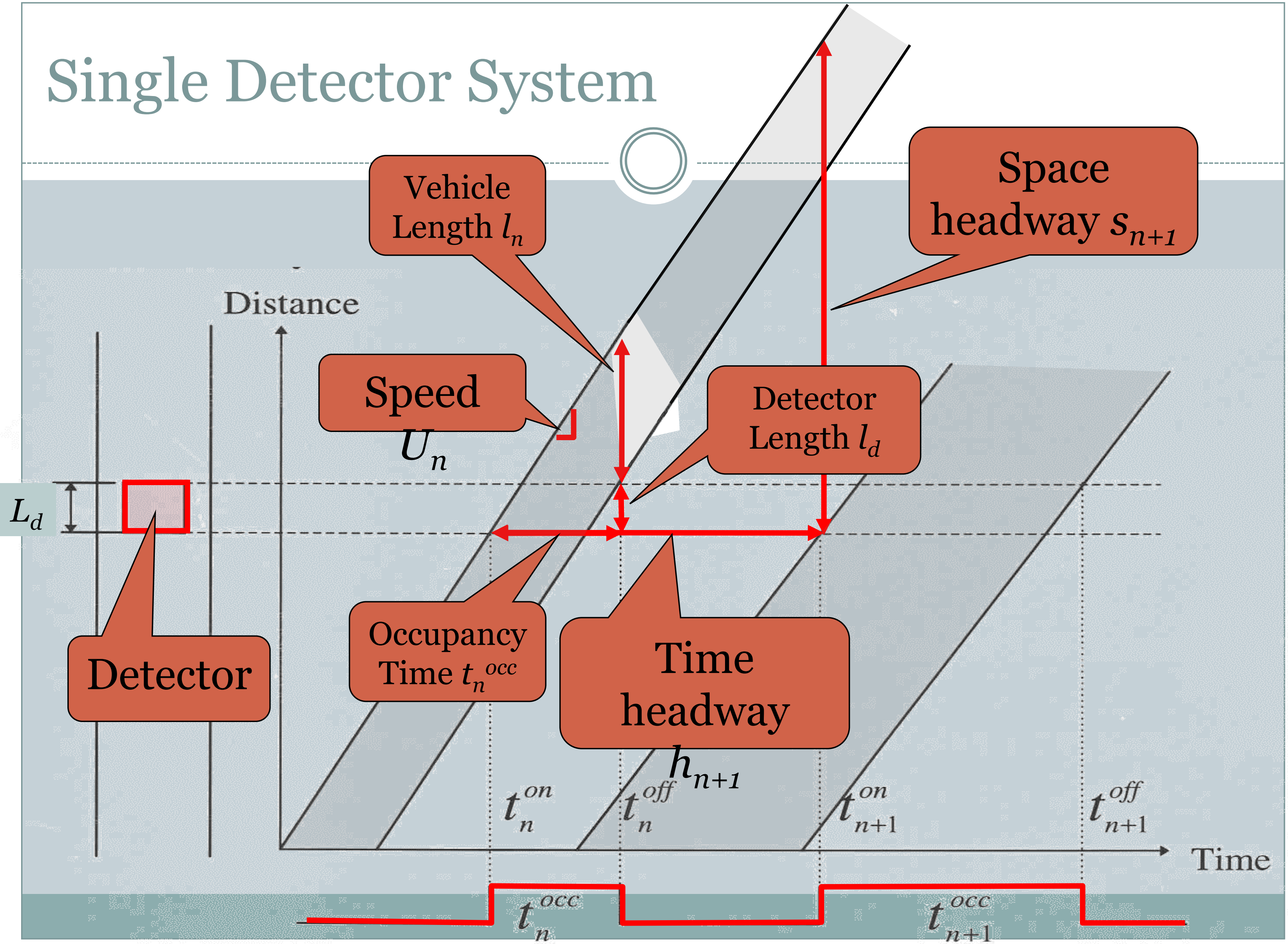
CIVL - 7904/8904

TRAFFIC FLOW THEORY

LECTURE -20

Measurements with Presence of Detectors

Single Detector System



Single Detector System-Time Headway

- A number of microscopic flow characteristics can be obtained.

$$h_{n+1} = (t_{on})_{n+1} - (t_{on})_n$$

Where,
h-> time headway in seconds
t_{on}-> Time instant that vehicle is detected

Single Detector System-Vehicle Occupancy

- Vehicle occupancy can be determined as

$$(t_{occ})_n = \left(t_{off}\right)_n - (t_{on})_n$$
$$(t_{occ})_{n+1} = \left(t_{off}\right)_{n+1} - (t_{on})_{n+1}$$

Where,
t_{occ} is the individual occupancy time (sec)

Single Detector System-Vehicle Speed

- Vehicle occupancy time is a function of
  - Vehicle speed,
  - Vehicle length
  - Detection zone length

$$\dot{x}_n = \frac{L_n + L_D}{(t_{occ})_n} \quad and \quad \dot{x}_{n+1} = \frac{L_{n+1} + L_D}{(t_{occ})_{n+1}}$$

Where,
$\dot{x}_n$-> Speed of the vehicle (feet/sec)
L_n-> vehicle length (feet)
L_D->detection zone length (feet)

Single Detector System-Calibration

- With specific sensitivity adjustment setting, the detection zone length is constant and can numerically be determined through calibration
- If all vehicles passing over the detector have approximately the same length, then vehicle speed can be defined as

$$\dot{x}_n = \frac{K}{(t_{occ})_n} \quad and \quad \dot{x}_{n+1} = \frac{K}{(t_{occ})_{n+1}}$$

Where,
K-> constant representing average vehicle length detection zone (ft)

Single Detector System-Distance Headway

- Distance headway can be determined with vehicle speed and time headway as follows

$$\dot{x}_n = \frac{(x)_n - (x)_{n+1}}{(h)_{n+1}}$$

$$Or$$

$$(x)_n - (x)_{n+1} = \dot{x}_n (h)_{n+1}$$

- The accuracy of this estimation is dependent on the
  - Assumed constant vehicle length
  - The constant speed of vehicle n during the time period of $(t_{on})_n$ and $(t_{on})_{n+1}$

Nomograph Estimation

- See Figure 6.13

Macroscopic Flow Characteristics-Flow rate

- Flow rate can be estimated as following

$$q = \frac{3600}{\left({}^{1}/_{(N-1)}\right)\sum_{n=n+1}^{N} h_n}$$

$$q = \frac{\sum_{t=0}^{T} \delta}{T}$$

Where,
δ->change is detector signal from state 0 to 1
T-> time period during detector signal change count (hours)

Macroscopic Flow Characteristics-Speed

- Average traffic speed (time mean speed) at a point can be obtained by averaging the individual vehicle speeds:

$$\overline{\dot{x}} = \frac{\sum_{n=1}^{N} \dot{x}_n}{N} = \frac{K}{\frac{1}{N}\sum_{n=1}^{N}(t_{occ})_n}$$

$$k = \frac{q}{\mu}$$

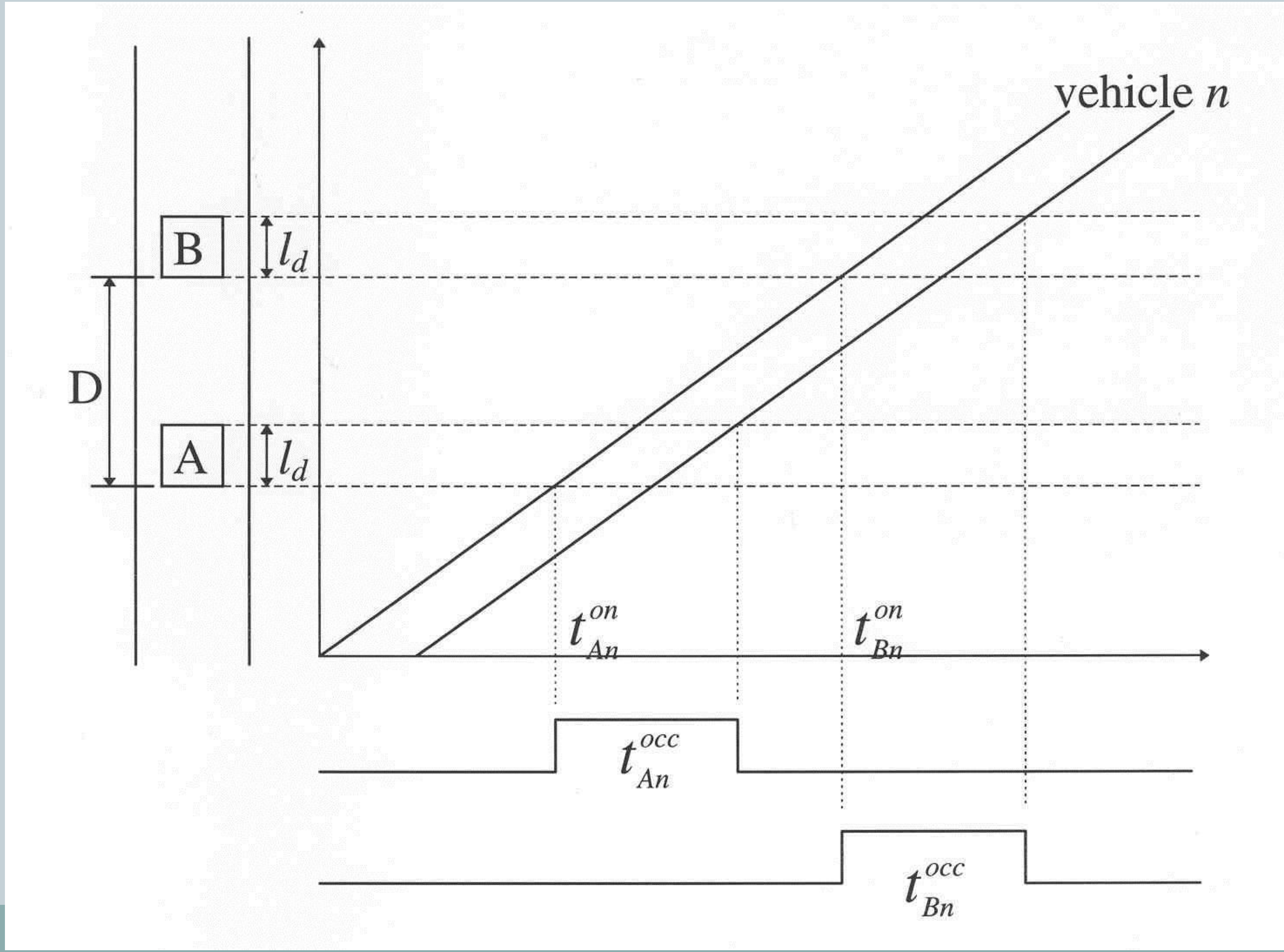
Macroscopic Flow Characteristics-% Occupancy

- % occupancy time is a surrogate for density and is obtained as follows:

$$\% \text{ occ} = \frac{\sum_{n=1}^{N}(t_{occ})_n}{T} \times 100$$

% occ = percent occupancy time
N= Number of vehicles detected in time period T
T = selected time period

Measurements with Two Detectors



Two Detectors: Speed

- The assumption of a known constant vehicle length can be ignored.
  - If two closely-spaced detectors are employed

$$\dot{x}_n = \frac{D}{[(t_{on})_n]_B - [(t_{on})_n]_A}$$

$$\dot{x}_{n+1} = \frac{D}{[(t_{on})_{n+1}]_B - [(t_{on})_{n+1}]_A}$$

Where,
D-> distance from the upstream edge of the detection zone A to the upstream edge of the detection zone B

Two Detectors: Length of Vehicle

$$[L_n]_A = \dot{x}_n [(t_{occ})_n]_A - [L_D]_A$$

or

$$[L_n]_B = \dot{x}_n [(t_{occ})_n]_B - [L_D]_B$$

$$[L_{n+1}]_A = \dot{x}_{n+1} [(t_{occ})_{n+1}]_A - [L_D]_A$$

or

$$[L_{n+1}]_B = \dot{x}_{n+1} [(t_{occ})_{n+1}]_B - [L_D]_B$$

Two Detectors: Distance Headway

$$(x)_n - (x)_{n+1} = [h_{n+1}]_A \, \dot{x}_n$$

or

$$(x)_n - (x)_{n+1} = [h_{n+1}]_B \, \dot{x}_n$$

Shock Wave Analysis

What is Shock Wave?

- Flow-speed-density changes over space and time.
- When these changes of state occur a boundary is established that demarks the time-space domain of one flow from another ->
  - This boundary is referred as a shock wave
- Alternatively, shock waves are defined as boundary conditions in the time-space domain that demark a discontinuity in flow-density conditions.

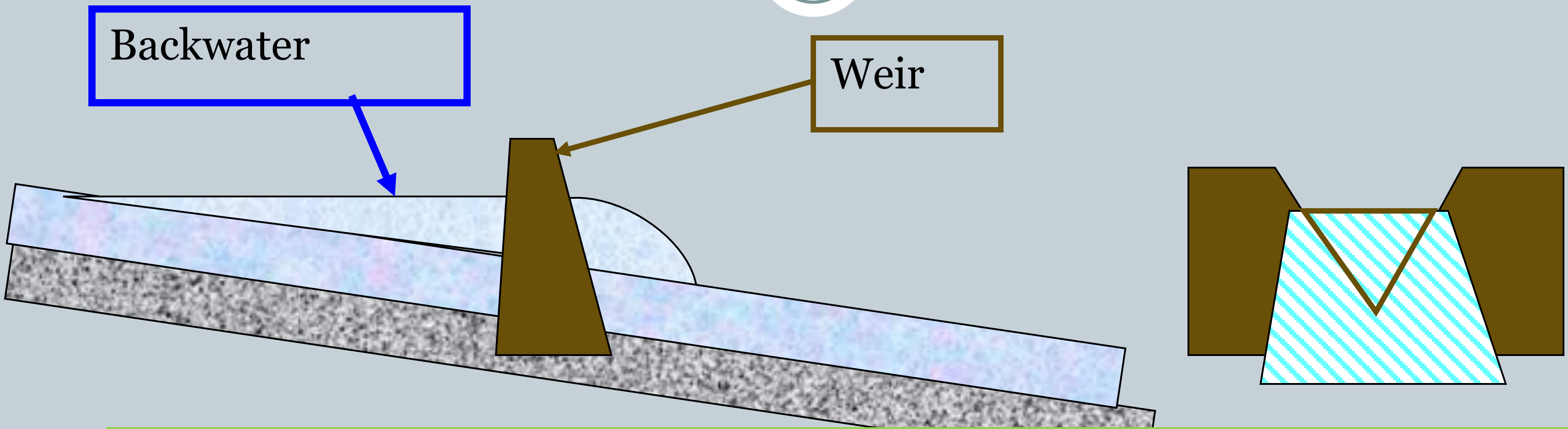
Shock Wave – Examples (1)

- In some situation shock wave can be very mild
  - Ex- a platoon of high speed vehicles catching up to a slightly slowing moving vehicles
- In other situations shock waves can be very significant
  - Ex-High speed vehicles approach a queue of stopped vehicles

Shock Wave – Examples (2)

- Shock waves can be formed because of
  - Traffic crashes
  - Reduction in number of lanes
  - Restricted bridge size
  - Work zones
  - Signal turning red
- These situations are related to cases where capacity on the highway suddenly changes, with changes in
  - Density
  - Flow
  - Speed

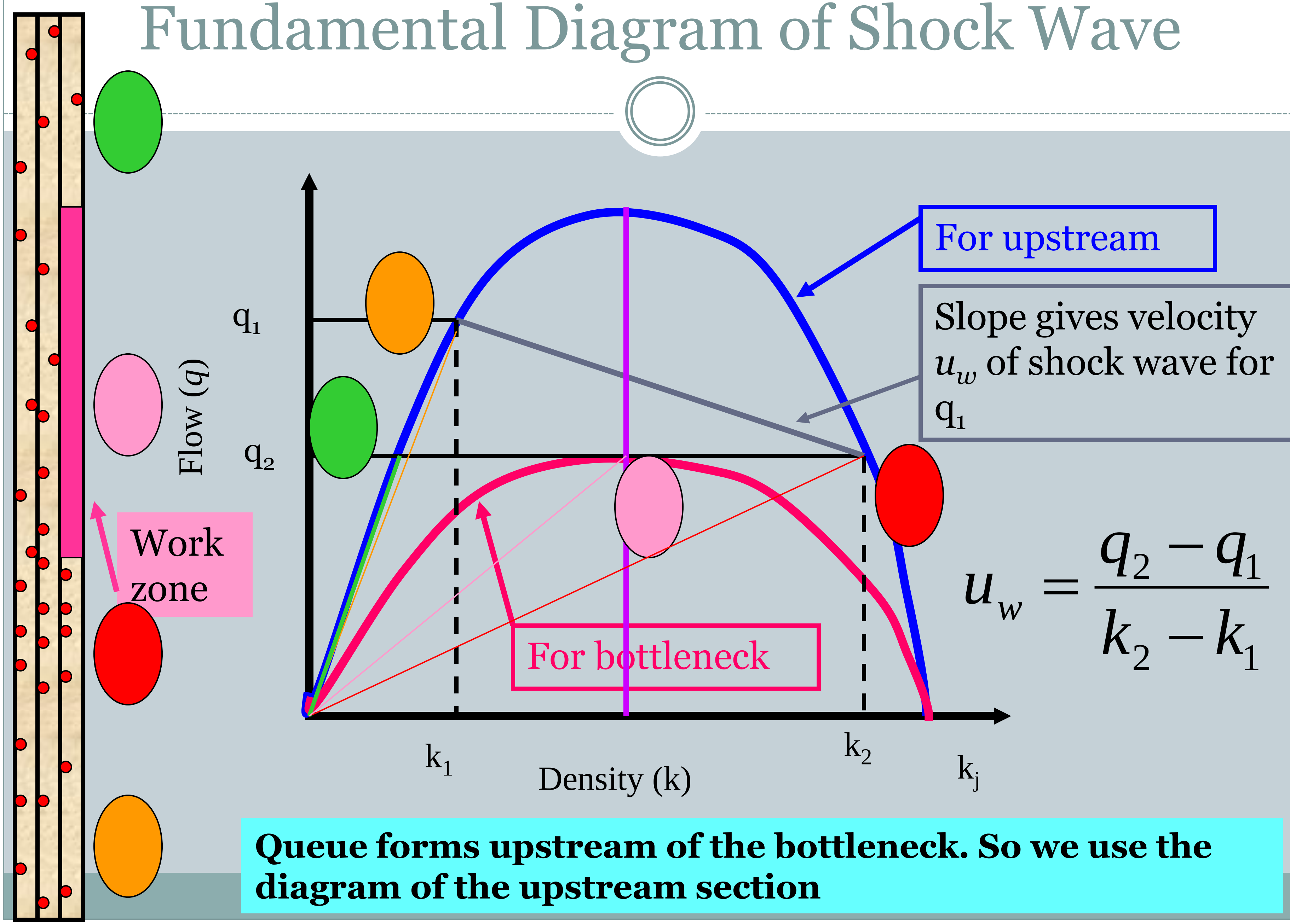
Analogy to Fluid Flow



When the capacity of the weir opening is smaller than the upstream flow, what would happen?

Backwater is created. This is like a queue created upstream of a bottleneck. The tail of the queue moves upstream until the upstream in flow and the outflow from the opening become equal. This phenomenon is called "shock wave." So, the formation of queue upstream of a bottleneck is like the formation of backwater caused by a weir.

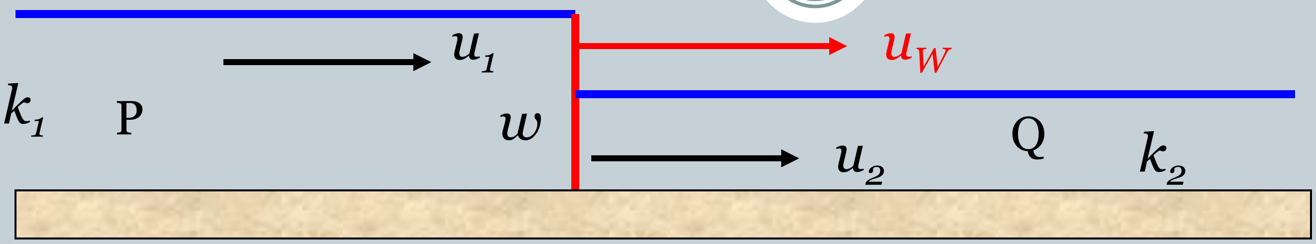
Fundamental Diagram of Shock Wave



$$u_w = \frac{q_2 - q_1}{k_2 - k_1}$$

Queue forms upstream of the bottleneck. So we use the diagram of the upstream section

Shock Wave from Flow Conservation



$$u_{r_1} = (u_1 - u_w)$$

Speed relative to w

$$u_{r_2} = (u_2 - u_w)$$

$$N_1 = u_{r_1} k_1 t$$

No. of veh's crossing line w during t

$$N_2 = u_{r_2} k_2 t$$

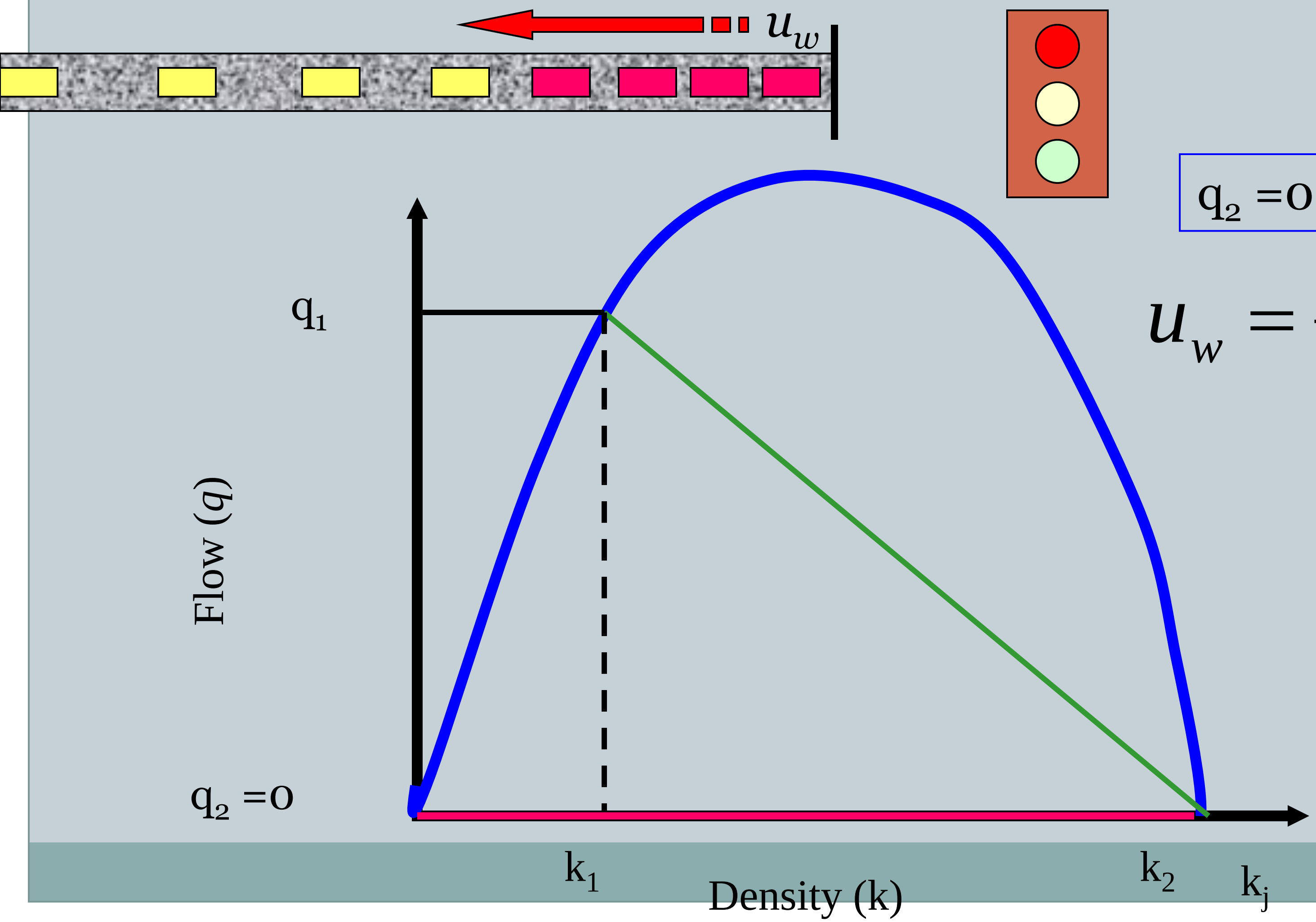
Conservation of flow says ➔ $N_1 = N_2$. Hence,

$$u_2 k_2 - u_1 k_1 = u_w (k_2 - k_1)$$

➡ $$q_2 - q_1 = u_w (k_2 - k_1)$$ ➡

$$u_w = \frac{q_2 - q_1}{k_2 - k_1}$$

Stopping waves (like “stopped at the red light”)

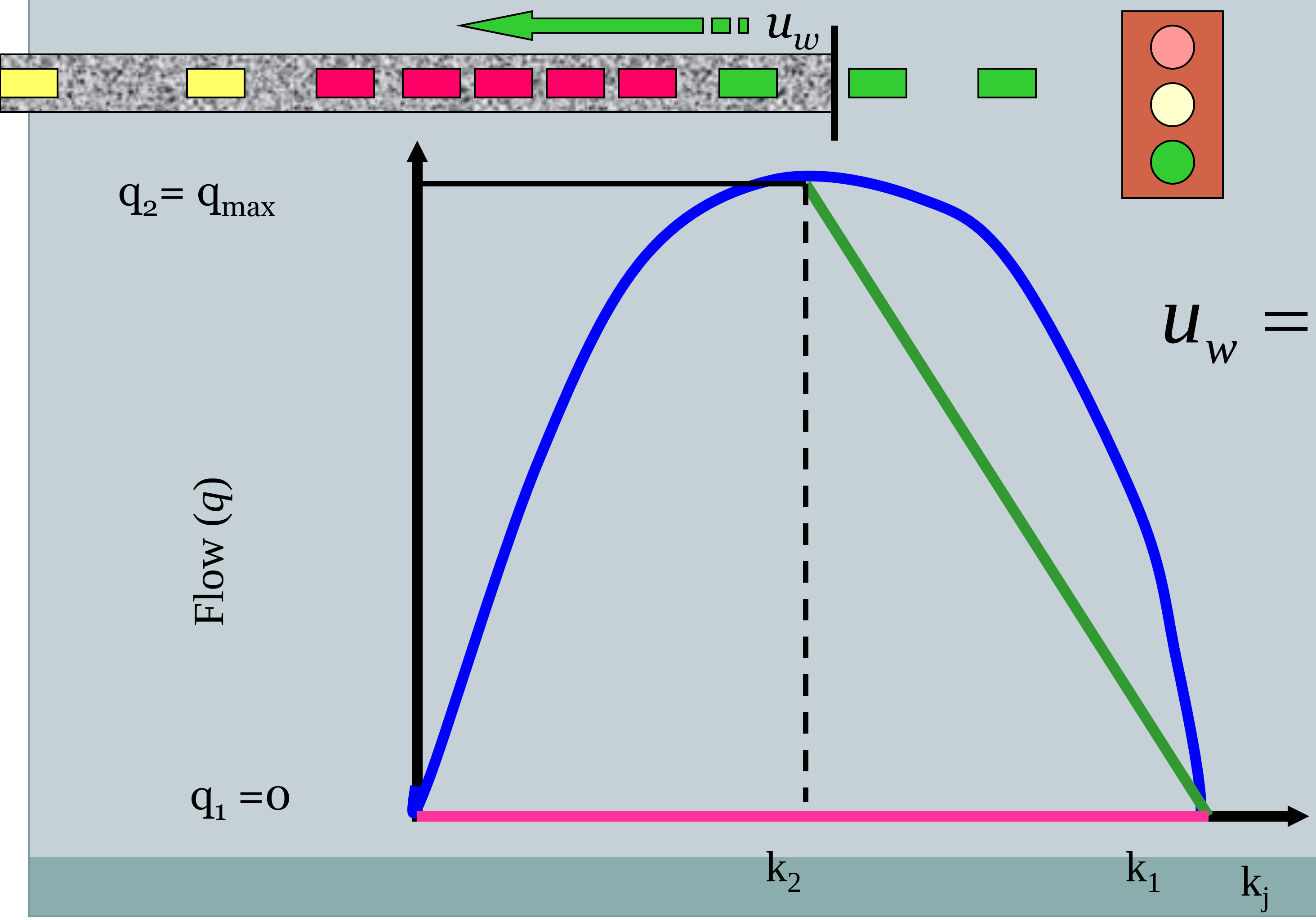


$q_2 = 0$

$$u_w = \frac{q_2 - q_1}{k_2 - k_1}$$

$k_2 = k_j$

Starting Waves (like "leaving the stop line given green light)



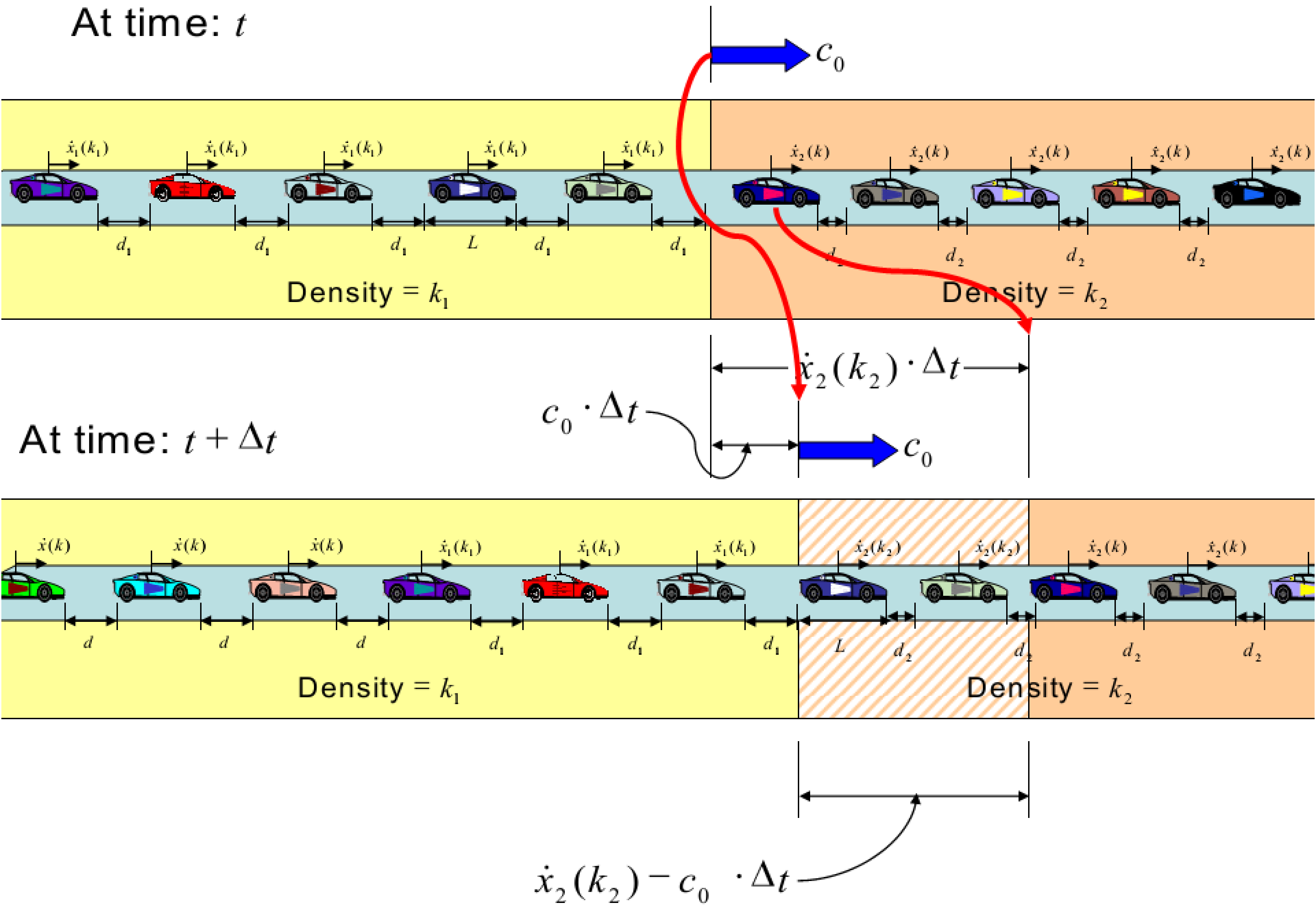
$q_1 = 0$

$$u_w = \frac{q_2 - q_1}{k_2 - k_1}$$

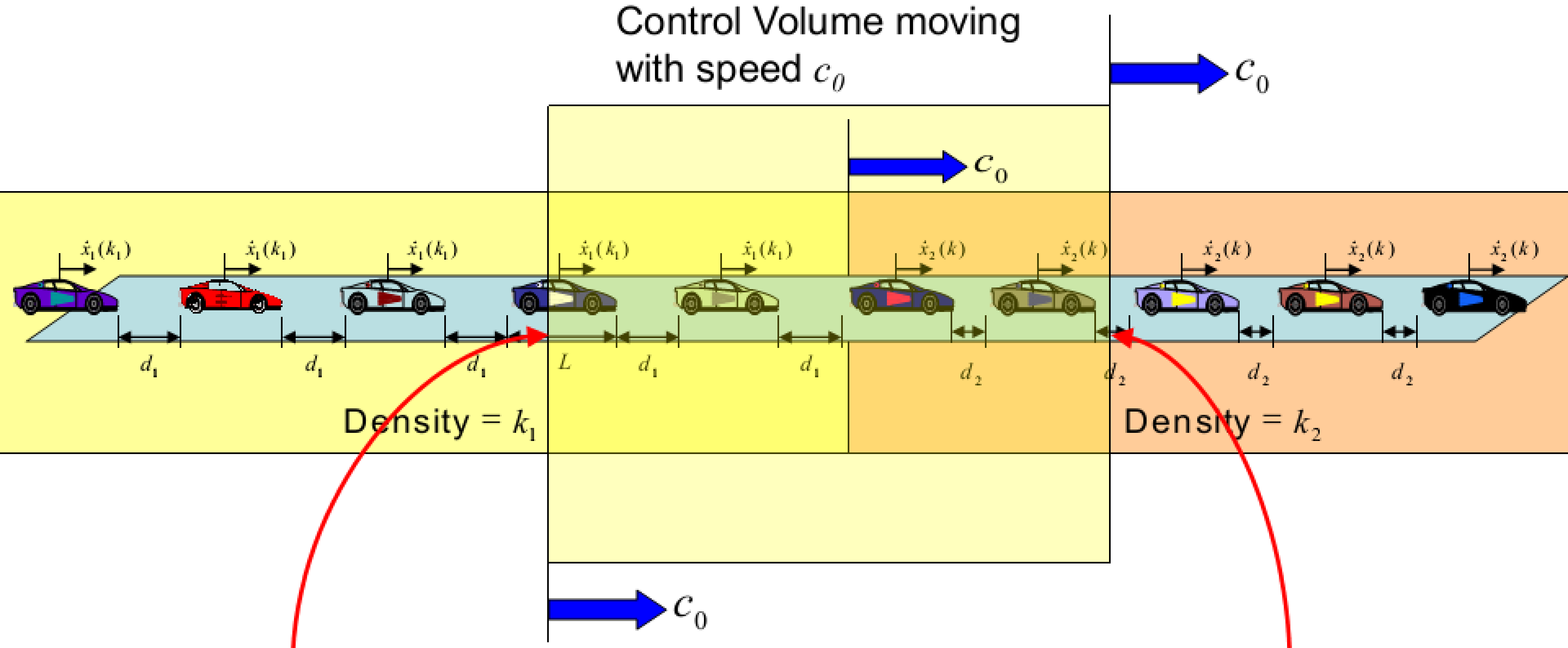
$k_1 = k_j$

$k_2 = k_o$

Examples and Relationship with Real Life



At time: t
c_0
$\dot{x}_1(k_1)$
$\dot{x}_2(k)$
d_1
L
d_2
Density = k_1
Density = k_2
$\dot{x}_2(k_2)\cdot\Delta t$
$c_0\cdot\Delta t$
At time: $t+\Delta t$
c_0
$\dot{x}(k)$
$\dot{x}_1(k_1)$
$\dot{x}_2(k_2)$
$\dot{x}_2(k)$
d
d_1
L
d_2
Density = k_1
Density = k_2
$\dot{x}_2(k_2)-c_0\ \cdot\Delta t$



Flow into Control Volume:

$$\hat{q}_1 = k_1 \cdot (\dot{x}_1 - c_0)$$

Flow out of Control Volume:

$$\hat{q}_2 = k_2 \cdot (\dot{x}_2 - c_0)$$

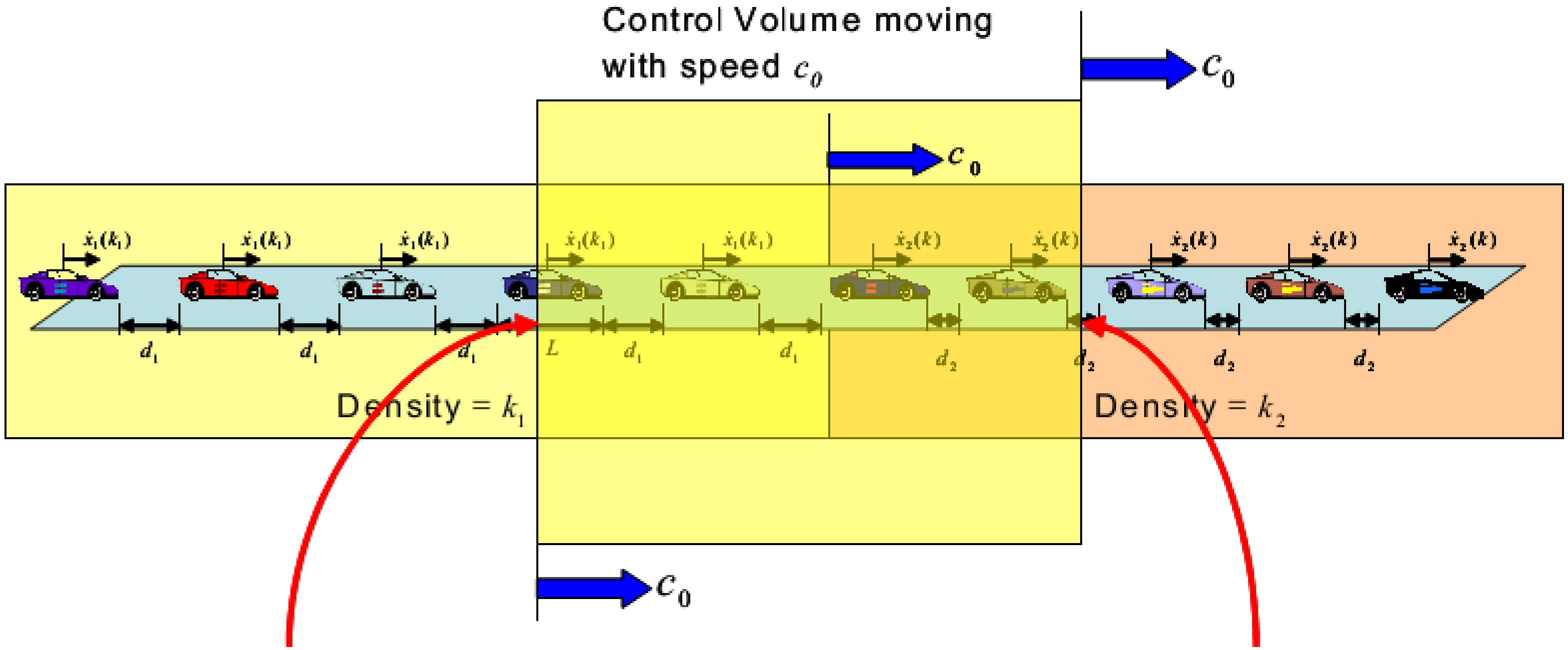
Conservation Equation

$$\hat{q}_1 = \hat{q}_2$$

$$c_0 = \frac{k_1 \cdot \dot{x}_1 - k_2 \cdot \dot{x}_2}{k_1 - k_2}$$

$$k_1 \cdot (\dot{x}_1 - c_0) = k_2 \cdot (\dot{x}_2 - c_0)$$

$$c_0 = \frac{q_1 - q_2}{k_1 - k_2}$$

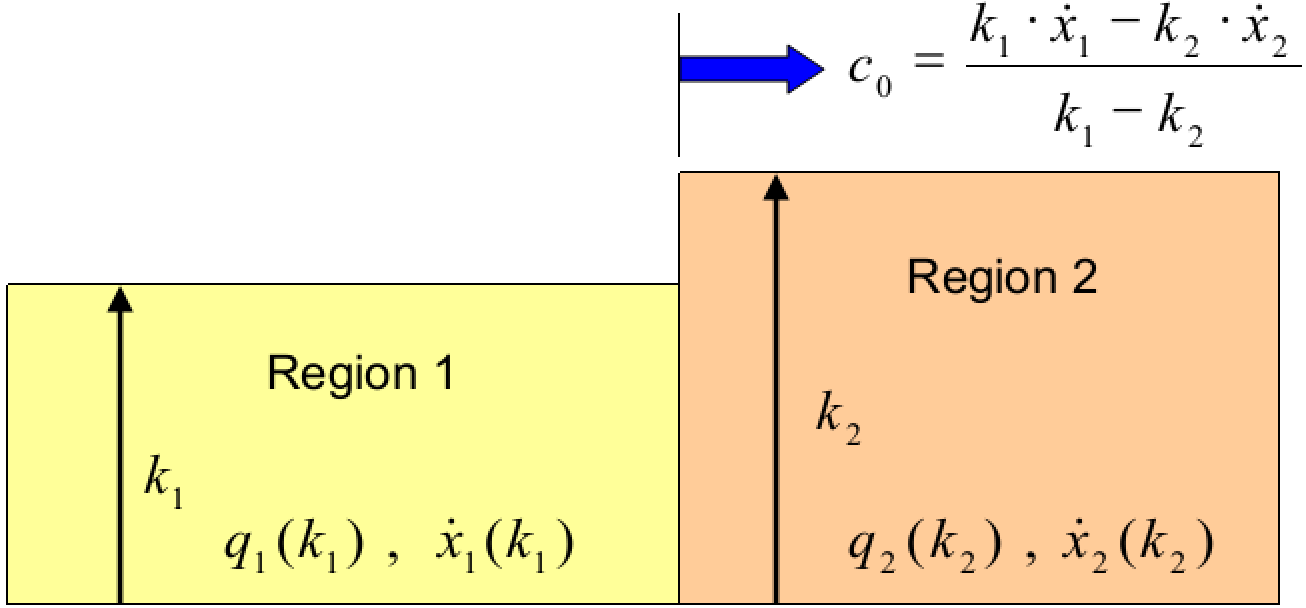


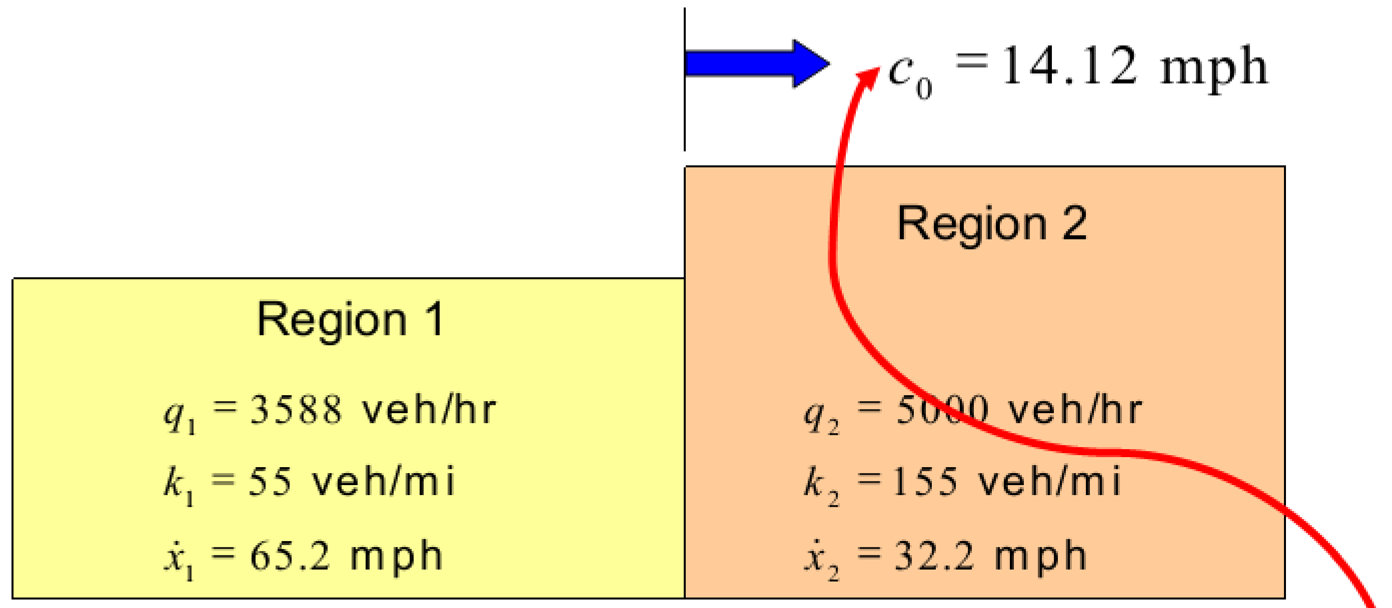
Flow into Control Volume:

$$\hat{q}_1 = k_1 \cdot (\dot{x}_1 - c_0)$$

Flow out of Control Volume:

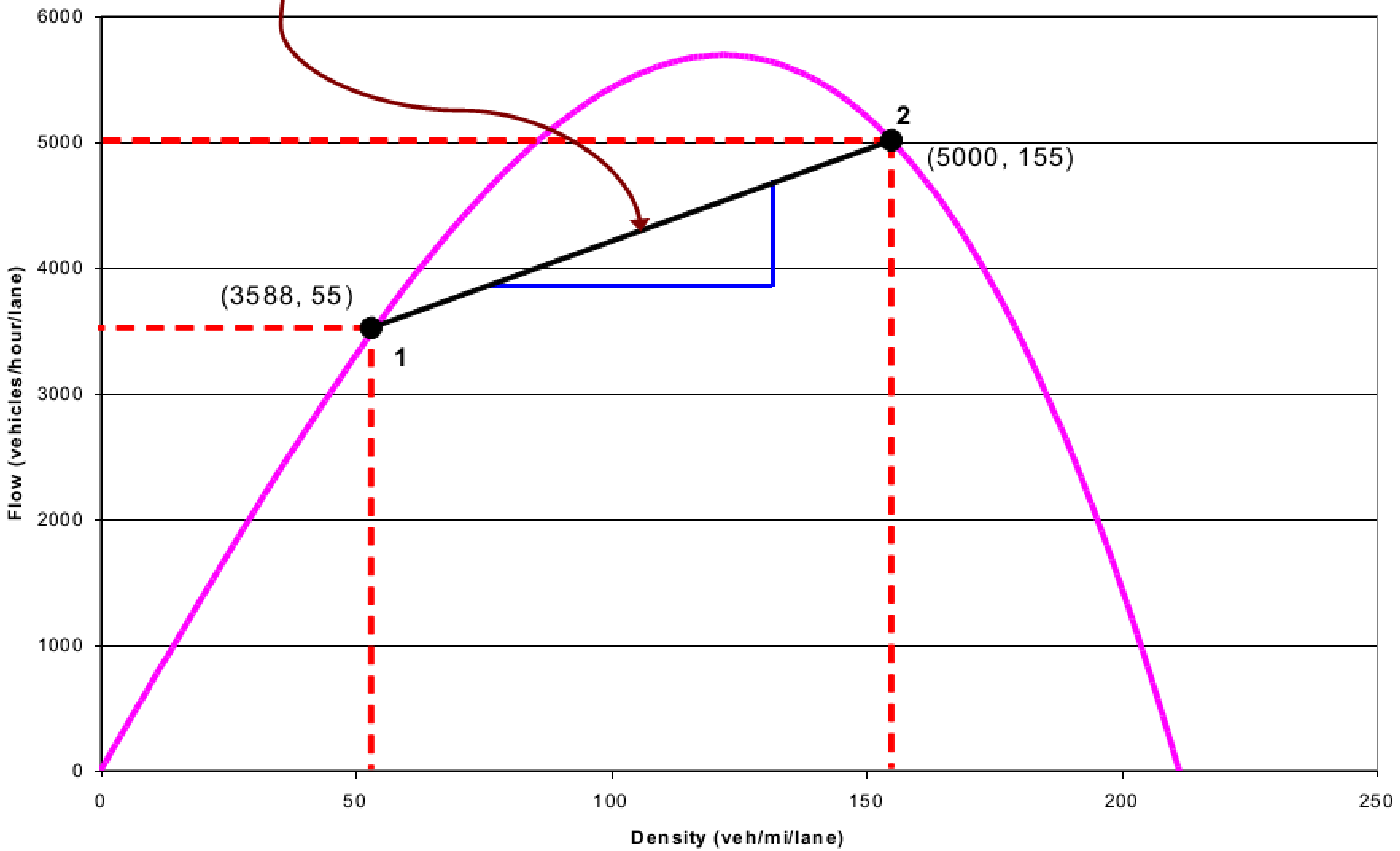
$$\hat{q}_2 = k_2 \cdot (\dot{x}_2 - c_0)$$


$c_0 = \frac{k_1 \cdot \dot{x}_1 - k_2 \cdot \dot{x}_2}{k_1 - k_2}$
Region 2
k_2
$q_2(k_2)$, $\dot{x}_2(k_2)$
Region 1
k_1
$q_1(k_1)$, $\dot{x}_1(k_1)$

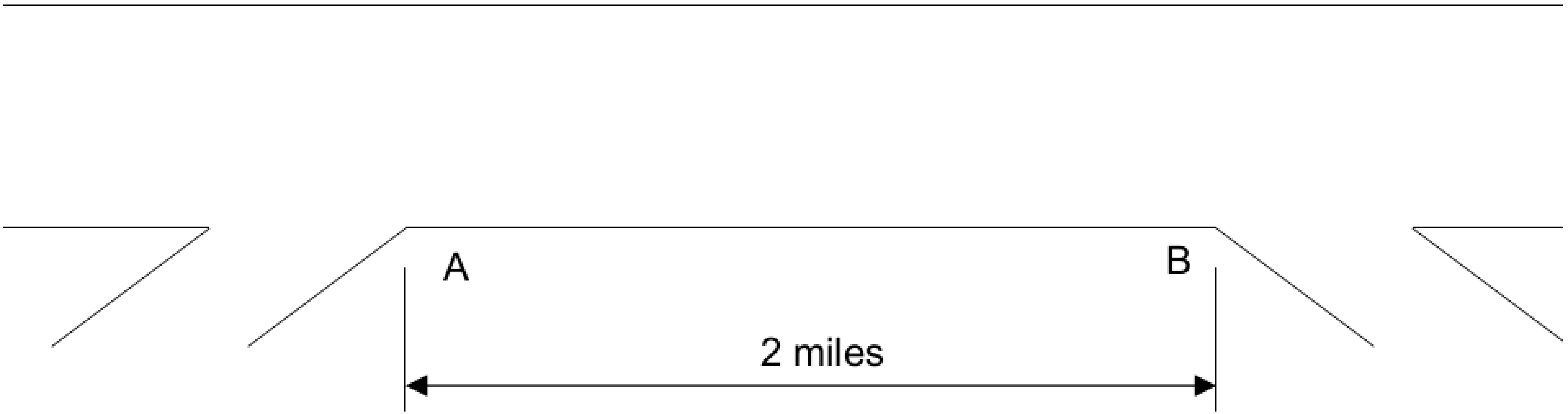


$$c_0 = \frac{5000 - 3588}{155 - 55} = \frac{1412}{100} = 14.12\,\text{mph}$$

$$c_0 = \frac{q_1 - q_2}{k_1 - k_2} =$$ slope of chord between Point 1 and Point 2

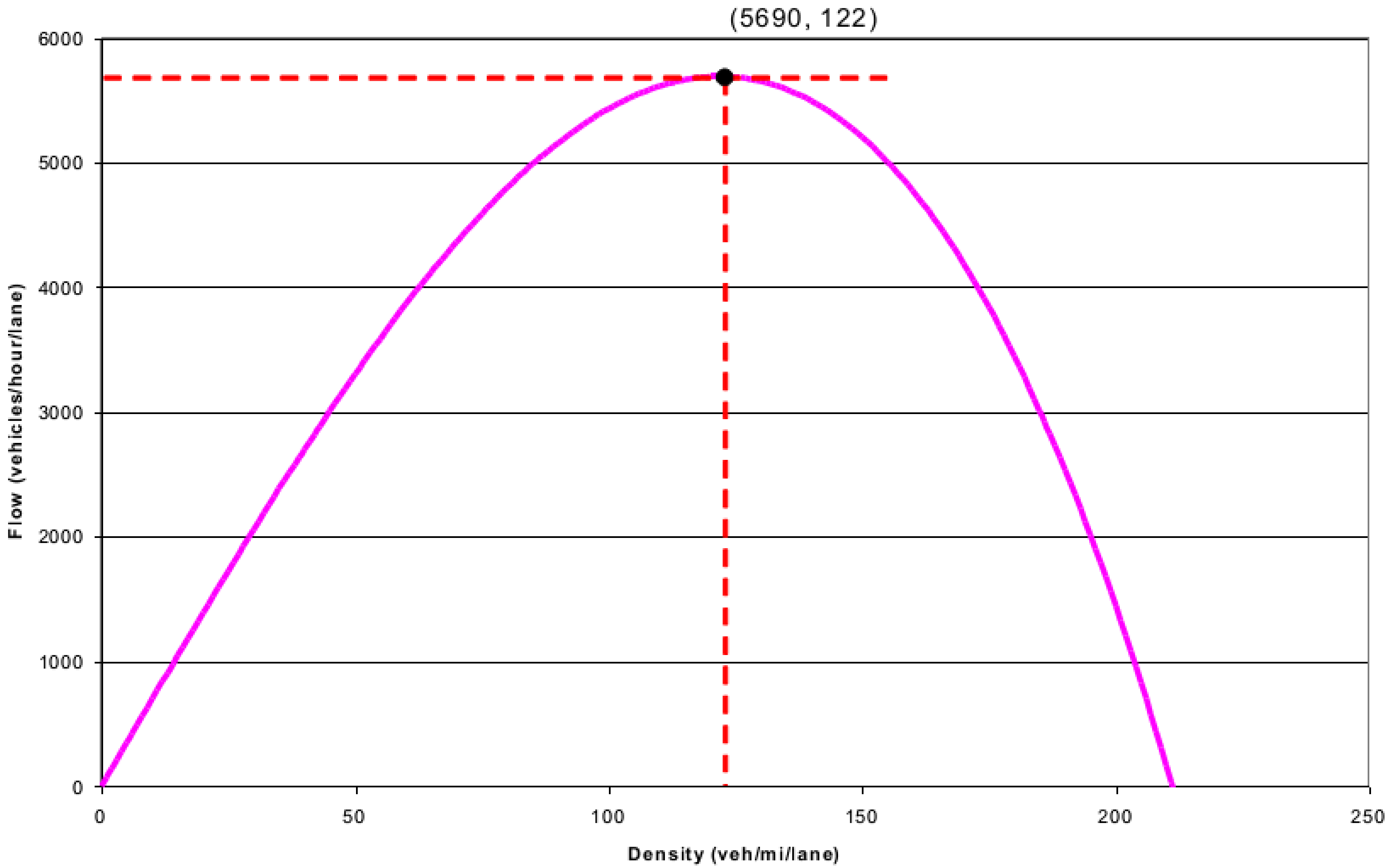


Example 1: Slow-Moving Traffic



At time *t = 0 a truck, traveling at 12 mph, enters the freeway at* Point A and travels along the freeway until exiting at point B, 2 miles

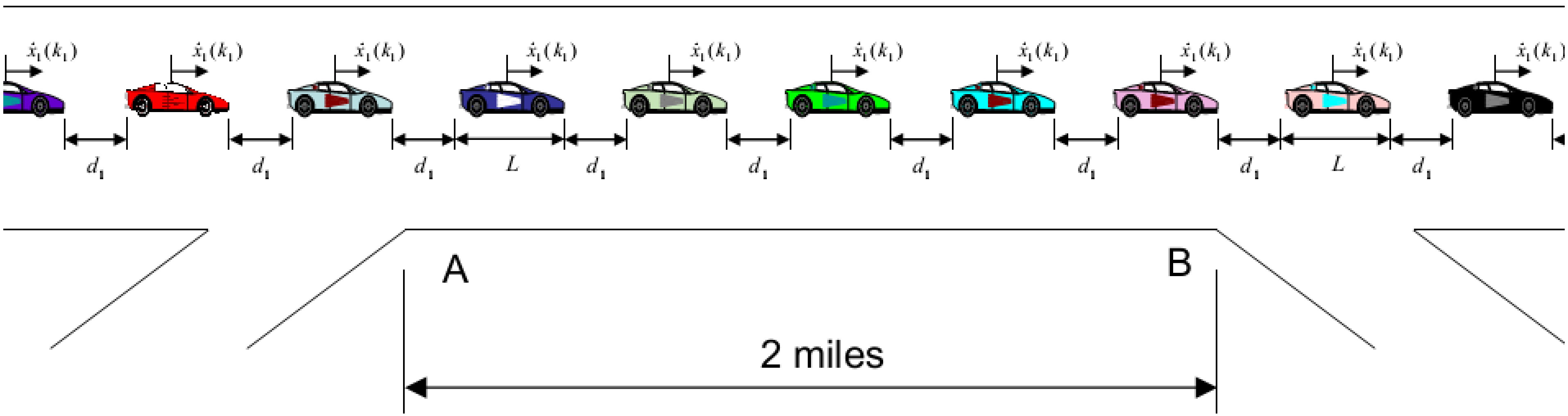
Flow Density Relationship



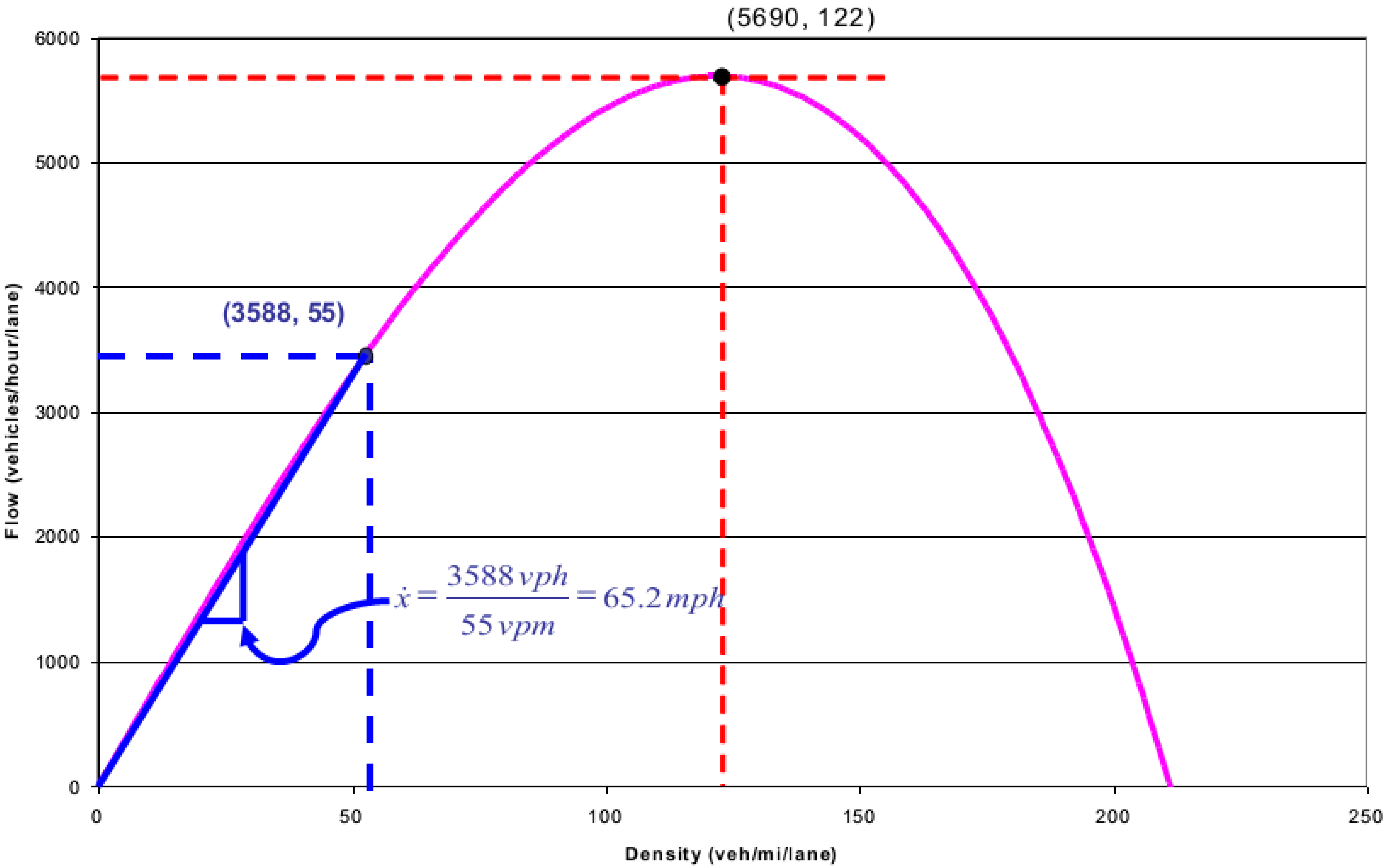
$$q_1 = 3588 \text{ veh/hr}$$

$$k_1 = 55 \text{ veh/mi}$$

$$\dot{x}_1 = 65.2 \text{ mph}$$



Flow Density Relationship

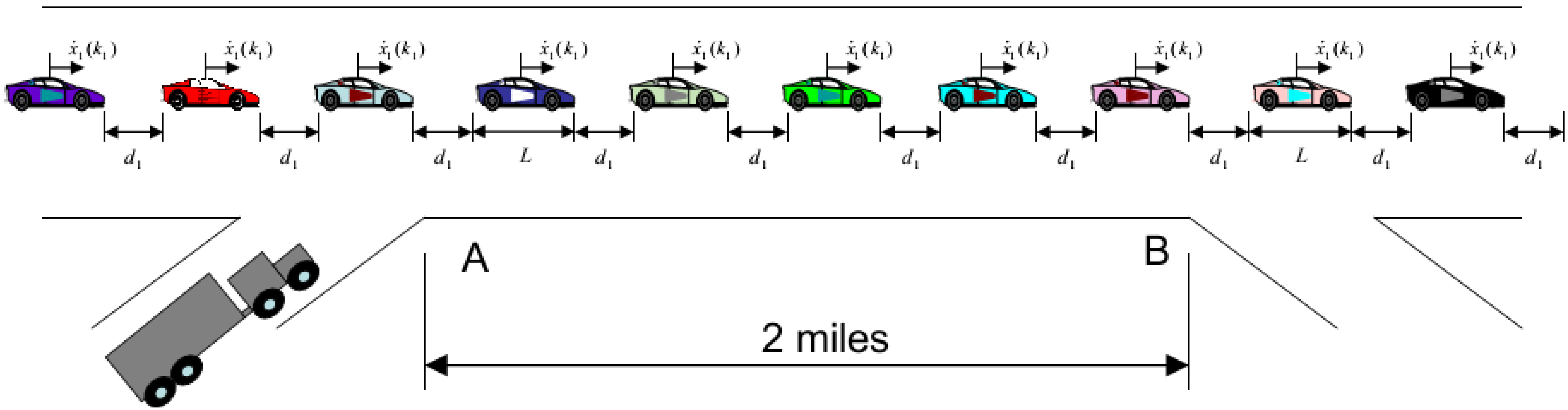


At t=0

$q_1 = 3588$ veh/hr

$k_1 = 55$ veh/mi

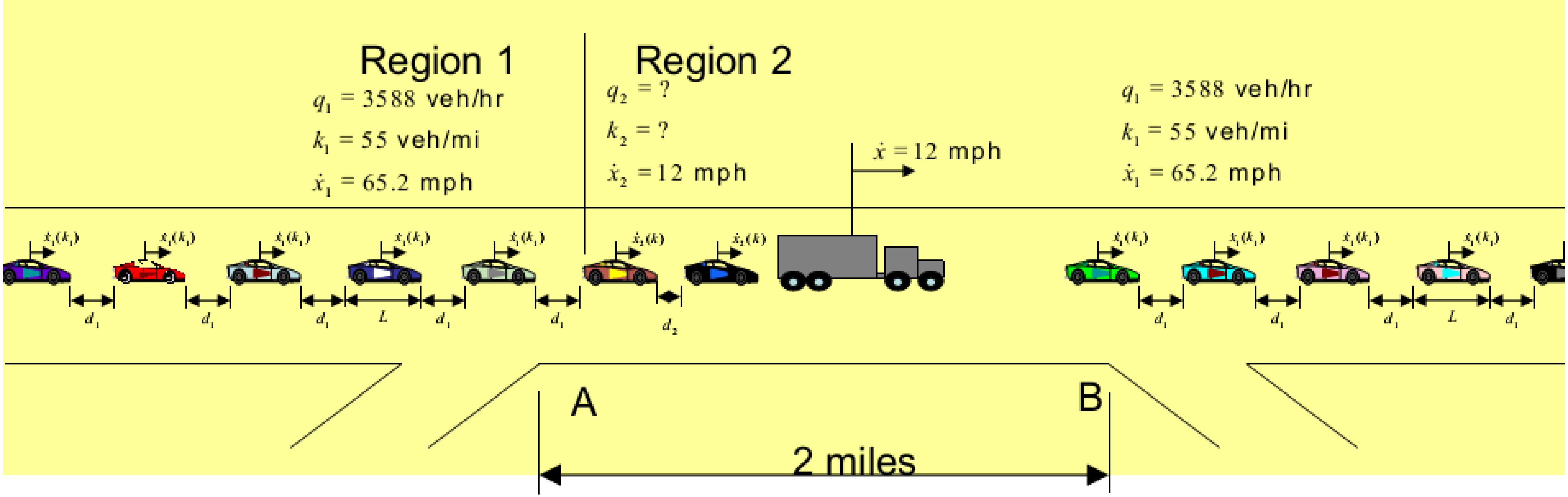
$\dot{x}_1 = 65.2$ mph



At time *t = 0 a truck, traveling at 12 mph, enters the freeway at* Point A and travels along the freeway until exiting at point B, 2 miles

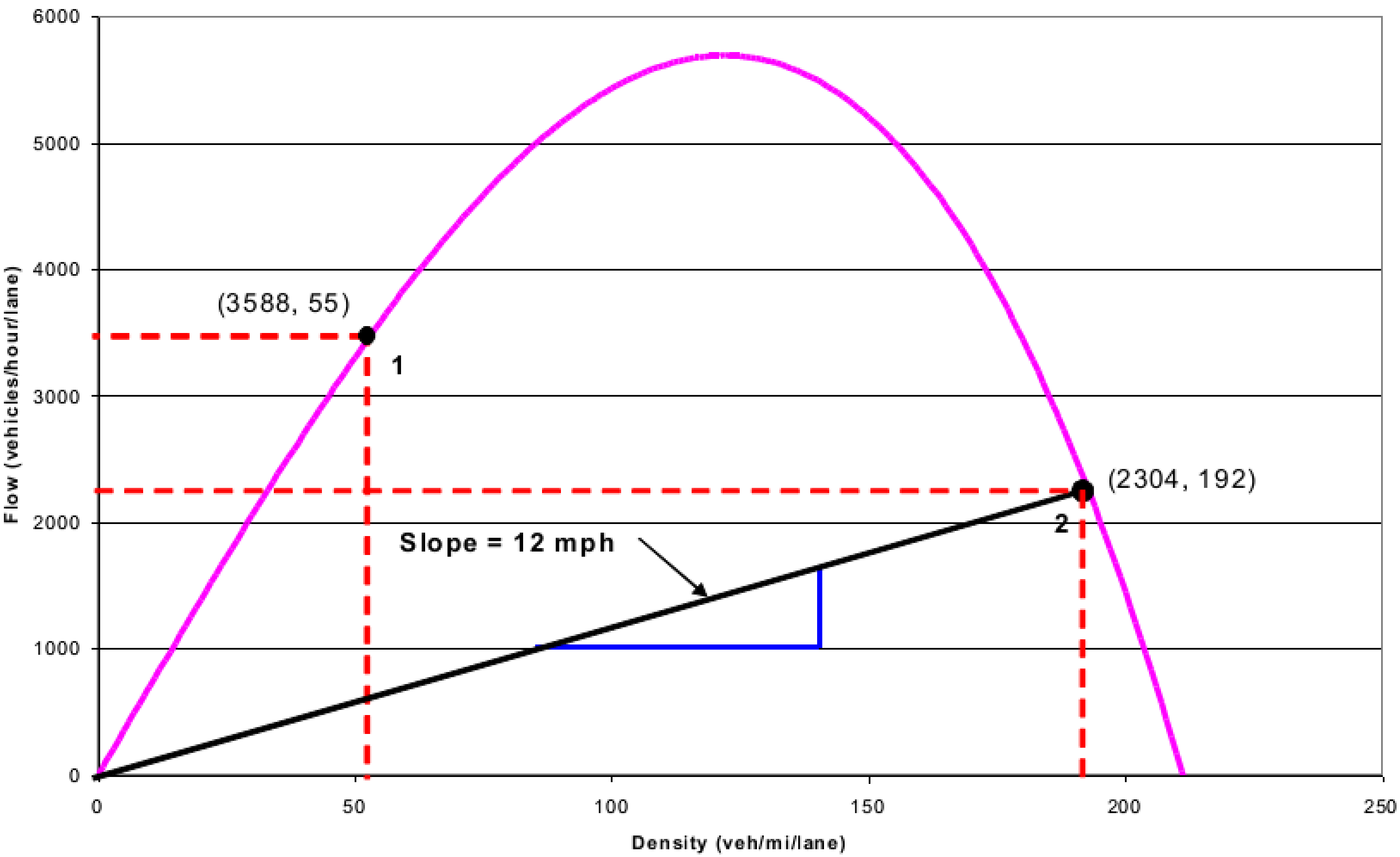
When the truck pulls off the road, how long is the queue behind the truck?

After the truck pulls off the road, how long it takes for the queue to disappear?

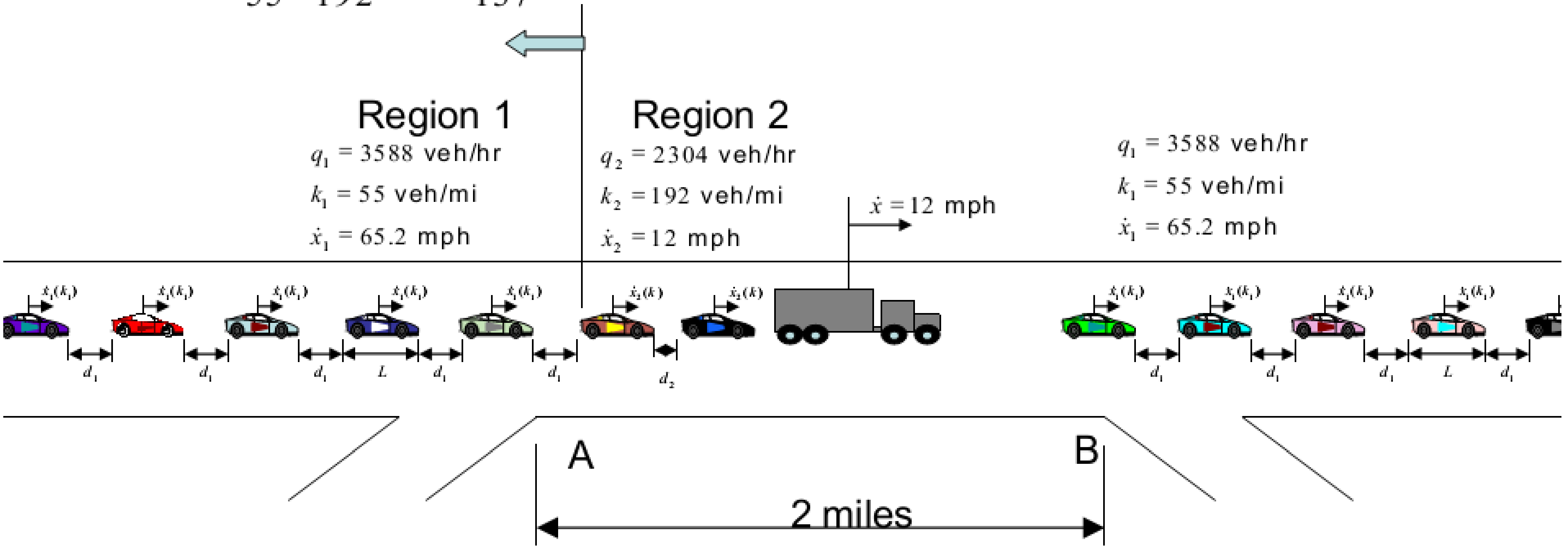


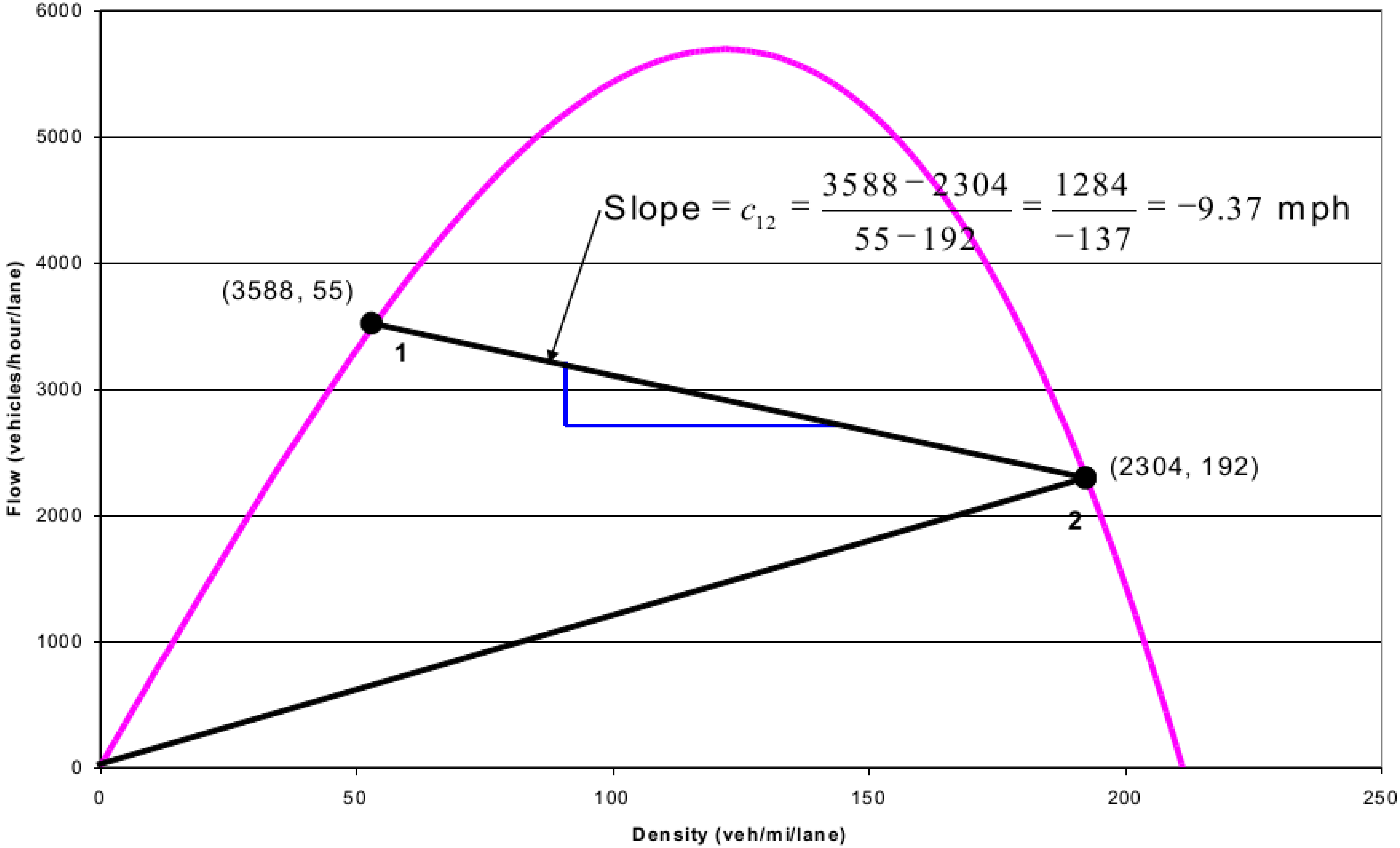
Region 1
q_1 = 3588 veh/hr
k_1 = 55 veh/mi
$\dot{x}_1$ = 65.2 mph
Region 2
q_2 = ?
k_2 = ?
$\dot{x}_2$ = 12 mph
$\dot{x}$ = 12 mph
q_1 = 3588 veh/hr
k_1 = 55 veh/mi
$\dot{x}_1$ = 65.2 mph
$\dot{x}_1(k_1)$
$\dot{x}_2(k)$
d_1
d_2
L
A
B
2 miles

Determining Conditions Behind Truck

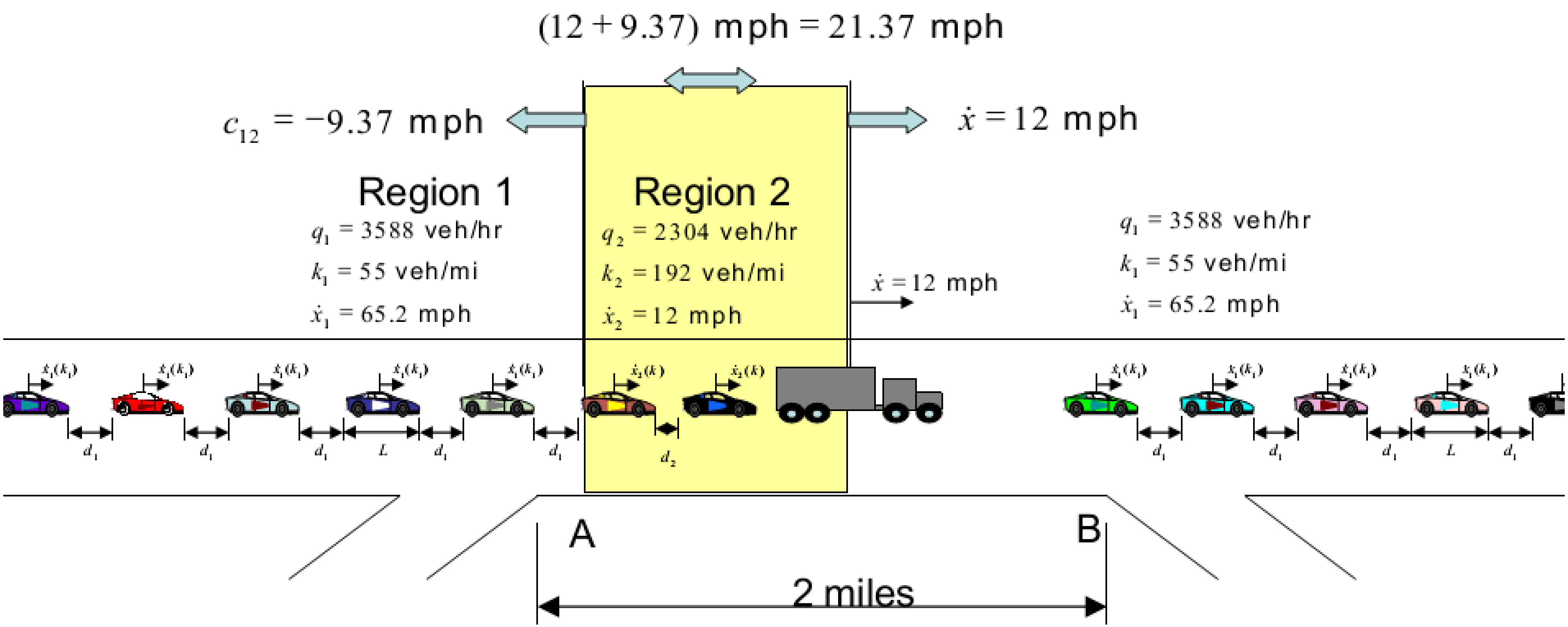


$$c_{12} = \frac{3588 - 2304}{55 - 192} = \frac{1284}{-137} = -9.37 \text{ mph}$$





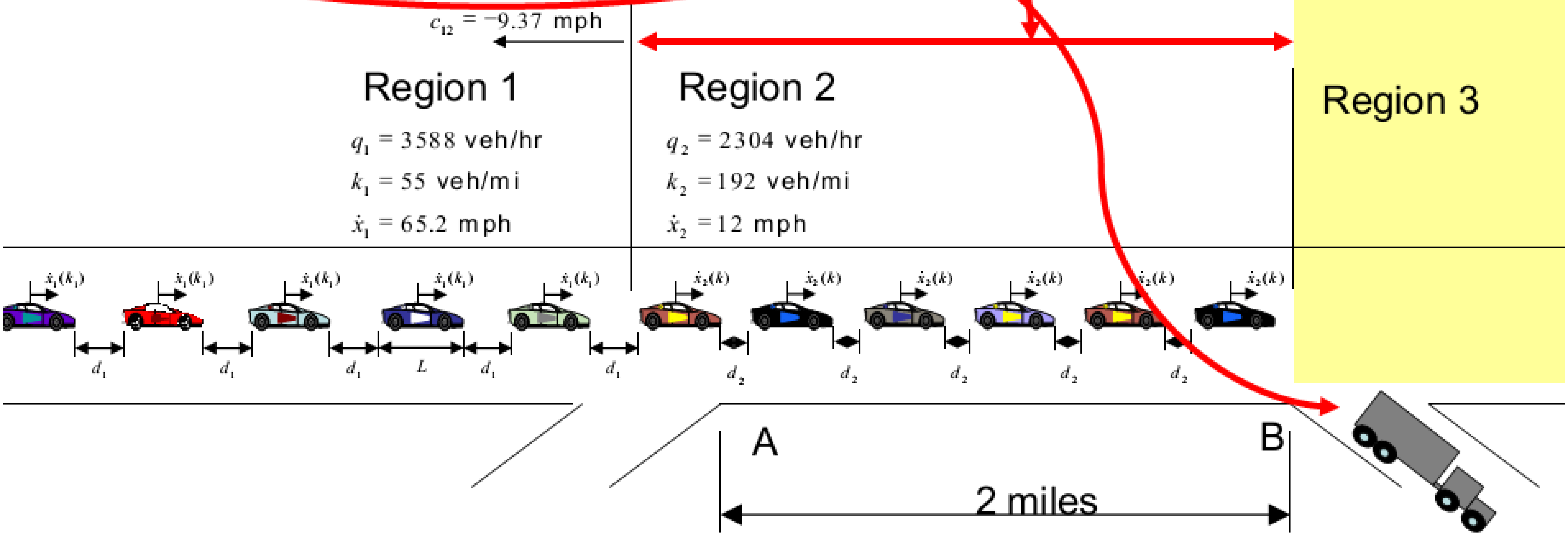
6000
5000
4000
3000
2000
1000
0
0
50
100
150
200
250
Flow (vehicles/hour/lane)
Density (veh/mi/lane)
Slope = $c_{12} = \frac{3588 - 2304}{55 - 192} = \frac{1284}{-137} = -9.37$ mph
(3588, 55)
1
(2304, 192)
2

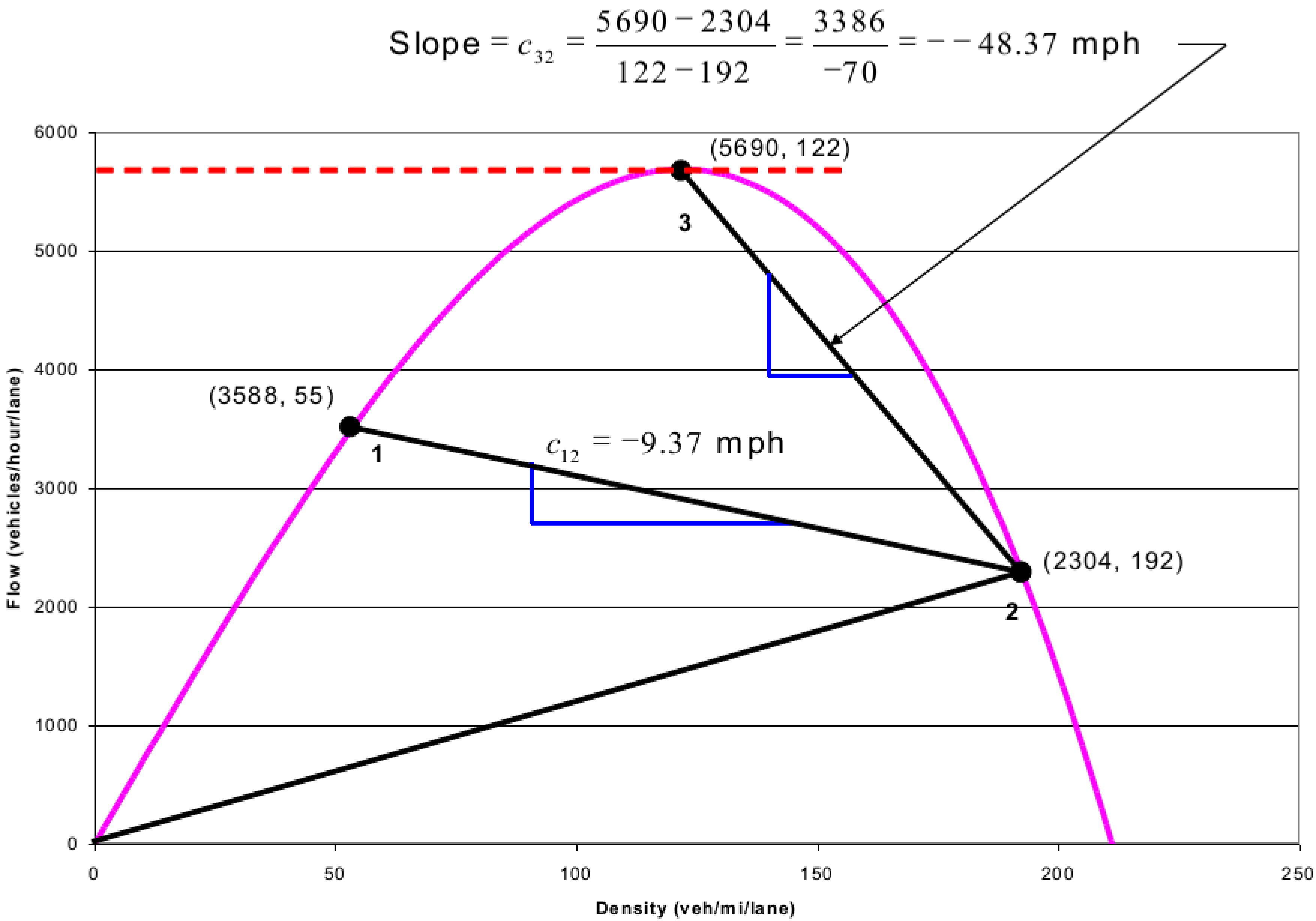


$(12+9.37)$ mph $= 21.37$ mph
$c_{12} = -9.37$ mph
$\dot{x} = 12$ mph
Region 1
$q_1 = 3588$ veh/hr
$k_1 = 55$ veh/mi
$\dot{x}_1 = 65.2$ mph
Region 2
$q_2 = 2304$ veh/hr
$k_2 = 192$ veh/mi
$\dot{x}_2 = 12$ mph
$\dot{x} = 12$ mph
$q_1 = 3588$ veh/hr
$k_1 = 55$ veh/mi
$\dot{x}_1 = 65.2$ mph
A
B
2 miles

$$\text{QueueLength} = (21.37\text{mph}) \cdot (0.167\text{hr}) = 3.57\text{mi}$$

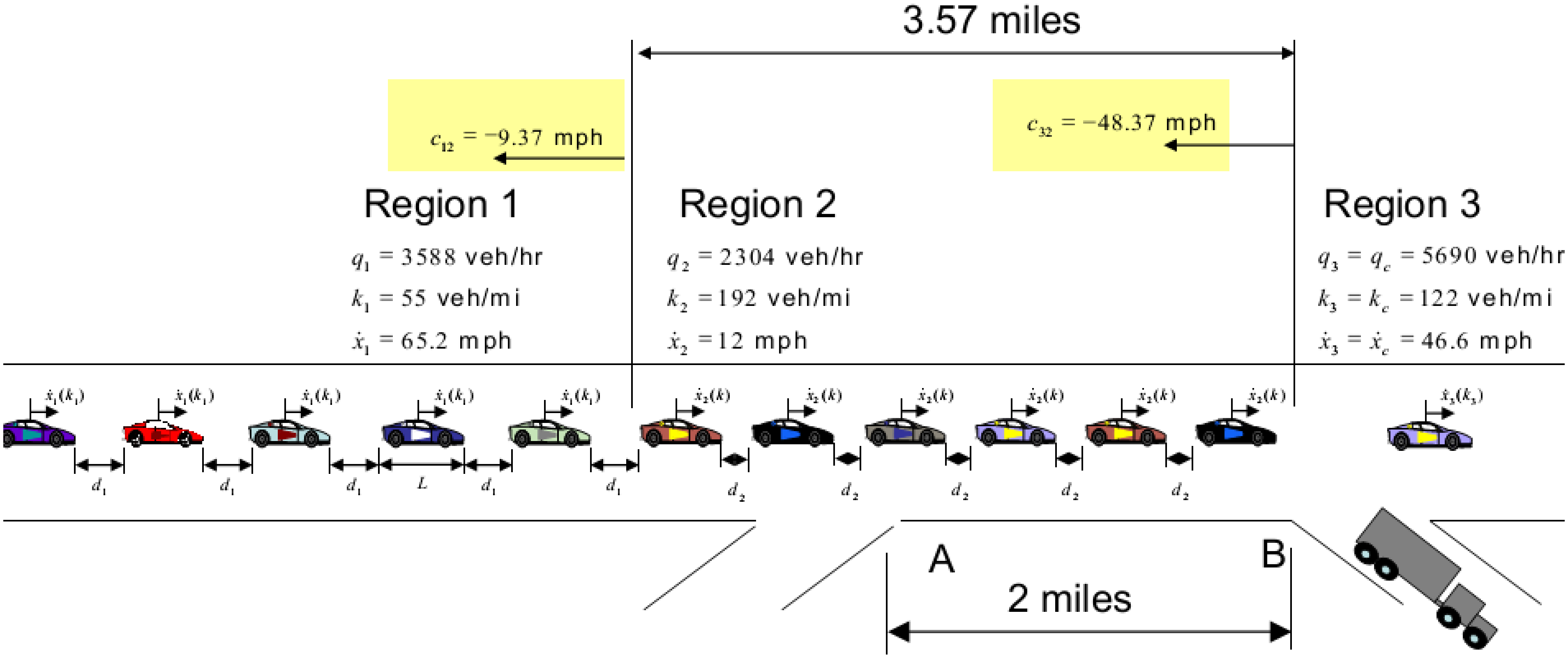
$$\text{At time } t = \frac{2 \text{ mi}}{12 \text{ mph}} = 0.167 \text{ hr} = 10 \text{min}$$





$\text{Slope} = c_{32} = \frac{5690 - 2304}{122 - 192} = \frac{3386}{-70} = -48.37$ mph
(5690, 122)
3
(3588, 55)
1
$c_{12} = -9.37$ mph
(2304, 192)
2
Flow (vehicles/hour/lane)
Density (veh/mi/lane)
0
1000
2000
3000
4000
5000
6000
0
50
100
150
200
250

At time $t = 0.167$ hr $= 10$ min



Once the truck leaves the freeway, the “unloading” shock wave catches up to and annihilates the slow-moving queue when

$$t(48.37\ \text{mph}) = 3.57\ \text{mi} + t(9.37\ \text{mph})$$

or

$$t = \frac{3.57\ \text{mi}}{48.37\ \text{mph} + 9.37\ \text{mph}} = 0.062\ \text{hr} = 3.71\ \text{min}$$

after the truck leaves.