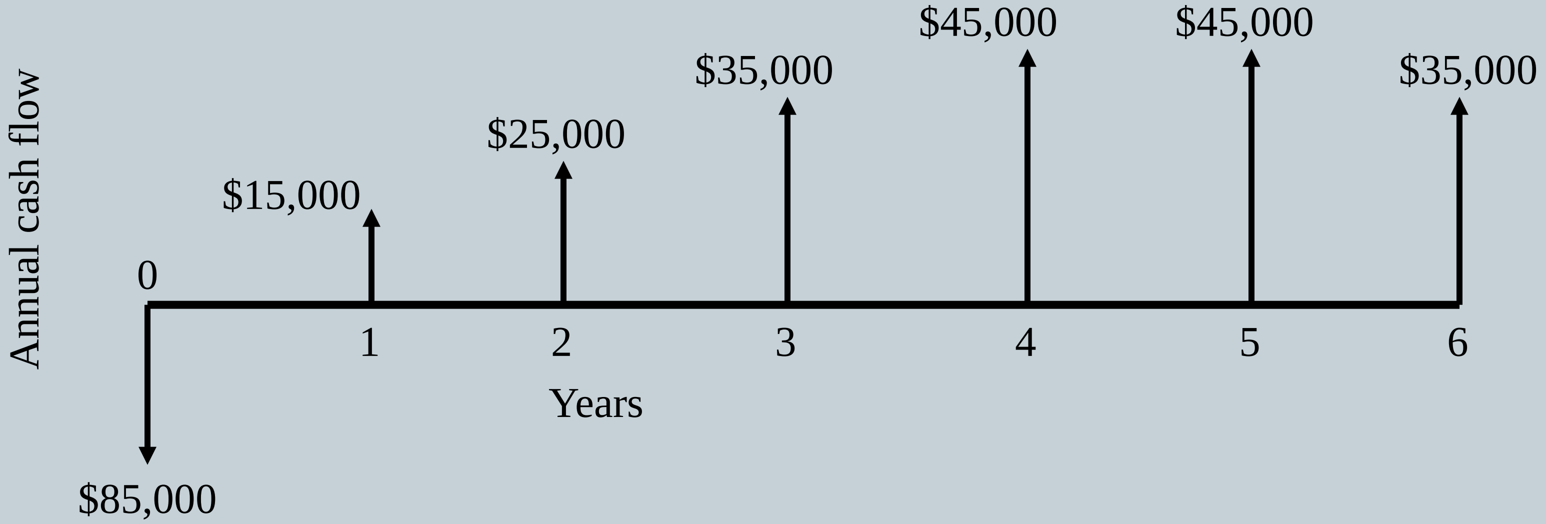
Transportation Economics and Decision Making

Lecture-5

Pay Off or Pay Back Period

- How long does it take the project to “pay back” its initial investment?
- Payback Period = number of years to recover initial costs
- By the time you have discounted the cash flows, you might as well calculate the NPV.
- When benefits = costs

Example-1

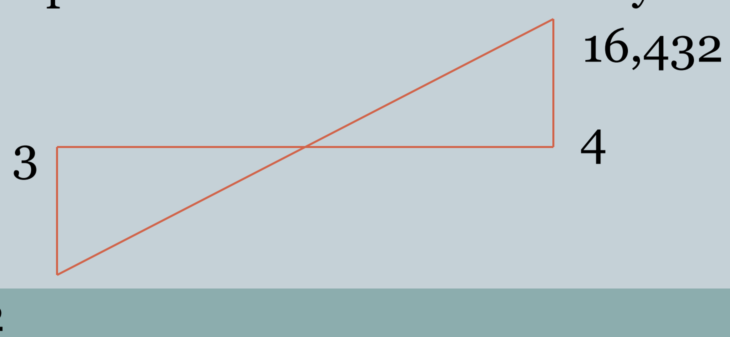


MARR = 6%

Example-1

Year	Abs. Cost	Abs. Benefit	PW Cost	PW Benefit	Cummulative Revenue
0	85,000		-85,000	0	-85,000
1		15,000		14,151	-70,849
2		25,000		22,250	-48,599
3		35,000		29,387	-19,212
4		45,000		35,644	16,432
5		45,000		33,627	50,058
6		35,000		24,674	74,732

Change in sign occurs between 3 to four years
Use interpolation to assess the exact year (say up to one decimal place)



Example-2

- Assume you have the following information on Project X:
  - Initial outlay -$1,000
  - Required return = 10%
- Annual that cash flows and their PVs are as follows:

Year	Cash flow	PV of Cash flow
1	$ 200	$ 182
2	400	331
3	700	526
4	300	205

Year	Cummulative discounted CF
1	$ 182
2	513
3	1,039
4	1,244

- Discounted payback period is just under 3 years

Nominal and Effective Rates of Interest

- Nominal and effective interest rates have similar relationship to that of simple and compound interest rates.
- The difference is that nominal and effective interest rates are used when compounding period (or interest period) is less than one year.
- Let i= interest rate per base period conversion; quoted interest rate

  r = nominal rate per annum

  j = effective rate per annum

  m= times per year, or base period, the nominal rate is converted

Effective Rates of Interest

- Let i= interest rate per base period conversion; quoted interest rate

    r = nominal rate per annum

    j = effective rate per annum

    m= times per year, or base period, the nominal rate is converted

Effective Rates of Interest

Since $i = \frac{r}{m}$

Effective Interest Rate $j = \left[1 + \frac{r}{m}\right]^m - 1$

Example:

Find the effective rate of interest for $100 for 1 year at nominal interest of 12% per year, interest payable monthly:

$$F = 100 * \left[1 + \frac{0.12}{12}\right]^{12} = 100 * (1.01)^{12} = 112.6825$$

Effective Interest Rate $= (1 + .01)^{12} - 1 = 0.1268 = 12.68\%$

Example

- A bank pays 6% nominal interest rate. Calculate the effective interest with
- a) monthly, b) daily, c) hourly d) secondly compounding
  - $i = (1 + r/m)^m - 1$
  - i monthly $= (1 + .06/12)^{12} - 1 = 6.1678$ %
  - i daily $= (1 + .06/365)^{365} - 1 = 6.183$ %
  - i hourly $= (1 + .06/8760)^{8760} - 1 = 6.1836$ %
  - i secondly $= (1 + .06/31.5M)^{31.5M} - 1 = 6.18365$ %

Kraft Demand Model

- We occasionally come across a demand function where the elasticity of demand of travel with respect to price is constant. The demand function is represented as

$$q = \alpha(p)^{\beta}$$

$$e_p = \frac{\delta q}{\delta p}\,\frac{p}{q}$$

$$= \alpha\beta p^{\beta-1}\,\frac{p}{q}$$

$$= \alpha\beta p^{\beta-1}\,\frac{p}{\alpha(p)^{\beta}}$$

$$= \beta$$

Example

- The elasticity of transit demand with respect to price has been found to be -2.75. A transit line on this system carries 12,500 passengers per day with a flat fare of 50 cents/ride. The management would like to rise the fare to 70 cents/ride. Will this be a prudent decision?

$q = \alpha(p)^{\beta}$
$12{,}500 = \alpha(50)^{-2.75}$
$\alpha = 5.876 \times 10^{8}$

Hence q= $5.876 \times 10^{8} (50)^{-2.75}$

Fare 70 cents would result in demand = $5.876 \times 10^{8} (70)^{-2.75}$ = 4995 passengers

Revenue @ 50cents/ride = 50 * 12, 50 = $6,250

Revenue @ 70cents/ride = 70 * 4,995 = $3,486.50

It would not be prudent to increase the fare.

Example

- The demand function from suburbs to university of Memphis is given by

$$Q = T^{-0.3}\, C^{-0.2} A^{0.1} I^{-0.25}$$

Where
Q-> number of transit trips
T-> travel time on transit (hours)
C-> Fare on transit (dollars)
A-> Average cost of automobile trip (dollars)
I-> Average income (dollars)

- There are currently 10,000 persons per hour riding the transit system, at a flat fare of $1 per ride. What would be the change in ridership with a 90 cent fare?
- By auto the trip costs $3 (including parking). If the parking fees are raised by 30 cents, how would it affect the transit ridership?

Solution

- This is essentially a modified kraft demand model. The price elasticity of demand for transit trips is $\frac{\delta Q/Q}{\delta C/C} = 0.2$
- This means 1% reduction in fare would lead to a 0.2% increase in transit ridership.
- Because the fare reduction is (100-90)/100 = 10%, one would expect 2% increase in ridership.
- New ridership will be 10,000 * 1.02 = 10,200
- Revenue @$1/ride = 10,000 * 1 = $10,000
- Revenue @$0.9/ride = 10,200*0.9 = $9,180
- The company will loose $820

Solution

- The automobile elasticity of demand is 0.1, i.e. $\frac{\delta Q/Q}{\delta C/C} = 0.1$,
- 1% rise in auto costs will lead to a 0.1% rise in transit trips,
- 10% rise in auto cost (0.3 is 10% of $3) would result in 1% increase in transit ridership, i.e. 1.1*10,000 = 10,100

Direct and Cross Elasticity

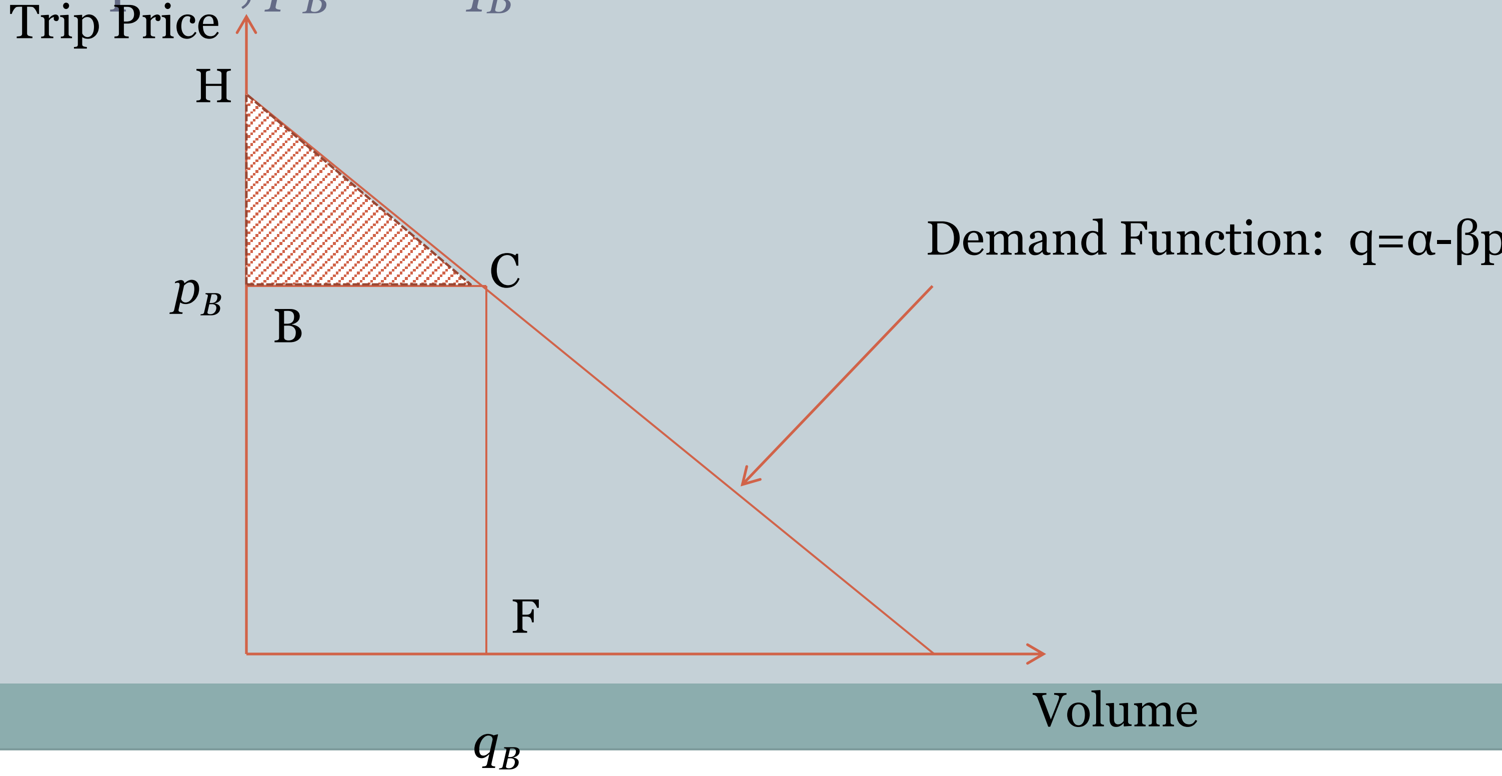
- Direct elasticity
  - The effect of change in the price of a good on the demand for the same good is referred as direct elasticity
- Cross elasticity
  - The measure of responsiveness of the demand for a good to the price of another good is referred as cross elasticity

Consumer Surplus (CS)

- Demand = f^n(quantity of trip making, price of travel)
- Price of travel includes all private time, effort, and expenses incurred by the traveller.
- Demand is related to “Willingness to Pay (WTP)” (choice of making a trip or not)
- WTP is the primary measure of (individual) value or benefit derived from a particular trip

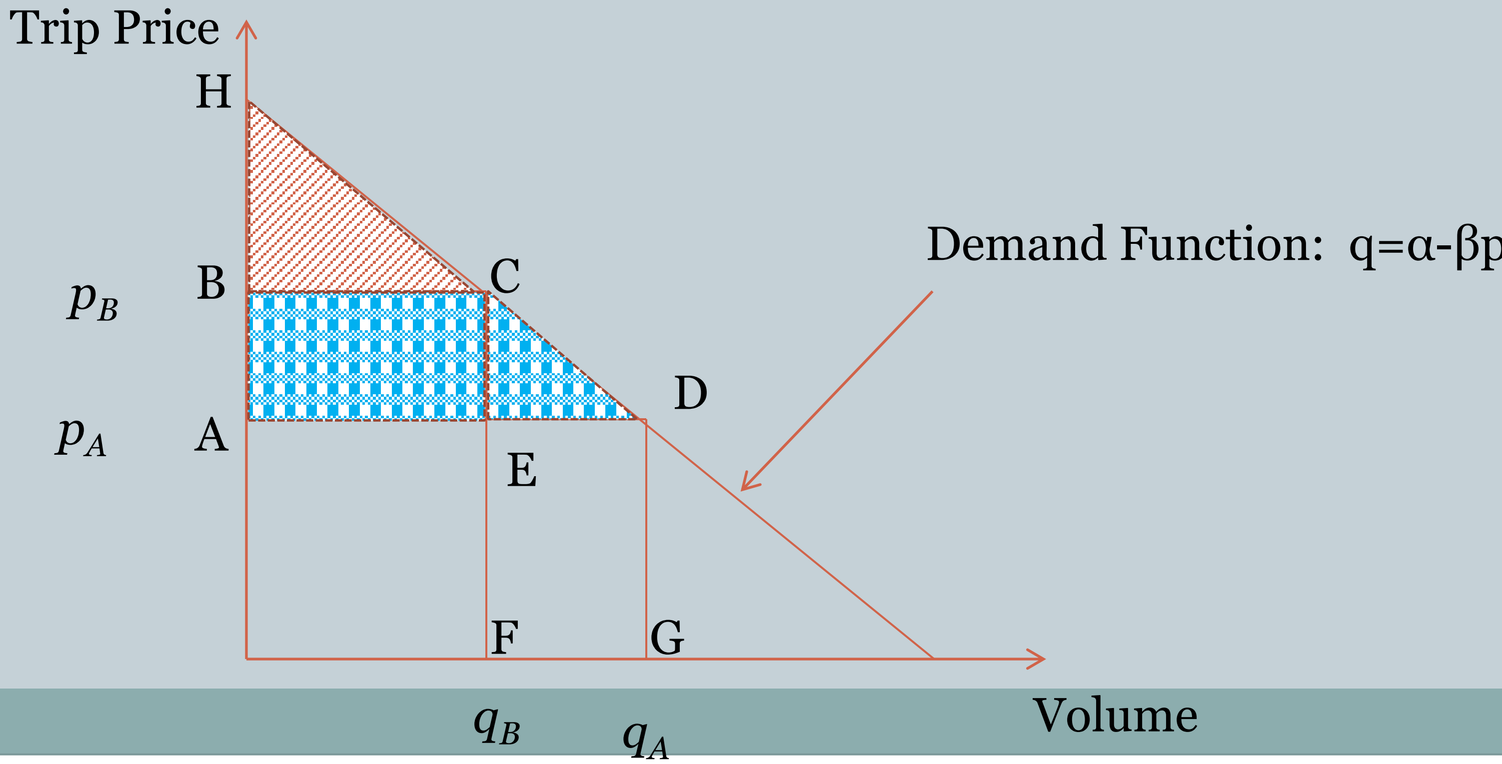
CS-WTP

- First, we must distinguish what
  - Users do actually pay
  - Users are willing to pay
  - CS from price, p_B and q_B users = area HBC



CS-WTP

- If the price is lowered then consumer surplus would increase to HAD
- Additional CS resulted = BADC



Benefit

- Benefit is equivalent to the value which travellers are expected to receive from making trips as measured by the gross amount which travellers would be willing to pay.
- In the example,
  - for price p_B, total benefit = HOFC
  - For price p_A, total benefit = HOGD

Benefit

- Net user benefit before change of price
  - Travel benefits at p_B = value derived from q_B trips = HOFC
  - User costs at price p_B = user payments for trips = BOFC
  - net user benefit at price p_B = CS = HBC = NUB_B
- Net user benefit after change of price
  - Travel benefits at p_A = value derived from q_A trips = HOGD
  - User costs at price p_A= user payments for trips = AOGD
  - net user benefit at price p_A = CS = HBD = NUB_A
- Change in net user benefit p_B = user payments for trips = BADC = NUB_A- NUB_B

User Cost

- Reduction in user cost accompanies a price change , where the price is construed broadly in terms of time, effort, and expenses of travel.
- Initially, when price is p_B, then cost to each user is same as what is actually paid, i.e. p_B, of course all trip makers except the one at the margin would be willing to pay more than p_B.
- When the price is p_A dropped to user cost difference (p_B- p_A)
  - Benefit accrued per trip
- So the benefit is BADC

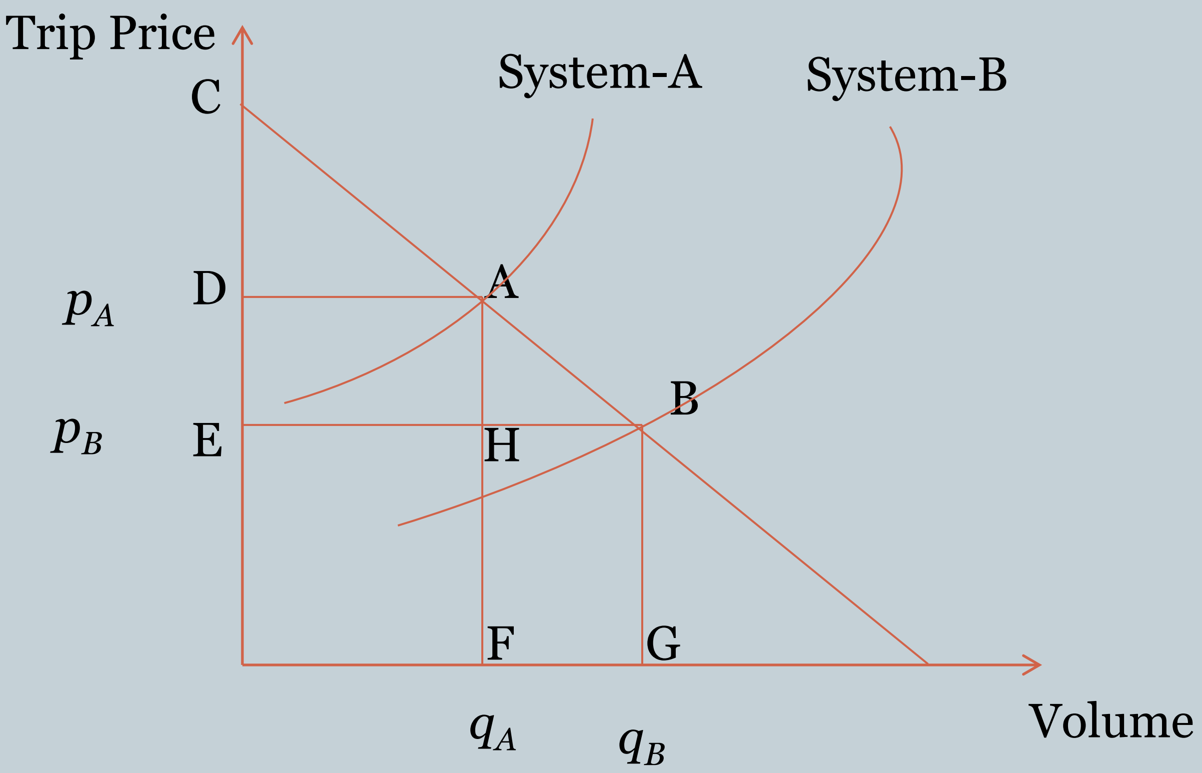
Benefit and User Cost Measure

- Considering the benefit measure or road user cost measure will lead to the same CS.
  - Benefit measure: BACD = HOGD-HOFC
  - User Cost measure: BACD = BAEC + CED

Benefits for Different Facilities

- Benefits are associated when there is a change in facility capacity.
- Example: highways or transit.
- There exists a relationship
  - between price and volume
  - average variable cost and price paid by the travellers during tripmaking

Benefits for Different Facilities



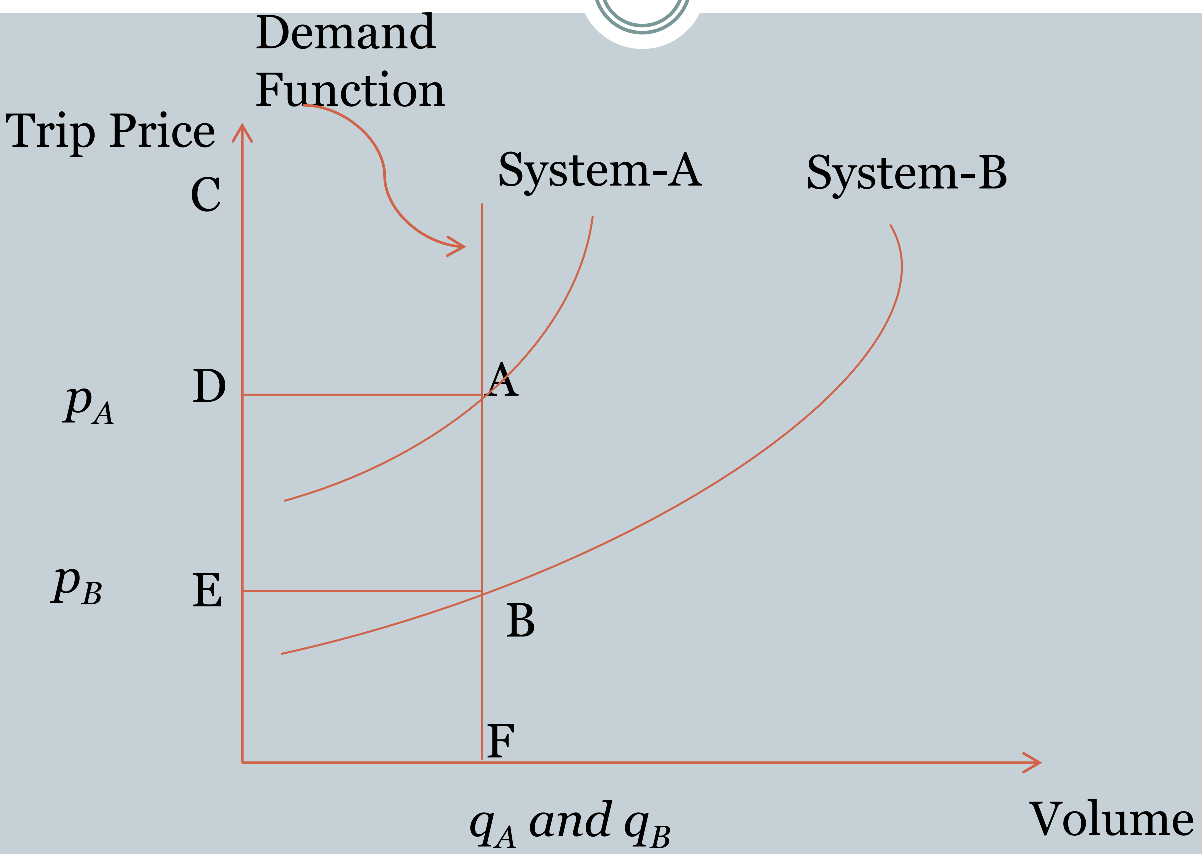
Benefits for Different Facilities

- Improvement in facility from A to B
  - will drop the equilibrium price from p_A to p_B.
  - induce more trip making
  - (i.e. quantity of travel demand will increase from volume q_A to q_B.)
  - the difference: q_B- q_A, is the diverted volume
  - Often called as "induced" or "generated" demand
  - Note that the change is *NOT* called as *increase in demand* (because increase in demand relates to shift in the demand function)

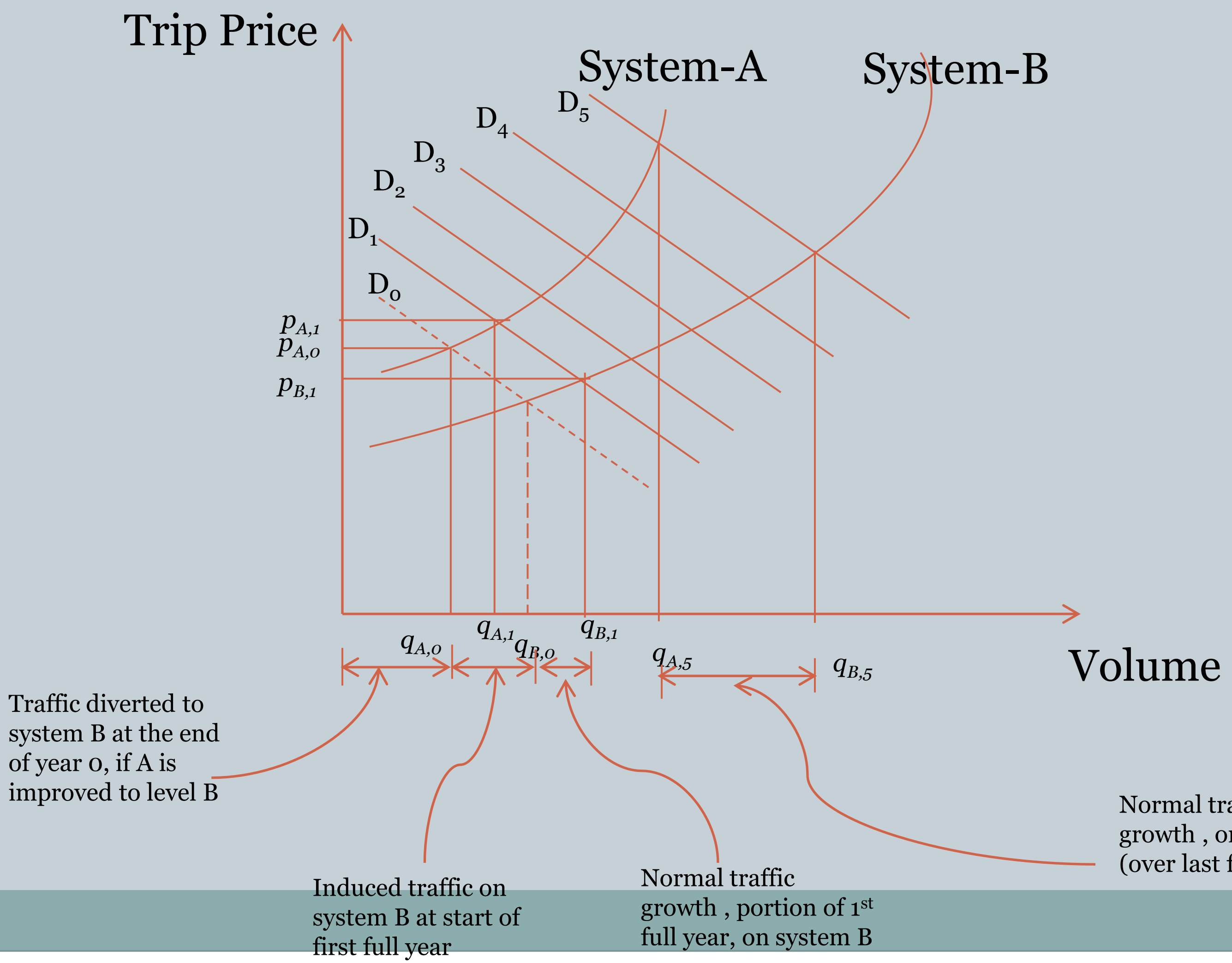
- Total benefit
  - Benefit from facility A: AFOC
  - Benefit from facility B: BGOC
  - Additional or extra benefit accrued because of improvement is: ABGF = BGOC - AFOC
- User Cost
  - User cost or Net benefit (i.e. difference between total user benefit, and the user payments)
  - Net User Benefit from facility A: ADC
  - Net User Benefit from facility B: BEC
  - Net user travel benefit (consumer surplus) because of improvement is: ABED = BEC-ADC
- ABGF is the change in consumer surplus

- Diverted travellers accrue larger increment in net benefit than new/induced travellers (Area AHED): $(p_B - p_A)\, q_A$
- The induced travellers will receive benefits= $(q_B - q_A)\, 0.5 * (p_A - p_B)$
  - *Assuming the demand curve is linear or nearly linear in this range*
- Observation: The first induced traveller will receive benefits $(p_A - p_B)$, the last traveller (q_B *th*) will receive no increase in benefit.

Perfectly Inelastic Demand



Long Term Effects of Demand Shift



Long Term Effects of Demand Shift

- If facility A were improved to the level of B and opened for usage at end of year 0, $q_{A,0}$ is the diverted traffic
- $q_{B,0} - q_{A,0}$ is the induced traffic
- The additional increase in traffic flow from year to year can be regarded as normal traffic growth for facility B
  - During the first full year, the normal traffic growth for B would be $q_{B,1} - q_{A,1}$
  - During year 0 to 5, increased growth is $q_{B,5} - q_{A,5}$

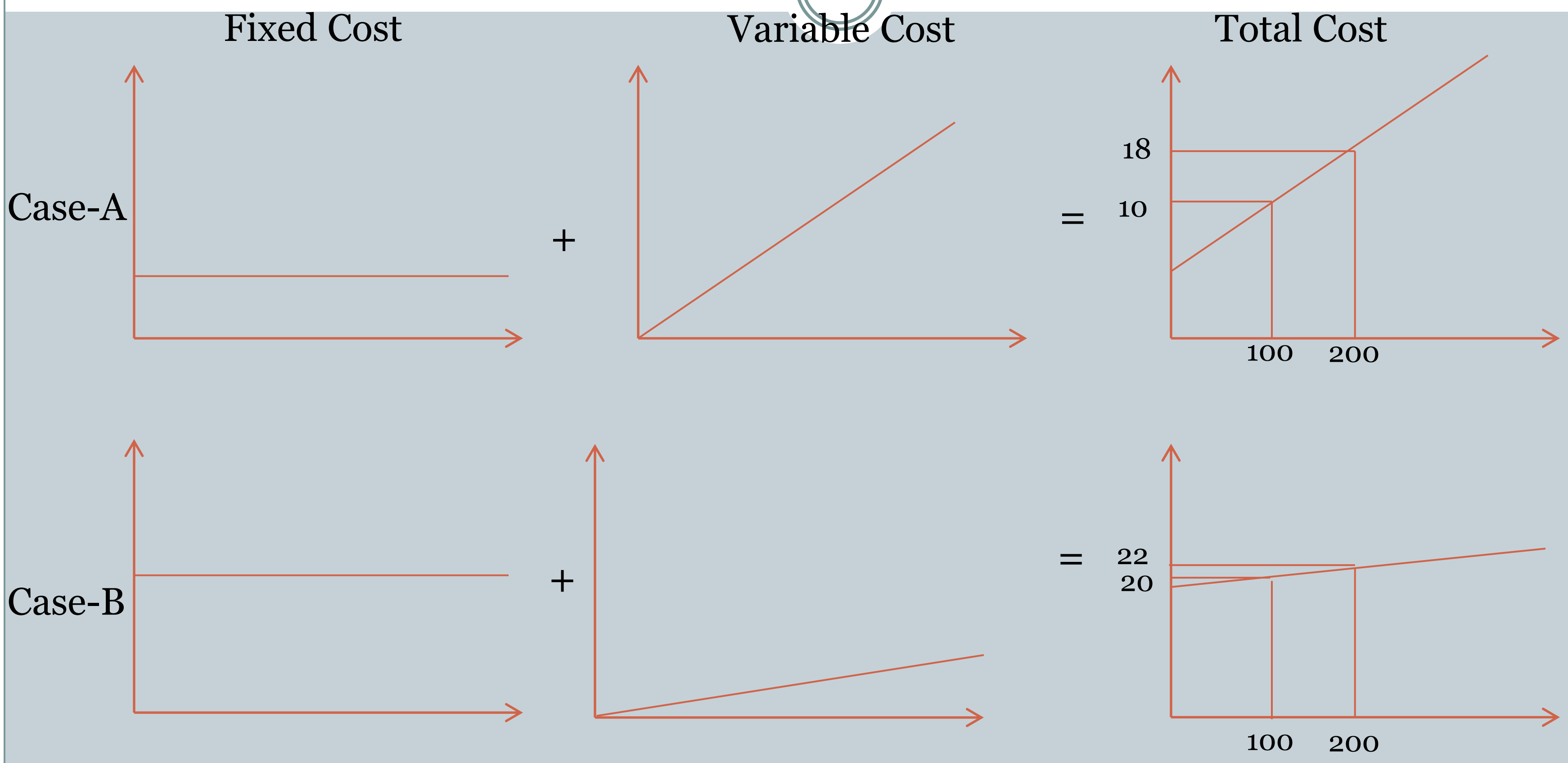
Consumer Surplus

- If we would like to measure changes in CS over five years then
  - We should measure changes year by year and then accumulate over the planning horizon.

Costs

- The total cost of owning and operating a facility is broken into
  - Fixed cost
  - Variable cost
  - Total cost = Fixed Cost + Variable Cost
- Fixed cost does not depend on production levels or degree of utilization
  - Purchase price
- Variable cost depends on degree of production or utilization
  - Depends on use (increased wear and tear)

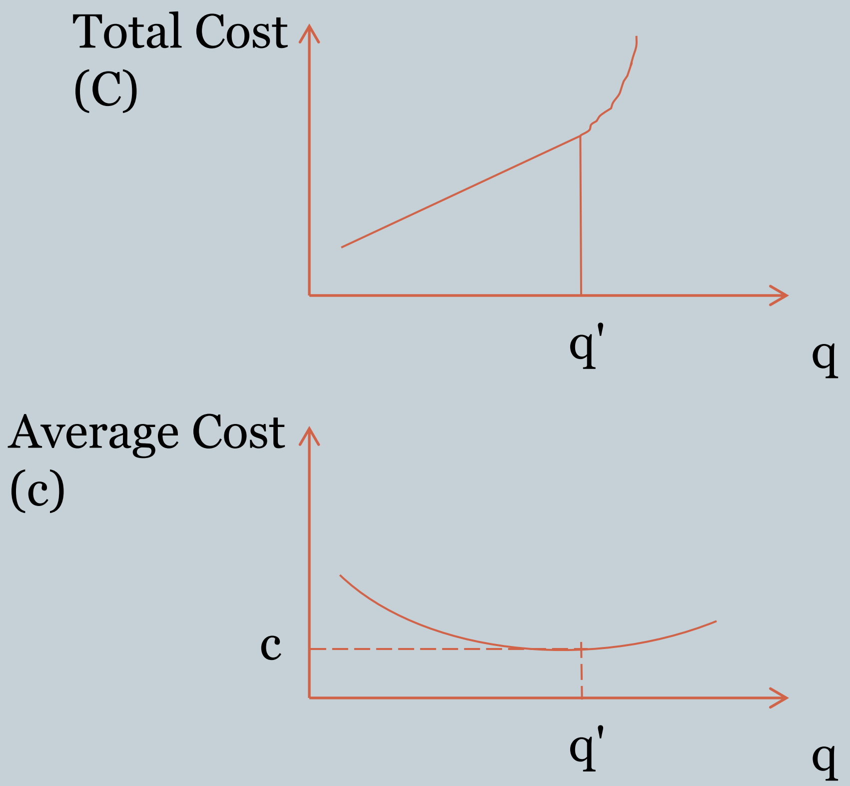
Fixed and Variable Cost



Case-A: When production is doubled unit cost drops from $0.10 to $0.09 (10% reduction)
Case-B: When production is doubled unit cost drops from $0.20 to $0.11 (45% reduction)

Economy of Scale

- Economy of scale is defined as the decrease in average cost as the output increases.



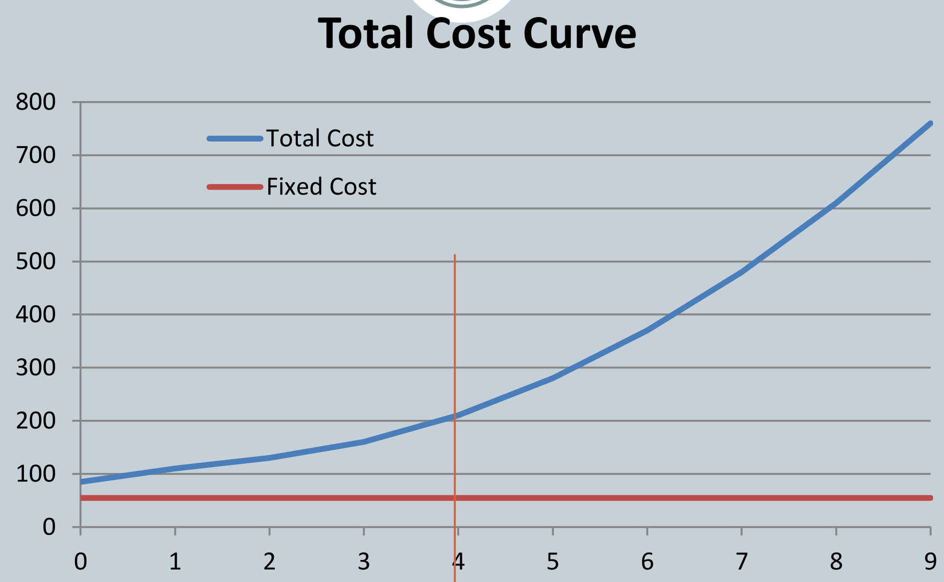
Example-1

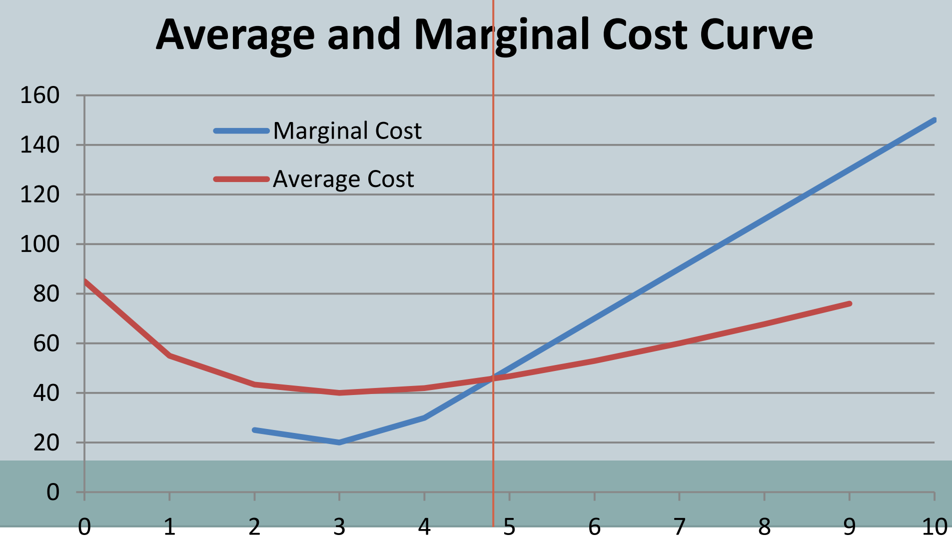
Number of Wagons	Fixed Cost	Varibale Cost
0	0	0
1	55	30
2	55	55
3	55	75
4	55	105
5	55	155
6	55	225
7	55	315
8	55	425
9	55	555
10	55	705

Example-1

Number of Wagons	Fixed Cost	Varibale Cost	Total Cost	Average Cost	Marginal Cost
0	0	0	0	0	0
1	55	30	85	85.00	
2	55	55	110	55.00	25
3	55	75	130	43.33	20
4	55	105	160	40.00	30
5	55	155	210	42.00	50
6	55	225	280	46.67	70
7	55	315	370	52.86	90
8	55	425	480	60.00	110
9	55	555	610	67.78	130
10	55	705	760	76.00	150

Example-1


Total Cost Curve
Total Cost
Fixed Cost
800
700
600
500
400
300
200
100
0
0 1 2 3 4 5 6 7 8 9


Average and Marginal Cost Curve
Marginal Cost
Average Cost
160
140
120
100
80
60
40
20
0
0 1 2 3 4 5 6 7 8 9 10

Example-2

- A city is considering building one of the following two types of transit systems, Type A and Type B. Type A is a conventional high-speed bus system on a freeway network, and Type B is an advanced, energy-efficient, light-trail transit system on a fully controlled-access network.

	Type A	Type B
Initial Cost	$\$45 \times 10^6$	$\$80 \times 10^6$
Project Life	20 years	20 years
Operating Cost	$0.25/pass. mile	$0.18/pass. mile
Ridership	180,000 Pass. Miles/day	216,000 Pass. Miles/day

(a) Using a discount rate of 6% per year, compute the fixed cost, variable cost and total cost of the two systems on an annualized basis, as well as the unit cost/pass. mile. Which system should be built?

(b) Assuming an average 15-mile trip length per passenger, and a 30% subsidy, what should be the fare/passenger for the recommended system? Also compute the minimum demand for the Type B system in order for it to be more cost efficient than Type A. Assume demand for Type B to be 20% higher than that for Type A.