Agenda for Today

- Introduction
- Review of Syllabus
- Traffic Flow Theory Basics
- Text Book Example
Background

- Traffic Engineering Course
- Math Background
  - Algebra
  - Calculus
  - Introductory Probability / Statistics
  - Introductory Optimization
- Field Surveys
By restricting the scope of analysis, we can take a much more detailed look at how transportation facilities function.
Few questions come to mind!

Typical operations questions:

- How long will people have to wait to turn left from a driveway?
- How much delay will people face at this signal?
- How do heavy vehicles affect traffic speeds?
- What is the “capacity” of a freeway?
Consider a long, uninterrupted, single-lane roadway:

No passing, no opposing traffic, no intersections
Traffic Flow Basics (2)
Traffic Flow Basics - Speed

Distance ($x$) vs. Time ($t$)

$\Delta x$, $\Delta t$
Traffic Flow Basics - Trajectories

Distance ($x$)

Time ($t$)
This is called a **time-space** diagram.
Consider a horizontal “slice” of the diagram
The number of trajectories crossing this line is the number of vehicles passing a fixed point on the road.

This is called the **volume** or **flow**, and has units of vehicles per time (usually veh/hr).
Traffic Flow Basics - Volume

What does a vertical slice tell us?

Distance ($x$)  
Time ($t$)
The number of trajectories crossing this line is the number of vehicles on the road at one instant in time.

This is called the **density**, and has units of vehicles per distance (usually veh/mi).
<table>
<thead>
<tr>
<th>Flow</th>
<th>Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>veh/hr</td>
<td>veh/mi</td>
</tr>
<tr>
<td>Measured over time at a fixed point</td>
<td>Measured over space at a fixed time</td>
</tr>
<tr>
<td>How many vehicles are getting somewhere?</td>
<td>How crowded is the roadway?</td>
</tr>
<tr>
<td>Can measure with a point detector</td>
<td>Can measure with an aerial photo</td>
</tr>
<tr>
<td>$q$</td>
<td>$k$</td>
</tr>
</tbody>
</table>
Traffic Flow Basics-Summary (2)

<table>
<thead>
<tr>
<th>Individual vehicle</th>
<th>Traffic stream</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed [L/T]</td>
<td>Flow [V/T]</td>
</tr>
<tr>
<td></td>
<td>Density [V/L]</td>
</tr>
</tbody>
</table>
Traffic Flow Basics-Summary (3)

Classify the quantities

<table>
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<td></td>
</tr>
<tr>
<td>Flow [V/T]</td>
<td></td>
</tr>
<tr>
<td>Density [V/L]</td>
<td></td>
</tr>
</tbody>
</table>

Brackets describe units... L = length, T = time, V = vehicles
Let’s try to fill in the rest of the table.

<table>
<thead>
<tr>
<th><strong>Individual vehicle</strong></th>
<th><strong>Traffic stream</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed [L/T]</td>
<td>Flow [V/T]</td>
</tr>
<tr>
<td>Time Headway [T]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Density [V/L]</td>
</tr>
</tbody>
</table>
The **time headway** is the time between two vehicles passing a point.
On a space-time diagram, it is the **horizontal distance** between two adjacent trajectories.
Let’s try to fill in the rest of the table.

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<thead>
<tr>
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<tbody>
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<td>Flow [V/T]</td>
</tr>
<tr>
<td>Time Headway [T]</td>
<td>Density [V/L]</td>
</tr>
<tr>
<td>Space Headway [L]</td>
<td></td>
</tr>
</tbody>
</table>
The **space headway** is the distance between two vehicles.
On a space-time diagram, it is the **vertical distance** between two adjacent trajectories.
Traffic Flow Basics-Summary

Let’s try to fill in the rest of the table.

<table>
<thead>
<tr>
<th>Individual vehicle</th>
<th>Traffic stream</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed [L/T]</td>
<td>Average Speed [L/T]</td>
</tr>
<tr>
<td>Time Headway [T]</td>
<td>Flow [V/T]</td>
</tr>
<tr>
<td>Space Headway [L]</td>
<td>Density [V/L]</td>
</tr>
</tbody>
</table>
Three Parameters of Traffic Flow

- Macroscopic:
  - Speed (V)
  - Density (K)
  - Flow (Q)

\[ Q = KV \]
Spacing

- **Spacing** is defined as the distance between successive vehicles in a traffic lane; measured from common reference
  - Front bumper or
  - Front wheels
- **Average spacing** in a traffic lane is related to density

\[ d_a = \frac{5280}{k} \]

Where,
- \( k \) = density in veh/mile/lane
- \( d_a \) = *Average spacing between vehicles* in ft
Headway

- **Headway** is defined as the time interval between successive vehicles as they pass along a lane.
- Also measured between common point of reference.

\[ h_a = \frac{3,600}{q} \]

Where,
- \( q \) = traffic volume in veh/hour/lane
- \( h_a \) = *Average headway in the lane* in sec
Example

Traffic in an interstate at 7:15 AM is observed to have spacing of 250 feet; and average headway of 3 sec. Estimate

- Volume
- Density
- Speed
Step 1: Calculate flow
\[ q = \frac{3,600}{h_a} = \frac{3,600}{3} = 1,200 \text{ veh/hour/lane} \]

Step 2: Calculate density
\[ k = \frac{5,280}{d_a} = \frac{5,200}{250} = 20.8 \text{ veh/mile/lane} \]

Step 3: Calculate Speed
\[ q = uk \Rightarrow u = \frac{q}{k} = \frac{1200}{20.8} = 57.69 \text{ miles/hour} \]
Speed

- **Time mean speed (spot speed)**
  - Arithmetic mean of all instantaneous vehicle speeds at a given “spot” on a roadway section

- **Space mean speed (u)**
  - The mean travel speed of vehicles traversing a roadway segment of a known distance (d)
  - More useful for traffic applications
Time Mean Speed

- Time mean speed is the average of all vehicles passing over a point over a duration of time.
- It is simple average of spot speed
- Mean speed is given by
  \[ v_t = \frac{1}{n} \sum_{i=1}^{n} v_i \]
- Often speeds are given as frequency tables. Then TMS is
  \[ v_t = \frac{\sum_{i=1}^{n} q_i v_i}{\sum_{i=1}^{n} q_i} \]
- Where \( q_i \) is the number of vehicles having speed \( v_i \), and \( n \) is number of such speed categories
Space Mean Speed

- Space mean speed also averages spot speeds but spatial weights is given instead of temporal.
- Consider a unit length of road and let $v_i$ is the spot speed, and let $t_i$ is the time vehicles takes to complete unit distance and is given by $1 / v_i$.
- If we have $n$ such vehicles, then average travel time is given by
  \[ t_s = \frac{\sum t_i}{n} = \frac{1}{n} \sum \frac{1}{v_i} \]
  
  or
  \[ \frac{1}{t_s} = v_s = \frac{n}{\sum_{i=1}^{n} \frac{1}{v_i}} \]
  
  or
  \[ v_s = \frac{\sum_{i=1}^{n} q_i}{\sum_{i=1}^{n} q_i / v_i} \]
Example

• Question: If the spot speeds are 50, 40, 60, 54 and 45, then find the time mean speed and space mean speed.

• Sol.
  - Time mean speed is the average of spot speed. Therefore, \( v_t = \frac{1}{n} \sum_{i=1}^{n} v_i \) 
  - Space mean speed is the harmonic mean of spot speed. Therefore, \( t_s = \frac{\sum t_i}{n} = \frac{1}{n} \sum \frac{1}{v_i} \)
Proof:

\[ v_t = v_s + \frac{\sigma^2}{v_s} \]