Transportation Economics and Decision Making
Pricing and Subsidy Policies

- One of the most difficult problems in transportation
  - Congestion of motor vehicles in urbanized areas

- Congestion creates
  - Loss in private and societal cost
  - Higher operating cost
  - Losses in valuable time
  - More highway crashes
  - More air pollution
  - More discomfort, inconvenience
  - Noise pollution
What are the choices

Naturally, the society’s choices are to
- Do nothing
- Reduce inconvenience by increasing capacity
- Restrict road use to congested areas in the network
- Long standing experience suggests that the first two choices further add congestion and leading to greater inefficiency

The third choice offers the possibility to solve the problem through pricing.
How pricing is considered

- Three broad category of choices
  - Taxes on suburban or dispersed living’
  - Subsidies to public transportation
  - Increased cost of driving including road and parking pricing
- One of the important methods of reducing congestion is through taxing the motorist (congestion pricing)
What is congestion pricing?

- A market-based traffic management strategy
- Charges drivers for the use of roads
- A method of both managing traffic congestion and generating revenue
Charges for use of congested areas during times of peak use provides an incentive for people who do not need to be on the road to postpone trips to non-peak hours or shift modes.

These trips would be more efficient during off-peak hours.
Types of Congestion Pricing

- Cordon Pricing

London
Congestion Pricing in the U.S.?

- FHWA funds available under SAFETEA-LU for implementing congestion pricing
- Urban Partnership Agreements – U.S. Department of Transportation
- Public-Private Partnership (PPP)
Benefits

- Reduction of peak-period and total roadway congestion
- Better mass transit
- Reduction of greenhouse gas emissions and energy consumption
- Increased traffic safety?
How it works

How it works:

- Better Transit
- Fewer Cars
- More Space for People

- Costs Money
- Paid thru Road Pricing
- Fewer Delays for Essential Traffic

Boost City’s Economy and Livability

(Transportation Alternatives)
A Transport Network Model

- \( v \) is the speed in km/h on the network.
- \( q \) is number of vehicles = the traffic flow.
- \( v(q) \) is the speed-flow relationship, \( v' < 0 \).
- \( c \) is the travel cost per km of a representative vehicle: \( c(q) = a + \frac{b}{v(q)} \)
  - \( a \) is “fixed” cost of travel per vehicle
  - \( b \) is cost per vehicle (including opportunity cost of time);
- Total social cost is \( C(q) = c(q)q \)
- The (inverse) demand (the marginal social benefit, MSB) of an extra vehicle on the network is \( D(q) \) with \( D' < 0 \).
Costs

Marginal social cost of an extra vehicle on the network:

\[
MSC = \frac{dC}{dq} = c(q) + q \frac{dc}{dq} = c(q) - q \frac{b}{v^2} \frac{\partial v}{\partial q}
\]

- Private marginal cost
- Marginal external cost borne by other users.
- Reduction in speed

Average social cost of an extra vehicle on the network:

\[
ASC(q) = \frac{C}{q} = c(q) = MPC(q)
\]

Average Social Cost (ASC) = Marginal private cost (MPC)
Vehicles enter the network until the private marginal benefit is equal to the private marginal cost:

\[ D(q^a) = MPC(q^a) = ASC(q^a) \]

The socially optimal number of vehicles entering the network is determined by:

\[ D(q^e) = MSC(q^e) \]

\[ MSC(q) > ASC(q) \Rightarrow q^a > q^e \]
Congestion Pricing (2)

- $q^f$ is the free flow speed per hour and cost is lower than $G$
- (includes time and operating expenses)
- Beyond $q^f$, speed falls, therefore cost per trip increases for each additional user
- Additional vehicle increases the operating cost of all other vehicles in the stream of flow.
Congestion Pricing (3)

- Pricing should be in accord with marginal cost to give rise to a flow equal to $q_e$
- This could be achieved by charging a toll of ED
- Thus rising the average cost curve to achieve optimal traffic flow.
Congestion Pricing (4)

- The benefit from this action is the reduction
  - operating cost of all the remaining vehicles and
  - the loss of benefit from the additional trips beyond optimal \((q^a - q^e)\)
Optimal congestion charges

- First-best corrective (Pigouvian) charge = MSC-MPC at traffic flow $q^e$.

- Levied directly on the congestion externality but should vary by link, junction and time.

- “Electronic plate” that records where each vehicle is at any point in time and charge the owner accordingly.
It is costly to collect charges that vary by link, junction and time => cannot tax the congestion externality directly.

How then to design an optimal second-best road pricing system?

- **Tax fuel?** Fairly good proxy for road damage, but not for congestion.
- **Subsidise public transport?** Costly to do, but bus lanes, raising bollards, bike paths can be seen as an attempt to shift the balance.
- Use a **cordon** system to price congestion.
The Federal Highway Administration has established the following relationship between travel time and flow on a 10 mile length of a highway:

\[ t = 10 \left[ 1 + 0.15 \left( \frac{V}{2000} \right)^4 \right] \]

The demand function is given by \( d = 4000 - 100t \)

The value of travel time of users is $8 per vehicle hour. What should be the congestion toll on this section of highway.

Notations:
- \( t \rightarrow \) travel time (min)
- \( V \rightarrow \) flow (veh/hour)
- \( d \rightarrow \) demand
- Time taken by all the vehicles travelling this section

\[ tV = 10 \left[ 1 + 0.15 \left( \frac{V}{2000} \right)^4 \right] V \]
\[ = 10 \left[ V + 0.15 \left( \frac{V}{2000} \right)^5 \right] \]

Marginal time \( \frac{d(tV)}{dV} = 10 \left[ 1 + 0.75 \left( \frac{V}{2000} \right)^4 \right] \)

- Let us form a table and plot results

<table>
<thead>
<tr>
<th>Volume (veh/hr)</th>
<th>Time (min)</th>
<th>Demand D=4000-100t</th>
<th>Speed (mph)</th>
<th>Marginal Time (min)</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
</tbody>
</table>
Toll = 19.12 \(-\) 11.82 = 7.3 min = 7.3*8/60=$0.97
Length of section = 10 miles
Toll/mile = 9.7 cents
Personal time is the average cost,
- Average cost curve represents flow on highway when each trip-maker is only aware of his/her own personal time

The additional time of adding one extra vehicle to the traffic stream is referred as marginal time and is shown in marginal curve
- (additional time is taken over the 10 mile span)

The intersection of demand curve and the marginal curve shows the optimal flow (2100 veh/hr)
Solution (3)

- Optimal flow on the highway occurs when a trip is made only if benefit of a trip to the trip maker exceeds the additional time imposed on the trip maker.
- The differential cost of average and marginal cost would show what would be the pricing.
Depreciation
Depreciation Concept

- Common usage of depreciation is in the sense of decrease in value of a machine, property, etc.
- A state of physical wear and tear
- Replacement is higher than the amount of depreciation of the real estate.
- Income generating property
Use of Depreciation

- Calculating net operating profits, especially when profits are distributed to stockholders.
- Estimating bid prices as does a contractor.
- Calculating production cost as a basis of price setting as is done in regulating of public utilities.
- Deciding property valuation for tax purposes, settlement of estates, including, determining the fair value rate base in utility regulations.
- Analyzing investment securities for their investment merit.
Depreciation and Value

- Used in the sense of decrease in value.
- The lessening of value is a result of increasing age and obsolescence and decreasing usefulness of the property.
- Man made physical properties generally do decrease in value with age and use as long as the general price level remains substantially stable.
- Value of a property depends upon the economic laws of supply and demand, hence depreciation or appreciation both can occur.
Depreciation and Usefullness

- Impaired service usefulness of physical property or the state of wear and tear
- The intent is to give some indication of how the remaining service usefulness of a property compares to a similar or identical property that is new.
- In this sense, there is no direct reference to the value of the property, although it is inferred that any property highly depreciated (in the physical sense) is probably worth considerably less than it would be were it new.
Methods of Allocating Depreciation Expense

- Straight Line Method
- Declining Balance Method
- Double-Declining Balance Method
- Sum of Digits Method
- Sinking Fund Method
- Present Worth Method
## Notations

- $B =$ Depreciation base including terminal value
- $B_d =$ Depreciable base $= B - T$
- $B_x =$ Unallocated portion of Dep. Base yet unallocated at age $X$
- $T =$ Terminal value
- $B - B_x =$ Total depreciation at age $X$
- $D =$ Annual depreciation allocation
Notations (2)

- $D_x$ = Accumulated Dep. Allocation at age $X$
- $X$ = Age of the property
- $n$ = Probable service life
- $f$ = Depreciation rate per year
The annual depreciation allocation \((D)\) may be expressed as

\[
D = \frac{B - T}{n} = \frac{B_d}{n}
\]

where \(B_d = \text{Depreciable Base}\)

\(B = \text{Depreciation base including Terminal Value}\)

The total allocation to age \(x\) \((D_x)\) would be

\[
D_x = B_d \left(\frac{x}{n}\right)
\]

where \(n\) is probable service life

The unallocated base \((B_x)\) at age \(x\) may be given as

\[
B_x = B - D_x = B - B_d \left(\frac{x}{n}\right)
\]
Straight Line Method (2)

\[ B_x = (B_d + T) - B_d \left( \frac{x}{n} \right) \]

\[ B_x = B_d \left( 1 - \frac{x}{n} \right) + T \]

\[ B_x = B_d \left( \frac{n - x}{n} \right) + T \]

Where \( B_x \) = Unallocated portion of Depreciation Base

\( B_d \) = Depreciable Base

\( T \) = Terminal Value

\( x \) = Age of property
Straight Line Method—Example

- **Original Amount**: 50,000
- **Salvage Value**: 10,000
- **Time Period**: 5

<table>
<thead>
<tr>
<th>Year</th>
<th>D</th>
<th>Bx</th>
<th>B-Bx</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>50000</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>8000</td>
<td>42000</td>
<td>8000</td>
</tr>
<tr>
<td>2</td>
<td>8000</td>
<td>34000</td>
<td>16000</td>
</tr>
<tr>
<td>3</td>
<td>8000</td>
<td>26000</td>
<td>24000</td>
</tr>
<tr>
<td>4</td>
<td>8000</td>
<td>18000</td>
<td>32000</td>
</tr>
<tr>
<td>5</td>
<td>8000</td>
<td>10000</td>
<td>40000</td>
</tr>
</tbody>
</table>

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The table above shows the depreciation calculation using the straight line method. The depreciation for each year is calculated by subtracting the salvage value from the original amount and dividing by the time period. The graph illustrates the unallocated portion of depreciation over time.
Straight Line Method

- Simple, easily applied method that distributes the depreciable base according to a positive system that requires the exercise of judgment only in estimating the service life and the terminal value
- Has a long record of acceptance in industry and business
Declining Balance Method

- In this method, a fixed percentage depreciation expense is used for each allocation period applied to the remaining, or unallocated, cost balance at the beginning of the period.
Declining Balance Method (2)

Since the rate of depreciation in the declining balance method is constant, the unallocated portion to the base is given by

$$B_x = B(1-f)^x$$

Where $B_x$ = Unallocated portion of Dep. Base

$B = \text{Dep. base}, \quad f = \text{Dep. rate/year}$

Since the factor $(1-f)^x$ is always less than unity and greater than zero, the unallocated base $B_x$ will not reach zero

When a terminal value is chosen at endpoint, the depreciation rate $f$ can be computed for any chosen probable life $n$: 

Declining Balance Method (3)

\[ T = B(1 - f)^n \quad \text{Where, } B = \text{Dep. base} \]
\[ T = \text{Terminal Value, } f = \text{Dep. rate/yr} \]

\[ f = 1 - \frac{n}{\sqrt[n]{B}} \]
Declining Balance Method - Example

Original Amount: 50,000
Salvage Value: 10,000
Time Period: 5

<table>
<thead>
<tr>
<th>Year</th>
<th>D</th>
<th>Bx</th>
<th>B-Bx</th>
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<tr>
<td>1</td>
<td>13,761</td>
<td>36,239</td>
<td>13,761</td>
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<tr>
<td>2</td>
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</tr>
<tr>
<td>3</td>
<td>7,229</td>
<td>19,037</td>
<td>30,963</td>
</tr>
<tr>
<td>4</td>
<td>5,239</td>
<td>13,797</td>
<td>36,203</td>
</tr>
<tr>
<td>5</td>
<td>3,797</td>
<td>10,000</td>
<td>40,000</td>
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Unallocated Portion of Depreciation

DB
Double Declining Balance Method

- Depreciation rate, \( f = \frac{2}{n} \)

<table>
<thead>
<tr>
<th>Year</th>
<th>D</th>
<th>Bx</th>
<th>B-Bx</th>
</tr>
</thead>
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<tr>
<td>0</td>
<td></td>
<td>50000</td>
<td>0</td>
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<td>1</td>
<td>20000</td>
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<td>2</td>
<td>12000</td>
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<td>7200</td>
<td>10800</td>
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<td>40000</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Original Amount: 50,000
Salvage Value: 10,000
Time Period: 5
Plotting the results

Unallocated Portion of Depreciation vs Time (Years)

- DDB
- DB
- SL
Sum of the digit method uses a definite method of calculating the depreciation rate.
It gives decreasing annual depreciation each following year.
The decreasing charges are obtained by the arithmetical concept used.
The depreciation rate is applied to the depreciable base, $B-T$.
It is a scheme, which allocates the largest annual charge to the first year of service and the smallest to the last year.
### Sum of Years Digit Method (2)

<table>
<thead>
<tr>
<th>Year</th>
<th>Factor</th>
<th>Depreciation</th>
<th>Bx</th>
<th>B-Bx</th>
</tr>
</thead>
<tbody>
<tr>
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<td>13,333.33</td>
<td>50,000</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.27</td>
<td>10,666.67</td>
<td>26,000.00</td>
<td>24,000</td>
</tr>
<tr>
<td>2</td>
<td>0.20</td>
<td>8,000.00</td>
<td>18,000.00</td>
<td>32,000</td>
</tr>
<tr>
<td>3</td>
<td>0.13</td>
<td>5,333.33</td>
<td>12,666.67</td>
<td>37,333.33</td>
</tr>
<tr>
<td>4</td>
<td>0.07</td>
<td>2,666.67</td>
<td>10,000.00</td>
<td>40,000</td>
</tr>
</tbody>
</table>

**Unallocated Portion of Depreciation**

**SumOfYears**

![Graph showing the decline in unallocated portion of depreciation over time (Years)](image)
Plotting the results (2)
Sinking Fund Method

- The method is based on the compound interest theory.
- By the sinking fund theory, the equal annual year-end deposits in a sinking fund would accumulate with compound interest thereon to the total depreciation allocation to any given date.
- The allocation for a specific year is, therefore, the annual deposit, or annuity, plus the compound interest increment to the fund for the year.
Sinking Fund Method (2)

- Annual year end deposit to a fund to accumulate to “F” in ‘n’ years at ‘i’ interest rate is:

\[
A = F \frac{i}{(1+i)^n - 1}
\]

\[
A = (B - T) \frac{i}{(1+i)^n - 1}
\]

The accumulation to any date \( n \) is given by

\[
\therefore F = A \frac{(1+i)^n - 1}{i}
\]
In terms of depreciation symbols

\[ D_x = A \frac{(1+i)^x - 1}{i} \]

\[ D_x = (B - T) \left[ \frac{i}{(1+i)^n - 1} \right] \left[ \frac{(1+i)^x - 1}{i} \right] \]

\[ D_x = (B - T) \left[ \frac{(1+i)^x - 1}{(1+i)^n - 1} \right] \]

\[ D_x = B_d \left[ \frac{(1+i)^x - 1}{(1+i)^n - 1} \right] \]
The unallocated base at age $x$ would be then be given by

$$B_x = (B_d + T) - \left[B_d \left(\frac{(1+i)^x - 1}{(1+i)^n - 1}\right)\right]$$

$$B_x = B_d \left[\frac{(1+i)^n - (1+i)^x}{(1+i)^n - 1}\right] + T$$
Sinking Fund Method - Example

Original Amount | 50,000
Salvage Value  | 10,000
Time Period    | 5
Interest rate  | 5%

\[
B_x = B_d \left[ \frac{(1+r)^n - (1+r)^x}{(1+r)^n - 1} \right] + T
\]

<table>
<thead>
<tr>
<th>Year</th>
<th>Factor</th>
<th>Bx</th>
<th>B-Bx</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>1.00</td>
<td>50,000.00</td>
<td>0.00</td>
</tr>
<tr>
<td>1.00</td>
<td>0.82</td>
<td>42,761.01</td>
<td>7,238.99</td>
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<tr>
<td>2.00</td>
<td>0.63</td>
<td>35,160.07</td>
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<td>3.00</td>
<td>0.43</td>
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<td>4.00</td>
<td>0.22</td>
<td>18,799.04</td>
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<td>5.00</td>
<td>0.00</td>
<td>10,000.00</td>
<td>40,000.00</td>
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Unallocated Portion of Depreciation

Sinking Fund

Depreciation Coefficient
Present Worth Method

• Variant of the sinking fund method
• The concept is that the decrease in the value of the property in any given year, and therefore its depreciation for the year, is equal to the decrease for that year in the present value of its probable future returns.
• In terms of depreciation symbols and say rate of return ‘r’ is analogous to ‘i’ then the following can be written:
  \[ B = B_d + T \]
Present Worth Method

- Solving for $A$ gives:

$$A = B_d \left[ \frac{r(1+r)^n}{(1+r)^n - 1} \right] + rT$$

The present value at any age $x$, i.e. the depreciated value, would be:

$$B_x = A \left[ \frac{(1+r)^{n-x} - 1}{r(1+r)^{n-x}} \right] + \frac{T}{(1+r)^{n-x}}$$

Substituting the value of ‘$A$’

$$B_x = \left[ B_d \frac{r(1+r)^n}{(1+r)^n - 1} + rT \right] \left[ \frac{(1+r)^{n-x} - 1}{r(1+r)^{n-x}} \right] + \frac{T}{(1+r)^{n-x}}$$

Where $r =$ rate of return

$B =$ Depreciation Base (including Terminal Value)

$B_d =$ Depreciable base

$A =$ uniform annual return
Present Worth Method

\[ B_x = \left[ B_d \frac{(1+r)^n - (1 + \frac{r}{2})^x}{(1+r)^n - 1} \right] + T \]

Capital Recovery Factor = \[ \frac{r(1+r)^n}{(1+r)^n - 1} \]

Where \( r \) = rate of return

\( B_x \) = Unallocated portion of depreciable Base at age x

\( B_d \) = Depreciable base (Depreciation base-Terminal value)

\( A \) = uniform annual return

Note that the formula for computing the depreciation using Sinking Fund Method and Present Worth Method is same except \( r \) (rate of return) is used in Sinking Fund Method instead of \( i \) (interest rate) in Present Worth Method.
Complete the Example with PW Method
Comparison of All Methods

- A = Present Worth at i %
- B = Straight Line Method
- C = Multiple Straight Line Method
- D = Sum of the Year Digit Method
- E = Declining Balance Method