Agenda for Today

- Headway Distributions
  - Pearson Type –III
  - Composite
- Goodness of fit
- Visit to the Traffic Management Center (April **)
Pearson Type III Distribution

- Pearson Type III Distribution is a generalized mathematical approach
- The probability density function is given by

\[ f(t) = \frac{\lambda}{\Gamma(K)} [\lambda (t - \alpha)]^{K-1} e^{-\lambda(t-\alpha)} \]

Where
- \( f(t) \) -> Probability distribution
- \( \lambda \) -> Parameter that is a function of K and
- \( K \) -> Parameter between 0 and \( \infty \) (that affects shape of the distribution)
- \( \alpha \) -> Parameter greater than zero (that affects shift of the distribution)
- \( t \) -> time headway being investigated
- \( e \) -> Napier's constant 2.718
- \( \Gamma(K) \) -> Gamma function, equivalent to \((K-1)!\)
Pearson Type III Distribution (2)

- Integration of $F(t)$ between 0 and $\infty$

\[ \int_0^{\infty} f(t) \, dt = 1.0 \]
Probability of headway greater than $t$

$$P(h \geq t) = \int_t^\infty f(t) \, dt$$
Pearson Type III Distribution (3)

- Probability of headway lying between $t$ and $t + \Delta t$
- \[ P(h \geq t) = \int_t^\infty f(t) \, dt - \int_{t+\Delta t}^\infty f(t + \Delta t) \, dt \]
Pearson Type III Distribution (3)

- If $\Delta t$ is very small
- And an approximate solution is possible
- The probability can be defined as

$$P(t \leq h < t + \Delta t) = \left[ \frac{f(t) + f(t + \Delta t)}{2} \right] \Delta t$$
### Other Distributions

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Estimating $K$</th>
<th>Calculating $\lambda$</th>
<th>Probability Density Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pearson Type-III (K and $\alpha$)</td>
<td>$\frac{\bar{t} - \alpha}{s}$</td>
<td>$\frac{K}{\bar{t} - \alpha}$</td>
<td>$\frac{\lambda}{\Gamma(K)} [\lambda (t - \alpha)]^{K-1} e^{-\lambda(t-\alpha)}$</td>
</tr>
<tr>
<td>Gamma (K and $\alpha = 0$)</td>
<td>$\frac{\bar{t}}{s}$</td>
<td>$\frac{K}{\bar{t}}$</td>
<td>$\frac{\lambda}{\Gamma(K)} [\lambda t]^{K-1} e^{-\lambda t}$</td>
</tr>
<tr>
<td>Erlang (K = +ve integer, $\alpha = 0$)</td>
<td>$\frac{\bar{t}}{s}$</td>
<td>$\frac{K}{\bar{t}}$</td>
<td>$\frac{\lambda}{(K - 1)!} [\lambda t]^{K-1} e^{-\lambda t}$</td>
</tr>
<tr>
<td>Neg. Exponential (K =1 and $\alpha = 0$)</td>
<td>$\frac{\bar{t}}{s}$</td>
<td>$\frac{1}{\bar{t}}$</td>
<td>$\lambda e^{-\lambda t}$ or $\lambda e^{-t/\bar{t}}$</td>
</tr>
<tr>
<td>Shifted Neg. Exponential (K =1 and $\alpha &gt; 0$)</td>
<td>$\frac{\bar{t} - \alpha}{s}$</td>
<td>$\frac{1}{\bar{t} - \alpha}$</td>
<td>$\lambda e^{-\lambda(t-\alpha)}$ or $\lambda e^{-(t-\alpha)/(\bar{t}-\alpha)}$</td>
</tr>
</tbody>
</table>
Eight Steps for Pearson Type-III

- **Step-1**
  - Calculate mean time headway and standard deviation of headway

- **Step-2**
  - Select appropriate value of $\alpha$

- **Step-3**

- Calculate approximate value of $K$

\[ \hat{K} = \frac{\bar{t} - \alpha}{s} \]
Eight Steps for Pearson Type-III

- **Step-4**
  - Calculate $\lambda$
  
  $$\lambda = \frac{K}{t - \alpha}$$

- **Step-5**
  - Calculate Gamma Function
  
  $$\Gamma(K) = (K-1)!$$
  
  - Check gamma value for fractional number
Eight Steps for Pearson Type-III

- **Step-6**
  - Solve for f(t)
- **Step-7**
  - Solver for P(t)
- **Step-8**
  - Solver for F(t)
Derive Table

- Apply Pearson Type-III to the lowest flow condition
- Derive Table in-class
Observations

- The probabilities decrease as the time headway increases.
- Appears that Pearson Type III matches with the observed distribution well.
- As headway > 4 sec, theoretical headways are less than observed.
- For headways less than 1 sec there is little mismatch.
- Pearson Type III involves assumption of K and $\alpha$, but these values are approximate.
- Sensitivity analysis should be conducted to determine best Pearson Type-III distribution.
Composite Model Approach

- **Vehicles in platoon**
  - Normal distribution

- **Vehicles not in platoon**
  - Shifted negative exponential distribution
Composite Model Approach

- **Overall four parameters are needed**
  - Normal distribution
    - Mean and standard deviation of headway
- **Shifted negative exponential distribution**
  - Minimum time headway for vehicles not in platoon
  - Proportion of vehicles not in platoon
Composite Model Approach

- Proportion of vehicles in platoon + Proportion of vehicles not in platoon = 1.00
- Mean time headway of the vehicles not in platoon can be computed from the following equation

$$\bar{t} = \bar{t}_p P_P + \bar{t}_{Np} P_{Np}$$

- $P_P$ -> Proportion of vehicles in platoon
- $P_{Np}$ -> Proportion of vehicles not in platoon
- $\bar{t}_p$ -> Time headway for vehicles in platoon
- $\bar{t}_{Np}$ -> Time headway for vehicles not in platoon
If we assume all vehicles at headways of less than 1.5 sec. are in platoons and others not in platoons.

Then proportion of vehicles in platoon

\[ P_p = \frac{1.5}{t} \]

For figure 2.2 (a), percentage of vehicles in platoon is

\[ P_p = \frac{1.5}{5} = 30\% \]
Proportion of Vehicles not in Platoon

- Proportion of vehicles not in platoon = 70%
- Headway for vehicles not in platoon can be computed as following

\[
t_{NP} = \frac{t_p (P - \bar{t})}{P_{NP}} = \frac{1.5 \times 0.3 - 5}{0.7} = 6.5
\]

- The standard deviation is \( \bar{t} - \alpha = 6.5 - 2.0 = 4.5 \) sec
## Input Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Platoon</th>
<th>Not in Platoon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Headway</td>
<td>1.5</td>
<td>6.5</td>
</tr>
<tr>
<td>S.D. Headway</td>
<td>0.5</td>
<td>4.5</td>
</tr>
<tr>
<td>Proportion</td>
<td>30%</td>
<td>70%</td>
</tr>
</tbody>
</table>

\[ \alpha = 2 \text{ sec} \]
Create Table

- Derive Composite Distribution
Observations

- Two distributions appear to have the same general shape
- The two distributions are most different under low flow conditions
- But becomes similar as flow level increases
- Theoretical distribution > measured when headway is more than 4 sec
- Larger differences when headway is between 1 and 2.5 sec
Other Approaches

- Schuhl’s model
- Two classes of vehicles
  - Constrained vehicles
  - Free-moving vehicles
- Constrained vehicles:
  - Shifted negative exponential distribution
- Free flowing vehicles
  - Negative exponential distribution

\[
P(h \geq t) = P e^{-(t-\alpha)/(\bar{t}-\alpha)} + (1 - P) e^{-t/\bar{t}}
\]