CIVL - 7904/8904

TRAFFIC FLOW THEORY

LECTURE -10

Car Following Theory

Microscopic Density Characteristics

- Minimum space must be available in front of every vehicle so that
  - The driver can control his/her vehicle without colliding with the vehicle ahead
  - Avoid collision with fixed objects
- As spacing decreases drivers are required to give more attention to the driving task
- May have to reduce operating speed
- Lower speeds reduces level of service

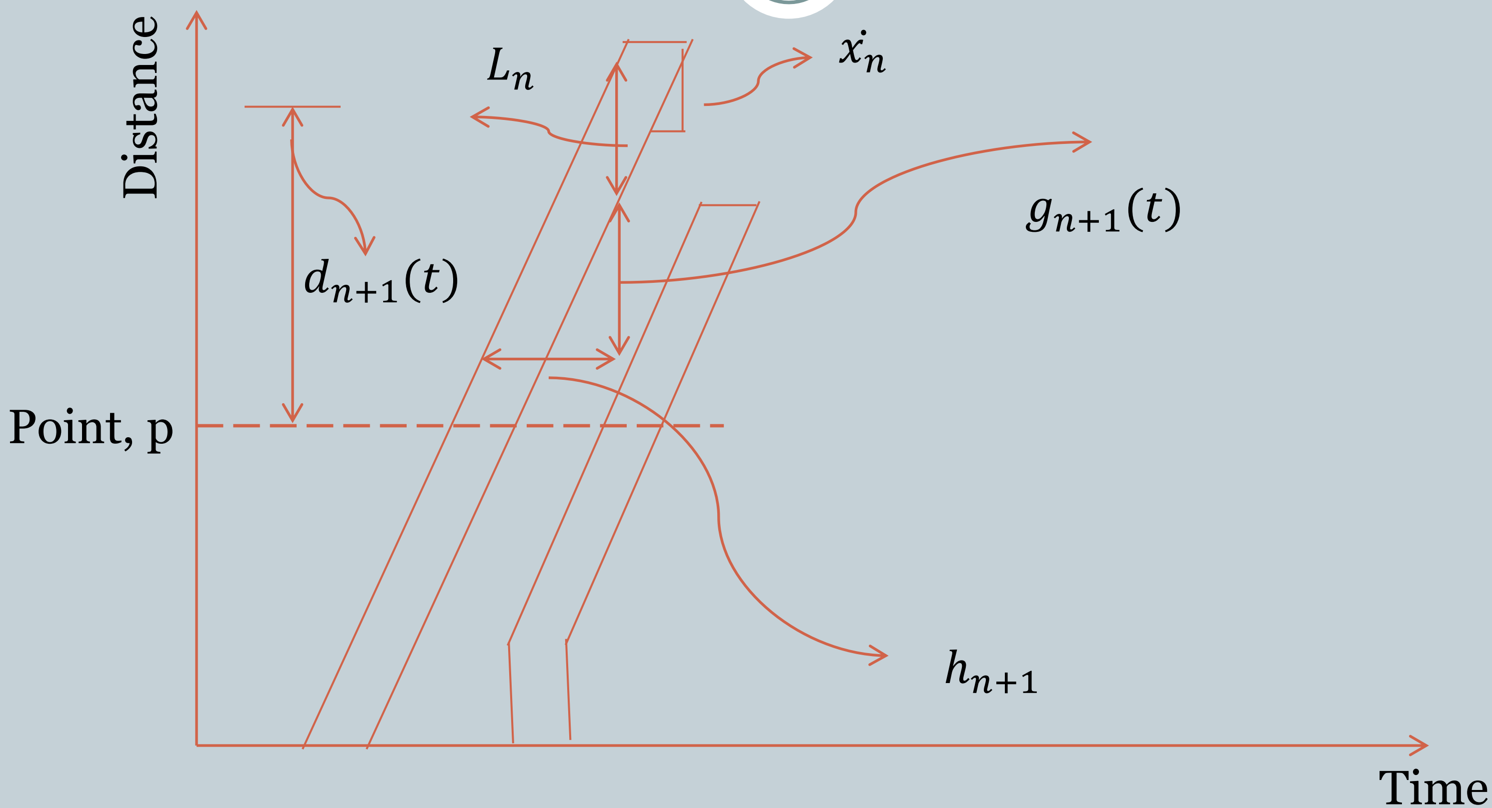
Distance Headway (1)

- The longitudinal space occupied by individual vehicles in the traffic stream consists of space occupied by the physical vehicles and the gaps between vehicles
- The two microscopic measures considered are:
  - Distance headway
  - Distance gap

Distance Headway (2)

- Distance headway is defined as the distance from a selected point on the lead vehicle to the same point on the following vehicle
- Usually the front edge or bumpers are selected since they are often detected in the automatic detection system.
- Hence distance headway includes the length of the lead vehicle and the gap length between the lead and following vehicle

Distance Headway (3)



Distance gap is defined as the gap length between the rear edge of the lead vehicle and the front edge of the following vehicle

Distance Headway (4)

$$d_{n+1}(t) = L_n + g_{n+1}(t)$$

Where,

$d_{n+1}(t)$-> Distance headway of vehicle n+1 at time t (feet)

$L_n \rightarrow$ Physical length of vehicle n (feet)

$g_{n+1}(t)$-> Gap length of between vehicle n and n+1 at time t (feet)

Distance Headway (4)

- Often distance gap is used as the primary microscopic characteristics of density because of its more direct relationship to time headway and density
- Time headway

$$d_{n+1}(t) = h_{n+1}\dot{x_n}$$

Where,

$d_{n+1}(t)$-> Distance headway of vehicle n+1 at time t (feet)

h_{n+1} → Time headway of vehicle n+1 at point p (feet)

$\dot{x_n}$-> Speed of vehicle n during the time period of h_{n+1} (feet/sec)

- If the average distance headway is known then density can be determined as following

$$k = \frac{5280}{\bar{d}}$$

k-> Density (veh/mile/lane)
$\bar{d}$-> Average distance headway (ft/veh)
5280-> Constat, # of feet / mile

- The average distance headway can be determined as following

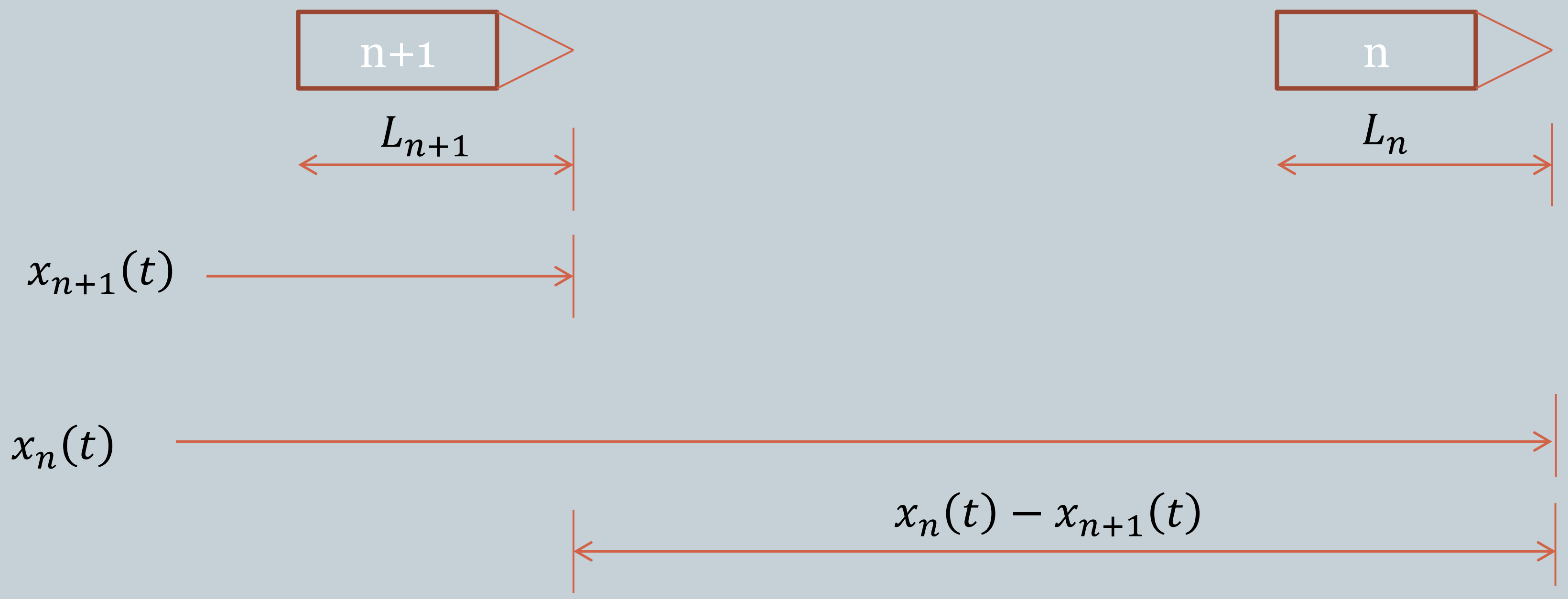
$$\bar{d} = \frac{\sum_{n=1}^{N} d_n}{N}$$

d_n-> Individual distance headway (ft/veh)
N-> Number of observed distance headways

Car Following Theories

- Theories describing how one vehicle follows another were developed primarily in 1950s and 1960s.
- Theories of significant importance
  - Michigan State University
  - Pipes
  - Forbes
  - General Motors

Notations and Definitions



Notations

$n \rightarrow Lead\ vehicle$
n+1-> Following vehicle
L_n->Length of the lead vehicle (ft)
L_{n+1} → Length of the following vehicle (ft)
x_n->Position of the lead vehicle (ft)
x_{n+1}-> Position of the following vehicle (ft)
$\dot{x_n}$-> Speed of the lead vehicle (ft/sec)
$\dot{x_{n+1}}$-> Speed of the following vehicle (ft/sec)
$\ddot{x_{n+1}}$-> Acceleration/Decceleration rate of the following vehicle (ft/sec^2)
T->At time t
$t + \Delta t$-> Δt time after t

Few points on Car Following Notations (1)

- The acceleration / deceleration rate of the following vehicle is specified as occurring at time $t + \Delta t$
- Δt refers represents a interval of time between the time of a unique car following situation occurs and the time driver of the following vehicle decides to apply a specified acceleration/deceleration rate at t+Δt
- This interval is often referred as the reaction time

Few points on Car Following Notations (2)

- The distance between the lead vehicle and the following vehicle is denoted as
- The relative velocity between the lead vehicle and the following vehicle is denoted as
  - If the relative velocity is +ve: Lead vehicle has higher speed
  - If the relative velocity is –ve: Following vehicle has higher speed
- Acceleration rate can be +ve or –ve
  - +ve indicating that the following vehicle is accelerating
  - -ve indicates the reverse

Pipes Theory

- Pipes characterized the motion of vehicles in a traffic stream as following rules suggested in California Motor Vehicle Code

"A good rule for following another vehicle at a safe distance is to allow yourself at least the length of a car between your vehicle and the vehicle ahead for every 10mph of speed at which you are travelling"

Pipes Theory (2)

$$d_{MIN} = [x_n(t) - x_n(t)\ ]_{MIN} = L_n\left[\frac{x_{\dot{n}+1}}{(1.47)(10)}\right] + L_n$$

Assuming length of the vehicle as 20ft

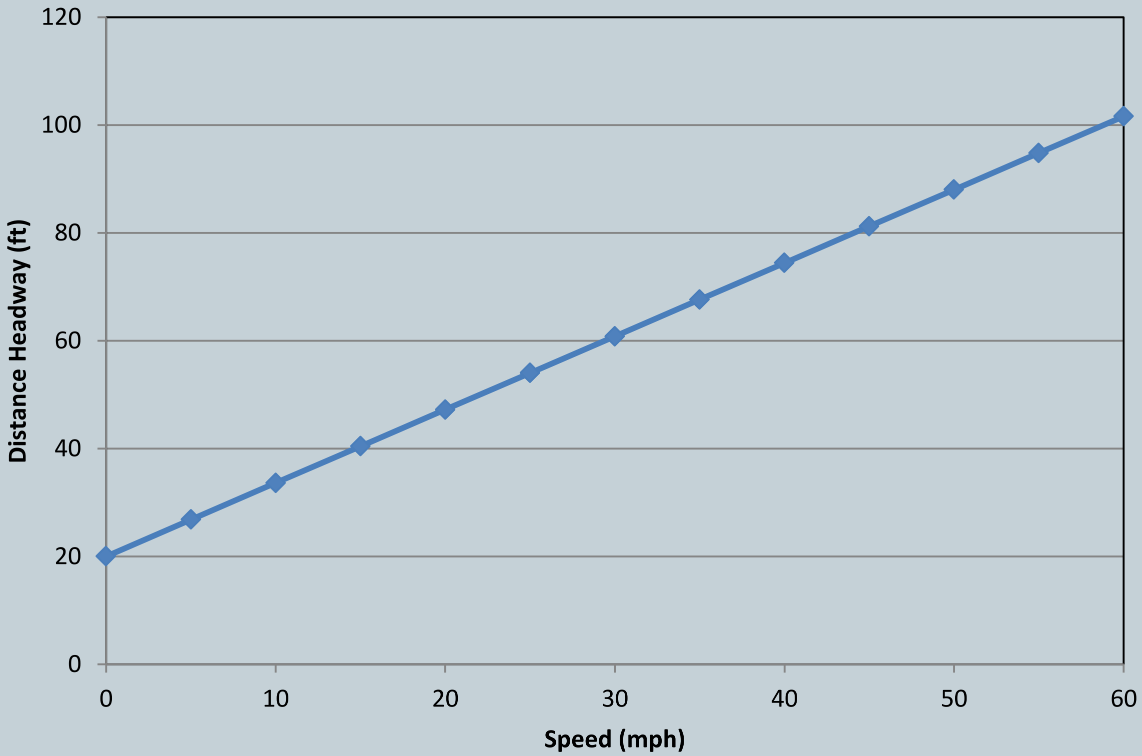
$$d_{MIN} = 1.36L_n + 20$$

Alternatively,

$$h_{MIN}{=}1.36{+}\frac{20}{x_{\dot{n}+1}}$$

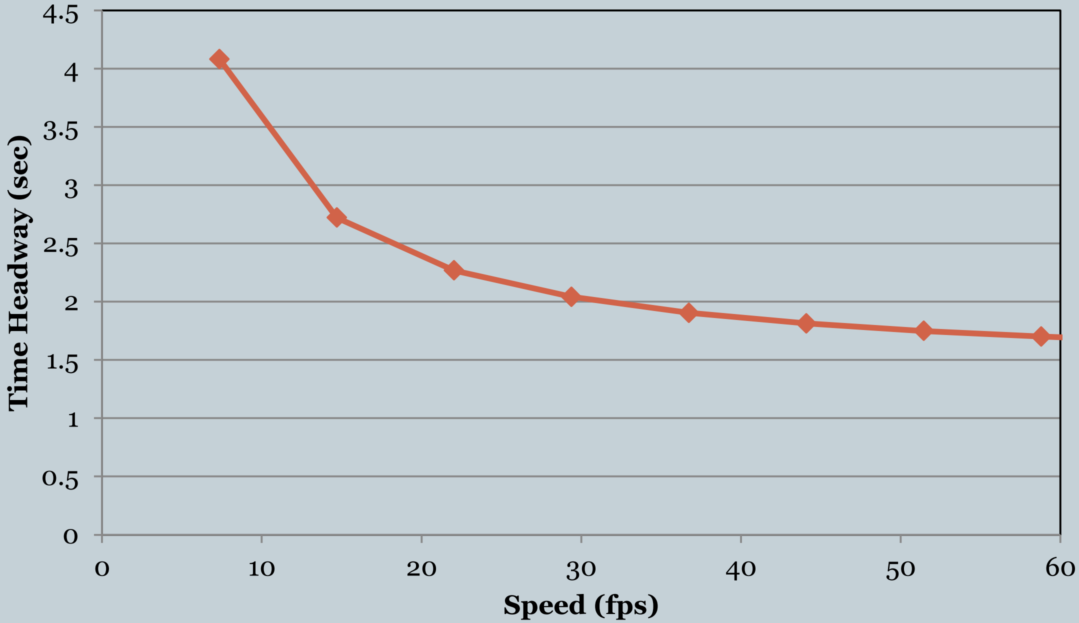
Pipes Theory (3)

- From Pipes model, minimum safe distance headway linearly increases with speed



Pipes Theory (4)

- From Pipes model, minimum safe time headway continuously decreases with speed and theoretically reaches an absolute minimum time headway of 1.36 sec at speed of infinity.



Forbes' Theory

- Forbes approached car-following behavior by considering the reaction time needed for the following vehicle to perceive the need to decelerate and apply the brakes.
  - i.e. the time gap between the rear of the lead vehicle and the front of the following vehicle should always be equal to or greater than the reaction time.
- Therefore, the minimum time headway is equal to
  - The reaction time, plus
  - The time required for the lead vehicle to traverse a distance equivalent to its length

$$h_{MIN} = \Delta t + \frac{L_n}{\dot{x}_{n+1}}$$

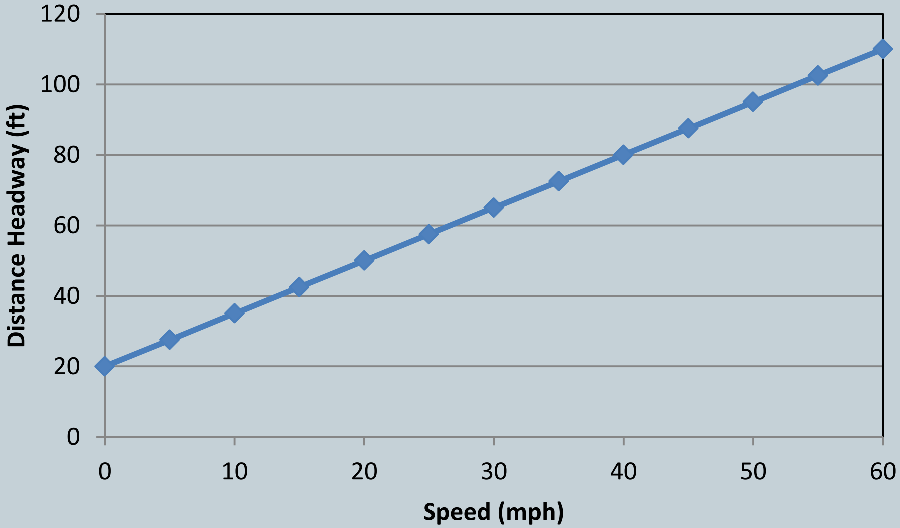
Forbes conducted many field studies of minimum time gaps and found considerably variability between drivers and sites.
Minimum time gaps varied from 1, 2, or 3 seconds.
Assuming a reaction time of 1.5 sec and a vehicle length of 20 ft, the equation can be rewritten as follows:

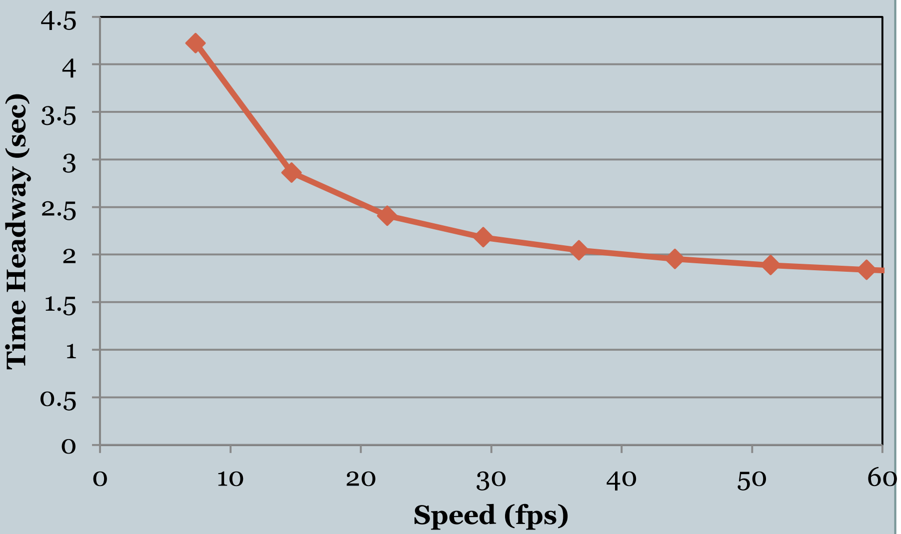
$$h_{MIN} = 1.5 + \frac{L_n}{\dot{x}_{n+1}}$$

Alternatively,

$$d_{MIN} = 1.5[\dot{x}_{n+1}] + 20$$

- Results of Forbes theory is very similar to that of Pipes.





General Motor's Theory (1)

- Car-following theory developed at GM were very much extensive and are of particular importance
- The research bridged the gap between microscopic and macroscopic theories of traffic flow
- Large number of researchers were involved both inside and outside GM

General Motor's Theory (2)

- The research team developed five generations of car-following models, all of which took form
- Response = function (sensitivity, stimuli)
- The response was always represented by acceleration (or deceleration) of the following vehicle
- Stimuli was represented by the relative velocity of the lead and following vehicle.

General Motor's Theory –First Model

- The first model assumed that the sensitivity term was a constant and the model formulation is

$$\ddot{x}_{n+1}(t + \Delta t) = \alpha\left[\dot{x}_n(t) - \dot{x}_{n+1}(t)\right]$$

- The stimuli term can be positive, negative, and zero which could cause the response to be
  - Acceleration
  - Deceleration
  - Constant speed

General Motor's Theory –First Model

Measured Value	Reaction Time (Δt) (sec)	Sensitivity (α) (sec^{-1})
Minimum	1.0	0.17
Average	1.55	0.37
Maximum	2.2	0.74

General Motor’s Theory –Second Model

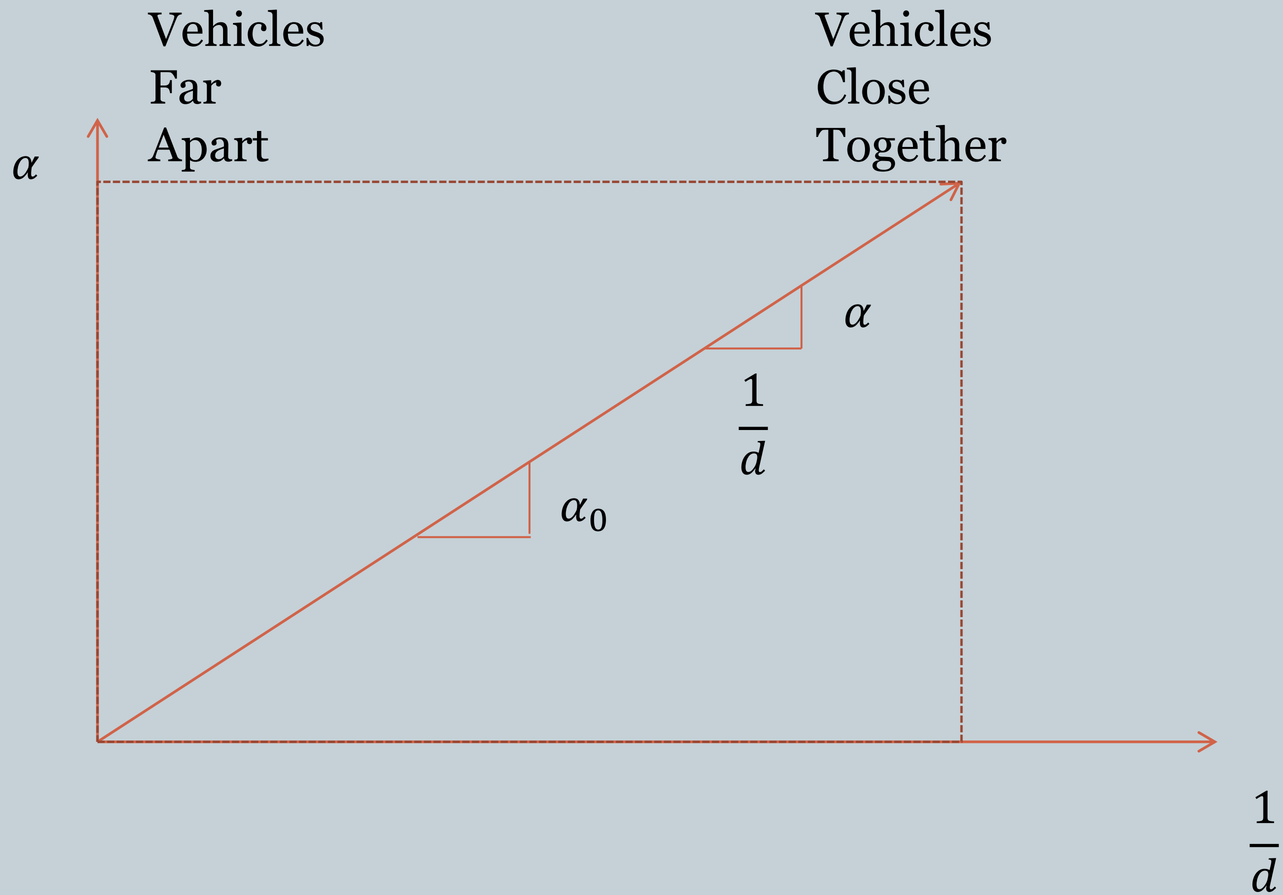
- Significant ranges in the sensitivity value alerted the investigators that the spacing between vehicles should be introduced into the sensitivity term
- This led to the development of the second model, which proposed sensitivity parameter should have two states
- A high sensitivity value should be employed
  - If two vehicles are close to each other
- A low sensitivity value should be employed
  - If two vehicles are far apart

General Motor's Theory –Second Model

$$\ddot{x}_{n+1}(t+\Delta t) = \alpha_1 or \alpha_2 \left[\dot{x}_n(t) - \dot{x}_{n+1}(t)\right]$$

- Very quickly the investigators saw the difficulty in selecting $\alpha_1 and\ \alpha_2$
- This led to further field experiments to determine means of incorporating distance headway into the sensitivity term
- The experimental results provided significant breakthrough

General Motor's Theory –Second Model



General Motor's Theory –Second Model

$$\alpha_0 = \frac{\alpha}{1/d} = \alpha d$$

$$\alpha = \frac{\alpha_0}{d} = \frac{\alpha_0}{x_n(t) - x_{n+1}(t)}$$

$$\ddot{x}_{n+1}(t + \Delta t) = \frac{\alpha_0}{x_n(t) - x_{n+1}(t)} \left[\dot{x}_n(t) - \dot{x}_{n+1}(t)\right]$$

General Motor's Theory –Third Model

- The sensitivity parameter is a function of ($\alpha = \frac{\alpha_0}{d}$) α_0 and d
- As the vehicles come closer and closer together the sensitivity term becomes larger and larger.

Measured Value	**Reaction Time (Δt) (sec)**	**Sensitivity (α_o) (ft/sec)**
GM Test Track	1.5	40.3
Holland Tunnel	1.4	26.8
Lincoln Tunnel	1.2	29.8

General Motor's Theory –Fourth Model

- The fourth model was a further development toward improving the sensitivity term by introducing the speed of the vehicle.
- The concept was that

"as the speed of the traffic stream is increased, the driver of the following vehicle would be more sensitive to the relative velocity between lead and the following vehicle"

General Motor's Theory –Fourth Model

$$\ddot{x}_{n+1}(t+\Delta t) = \frac{\alpha[\dot{x}_{n+1}(t+\Delta t)]}{x_n(t) - x_{n+1}(t)}\left[\dot{x}_n(t) - \dot{x}_{n+1}(t)\right]$$

General Motor's Theory –Fifth Model

- The fifth and final model was continued effort to improve and generalize the sensitivity term

$$\ddot{x}_{n+1}(t+\Delta t) = \frac{\alpha_{l,m}[\dot{x}_{n+1}(t+\Delta t)]^m}{[x_n(t) - x_{n+1}(t)]^l}\left[\dot{x}_n(t) - \dot{x}_{n+1}(t)\right]$$

- First and second model were m=0, l=0
- Third and fourth model were, m=0, l=1 and m=1 ; l=1 respectively

Car-Following Theory Application

- The example is based on General Motors research
- The example is based on two vehicles
  - Lead vehicle
  - Following vehicle
- Initial distance headway between two vehicles is 25ft
- The lead vehicle is automatically controlled to accelerate @3.3 ft/sec2 until a speed of 44ft/sec (30mph). Constant speed is maintained for 10 sec.

- Following a period of constant speed the lead vehicle decelerates at a constant rate of $4.6 ft/sec^2$ until it is stopped.
- The driver of the following vehicle is instructed to follow the lead vehicle with a safe minimum distance headway.

- The objective of this example is to track the trajectories of the vehicles over space and time through the constant driving cycle.
  - Acceleration
  - Constant speed
  - Deceleration

- The first task is to track the lead vehicle since its trajectory is pre-specified.
- The second task is to track the following vehicle as the path of the following vehicle is a function of the lead vehicle
- The trajectory of the lead vehicle

- Speed for each second of the lead vehicle is

$$\dot{x}_1(t+T) = \dot{x}_1(t) + \left[\frac{\ddot{x}_1(t)+\ddot{x}_1(t+T)}{2}\right]\mathrm{T}$$

- If T is selected as 1 second , and acceleration/deceleration rate is constant for the lead vehicle then,

$$\dot{x}_1(t+1) = \dot{x}_1(t) + \ddot{x}_1$$

- The distance position can be calculated as

$$x_1(t+T) = x_1(t) + \dot{x}_1(t)T + \left[\frac{\ddot{x}_1(t) + \ddot{x}_1(t+T)}{2}\right]\frac{T^2}{2}$$

- Similarly, for T=1,

$$x_1(t+1) = x_1(t) + \dot{x}_1(t) + \left[\frac{\ddot{x}_1}{2}\right]$$