CIVL - 7904/8904

TRAFFIC FLOW THEORY

LECTURE -4

Agenda for Today

- Headway Distributions
  - Random (last class)
  - Constant
  - Intermediate
  - Mixed

Constant Headway State (1)

- Normal distribution can be used
  - When headway is nearly constant
  - Drivers attempt to drive at a constant headway
  - In reality there exist driver error which make time headways not exactly constant but nearly constant
- For normal distribution two parameters are needed
  - Mean time headway
  - Standard deviation of headway

Constant Headway State (2)

- In constant headway state, mean headway ($\bar{t}$) is

$$\bar{t} = \frac{3600}{V}$$

- Where *V* is volume in veh/hr

- Standard deviation >0
- How to estimate Standard Deviation?

Standard Deviation (1)

- If the capacity of the stream of traffic is in order of 1800 veh/hr.,
  - Then mean time headway would be 2 sec/veh
- Since time headways can not be negative (normally not less than 0.5 sec), the lowest theoretical headway should be approximately

$$\alpha = \bar{t} - 2s$$

α-> Minimum time headway
$\bar{t}$->Mean time headway (sec/veh)
s-> Standard deviation of the headway distribution
2-> Constant, specifying that 2 standard deviations below the mean time headway should be very near the lowest theoretical headway

Standard Deviation (2)

- The standard deviation yields

$$s = \frac{\bar{t} - \alpha}{2}$$

- If $\bar{t}$ = 2 sec, and α = 0.5 sec, then standard deviation is 0.75 sec
- The corresponding variance is 0.56

Application of Normal Distribution

- Applied to highest flow condition, Fig. 2.2(d)
- Mean time headway = 2.2 sec
- Standard deviation = 0.85
- Step-1: Calculate z
- Step-2: Calculate z/s
- Step-3: Calculate f(t)
- Step-4: Calculate P(t)
- Step-5: Calculate F(t)

Comparison with Observed Data

- Normal distribution is inherent being symmetrical
- Two distributions are quite different, under low flow conditions
- Two distributions match well under higher flow conditions
- Normal distribution is shifted to the right of measured distribution 0.5 to 1.0 sec
- S.D. of measured distribution is always greater than S.D of normal distribution

Intermediate Headway State

- Lies between two headway state
  - Random state
  - Constant headway state
- Other approaches
  - Generalized mathematical model
  - Composite model approach
  - Variety of other approaches

Pearson Type III Distribution

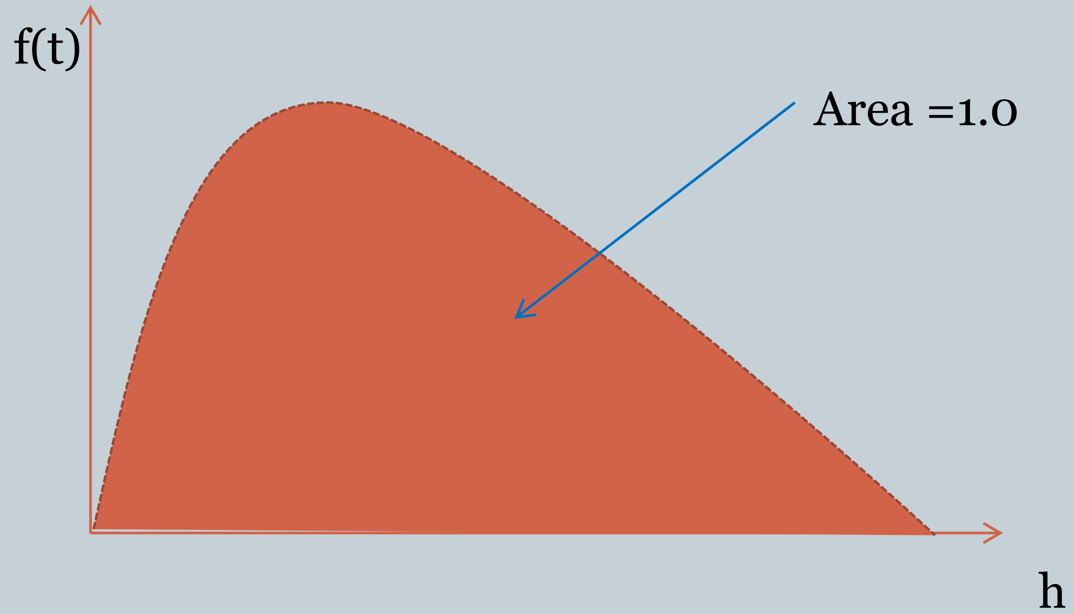
- Pearson Type III Distribution is a generalized mathematical approach
- The probability density function is given by

$$f(t) = \frac{\lambda}{\Gamma(K)}[\lambda\,(t-\alpha)]^{K-1} e^{-\lambda(t-\alpha)}$$

Where
f(t)-> Probability distribution
λ -> Parameter that is a function of K and
K-> Parameter between 0 and ∞ (that affects shape of the distribution)
α -> Parameter greater than zero (that affects shift of the distribution)
t-> time headway being investigated
e-> Napier's constant 2.718
$\Gamma(K)$-> Gamma function, equivalent to (K-1)!

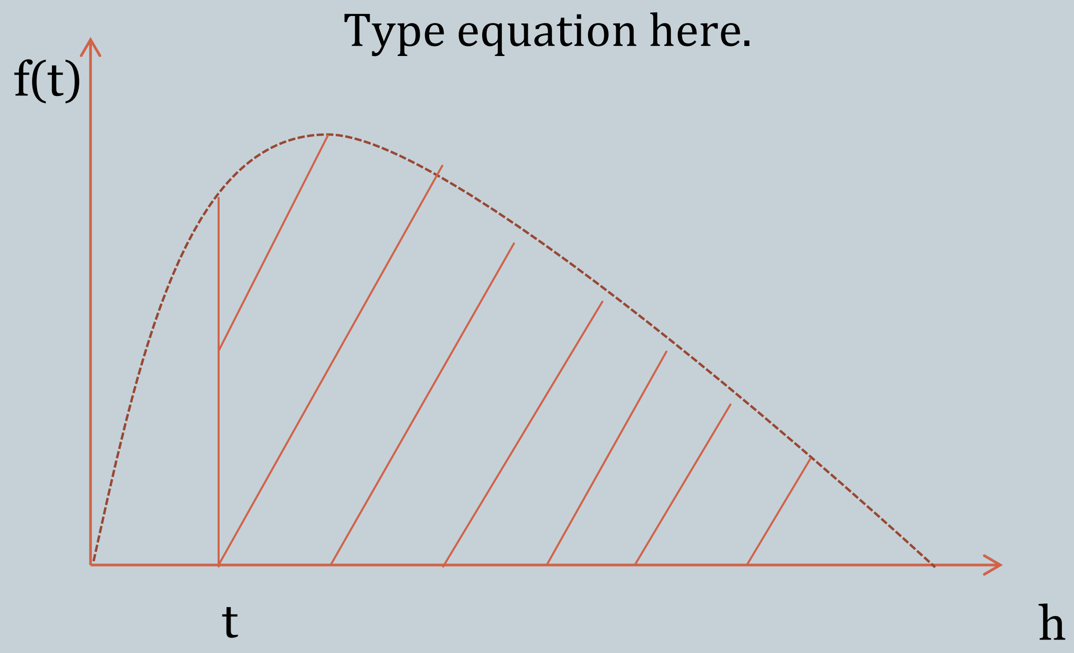
Pearson Type III Distribution (2)

- Integration of F(t) between 0 and ∞



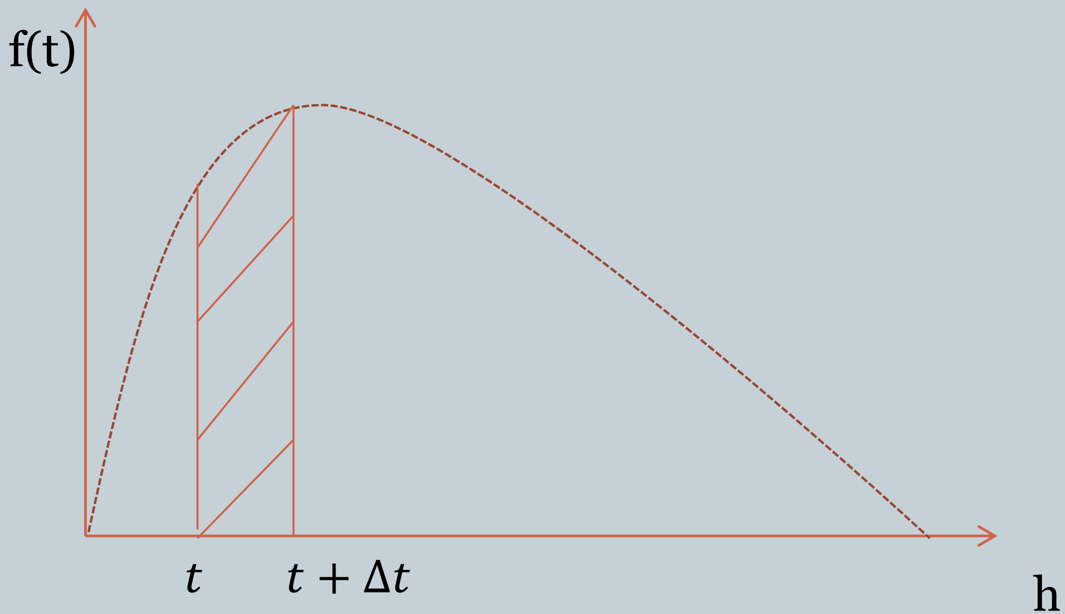
Pearson Type III Distribution (2)

- Probability of headway greater than t
- $P(h \geq t) = \int_t^{\infty} f(t)\,dt$



Pearson Type III Distribution (3)

- Probability of headway lying between t and and $t + \Delta t$
- $P(h \geq t) = \int_{t}^{\infty} f(t)\, dt - \int_{t+\Delta t}^{\infty} f(t + \Delta t)\, dt$



Pearson Type III Distribution (3)

- If Δt is very small
- And an approximate solution is possible
- The probability can be defined as

$$P(t \leq h < t + \Delta t) = \left[\frac{f(t) + f(t + \Delta t)}{2}\right] \Delta t$$

Other Distributions

Distribution	Estimating K	Calculating λ	Probability Density Function
Pearson Type-III (K and α)	$\frac{\bar{t}-\alpha}{s}$	$\frac{K}{\bar{t}-\alpha}$	$\frac{\lambda}{\Gamma(K)}[\lambda\,(t-\alpha)]^{K-1}e^{-\lambda(t-\alpha)}$
Gamma (K and $\alpha=0$)	$\frac{\bar{t}}{s}$	$\frac{K}{\bar{t}}$	$\frac{\lambda}{\Gamma(K)}[\lambda t]^{K-1}e^{-\lambda t}$
Erlang (K = +ve integer, $\alpha=0$)	$\frac{\bar{t}}{s}$	$\frac{K}{\bar{t}}$	$\frac{\lambda}{(K-1)!}[\lambda t]^{K-1}e^{-\lambda t}$
Neg. Exponential (K =1 and $\alpha=0$)	$\frac{\bar{t}}{s}$	$\frac{1}{\bar{t}}$	$\lambda e^{-\lambda t}\ or \lambda e^{-t/\bar{t}}$
Shifted Neg. Exponential (K =1 and $\alpha>0$)	$\frac{\bar{t}-\alpha}{s}$	$\frac{1}{\bar{t}-\alpha}$	$\lambda e^{-\lambda(t-\alpha)} or \lambda e^{-(t-\alpha)/(\bar{t}-\alpha)}$

Eight Steps for Pearson Type-III

- Step-1
  - Calculate mean time headway and standard deviation of headway
- Step-2
  - Select appropriate value of α
- Step-3
- Calculate approximate value of K

$$\widehat{K} = \frac{\bar{t} - \alpha}{s}$$

Eight Steps for Pearson Type-III

- Step-4
  - Calculate λ

$$\lambda = \frac{K}{\bar{t} - \alpha}$$

- Step-5
  - Calculate Gamma Function

$$\Gamma(K) = (K-1)!$$

  - Check gamma value for fractional number

Eight Steps for Pearson Type-III

- Step-6
  - Solve for f(t)
- Step-7
  - Solver for P(t)
- Step-8
  - Solver for F(t)

Derive Table

- Apply Pearson Type-III to the lowest flow condition
- Derive Table in-class