2.1 Setup of the economy

The basic assumption of the simple CGE model is twofold. The first is a static economy in the sense that no time-related elements such as investment and savings are included. The second is a closed economy; that is, no international trade is included. Then, it is assumed that two goods are produced, bread and milk (denoted with indices i or j interchangeably) and that two factors exist, capital and labour (denoted with indices h or k interchangeably), in this economy. One representative household exists and consumes two kinds of goods to maximize its utility. There are two representative firms, each of which produces one commodity, either bread or milk. The household, endowed with two factors, provides them to the firms in return for income payments. The firms employ these factors in their production. Household and firm demand and supply of these goods and factors are equilibrated in the markets with flexible price adjustments. We assume that the markets are perfectly competitive. That is, no agent has any market power to control prices; in other words, all the agents are 'price takers'.

Figure 2.1 summarizes the commodity flows in the model economy. (1) The goods Z_j are produced by the firms with factor inputs $F_{h,j}$. Then, (2) they are shipped to the goods markets, where they are sold to the household for their consumption X_j. (3) At the household, the goods are consumed and generate utility UU. The payments occur in the opposite direction. The factor income is generated by the firms and paid back to

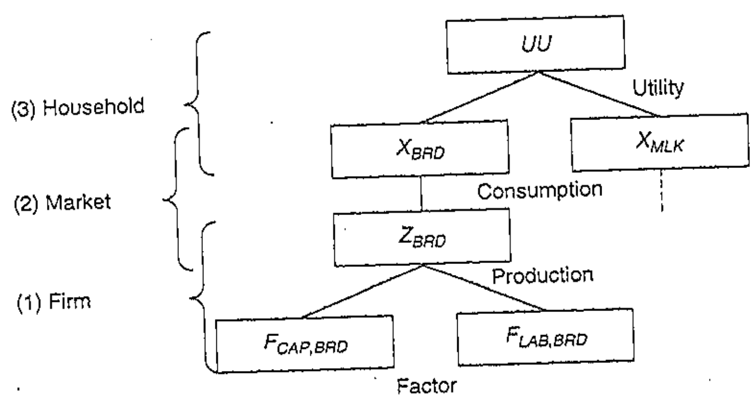

Figure 2.1 Model structure

the provider of the factors; i.e., the household. The household uses the received factor income to purchase the goods. In the following sections, household optimization behaviour, firm optimization behaviour and the market-clearing conditions are presented.

2.2 Household behaviour

The household aims to sell all of its endowed factors to the firms to earn income. (We do not assume self-consumption of factors such as consumption of leisure or voluntary unemployment.) To simplify the notation, capital and labour are abbreviated as CAP and LAB. The household expends its factor income on the consumption of bread and milk, which are also abbreviated as BRD and MLK. The household is assumed to choose the consumption of bread and milk to maximize its utility. It is assumed that the utility function is the Cobb–Douglas type. At this stage, prices of goods and factors are assumed to be given in the household utility maximization problem.

The household maximizes its utility subject to its budget constraint in the following manner:[3]

$$\underset{X_i}{\text{maximize}}\ UU = \prod_i X_i^{\alpha_i} \quad (2.a)$$

subject to

$$\sum_i p_i^x X_i = \sum_h p_h^f FF_h \quad (2.b)$$

where:
i, j: goods (BRD, MLK),
h, k: factors (CAP, LAB),
UU: utility,
X_i: consumption of the i-th good ($X_i \geq 0$),
FF_h: endowments of the h-th factor for the household,
p_i^x: demand price of the i-th good ($p_i^x \geq 0$),
p_h^f: price of the h-th factor ($p_h^f \geq 0$),
α_i: share parameter in the utility function ($0 \leq \alpha_i \leq 1$, $\sum_i \alpha_i = 1$).

The first equation (2.a) is the utility function to be maximized. The second one (2.b) is the budget constraint to equilibrate total expenditure and total income.

To solve this problem analytically, let us employ the Lagrange multiplier method. With a Lagrange multiplier φ, the Lagrangian is defined as follows:

$$L(X_i;\varphi) = \prod_i X_i^{\alpha_i} + \varphi\left(\sum_h p_h^f FF_h - \sum_i p_i^x X_i\right)$$

As this problem with a Cobb–Douglas specification usually has an interior solution, the first-order optimality conditions are:

$$\frac{\partial L}{\partial X_i} = \alpha_i \frac{\prod_j X_j^{\alpha_j}}{X_i} - \varphi p_i^x = 0 \quad \forall i \tag{2.c}$$

$$\frac{\partial L}{\partial \varphi} = \sum_h p_h^f FF_h - \sum_i p_i^x X_i = 0 \tag{2.d}$$

Eliminating the Lagrange multiplier φ by solving the system (2.c)–(2.d) for the demand X_i, we obtain the demand function for the i-th good:[4]

$$X_i = \frac{\alpha_i}{p_i^x} \sum_h p_h^f FF_h \quad \forall i \tag{2.1}$$

The derived demand function (2.1) implies that the demand for the i-th good X_i increases with a decline of its price p_i^x (i.e., a downward-sloping demand function) or with an increase of income $\sum_h p_h^f FF_h$.

2.3 Firm behaviour

This economy has two representative firms; one produces bread and the other milk. Each firm uses only capital and labour to produce bread or milk, and is supposed to maximize its profits subject to the given production technology.

The above-mentioned setup for firm behaviour may appear too simple to capture reality in the following two ways. First, in addition to capital and labour, the bread-producing firm might need to use intermediate inputs such as butter, salt and yeast. The same is also the case for the milk-producing firm. Such intermediate inputs in the production process are only used in Chapter 6 onward. Second, each of these firms is assumed to produce only one product with no by-products. That is, the bread firm produces only bread, and not milk as a by-product. Although by-products are important in the real world, they are not dealt with in this book.

Let us formulate the j-th firm's behaviour. It is assumed to maximize its profits subject to its production technology constraint under given output and input prices as follows:

$$\underset{Z_j, F_{h,j}}{\text{maximize}}\ \pi_j = p_j^z Z_j - \sum_h p_h^f F_{h,j}$$

subject to

$$Z_j = b_j \prod_h F_{h,j}^{\beta_{h,j}} \tag{2.2}$$

where:
i, j: firm (BRD, MLK),
h, k: factor (CAP, LAB),
π_j: profit of the j-th firm,
Z_j: output of the j-th firm,
$F_{h,j}$: the h-th factor used by the j-th firm,
p_j^z: supply price of the j-th good,
p_h^f: price of the h-th factor,
$\beta_{h,j}$: share coefficient in the production function ($0 \le \beta_{h,j} \le 1$, $\sum_h \beta_{h,j} = 1$),
b_j: scaling coefficient in the production function.

The j-th firm determines the amounts of factor input $F_{h,j}$ and its output Z_j that maximize its profits. The first term on the right-hand side of the profit function represents its revenues accruing from sales of the j-th good. The second term corresponds to its payments for employed factors. The constraint (2.2) is the production function, representing its production technology, which links factor $F_{h,j}$ with output Z_j. We employ a Cobb–Douglas-type function. It should be noted that, at this stage, the quantity of factors used by the j-th firm is not constrained by their availability but solved endogenously.

To solve this optimization problem for the j-th firm, we again employ the Lagrange multiplier method with the Lagrange multiplier ω_j as follows:

$$L_j(Z_j, F_{h,j}; \omega_j) = \left(p_j^z Z_j - \sum_h p_h^f F_{h,j}\right) + \omega_j \left(b_j \prod_h F_{h,j}^{\beta_{h,j}} - Z_j\right)$$

Usually, this problem with a Cobb–Douglas specification has an interior solution. The first-order conditions for the j-th firm optimization problem are:

$$\frac{\partial L_j}{\partial Z_j} = p_j^z - \omega_j = 0 \quad \forall j$$

$$\frac{\partial L_j}{\partial F_{h,j}} = -p_h^f + \omega_j \beta_{h,j} \frac{b_j \prod_k F_{k,j}^{\beta_{k,j}}}{F_{h,j}} = 0 \quad \forall h, j$$

$$\frac{\partial L_j}{\partial \omega_j} = b_j \prod_h F_{h,j}^{\beta_{h,j}} - Z_j = 0 \quad \forall j$$

Eliminating the Lagrange multiplier ω_j by solving this system for the demand for the h-th factor by the j-th firm $F_{h,j}$, we obtain the following demand function:

$$F_{h,j} = \frac{\beta_{h,j}}{p_h^f} p_j^z Z_j \quad \forall h, j \tag{2.3}$$

as well as the original production function (2.2).

The interpretation of this factor demand function is similar to that of the household demand function for goods in (2.3). Demand for the h-th factor input by the j-th firm increases when either the h-th factor price p_h^f falls, the supply price of the j-th good rises, or the production of the j-th good Z_j increases. The share parameter $\beta_{h,j}$ plays an important role in determining demand for the h-th factor input for production of the j-th good; in other words, the larger the parameter $\beta_{h,j}$ is, the more sensitive demand is to changes in the other variables.

2.4 Market-clearing conditions

Sections 2.2 and 2.3 explained how the household and firms determine their demand and supply of goods and factors as a result of their optimization behaviour. Those optimization problems are not dependent on other agents' decisions but only on the given good and factor prices. In other words, the optimization problems of the three agents (one household and two firms) have so far been solved separately. Therefore, there is no guarantee that the prices assumed by the household are the same as those assumed by the firms. More precisely, for the i-th good, the household assumes the demand price p_i^x, while the firms assume the supply price p_i^z, and these two prices are not necessarily the same in general. Furthermore, even if those prices are identical, supply is not necessarily equal to demand for each good and for each factor. In addition, the total demand for each factor by the two firms does not necessarily match its endowments. In sum, to ensure the market equilibrium of each good and factor in terms of quantity and price, we need to impose the following market-clearing conditions:

$$X_i = Z_i \quad \forall i \tag{2.4}$$

$$\sum_j F_{h,j} = FF_h \quad \forall h \tag{2.5}$$

$$p_i^z = p_i^x \quad \forall i \tag{2.6}$$

Equation (2.4) is the market-clearing condition for the i-th good, which ensures equality of its demand and supply quantities. As discussed in Section 2.1, one representative firm produces only one good, without any by-product. Therefore, the suffix j in Z_j, which stands for the j-th firm as in Section 2.3, also represents its product; namely, the i-th good without any confusion. Consequently, the suffix i for Z_i, instead of Z_j, is used in Equation (2.4).

Equation (2.5) is the market-clearing condition for factors indicating that the total demand for each factor must be equal to its given endowments. The left-hand side of (2.5) represents the sum of demand quantities for the h-th factor by both firms. The right-hand side stands for the total endowments of each factor, assumed to be given in the economy.

The last equation (2.6) is the market-clearing condition that equates the firm's supply price of the i-th good p_i^z to the corresponding demand price for the household p_i^x. In this chapter, no indirect tax is assumed; thus, there should be no gap between the supply and demand prices. In contrast to these goods' prices, the same variable p_h^f is used for both the supply and demand prices of the factors; thus, the equality condition (2.6) is not needed for factor prices.

2.5 Model system

The demand and supply equations of goods and factors, and the market-clearing conditions discussed above, yield the following system of simultaneous equations (2.1)–(2.6):

$$X_i = \frac{\alpha_i}{p_i^x} \sum_h p_h^f FF_h \quad \forall i \tag{2.1}$$

$$Z_j = b_j \prod_h F_{h,j}^{\beta_{h,j}} \quad \forall j \tag{2.2}$$

$$F_{h,j} = \frac{\beta_{h,j}}{p_h^f} p_j^z Z_j \quad \forall h, j \tag{2.3}$$

$$X_i = Z_i \quad \forall i \tag{2.4}$$

$$\sum_j F_{h,j} = FF_h \quad \forall h \tag{2.5}$$

$$p_i^z = p_i^x \quad \forall i \tag{2.6}$$

In this system, (2.1) is the demand function for the i-th good derived from the household utility maximization problem; (2.2) is the production function of the j-th good, set in the j-th firm profit-maximization problem; (2.3) is the demand function for the h-th factor by the j-th firm, derived from the j-th firm profit-maximization problem; (2.4)–(2.6) are the market-clearing conditions.

Solving the above system of simultaneous equations (2.1)–(2.6), we obtain a general equilibrium of this economy.[5] This system consists of 6 sets/14 equations (generally speaking $4i + h \cdot j + h$ equations) and the same number of endogenous variables.[6] It is notable that this system is homogeneous of degree zero in prices. As Walras's law always holds (even when some of the market-clearing conditions are not satisfied), one of the equations in the system is redundant.[7] Thus, we must choose one good or one factor as a numeraire and fix its price. Then, all other prices are expressed as relative prices in terms of the numeraire. Therefore, it should be noted that we cannot solve absolute prices but only relative prices as in other CGE and general equilibrium models with zero homogeneity in prices.

In concluding our discussion of the simple CGE model, we confirm the zero-profit condition for competitive firms in the following manner. Multiplying both sides of the factor demand function (2.3) by p_h^f and summing them with respect to the suffix h, we can derive the equation shown in (2.7). (In this mathematical manipulation, note that a constant-returns-to-scale production function (2.2) implies that the sum of the share coefficient $\beta_{h,j}$ with respect to the suffix h is equal to unity.)

$$\sum_h p_h^f F_{h,j} = p_j^z Z_j \quad \forall j \tag{2.7}$$

The left-hand side of (2.7) shows the cost incurred to the j-th firm, while the right-hand side represents its sales of the good. Equality of the total cost and sales in this simple CGE model proves that a firm cannot earn either excess profits nor suffer from excess losses in the equilibrium.[8] That is, the zero-profit condition for any firm is assured.

Notes

1. Although the real economy has multiple households, this fictitious one representative household is adequate as long as all households have homogeneous preferences. Section 10.1 introduces heterogeneous multiple households into the CGE model.
2. In this example, the names of the goods are set as bread and milk. However, these goods do not imply they are complements or substitutes, or essentials or luxuries.
3. We can rewrite the utility function without the product symbol Π as follows:

$$UU = \prod_i X_i^{\alpha_i} = X_{BRD}^{\alpha_{BRD}} \cdot X_{MLK}^{\alpha_{MLK}}$$

Similarly, the budget constraint can be rewritten without the summation symbol Σ as follows:

$$p_{BRD}^x X_{BRD} + p_{MLK}^x X_{MLK} = p_{CAP}^f FF_{CAP} + p_{LAB}^f FF_{LAB}$$

The symbol $\underset{X_i}{\text{maximize}}$ means to maximize the following objective function with respect to the variable X_i.

4. Detailed derivation of this household demand function (2.1) is shown in Appendix I.
5. Instead of directly solving this system of simultaneous equations, we can reformulate it into a social welfare maximization problem and solve it to obtain the same general equilibrium solution. Details are discussed in Appendix II.
6. We often verify whether a model has a solution by examining whether the number of equations is exactly equal to the number of endogenous variables. This rule is often used as a quick method of verifying the solvability of a model. However, it is not a necessary condition or a sufficient condition for the solvability of nonlinear systems either.
7. Walras's law implies that the sum of excess demand in all the markets is always equal to zero; that is:

$$p_{BRD}^x (X_{BRD} - Z_{BRD}) + p_{MLK}^x (X_{MLK} - Z_{MLK})$$
$$+ p_{CAP}^f \left(\sum_j F_{CAP,j} - FF_{CAP} \right) + p_{LAB}^f \left(\sum_j F_{LAB,j} - FF_{LAB} \right) = 0.$$

When the bread, milk and capital markets are in equilibrium (i.e., the first three terms on the left-hand side are zero), Walras's law implies that the labour market is also in equilibrium (i.e., the fourth term is zero). For details of Walras's law, see Varian (2006).

8. If the right-hand side of (2.7) exceeds its left-hand side (i.e., excess profits exist), an entry into the market takes place to reduce profits per firm. In contrast, excess losses lead to the exit of firms to increase profits per firm. Finally, entries and exits make profits per firm converge to zero as (2.7) indicates. The excess profit is different from normal profit (i.e., payments for capital $p^f_{CAP}F_{CAP,j}$ in our model). Positive normal profits accrue if the marginal productivity of capital is greater than zero.

3 Computation

Chapter 2 provided a mathematical presentation of the 'simple CGE model' with its theoretical underpinnings based in standard microeconomics. However, the general equilibrium of an economy cannot be 'computed' only with this mathematical model. The CGE modelling needs two more steps. The first step is to prepare a computer program of the CGE model, using GAMS syntax. This step is covered in this chapter. The second step is to construct a social accounting matrix (SAM) on the basis of empirical data and then to estimate coefficients and exogenous variables such as the share coefficient of the utility function α_i and the factor endowments FF_h. This second step is discussed in Chapters 4 and 5.

In this chapter, the household utility maximization problem, which is part of the simple CGE model explained in Chapter 2, is used to demonstrate GAMS programming. By reading this chapter, readers will become familiar with the use of GAMS for CGE modelling.[1] The entire computer program for the simple CGE model will be explained in Section 5.4. After that, Section 6.8 presents the computer program for the 'standard CGE model', which includes a government, investment and savings, intermediate inputs and outputs, indirect taxes and the external sector.

3.1 Example: the 'household utility maximization model'

To explain GAMS programming, we consider the household utility maximization problem (in Section 2.2), which is part of the simple CGE model. (Hereinafter, we call it the 'household utility maximization model'.) In this model, two goods (bread and milk) as well as two factors (capital and labour) are considered. As assumed in the previous chapter, this household does not have any power to affect market prices. That