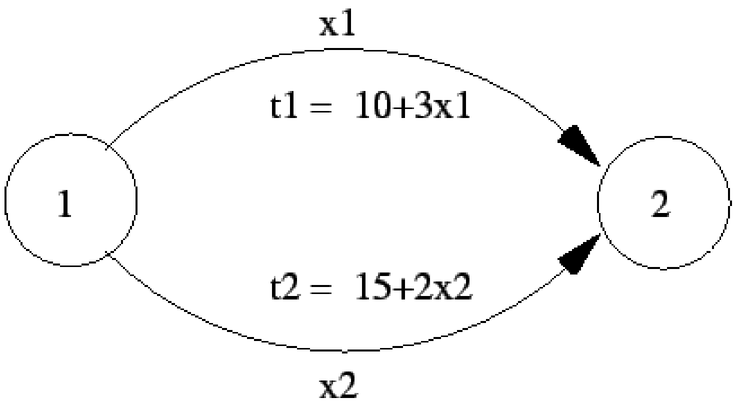


Figure 2: Two Link Problem with variable travel time function

Lets now take a case where travel time functions for both the links is given by:

$$t_1 = 10 + 3x_1$$
$$t_2 = 15 + 2x_2$$

and total flows from 1 to 2.

$$q_{12} = 12$$

Solving UE

$$\begin{aligned} min: \quad Z(x) &= \int_0^{x_1} (10 + 3x_1)\ dx_1 \\ &\quad + \int_0^{x_2} (15 + 2x_2)\ dx_2, \\ &= 10x_1 + \frac{3x_1^2}{2} + 15x_2 + \frac{2x_2^2}{2}, \\ st: \quad x_1 + x_2 &= 12. \end{aligned}$$

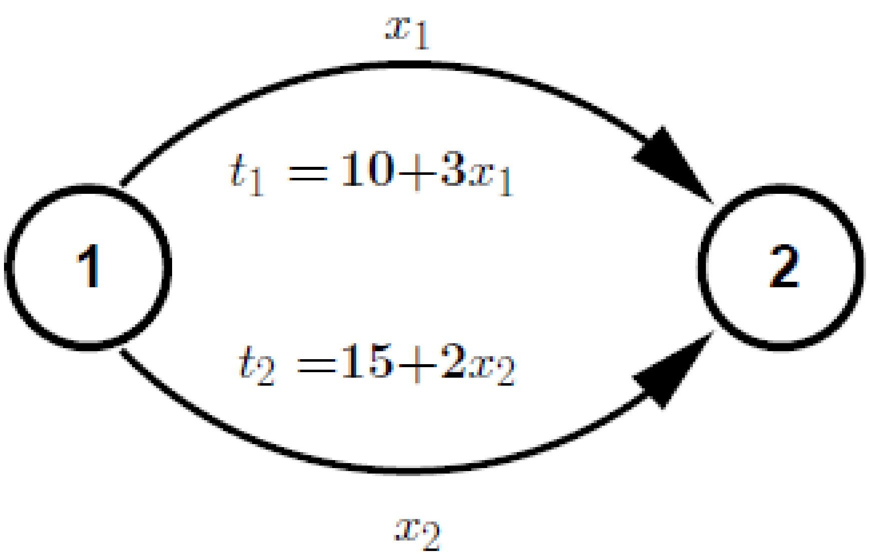
- Substitute x2= 12-x1

$$min\ Z(x) = 10x_1 + \frac{3x_1^2}{2} + 15(12 - x_1) + \frac{2(12 - x_1)^2}{2}$$

- Differentiate w.r.t x1 and equate to zero
- x1= 5.8, x2 = 6.2.

Solving SO

- Let us consider the same example



- For S $$\begin{aligned} min\ Z(x) &= x_1 * (10 + 3x_1) + x_2 * (15 + 2x_2) \\ &= 10x_1 + 3{x_1}^2 + 15x_2 + 2{x_2}^2 \end{aligned}$$

Solving SO

- Substitute x2= 12-x1

$$min\ Z(x) == 10x_1 + 3{x_1}^2 + 15(12 - x_1) + 2(12 - x_1)^2$$

- Differentiate the equation and set it to zero

- x1 = 5.3,x2= 6.7

Comparison of Methods

Type	t1	t2	x1	x2	UE Z(x)	SO Z(x)
AON	10.00	15.00	12.00	0.00	336.00	552.00
UE	27.40	27.40	5.80	6.20	239.90	328.80
SO	30.10	25.60	5.30	6.70	240.53	327.55

Stochastic Methods

- Emphasize the variability in driver perception of cost
- Need to consider second best routes
- No perfect information about network characteristics
- Different travel costs perception
- Eliminates “zero volume” links
- Requires large number of iterations and hence a longer run time
  - See more in Modeling Transport by Ortuzar and Williumsen, Chapter 10. or Sheffi (1984) Chapter 7

Stochastic Methods

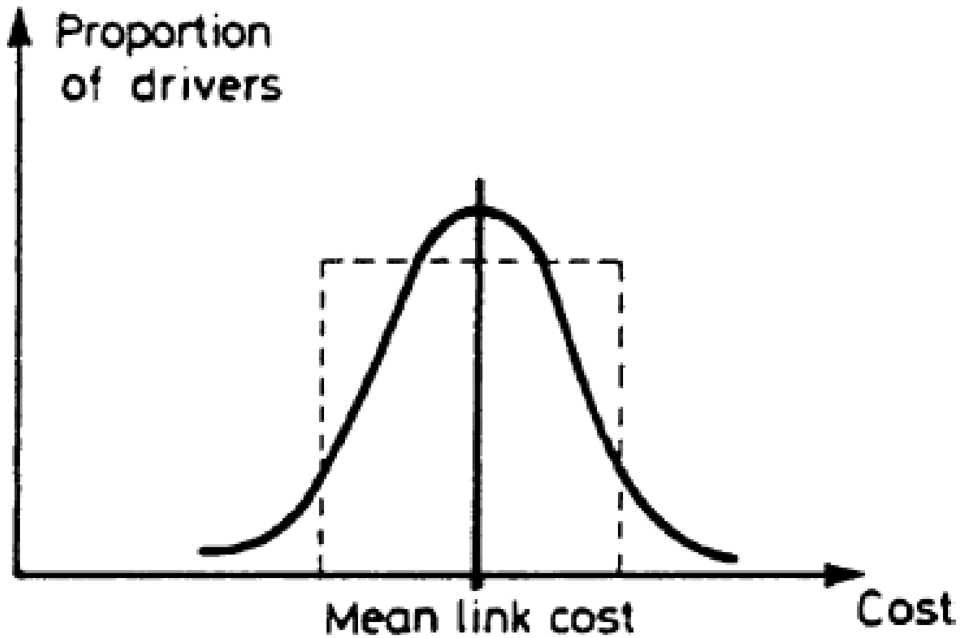
- Need to consider second-best routes (in terms of engineering or modelled costs);
- Generates additional problems as the number of alternative second-best routes between each O–D pairmay be extremely large

Stochastic Methods

- Several methods have been proposed to incorporate these aspects but only two have relatively widespread acceptance:
  - *simulation-based methods*
    - Uses ideas from stochastic (Monte Carlo) simulation to introduce variability in perceived costs.
  - *proportion-based methods*
    - allocates flows to alternative routes from proportions calculated using logit-like expressions.

Simulation Based Methods

- There is a distribution of perceived costs for each link with the engineering costs as the mean



Simulation Based Methods

- Assumptions
  - The distributions of perceived costs are assumed to be independent.
  - Drivers are assumed to choose the route that minimizes their perceived route costs, which are obtained as the sum of the individual link costs.

Proportional Stochastic Methods

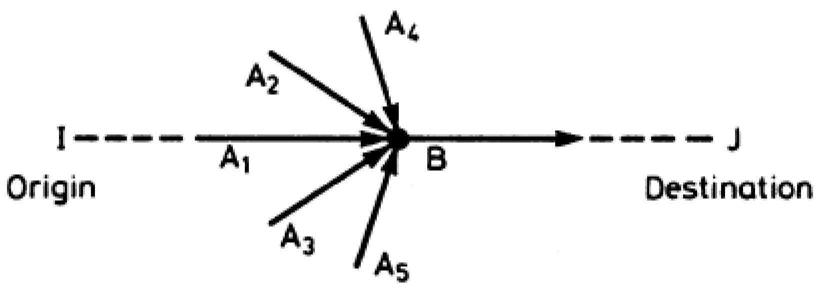
- Virtually all these methods are based on a loading algorithm which splits trips arriving at a node between all possible exit nodes,
  - as opposed to the all-or-nothing method which assigns all trips to a single exit node.
- Very often the implementation of these methods reverses the problem so that the division of trip at a node is actually based upon where the trips are coming from rather than where they are going to.

Proportional Based Methods

- Consider node B in Figure; there are a number of possible entry points denoted by A1, A2, A3, A4 and A5 for trips from I to J.
- Splitting functions

$$\begin{array}{ll} f_i = 0 & \text{if } d_{A_i} \geq d_B \\ 0 < f_i \leq 1 & \text{if } d_{A_i} < d_{B_i} \end{array}$$

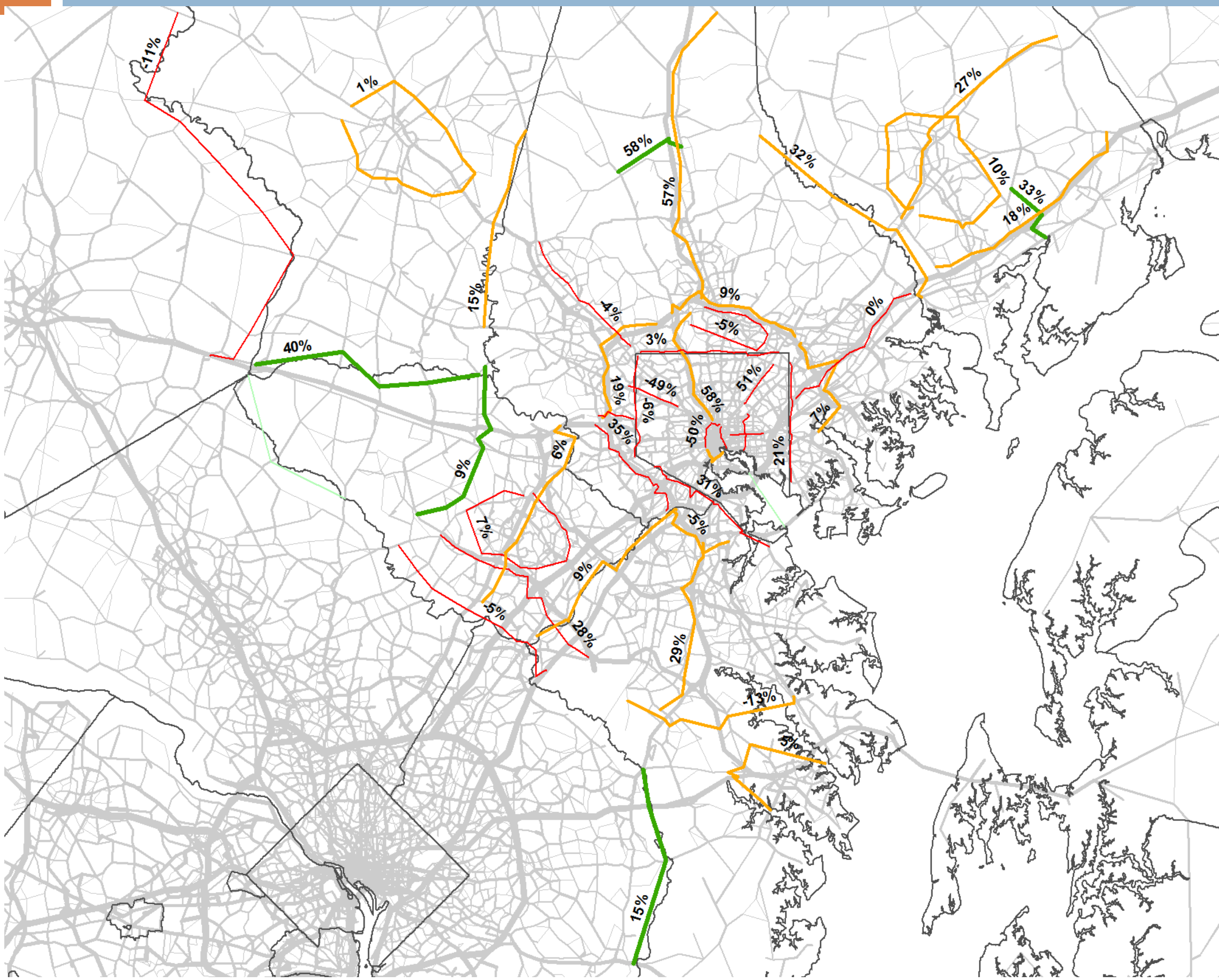
$$F(A_i, B) = \frac{T_B f_i}{\sum_i f_i}$$



Model Validation

- Truck counts
  - By vehicle class
  - By facility type
  - By time of day
  - Screen lines
  - Cordon lines
- Develop RMSE, R^2 or other goodness-of-fit measures

Screenlines Example



Share of counts per screenline

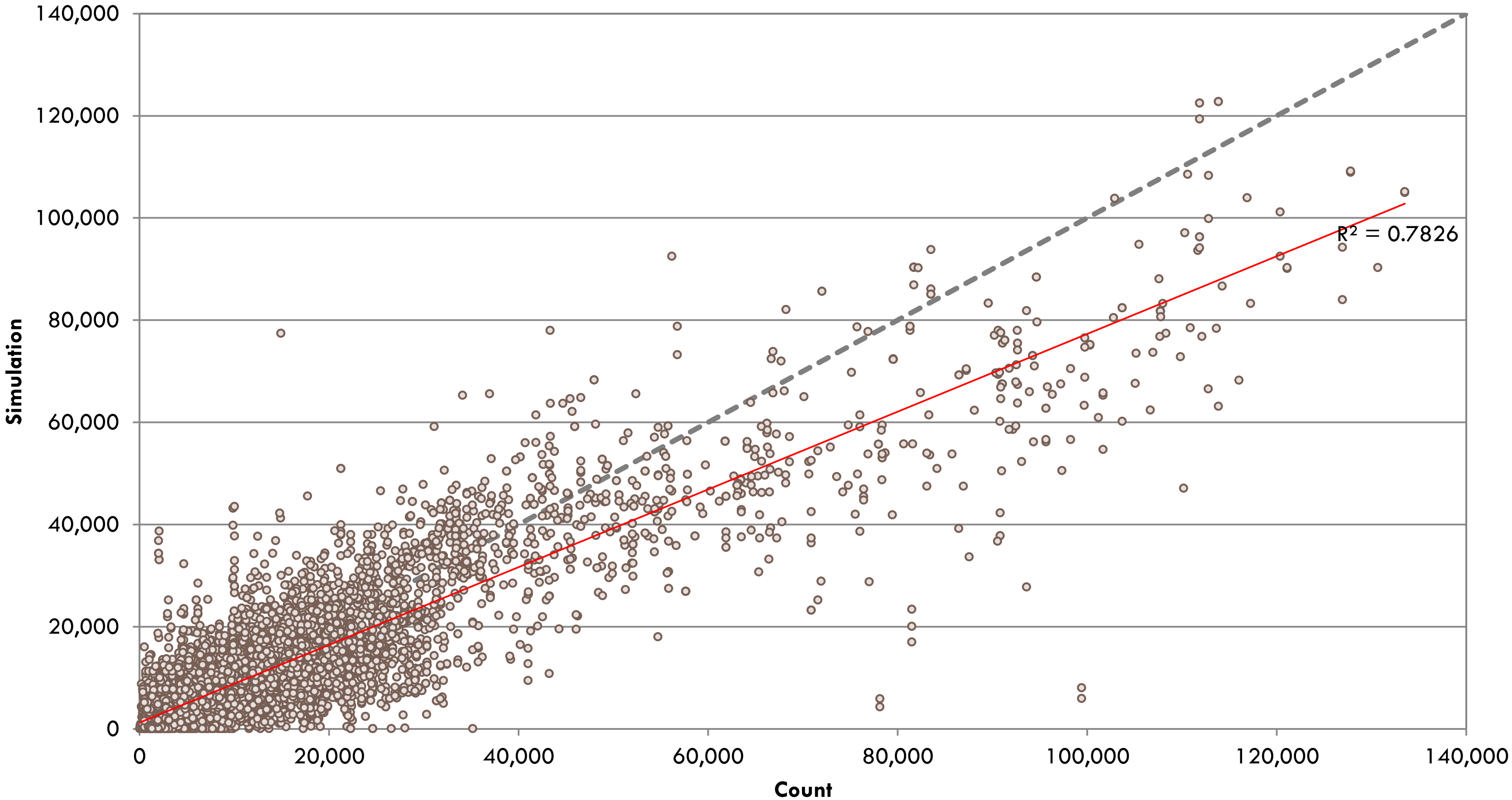
> 75%

50% - 75%

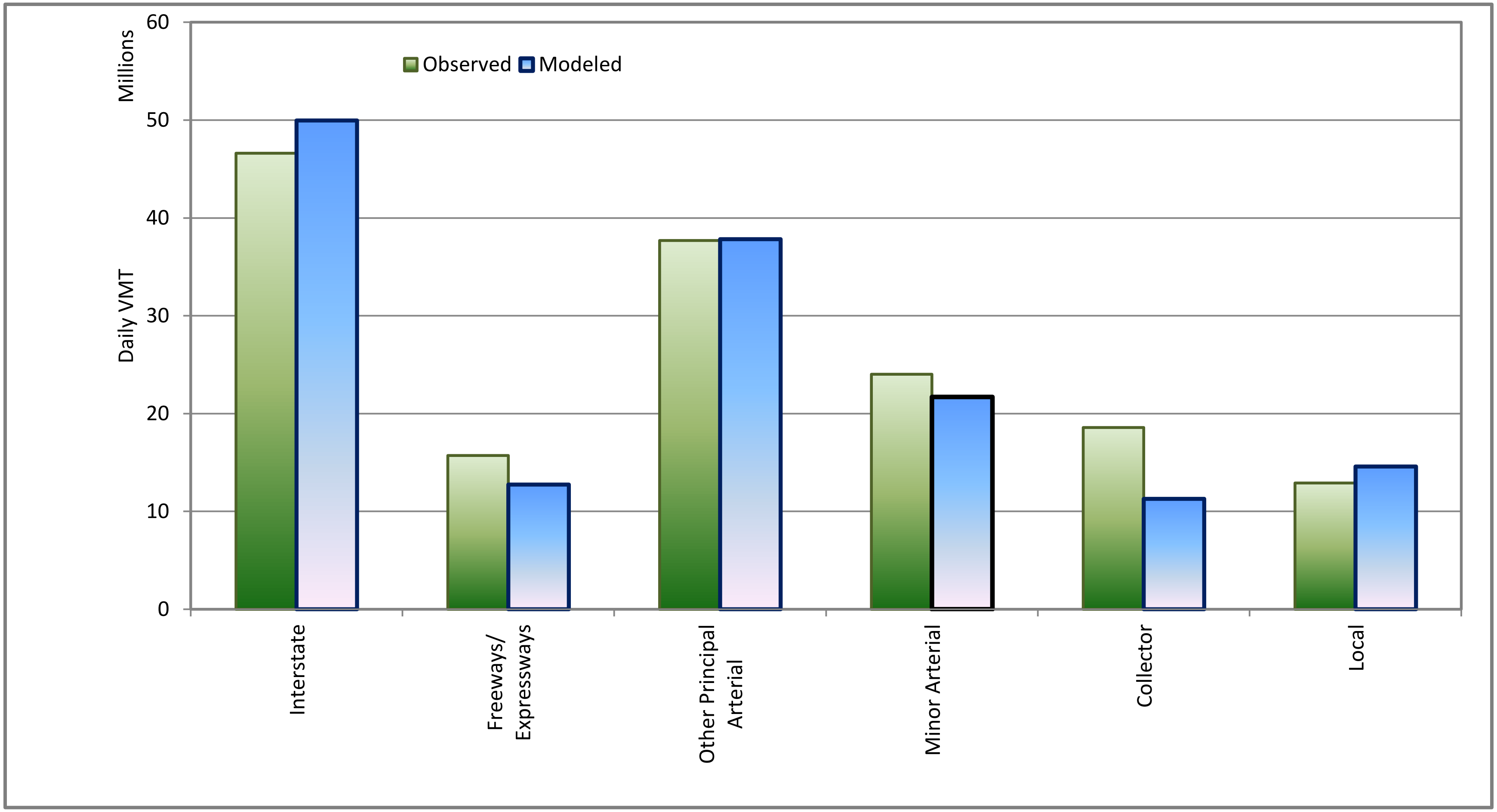
< 50%

Model Validation Example

- All vehicles all day



VMT based comparison



Volume class comparison