8. When we alternatively set the demand price of the i-th good p_i^{q0} at unity, the corresponding supply price p_i^{z0} becomes $1/(1 + \tau_i^z)$ rather than unity.
9. We discuss the elasticity parameters in Section 6.5.
10. The first line starting with "`$Title...`" is included only for file management purposes in the GAMS Model Library and is thus meaningless. This line can be omitted.
11. The uses of the `Parameter` directive demonstrated here are different from that in the household utility maximization model in Chapter 3. While we placed the directive to declare and to define the constants in one statement in List 3.1, these two procedures are done separately in two statements in List 5.1. For example, the declaration of `X0(i)` is done in line 22; its values are given in line 28. Following this practice, `F0(h,j)` is declared in line 23 and its value is given in line 29. When we express constants in two separate statements, the `Parameter` directive, not the `Table` directive, must be used even if they have two (or more) indices like `F0(h,j)`. Details about the uses of the `Parameter` directive are provided in Section A.2 in Annex A.
12. We indicate the rows and columns of the cells in the SAM using indices such as `i`, `j`, `h` and `k`, or individual elements such as 'HOH'. In the latter case, we enclose the element with double (or single) quotation marks.
13. This error message in the output file is '`**** 125 Set is under control already`'.
14. Given the model in List 5.1, we can solve the model either by initializing the variables or by setting lower bounds on the endogenous variables (discussed later) when we use the CONOPT solver. In contrast, when we use the MINOS solver, we cannot solve the model without variable initialization. When the solvers are updated, this result may differ.
15. The suffix '`.fx`' sets both the lower and the upper bounds of the domain of an endogenous variable at the same value. Therefore, when the '`.fx`' is followed by '`.lo`' or '`.up`' for the same endogenous variable, the latter statement takes effect and resets the lower or upper bound respectively.
16. For further information about the uses of GAMS IDE, see the online help or its printable file available at www.gams.com/dd/docs/tools/gamside.pdf.
17. There are some exceptions regarding the line numbers. Lines beginning with a '`$`' symbol (e.g., line 1 in List 5.1) are not printed in the output files. For a detailed discussion of their uses, see Appendix C in *GAMS – A User's Guide*. If GAMS finds a syntax error, the line with an error is indicated by four asterisks '`****`' with an error code inserted in the echo print part. For a discussion about syntax errors, see Section B.1 in Annex B of this book.
18. When the `Display` directive is put after the `Solve` statement in the program, the printout by the `Display` directive appears after the `SOLVE SUMMARY` header, and vice versa.
19. See Appendix III for details about the Lagrange multipliers. See Appendix IV for details about the fictitious objective function for computation of a system of simultaneous equations.

6
The Standard CGE Model

The 'simple CGE model' presented in Chapter 2 is equipped with only the essential features of a very basic macroeconomic model, and thus it cannot be used for empirical analyses. Here, we extend the model by incorporating the following four features. First, we introduce intermediate inputs into the production process. Second, we introduce a government into the model, where its consumption, and direct and indirect taxes revenues including import tariffs, are considered. Third, we also introduce investment and savings. Finally, we extend the model to an open economy model, where international trade is considered. We call this model the 'standard CGE model', whose social accounting matrix (SAM) was previously shown in Table 4.2 in Chapter 4.[1]

Section 6.1 provides an overview of the standard CGE model. Intermediate inputs are included in the model in Section 6.2, a government in Section 6.3, investment and savings in Section 6.4 and international trade in Section 6.5. Section 6.6 explains the market-clearing conditions. Section 6.7 provides the complete system of simultaneous equations for the standard CGE model. In Section 6.8, we apply the computer program, explained in Chapter 5, covering installation of the SAM through to calibration in order to solve the standard CGE model. By understanding the structure of the model and the computer program presented in this chapter, readers can develop and solve their own models for their research objectives.

6.1 Overview of the standard CGE model

Figure 6.1 provides an overview of the standard CGE model from the viewpoint of the flows of goods and factors in an economy.[2] As the standard CGE model is an extension of the 'simple CGE model' described in

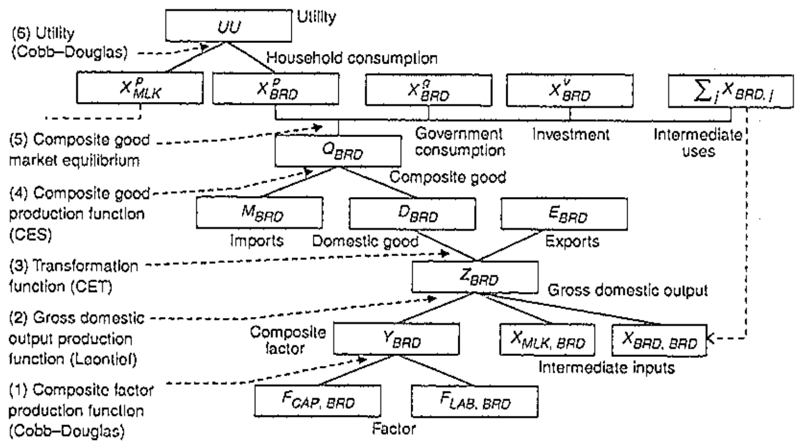

Figure 6.1 Overview of the standard CGE model
Note: Assumed functional forms are indicated in parentheses.

Figure 2.1, some variables reappear in Figure 6.1. These are UU, X^p_{BRD} and X^p_{MLK} of the utility function shown in the northwest corner (the latter two were denoted by X_{BRD} and X_{MLK} in Figure 2.1), and $F_{CAP,BRD}$, $F_{LAB,BRD}$ and Y_{BRD} (the last was Z_{BRD} in Figure 2.1) of the production function shown at the bottom.

Next, we explain the flows of goods and factors at each stage where they are combined for either production or consumption. The flows are explained from the bottom to the top in Figure 6.1, taking the bread sector as an example.

(1) Capital $F_{CAP,BRD}$ and labour $F_{LAB,BRD}$ are aggregated into the composite factor Y_{BRD} using the composite factor production function.
(2) This composite factor Y_{BRD} is combined with the intermediate inputs of bread $X_{BRD,BRD}$ and milk $X_{MLK,BRD}$ to produce the gross domestic output Z_{BRD} using the gross domestic output production function.
(3) The gross domestic output Z_{BRD} is transformed into the exports E_{BRD} and the domestic good D_{BRD} using the gross domestic output transformation function.
(4) The domestic good D_{BRD} is combined with the imports M_{BRD} to produce the composite good Q_{BRD} with the composite good production function.
(5) The composite good Q_{BRD} is distributed among household consumption X^p_{BRD}, government consumption X^g_{BRD}, investment X^v_{BRD} and intermediate uses by the bread and milk sectors $\sum_j X_{BRD,j}$.[3]

(6) Household utility UU is generated by consumption X^p_{BRD} and X^p_{MLK} as the utility function indicates.

Details of the composite factors, the composite goods and the functions newly mentioned here are explained below.

6.2 Intermediate inputs

In the simple CGE model, only capital and labour are assumed to be used for the production of goods. Here, in contrast, we make the model more realistic by assuming that firms use intermediate inputs in their production process. Following this extension, the behaviour of firms becomes more complicated; we divide the production process (or firms) into two stages.

In the first stage, capital and labour are used for the production of a composite factor (or value added). The production process of the composite factor can be regarded as the behaviour of a virtual factory, which maximizes its profit by choosing its output (composite factor) level and inputs (capital and labour) use, depending on their relative prices subject to its technology. In the second stage, the composite factor is combined with intermediates to produce the gross domestic output, as indicated by the gross domestic output production function.

As for the technology in this two-stage production process, we assume a Cobb–Douglas-type production function for the first stage and a Leontief-type production function for the second stage. They are both homogeneous of degree one and thus characterized as constant-returns-to-scale.[4] The Cobb–Douglas-type production function allows us to describe substitution between inputs, while the Leontief-type production function does not. As empirical CGE models are developed on the basis of the input–output (IO) tables, distinguishing dozens of sectors/goods, the number of endogenous variables, particularly for intermediate inputs, increases in accordance with the square of the number of sectors/goods. In this regard, the Leontief-type production function significantly reduces the complexity of the model and thereby the computational load.

The profit-maximization problems for the j-th firm can be written as follows:[5]

– For the first stage:

$$\underset{Y_j, F_{h,j}}{\text{maximize}} \quad \pi^y_j = p^y_j Y_j - \sum_h p^f_h F_{h,j}$$

subject to

$$Y_j = b_j \prod_h F_{h,j}^{\beta_{h,j}} \qquad (6.1)$$

– For the second stage:[6]

$$\underset{Z_j, Y_j, X_{i,j}}{\text{maximize}} \ \pi_j^z = p_j^z Z_j - \left(p_j^y Y_j + \sum_i p_i^q X_{i,j} \right)$$

subject to

$$Z_j = \min\left(\frac{X_{BRD,j}}{ax_{BRD,j}}, \frac{X_{MLK,j}}{ax_{MLK,j}}, \frac{Y_j}{ay_j} \right) \qquad (6.5')$$

Notations are:

π_j^y: profit of the j-th firm producing composite factor Y_j in the first stage,
π_j^z: profit of the j-th firm producing gross domestic output Z_j in the second stage,
Y_j: composite factor, produced in the first stage and used in the second stage by the j-th firm,
$F_{h,j}$: the h-th factor used by the j-th firm in the first stage,
Z_j: gross domestic output of the j-th firm,
$X_{i,j}$: intermediate input of the i-th good used by the j-th firm,
p_j^y: price of the j-th composite factor,
p_h^f: price of the h-th factor,
p_j^z: price of the j-th gross domestic output,
p_i^q: price of the i-th composite good,
$\beta_{h,j}$: share coefficient in the composite factor production function,
b_j: scaling coefficient in the composite factor production function,
$ax_{i,j}$: input requirement coefficient of the i-th intermediate input for a unit output of the j-th good,
ay_j: input requirement coefficient of the j-th composite good for a unit output of the j-th good.

In each stage of production, the objective value is the profits of the firm. In the first-stage profit function, the first term on the right-hand side represents the sales of the composite factor; the second term represents the input costs of capital and labour used for its production. The constraint (6.1) represents the technology of the composite factor production described by a Cobb–Douglas-type production function.

In the second-stage profit function, the first term on the right-hand side is the sales of the gross domestic output, which consists of ordinary goods such as bread and milk in this model; the second and third terms are the costs of the composite factor input and those of the intermediate inputs used in the second-stage production respectively. The constraint (6.5′) is a Leontief-type production function for production of the gross domestic output with the composite factor and intermediate inputs.

By solving these two problems, we obtain:[7]

$$Y_j = b_j \prod_h F_{h,j}^{\beta_{h,j}} \qquad \forall j \qquad (6.1)$$

$$F_{h,j} = \frac{\beta_{h,j} p_j^y}{p_h^f} Y_j \qquad \forall h, j \qquad (6.2)$$

$$X_{i,j} = ax_{i,j} Z_j \qquad \forall i, j \qquad (6.3)$$

$$Y_j = ay_j Z_j \qquad \forall j \qquad (6.4)$$

$$Z_j = \min\left(\frac{X_{BRD,j}}{ax_{BRD,j}}, \frac{X_{MLK,j}}{ax_{MLK,j}}, \frac{Y_j}{ay_j} \right) \qquad \forall j \qquad (6.5')$$

The production function (6.5′) generates rectangular isoquants (iso-output curves) as shown in Figure V.1 in Appendix V. The kinks in the isoquants often cause difficulty in numerical computations.[8] To work around such a computational problem, we replace (6.5′) with a zero-profit condition, which should always hold, as we explained with (2.7) in Section 2.5:[9]

$$\pi_j^z = p_j^z Z_j - \left(p_j^y Y_j + \sum_i p_i^q X_{i,j} \right) = 0 \qquad \forall j$$

We can include this zero-profit condition in the model, but it is more convenient to transform it into a simpler expression of a unit cost function. Using (6.3) and (6.4), we can eliminate $X_{i,j}$ and Y_j to obtain:

$$p_j^z Z_j - \left(ay_j p_j^y Z_j + \sum_i ax_{i,j} p_i^q Z_j \right) = 0 \qquad \forall j$$

and again by eliminating Z_j, we get the following unit cost function:

$$p_j^z = ay_j p_j^y + \sum_i ax_{i,j} p_i^q \qquad \forall j \qquad (6.5)$$

Replacing (6.5′) with (6.5), we can describe the firms' behaviour with (6.1)–(6.5).

6.3 Government

CGE models are often used for policy analysis. The main concern is the consequences of changes in government policy devices – typically tax rates. Thus, any realistic CGE model must include a government. In this section, we discuss how to model government behaviour.

In our CGE model, the government is supposed to collect taxes and consume goods. It should be noted that there is no single perfect way of modelling these government activities from the viewpoint of microfoundations, while we have modelled the behaviour of the household and the firms firmly based on their microfoundations. Therefore, we must develop our CGE models with a government, depending on the purpose of our analysis, the availability of data or sometimes the preference of the modeller. The model of the government presented here is one example among various possible specifications.

We assume that the government levies a direct tax on household income at the tax rate τ^d, an ad valorem production tax (an indirect tax) on gross domestic output at the tax rate τ_j^z and an ad valorem import tariff on imports at the rate τ_i^m. (Details about imports as well as exports are discussed later in Section 6.5.) At the same time, we assume that (1) the government spends all tax revenues on their consumption, and that (2) the government consumes each good (i.e., bread or milk) in fixed proportions in total government expenditure. For example, the government spends 40% of its total revenues on the purchase of bread and 60% on the purchase of milk.

The above assumptions can be written as follows:

$$T^d = \tau^d \sum_h p_h^f FF_h \tag{6.6}$$

$$T_j^z = \tau_j^z p_j^z Z_j \qquad \forall j \tag{6.7}$$

$$T_i^m = \tau_i^m p_i^m M_i \qquad \forall i \tag{6.8}$$

$$X_i^g = \frac{\mu_i}{p_i^q}\left(T^d + \sum_j T_j^z + \sum_j T_j^m\right) \qquad \forall i \tag{6.9'}$$

where:
T^d: direct tax,
T_j^z: production tax on the j-th good,
T_i^m: import tariff on the i-th good,
τ^d: direct tax rate,
τ_j^z: production tax rate on the j-th good,
τ_i^m: import tariff rate on the i-th good,
FF_h: endowments of the h-th factor for the household,
Z_j: gross domestic output of the j-th firm,
M_i: imports of the i-th good,
X_i^g: government consumption of the i-th good,
p_j^z: price of the j-th gross domestic output,
p_h^f: price of the h-th factor,
p_i^m: price of the i-th imported good,
p_i^q: price of the i-th composite good,
μ_i: share of the i-th good in government expenditure ($0 \leq \mu_i \leq 1$, $\sum_i \mu_i = 1$).
(The composite good is explained in Section 6.5.)

Although we assume that government expenditure is allocated among goods for consumption proportionately as (6.9′) indicates in this example, we can use other assumptions. For example, we can further simplify government behaviour by setting its consumption at the initial equilibrium level X_i^{g0}:[10]

$$X_i^g = X_i^{g0} \qquad \forall i$$

When the government sells its assets, such sales appear as negative consumption in statistical databases such as IO tables. (A similar observation can also take place in the investment account, when a decrease in stocks occurs.) An application of the proportionate government expenditure behaviour suggested above might not be suitable for such a negative consumption case. We can alternatively develop a model that allows negative values for some government consumption. That is, we can set a negative value for the consumption of some goods and assume positive proportional expenditure for the other goods.

6.4 Investment and savings

6.4.1 Introduction of investment and savings

The CGE model that we are developing here is a static model. Thus, strictly speaking, introducing dynamic factors like investment and savings is inconsistent with its original setup as a static model.[11] However, we cannot ignore investment because it has a significantly large share in final demand.[12] Although we cannot model investment in a manner that is perfectly consistent with economic theory, we have to incorporate it in some way.

Recall the discussion about the virtual investment agent in Subsection 4.1.2. The investment agent collects funds from the household, the government and the external sector and spends them on purchases of investment goods. Although the household and the government can make their own decisions about investment and savings, the present model assumes that a virtual agent absorbs all the savings of an economy and spends them on the purchase of goods proportionately with a constant share λ_i. We can describe its behaviour using the investment demand function (6.10). This is similar to the assumption about the government demand function for goods:

$$X_i^v = \frac{\lambda_i}{p_i^q}\left(S^p + S^g + \varepsilon S^f\right) \qquad \forall i \tag{6.10}$$

Notations are:
S^p: household savings,
S^g: government savings,
S^f: current account deficits in foreign currency terms (or equivalently foreign savings),
X_i^v: demand for the i-th investment good,
ε: foreign exchange rate (domestic currency/foreign currency),[13]
p_i^q: price of the i-th composite good,
λ_i: expenditure share of the i-th good in total investment ($0 \le \lambda_i \le 1$, $\sum_i \lambda_i = 1$).

The variables in parentheses on the right-hand side of (6.10) correspond to total savings consisting of savings by the household, the government and the external sector. It should be noted that, as the sum of the share parameter λ_i is equal to unity, (6.10) implies that the total savings in an economy are always equal to its total investment.

Then, let us assume that household and government savings are determined by constant average propensities for savings as follows:

$$S^p = ss^p \sum_h p_h^f FF_h \tag{6.11}$$

$$S^g = ss^g \left(T^d + \sum_j T_j^z + \sum_j T_j^m\right) \tag{6.12}$$

where:
ss^p: average propensity for savings by the household,
ss^g: average propensity for savings by the government.

In addition to S^p and S^g, the economy has other savings; namely, foreign savings S^f. In the present model, foreign savings S^f are assumed to be exogenous; however, we can alternatively assume any of these savings variables to be either endogenous or exogenous depending on our point of view of the real economy. This issue is closely related to the issue of the macro closure rules, which will be discussed in depth in the next chapter.

It should be noted that the investment determined by (6.10) implies abandoning goods, which contributes neither to household utility nor to firm production. In fact, the utility function is not dependent on the amount of investment X_i^v. Furthermore, the endowments of capital FF_{CAP} are predetermined in this economy and thus cannot be increased by the investment X_i^v in this static model.

6.4.2 Modification of household and government behaviour

The introduction of the government and investment and savings into the model requires us to modify the original model equations describing the behaviour of the household and the government. While we assume the same utility function, the household budget constraint has to be slightly modified. (Note that we have used X_i for household consumption in Chapter 2 but hereinafter use X_i^p for household consumption because other uses such as government consumption, investment uses and intermediate inputs are considered in the standard CGE model.) That is, the available funds for household consumption of goods are now reduced by the amount of household savings and direct tax payments; thus, the household problem is updated as follows:

$$\underset{X_i^p}{\text{maximize}} \; UU = \prod_i X_i^{p\,\alpha_i}$$

subject to

$$\sum_i p_i^q X_i^p = \sum_h p_h^f FF_h - S^p - T^d$$

where:
UU: utility,
X_i^p: household consumption of the i-th good,
FF_h: endowments of the h-th factor for the household,
S^p: household savings,

T^d: direct tax,
p_i^q: price of the i-th composite good,
p_h^f: price of the h-th factor,
α_i: share parameter in the utility function ($0 \leq \alpha_i \leq 1$, $\sum_i \alpha_i = 1$).

Solving this modified household problem in the same way as in Section 2.2, we obtain the household demand function for the i-th good:

$$X_i^p = \frac{\alpha_i}{p_i^q}\left(\sum_h p_h^f FF_h - S^p - T^d\right) \qquad \forall i \tag{6.13}$$

The government demand function for the i-th good is modified analogously by incorporation of government savings in a similar manner:

$$X_i^g = \frac{\mu_i}{p_i^q}\left(T^d + \sum_j T_j^z + \sum_j T_j^m - S^g\right) \qquad \forall i \tag{6.9}$$

6.5 International trade

6.5.1 Small-country assumption and balance of payments

The third major feature of the standard CGE model is the extension of the original closed economy model to an open economy model. For simplicity, we assume that this economy is so small that it does not have a significant impact on the rest of the world – even with an extreme activity such as export dumping.[14] The essence of the small-country assumption is that the export and import prices quoted in foreign currency terms are exogenously given for this economy.

Regarding international trade, we must distinguish between two types of price variables. One is prices in terms of the domestic currency p_i^e and p_i^m; the other is prices in terms of the foreign currency p_i^{We} and p_i^{Wm}. They are linked with each other as follows:

$$p_i^e = \varepsilon p_i^{We} \qquad \forall i \tag{6.14}$$

$$p_i^m = \varepsilon p_i^{Wm} \qquad \forall i \tag{6.15}$$

Furthermore, it is assumed that the economy faces balance of payments constraints, which can be described with export and import prices in foreign currency terms:

$$\sum_i p_i^{We} E_i + S^f = \sum_i p_i^{Wm} M_i \tag{6.16}$$

Notations are:
p_i^{We}: export price in terms of foreign currency (exogenous),
p_i^e: export price in terms of domestic currency,
ε: foreign exchange rate (domestic currency/foreign currency),
E_i: exports of the i-th good,
p_i^{Wm}: import price in terms of foreign currency (exogenous),
p_i^m: import price in terms of domestic currency,
M_i: imports of the i-th good,
S^f: current account deficit in terms of foreign currency (or equivalently foreign savings; exogenous).

As mentioned in Subsection 6.4.1, the current account deficit in foreign currency terms S^f is an exogenous variable. While in the present model the balance of payments constraints are expressed in terms of foreign currency, that constraint can be alternatively expressed in terms of domestic currency by replacing p_i^{We} and p_i^{Wm} with p_i^e and p_i^m using (6.14) and (6.15).

6.5.2 Armington's assumption

When we extend the model to an open economy model, we have to consider differences (or similarities) between goods domestically produced/consumed and those imported/exported. In this section, we find that it is necessary to assume that they are *imperfectly* substitutable with each other. That is, domestic-made bread is supposed to be similar to but is slightly different from imported bread.

Suppose that all the exported goods are perfectly substitutable with the corresponding imported goods, such that there cannot be both exports and imports for the same goods simultaneously.[15] It is nonsense to import 100 units of bread while exporting 20 units. Instead, we should import the net amount of 80 units. However, actual data often report both exports and imports for the same good. This is known as two-way trade or cross-hauling. To reconcile such a conflict between theory and practice, we distinguish an imported good from an exported one even when they are classified in the data as the same good. The degree of difference/similarity between them can be measured by a parameter such as the elasticity of substitution in constant elasticity of substitution (CES) functions. If they are significantly different from each other, the elasticity of substitution becomes small (i.e., inelastic) and vice versa.

In reality, it seems, however, that substitution (or competition) is more relevant between imports and domestic goods, and between exports and domestic goods, than between exports and imports. In CGE models, we assume substitution between imports and domestic goods, and between exports and domestic goods in a pairwise manner.[16] The assumption about imperfect substitution between imports and domestic goods is called Armington's (1969) assumption.

6.5.3 Substitution between imports and domestic goods

Armington's assumption implies that households and firms do not directly consume or use imported goods but instead a so-called 'Armington composite good', which comprises imports and the corresponding domestic goods. To describe the process of constructing Armington composite goods, we assume virtual firms that behave so as to maximize their profits by choosing a suitable combination of imported and domestic goods. The solution of their profit-maximization problem leads to their input demands for imports and that for domestic goods, and the output level by adjusting quantities of imported and domestic goods, depending upon all the prices involved.

For example, let us suppose that a firm mixes imported orange juice with domestic orange juice to produce orange juice bottled with its company's labels as we see at supermarkets. This production process can often be described by a CES function (6.17). This production function is an extension of the celebrated Cobb–Douglas and Leontief functions. The CES function is characterized by the parameter of elasticity of substitution, σ_i, which indicates the percentage changes in the input factor ratio caused by a 1% change in relative input prices. Graphically speaking, this parameter determines the curvature of isoquants. The larger the elasticity, the gentler the curvature of the isoquants or the more flexibly the input share is adjusted (see its isoquant in Figure 6.2; however, note that import tariff rates are omitted). The extreme case is the Leontief-type function, where $\sigma_i = 0$.

The optimization problem for the i-th Armington-composite-good-producing firm can be written as follows:

$$\underset{Q_i, M_i, D_i}{\text{maximize}} \ \pi_i^q = p_i^q Q_i - \left[(1 + \tau_i^m) p_i^m M_i + p_i^d D_i \right]$$

subject to

$$Q_i = \gamma_i \left(\delta m_i M_i^{\eta_i} + \delta d_i D_i^{\eta_i} \right)^{\frac{1}{\eta_i}} \qquad (6.17)$$

Notations are:

π_i^q: profit of the firm producing the i-th Armington composite good,
p_i^q: price of the i-th Armington composite good,

p_i^m: price of the i-th imported good in terms of domestic currency,
p_i^d: price of the i-th domestic good,
Q_i: the i-th Armington composite good,
M_i: the i-th imported good,
D_i: the i-th domestic good,
τ_i^m: import tariff rate on the i-th good,
γ_i: scaling coefficient in the Armington composite good production function,
$\delta m_i, \delta d_i$: input share coefficients in the Armington composite good production function ($0 \le \delta m_i \le 1$, $0 \le \delta d_i \le 1$, $\delta m_i + \delta d_i = 1$),
η_i: parameter defined by the elasticity of substitution, ($\eta_i = (\sigma_i - 1)/\sigma_i$, $\eta_i \le 1$),
σ_i: elasticity of substitution in the Armington composite good production function, $\left(\sigma_i = -\dfrac{d\,(M_i/D_i)}{M_i/D_i} \Big/ \dfrac{d(p_i^m/p_i^d)}{p_i^m/p_i^d} \right)$.

The first-order conditions for the optimality of the above problem imply the following demand functions for imports and the domestic good:[17]

$$M_i = \left[\frac{\gamma_i^{\eta_i} \delta m_i p_i^q}{(1 + \tau_i^m) p_i^m} \right]^{\frac{1}{1-\eta_i}} Q_i \qquad \forall i \qquad (6.18)$$

$$D_i = \left[\frac{\gamma_i^{\eta_i} \delta d_i p_i^q}{p_i^d} \right]^{\frac{1}{1-\eta_i}} Q_i \qquad \forall i \qquad (6.19)$$

It should be noted that the Armington-composite-good-producing firm faces tariff-inclusive import prices $(1 + \tau_i^m)p_i^m$ rather than the tariff-exclusive import prices p_i^m; therefore, the tariff rate τ_i^m appears in the definition of its profit π_i^q. Consequently, $(1 + \tau_i^m)p_i^m$ is also in the derived import demand function (6.18).

6.5.4 Transformation between exports and domestic goods

Let us consider the supply side; that is, exports and the domestic goods. We assume that the firms transform the gross domestic output into goods sold in international markets and in domestic markets. In this transformation process, we also assume imperfect substitution (strictly speaking, it should be called imperfect transformation) between exports and the domestic good supply.[18]

For example, electronic appliances are commonly used all over the world but are often customized by country considering the preferences of

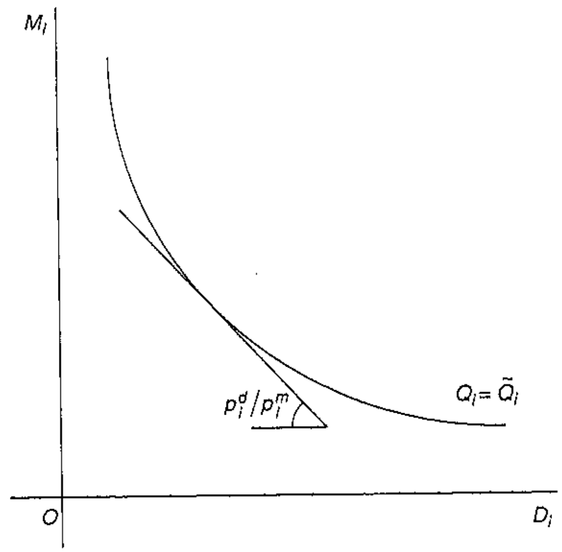

Figure 6.2 Isoquant of the CES function for the Armington composite good

targeted users. Those supplied to Japan are likely to have many functions in a small body, while those exported to the international markets are rather simple in function and of a larger size. For example, automobiles for domestic sales in Japan are often equipped with luxurious options, whereas those for exports have only essential ones.

Suppose that a firm (or the final stage of a production process in a firm), which is involved in shipping of the gross domestic output to international markets and to the domestic market, decides the supply ratio between these two markets and customizes its output to be suitable for these targeted markets. We express such a transformation process with a constant elasticity of transformation (CET) function. The isoquants (iso-input curves) for this transformation shown in Figure 6.3 are the mirror images of the isoquants (iso-output curves) of the CES function in Figure 6.2. Depending on the relative price between exports and domestic goods, the supply ratio changes. The larger the elasticity of transformation ψ_i, the gentler the curvature of the isoquants. That is, with a large elasticity of transformation, the export–domestic supply ratio tends to be more sensitive to a change in relative prices.

The profit-maximization problem for the i-th firm transforming the gross domestic output into exports and domestic goods can be expressed

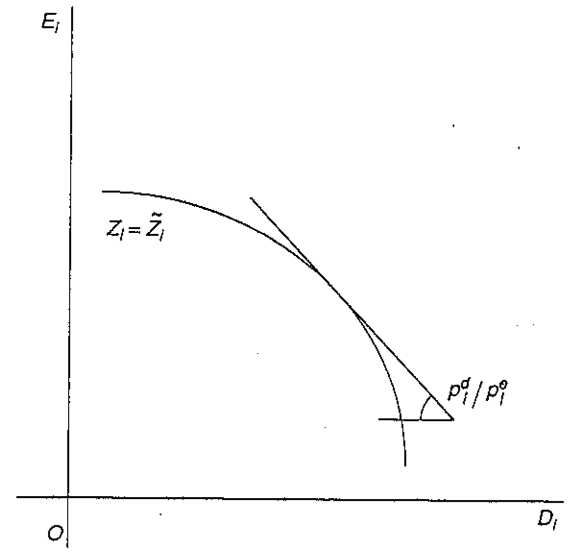

Figure 6.3 Isoquant of the CET function

as follows:

$$\underset{Z_i, E_i, D_i}{\text{maximize}}\ \pi_i = (p_i^e E_i + p_i^d D_i) - (1 + \tau_i^z)\, p_i^z Z_i$$

subject to

$$Z_i = \theta_i \left(\xi e_i E_i^{\phi_i} + \xi d_i D_i^{\phi_i} \right)^{\frac{1}{\phi_i}} \tag{6.20}$$

Notations are:

π_i: profit of the firm engaged in the i-th transformation,
p_i^e: price of the i-th export good in terms of domestic currency,
p_i^d: price of the i-th domestic good,
p_i^z: price of the i-th gross domestic output,
E_i: exports of the i-th good,
D_i: supply of the i-th domestic good,
Z_i: gross domestic output of the i-th good,
τ_i^z: production tax on the i-th gross domestic output,
θ_i: scaling coefficient of the i-th transformation,[19]
$\xi e_i, \xi d_i$: share coefficients for the i-th good transformation, $(0 \le \xi e_i \le 1, 0 \le \xi d_i \le 1, \xi e_i + \xi d_i = 1)$,

ϕ_i: parameter defined by the elasticity of transformation,
$(\phi_i = (\psi_i + 1)/\psi_i, \psi_i \geq 1)$,
ψ_i: elasticity of transformation of the i-th good transformation,
$\left(\psi_i = \dfrac{d\,(E_i/D_i)}{E_i/D_i} \Big/ \dfrac{d(p_i^e/p_i^d)}{p_i^e/p_i^d}\right)$.

By solving this maximization problem, we get the following supply functions for exports and for domestic goods:

$$E_i = \left[\frac{\theta_i^{\phi_i}\xi e_i\,(1+\tau_i^z)\,p_i^z}{p_i^e}\right]^{\frac{1}{1-\phi_i}} Z_i \qquad (6.21)$$

$$D_i = \left[\frac{\theta_i^{\phi_i}\xi d_i\,(1+\tau_i^z)\,p_i^z}{p_i^d}\right]^{\frac{1}{1-\phi_i}} Z_i \qquad (6.22)$$

Because the production tax τ_i^z is imposed on the gross domestic output Z_i, which is used as the input in this transformation process, τ_i^z appears in the equation defining the profit π_i and consequently also in the numerators of the above two supply functions.

6.6 Market-clearing conditions

We have described the behaviour of economic agents, such as the household, the firms, the government, the investment agent and the external sector, with a set of equations. Our final step of this modelling process is imposing the market-clearing conditions so that demand meets supply in all markets as follows:

$$Q_i = X_i^p + X_i^g + X_i^v + \sum_j X_{i,j} \quad \forall i \qquad (6.23)$$

$$\sum_j F_{h,j} = FF_h \qquad \forall h \qquad (6.24)$$

The market-clearing condition for the Armington composite goods is described by (6.23). As discussed in Subsection 6.5.3, the composite good Q_i is used by the household, the government and the investment agent as well as for intermediate input; we apply the same price p_i^q to all of them. Equation (6.24) is the factor market-clearing condition, which also appeared as (2.5) in the simple CGE model.

Recall that we have imposed the constraint (2.6) to equilibrate the supply price of each good with its demand price (p_i^z and p_i^x in Chapter 2), because we have assumed that the household and the firm must face the same price in each good market. In the present model, however, the price p_i^q, which the household faces, is not directly linked to the price p_i^z, which the firm faces. The CES and CET structures, which represent substitution between imports and domestic goods, and transformation between exports and domestic goods respectively, bring about equality between the demand and supply of goods by these agents but do not make a direct link between p_i^q and p_i^z. Therefore, we do not impose price equalization constraints between p_i^q and p_i^z such as (2.6) in this model.

6.7 Model system

As discussed above, we have developed a system of simultaneous equations for the standard CGE model consisting of (6.1)–(6.24).

– Domestic production:

$$Y_j = b_j \prod_h F_{h,j}^{\beta_{h,j}} \qquad \forall j \qquad (6.1)$$

$$F_{h,j} = \frac{\beta_{h,j} p_j^y}{p_h^f} Y_j \qquad \forall h, j \qquad (6.2)$$

$$X_{i,j} = ax_{i,j} Z_j \qquad \forall i, j \qquad (6.3)$$

$$Y_j = ay_j Z_j \qquad \forall j \qquad (6.4)$$

$$p_j^z = ay_j p_j^y + \sum_i ax_{i,j} p_i^q \qquad \forall j \qquad (6.5)$$

– Government:

$$T^d = \tau^d \sum_h p_h^f FF_h \qquad (6.6)$$

$$T_j^z = \tau_j^z p_j^z Z_j \qquad \forall j \qquad (6.7)$$

$$T_i^m = \tau_i^m p_i^m M_i \qquad \forall i \qquad (6.8)$$

$$X_i^g = \frac{\mu_i}{p_i^q}\left(T^d + \sum_j T_j^z + \sum_j T_j^m - S^g\right) \qquad \forall i \tag{6.9}$$

– Investment and savings:

$$X_i^v = \frac{\lambda_i}{p_i^q}\left(S^p + S^g + \varepsilon S^f\right) \qquad \forall i \tag{6.10}$$

$$S^p = ss^p \sum_h p_h^f FF_h \tag{6.11}$$

$$S^g = ss^g \left(T^d + \sum_j T_j^z + \sum_j T_j^m\right) \tag{6.12}$$

– Household:

$$X_i^p = \frac{\alpha_i}{p_i^q}\left(\sum_h p_h^f FF_h - S^p - T^d\right) \qquad \forall i \tag{6.13}$$

– Export and import prices and the balance of payments constraint:

$$p_i^e = \varepsilon p_i^{We} \qquad \forall i \tag{6.14}$$

$$p_i^m = \varepsilon p_i^{Wm} \qquad \forall i \tag{6.15}$$

$$\sum_i p_i^{We} E_i + S^f = \sum_i p_i^{Wm} M_i \tag{6.16}$$

– Substitution between imports and domestic goods (Armington composite):

$$Q_i = \gamma_i \left(\delta m_i M_i^{\eta_i} + \delta d_i D_i^{\eta_i}\right)^{\frac{1}{\eta_i}} \qquad \forall i \tag{6.17}$$

$$M_i = \left[\frac{\gamma_i^{\eta_i} \delta m_i p_i^q}{(1+\tau_i^m)\, p_i^m}\right]^{\frac{1}{1-\eta_i}} Q_i \qquad \forall i \tag{6.18}$$

$$D_i = \left[\frac{\gamma_i^{\eta_i} \delta d_i p_i^q}{p_i^d}\right]^{\frac{1}{1-\eta_i}} Q_i \qquad \forall i \tag{6.19}$$

– Transformation between exports and domestic goods:

$$Z_i = \theta_i \left(\xi e_i E_i^{\phi_i} + \xi d_i D_i^{\phi_i}\right)^{\frac{1}{\phi_i}} \qquad \forall i \tag{6.20}$$

$$E_i = \left[\frac{\theta_i^{\phi_i} \xi e_i (1+\tau_i^z)\, p_i^z}{p_i^e}\right]^{\frac{1}{1-\phi_i}} Z_i \qquad \forall i \tag{6.21}$$

$$D_i = \left[\frac{\theta_i^{\phi_i} \xi d_i (1+\tau_i^z)\, p_i^z}{p_i^d}\right]^{\frac{1}{1-\phi_i}} Z_i \qquad \forall i \tag{6.22}$$

– Market-clearing conditions:

$$Q_i = X_i^p + X_i^g + X_i^v + \sum_j X_{i,j} \qquad \forall i \tag{6.23}$$

$$\sum_j F_{h,j} = FF_h \qquad \forall h \tag{6.24}$$

The above system of simultaneous equations consists of 24 sets of equations/48 single equations and the same number of endogenous variables. (More precisely, the number of single equations and endogenous variables is $(h+i+17)\cdot j+h+4$.) The endogenous variables in this model are:

$$Y_j,\ F_{h,j},\ X_{i,j},\ Z_j,\ X_i^p,\ X_i^g,\ X_i^v,\ E_i,\ M_i,\ Q_i,\ D_i,\ p_h^f,\ p_j^y,\ p_j^z,\ p_i^q,\ p_i^e,\ p_i^m,\ p_i^d,\ \varepsilon,\ S^p,\ S^g,\ T^d,\ T_j^z \text{ and } T_i^m.$$

The exogenous variables are:[20]

$$FF_h,\ S^f,\ p_i^{We},\ p_i^{Wm},\ \tau^d,\ \tau_j^z \text{ and } \tau_i^m.$$

Before concluding our modelling of the standard CGE model, we have to reconsider Walras's law. According to Walras's law, we do not have to impose the market-clearing conditions on all the n markets because the general equilibrium of this model economy holds with only the $n-1$ market clearing conditions.[21] One of the equations of the model is redundant; therefore, all the prices cannot be solved. We have to choose a numeraire and fix its price at a certain level, and express all the other