CIVL - 7904/8904

TRAFFIC FLOW THEORY

LECTURE -5

Agenda for Today

- Headway Distributions
  - Pearson Type –III
  - Composite
- Goodness of fit
- Visit to the Traffic Management Center (April **)

Pearson Type III Distribution

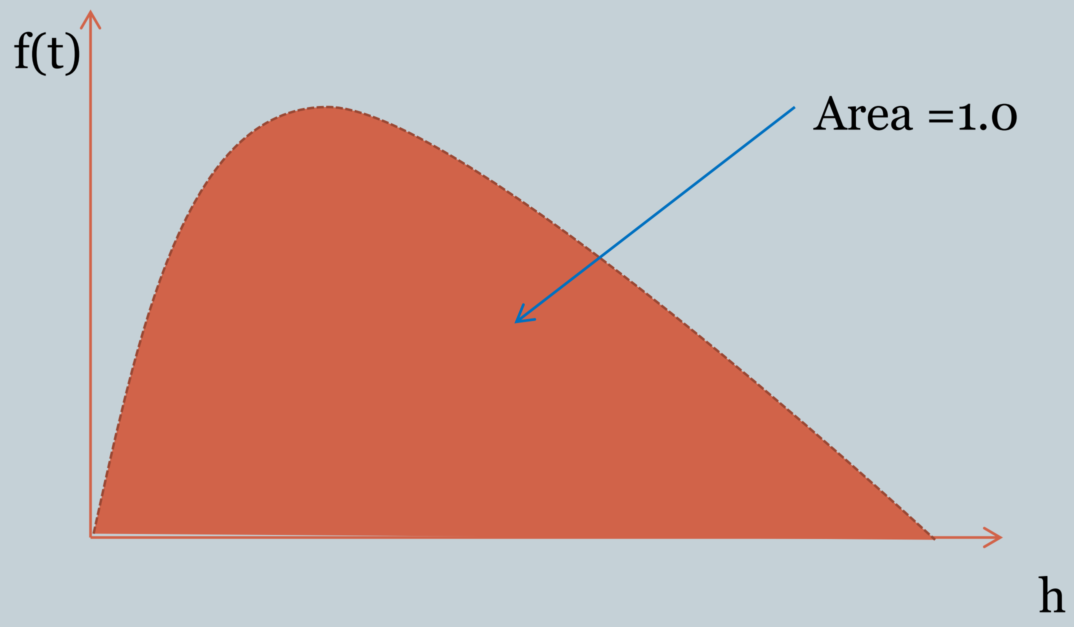
- Pearson Type III Distribution is a generalized mathematical approach
- The probability density function is given by

$$f(t) = \frac{\lambda}{\Gamma(K)}[\lambda\,(t-\alpha)]^{K-1}e^{-\lambda(t-\alpha)}$$

Where
f(t)-> Probability distribution
λ -> Parameter that is a function of K and
K-> Parameter between 0 and ∞ (that affects shape of the distribution)
α -> Parameter greater than zero (that affects shift of the distribution)
t-> time headway being investigated
e-> Napier's constant 2.718
$\Gamma(K)$-> Gamma function, equivalent to (K-1)!

Pearson Type III Distribution (2)

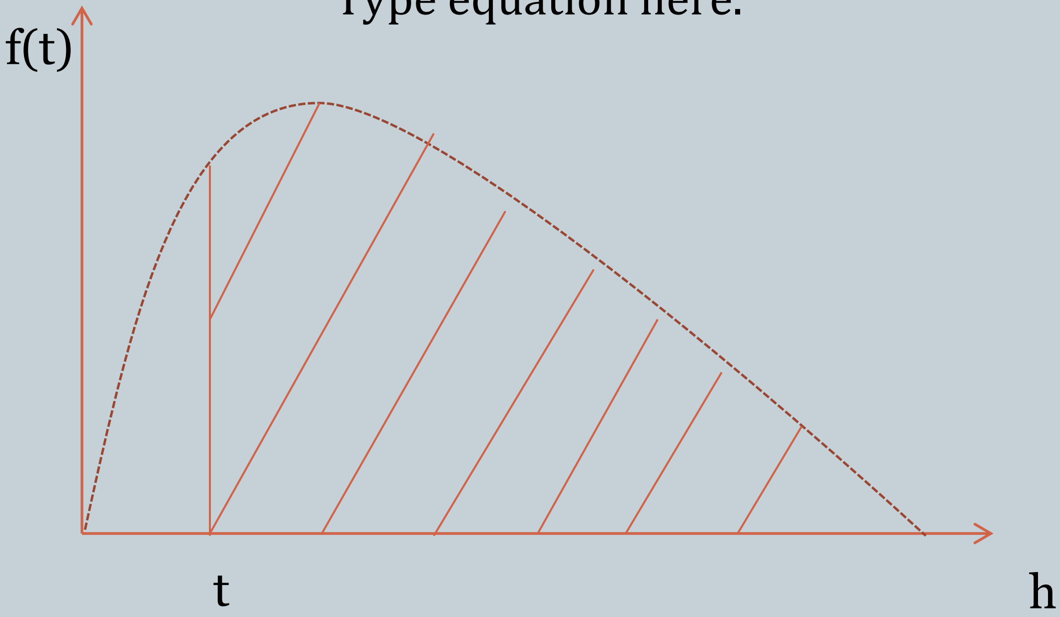
- Integration of F(t) between 0 and ∞



Pearson Type III Distribution (2)

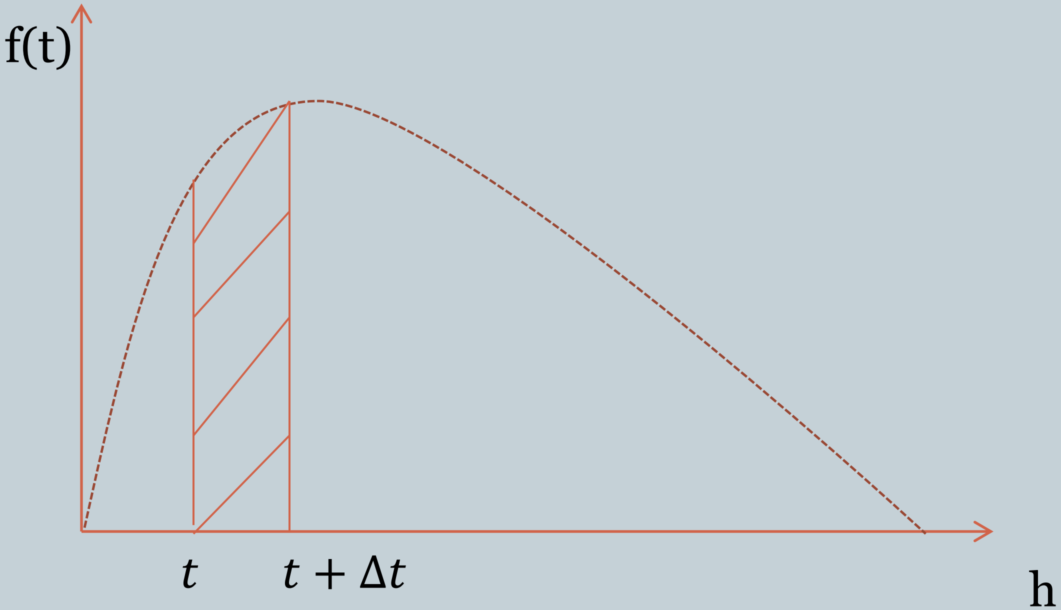
- Probability of headway greater than t
- $P(h \geq t) = \int_t^{\infty} f(t)\, dt$

Type equation here.



Pearson Type III Distribution (3)

- Probability of headway lying between t and and $t+\Delta t$
- $P(h \geq t) = \int_t^\infty f(t)\,dt - \int_{t+\Delta t}^\infty f(t+\Delta t)\,dt$



Pearson Type III Distribution (3)

- If Δt is very small
- And an approximate solution is possible
- The probability can be defined as

$$P(t \le h < t + \Delta t) = \left[\frac{f(t) + f(t + \Delta t)}{2}\right]\Delta t$$

Other Distributions

Distribution	Estimating K	Calculating λ	Probability Density Function
Pearson Type-III (K and α)	$\frac{\bar{t}-\alpha}{s}$	$\frac{K}{\bar{t}-\alpha}$	$\frac{\lambda}{\Gamma(K)}[\lambda\,(t-\alpha)]^{K-1}e^{-\lambda(t-\alpha)}$
Gamma (K and $\alpha=0$)	$\frac{\bar{t}}{s}$	$\frac{K}{\bar{t}}$	$\frac{\lambda}{\Gamma(K)}[\lambda t]^{K-1}e^{-\lambda t}$
Erlang (K = +ve integer, $\alpha=0$)	$\frac{\bar{t}}{s}$	$\frac{K}{\bar{t}}$	$\frac{\lambda}{(K-1)!}[\lambda t]^{K-1}e^{-\lambda t}$
Neg. Exponential (K =1 and $\alpha=0$)	$\frac{\bar{t}}{s}$	$\frac{1}{\bar{t}}$	$\lambda e^{-\lambda t}\ or \lambda e^{-t/\bar{t}}$
Shifted Neg. Exponential (K =1 and $\alpha>0$)	$\frac{\bar{t}-\alpha}{s}$	$\frac{1}{\bar{t}-\alpha}$	$\lambda e^{-\lambda(t-\alpha)} or \lambda e^{-(t-\alpha)/(\bar{t}-\alpha)}$

Eight Steps for Pearson Type-III

- Step-1
  - Calculate mean time headway and standard deviation of headway
- Step-2
  - Select appropriate value of α
- Step-3
- Calculate approximate value of K

$$\hat{K} = \frac{\bar{t} - \alpha}{s}$$

Eight Steps for Pearson Type-III

- Step-4
  - Calculate λ

$$\lambda = \frac{K}{\bar{t} - \alpha}$$

- Step-5
  - Calculate Gamma Function

$$\Gamma(K) = (K-1)!$$

  - Check gamma value for fractional number

Eight Steps for Pearson Type-III

- Step-6
  - Solve for f(t)
- Step-7
  - Solver for P(t)
- Step-8
  - Solver for F(t)

Derive Table

- Apply Pearson Type-III to the lowest flow condition
- Derive Table in-class

Observations

- The probabilities decrease as the time headway increases
- Appears that Pearson Type III matches with the observed distribution well
- As headway>4sec, theoretical headways are less than observed
- For headways less than 1 sec there is little mismatch
- Pearson Type III involves assumption of K and α, but these values are approximate
- Sensitivity analysis should be conducted to determine best Pearson Type-III distribution

Composite Model Approach

- Vehicles in platoon
  - Normal distribution
- Vehicles not in platoon
  - Shifted negative exponential distribution

Composite Model Approach

- Overall four parameters are needed
  - Normal distribution
    - Mean and standard deviation of headway
- Shifted negative exponential distribution
  - Minimum time headway for vehicles not in platoon
  - Proportion of vehicles not in platoon

Composite Model Approach

- Proportion of vehicles in platoon + Proportion of vehicles not in platoon =1.00
- Mean time headway of the vehicles not in platoon can be computed from the following equation

$$\overline{t} = \overline{t_p}\, P_P + \overline{t_{Np}}\, P_{Np}$$

P_P -> Proportion of vehicles in platoon
P_{Np}->Proportion of vehicles not in platoon
$\overline{t_p}$ ->Time headway for vehicles in platoon
$\overline{t_{Np}}$ ->Time headway for vehicles not in platoon

Proportion of Vehicles in Platoon

- If we assume all vehicles at headways of less than 1.5 sec. are in platoons and others not in platoons
- Then proportion of vehicles in platoon

$$P_p = \frac{1.5}{\bar{t}}$$

- For figure 2.2 (a), percentage of vehicles in platoon is

$$P_p = \frac{1.5}{5} = 30\%$$

Proportion of Vehicles not in Platoon

- Proportion of vehicles not in platoon = 70%
- Headway for vehicles not in platoon can be computed as following

$$\overline{t_{Np}} = \frac{\overline{t_p}\, P_P - \bar{t}}{P_{Np}} = \frac{1.5*0.3-5}{0.7} = 6.5$$

- The standard deviation is $\bar{t}$-α = 6.5-2.0 = 4.5 sec

Input Values

Parameter	Platoon	Not in Platoon
Mean Headway	1.5	6.5
S.D. Headway	0.5	4.5
Proportion	30%	70%

$$\alpha = 2\ sec$$

Create Table

- Derive Composite Distribution

Observations

- Two distributions appear to have the same general shape
- The two distributions
- are most different under low flow conditions
- But becomes similar as flow level increases
- Theoretical distribution > measured when headway is more than 4 sec
- Larger differences when headway is between 1 and 2.5 sec

Other Approaches

- Schuhl's model
- Two classes of vehicles
  - Constrained vehicles
  - Free-moving vehicles
- Constrained vehicles:
  - Shifted negative exponential distribution
- Free flowing vehicles
  - Negative exponential distribution

$$P(h \geq t) = Pe^{-(t-\alpha)/(\bar{t}-\alpha)} + (1 - P)\, e^{-t/\bar{t}}$$