Transportation Economics and Decision Making
Pay Off or Pay Back Period

- How long does it take the project to “pay back” its initial investment?
- Payback Period = number of years to recover initial costs
- By the time you have discounted the cash flows, you might as well calculate the NPV.
- When benefits = costs
Example-1

MARR = 6%

Annual cash flow

<table>
<thead>
<tr>
<th>Years</th>
<th>Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$85,000</td>
</tr>
<tr>
<td>1</td>
<td>$15,000</td>
</tr>
<tr>
<td>2</td>
<td>$25,000</td>
</tr>
<tr>
<td>3</td>
<td>$35,000</td>
</tr>
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<td>4</td>
<td>$45,000</td>
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<tr>
<td>5</td>
<td>$45,000</td>
</tr>
<tr>
<td>6</td>
<td>$35,000</td>
</tr>
</tbody>
</table>
### Example-1

<table>
<thead>
<tr>
<th>Year</th>
<th>Abs. Cost</th>
<th>Abs. Benefit</th>
<th>PW Cost</th>
<th>PW Benefit</th>
<th>Cumulative Revenue</th>
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</thead>
<tbody>
<tr>
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<td>85,000</td>
<td>-85,000</td>
<td>0</td>
<td>0</td>
<td>-85,000</td>
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<td>-48,599</td>
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<td>-48,599</td>
<td>-19,212</td>
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<td>29,937</td>
<td>-19,212</td>
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<td>50,058</td>
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<td>6</td>
<td>35,000</td>
<td>24,674</td>
<td>74,732</td>
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Change in sign occurs between 3 to four years  
Use interpolation to assess the exact year (say up to one decimal place)
Assume you have the following information on Project X:
- Initial outlay -$1,000
- Required return = 10%

Annual that cash flows and their PVs are as follows:

<table>
<thead>
<tr>
<th>Year</th>
<th>Cash flow</th>
<th>PV of Cash flow</th>
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<tr>
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<td>700</td>
<td>526</td>
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<td>4</td>
<td>300</td>
<td>205</td>
</tr>
<tr>
<td>Year</td>
<td>Cumulative discounted CF</td>
<td></td>
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<tr>
<td>------</td>
<td>--------------------------</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>$182</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>513</td>
<td></td>
</tr>
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<td>3</td>
<td>1,039</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1,244</td>
<td></td>
</tr>
</tbody>
</table>

- Discounted payback period is just under 3 years
Nominal and Effective Rates of Interest

- Nominal and effective interest rates have similar relationship to that of simple and compound interest rates.
- The difference is that nominal and effective interest rates are used when compounding period (or interest period) is less than one year.
- Let $i =$ interest rate per base period conversion; quoted interest rate
  
  $r =$ nominal rate per annum
  
  $j =$ effective rate per annum
  
  $m =$ times per year, or base period, the nominal rate is converted
Effective Rates of Interest

- Let $i =$ interest rate per base period conversion; quoted interest rate
  
  $r =$ nominal rate per annum
  
  $j =$ effective rate per annum
  
  $m =$ times per year, or base period, the nominal rate is converted
Effective Rates of Interest

Since \( i = \frac{r}{m} \)

Effective Interest Rate \( j = \left[ 1 + \frac{r}{m} \right]^m - 1 \)

Example:
Find the effective rate of interest for $100 for 1 year at nominal interest of 12% per year, interest payable monthly:

\( F = 100 \times \left[ 1 + \frac{0.12}{12} \right]^{12} = 100 \times (1.01)^{12} = 112.6825 \)

Effective Interest Rate = \((1 + .01)^{12} - 1 = 0.1268 = 12.68\%\)
Example

- A bank pays 6% nominal interest rate. Calculate the effective interest with
  - a) monthly, b) daily, c) hourly d) secondly compounding
    - \[ i = (1 + \frac{r}{m})^m - 1 \]
    - \( i \) monthly = \((1 + .06/12)^{12} -1 = 6.1678 \% \)
    - \( i \) daily = \((1 + .06/365)^{365} -1 = 6.183 \% \)
    - \( i \) hourly = \((1 + .06/8760)^{8760} -1 = 6.1836 \% \)
    - \( i \) secondly = \((1 + .06/31.5M)^{31.5M} -1 = 6.18365 \% \)
We occasionally come across a demand function where the elasticity of demand of travel with respect to price is constant. The demand function is represented as

\[ q = \alpha(p)\beta \]
\[ e_p = \frac{\delta q}{\delta p} \frac{p}{q} \]
\[ = \alpha \beta p^{\beta-1} \frac{p}{q} \]
\[ = \alpha \beta p^{\beta-1} \frac{p}{\alpha(p)\beta} \]
\[ = \beta \]
The elasticity of transit demand with respect to price has been found to be -2.75. A transit line on this system carries 12,500 passengers per day with a flat fare of 50 cents/ride. The management would like to rise the fare to 70 cents/ride. Will this be a prudent decision?

\[ q = \alpha (p)^\beta \]
\[ 12,500 = \alpha (50)^{-2.75} \]
\[ \alpha = 5.876 \times 10^8 \]

Hence \( q = 5.876 \times 10^8 (50)^{-2.75} \)

Fare 70 cents would result in demand = \( 5.876 \times 10^8 (70)^{-2.75} = 4995 \) passengers

Revenue @ 50 cents/ride = \( 50 \times 12,500 = $6,250 \)

Revenue @ 70 cents/ride = \( 70 \times 4,995 = $3,486.50 \)

It would not be prudent to increase the fare.
Example

- The demand function from suburbs to university of Memphis is given by

\[ Q = T^{-0.3} C^{-0.2} A^{0.1} I^{-0.25} \]

Where
- \( Q \rightarrow \) number of transit trips
- \( T \rightarrow \) travel time on transit (hours)
- \( C \rightarrow \) Fare on transit (dollars)
- \( A \rightarrow \) Average cost of automobile trip (dollars)
- \( I \rightarrow \) Average income (dollars)

- There are currently 10,000 persons per hour riding the transit system, at a flat fare of $1 per ride. What would be the change in ridership with a 90 cent fare?

- By auto the trip costs $3 (including parking). If the parking fees are raised by 30 cents, how would it affect the transit ridership?
This is essentially a modified kraft demand model. The price elasticity of demand for transit trips is \( \frac{\delta Q/Q}{\delta C/C} = 0.2 \). This means 1% reduction in fare would lead to a 0.2% increase in transit ridership. Because the fare reduction is \( \frac{100-90}{100} = 10\% \), one would expect 2% increase in ridership. New ridership will be \( 10,000 \times 1.02 = 10,200 \). Revenue @\$1/ride = \( 10,000 \times 1 = \$10,000 \). Revenue @\$0.9/ride = \( 10,200 \times 0.9 = \$9,180 \). The company will lose \$820.
The automobile elasticity of demand is 0.1, i.e. \( \frac{\delta Q/Q}{\delta c/c} = 0.1 \),

1% rise in auto costs will lead to a 0.1% rise in transit trips,

10% rise in auto cost (0.3 is 10% of $3) would result in a 1% increase in transit ridership, i.e. 1.1*10,000 = 10,100
Direct and Cross Elasticity

- Direct elasticity
  - The effect of change in the price of a good on the demand for the same good is referred as direct elasticity

- Cross elasticity
  - The measure of responsiveness of the demand for a good to the price of another good is referred as cross elasticity
Consumer Surplus (CS)

- Demand = f^n(quantity of trip making, price of travel)
- Price of travel includes all private time, effort, and expenses incurred by the traveller.
- Demand is related to “Willingness to Pay (WTP)” (choice of making a trip or not)
- WTP is the primary measure of (individual) value or benefit derived from a particular trip
First, we must distinguish what

- Users do actually pay
- Users are willing to pay
- CS from price, $p_B$ and $q_B$ users = area HBC

Demand Function: $q = \alpha - \beta p$
If the price is lowered then consumer surplus would increase to HAD.

Additional CS resulted = BADC.
Benefit

- Benefit is equivalent to the value which travellers are expected to receive from making trips as measured by the gross amount which travellers would be willing to pay.
- In the example,
  - for price \( p_B \), total benefit = HOFC
  - For price \( p_A \), total benefit = HOGD
Benefit

- **Net user benefit before change of price**
  - Travel benefits at $p_B = \text{value derived from } q_B \text{ trips} = \text{HOFC}$
  - User costs at price $p_B = \text{user payments for trips} = \text{BOFC}$
  - net user benefit at price $p_B = CS = HBC = \text{NUB}_B$

- **Net user benefit after change of price**
  - Travel benefits at $p_A = \text{value derived from } q_A \text{ trips} = \text{HOGD}$
  - User costs at price $p_A = \text{user payments for trips} = \text{AOGD}$
  - net user benefit at price $p_A = CS = HBD = \text{NUB}_A$

- **Change in net user benefit** $p_B = \text{user payments for trips} = BADC = \text{NUB}_A - \text{NUB}_B$
• Reduction in user cost accompanies a price change, where the price is construed broadly in terms of time, effort, and expenses of travel.

• Initially, when price is $p_B$, then cost to each user is same as what is actually paid, i.e. $p_B$, of course all trip makers except the one at the margin would be willing to pay more than $p_B$.

• When the price is $p_A$ dropped to user cost difference $(p_B - p_A)$
  - Benefit accrued per trip

• So the benefit is BADC
Benefit and User Cost Measure

- Considering the benefit measure or road user cost measure will lead to the same CS.
  - Benefit measure: $BACD = HOGD - HOFC$
  - User Cost measure: $BACD = BAEC + CED$
Benefits for Different Facilities

- Benefits are associated when there is a change in facility capacity.
- Example: highways or transit.
- There exists a relationship
  - between price and volume
  - average variable cost and price paid by the travellers during tripmaking
Benefits for Different Facilities

Trip Price

Volume

$p_A$ $p_B$

$q_A$ $q_B$
Benefits for Different Facilities

- **Improvement in facility from A to B**
  - will drop the equilibrium price from $p_A$ to $p_B$.
  - induce more trip making
  - (i.e. quantity of travel demand will increase from volume $q_A$ to $q_B$.)
  - the difference: $q_B - q_A$, is the diverted volume
  - Often called as “induced” or “generated” demand
  - Note that the change is **NOT** called as *increase in demand* (because increase in demand relates to shift in the demand function)
• **Total benefit**
  - Benefit from facility A: AFOC
  - Benefit from facility B: BGOC
  - Additional or extra benefit accrued because of improvement is: $ABGF = BGOC - AFOC$

• **User Cost**
  - User cost or Net benefit (i.e. difference between total user benefit, and the user payments)
  - Net User Benefit from facility A: ADC
  - Net User Benefit from facility B: BEC
  - Net user travel benefit (consumer surplus) because of improvement is: $ABED = BEC - ADC$

• **ABGF is the change in consumer surplus**
• Diverted travellers accrue larger increment in net benefit than new/induced travellers (Area AHED): 
  \((p_B - p_A) q_A\)

• The induced travellers will receive benefits = \((q_B - q_A) 0.5*(p_A - p_B)\)

  - Assuming the demand curve is linear or nearly linear in this range

• Observation: The first induced traveller will receive benefits \((p_A - p_B)\), the last traveller \((q_B \text{ th})\) will receive no increase in benefit.
Perfectly Inelastic Demand

Demand Function

Trip Price

$p_A$

$p_B$

$q_A$ and $q_B$

Volume

System-A

System-B
Traffic diverted to system B at the end of year 0, if A is improved to level B

Induced traffic on system B at start of first full year

Normal traffic growth, on system B (over last four years)

Normal traffic growth, portion of 1st full year, on system B
If facility A were improved to the level of B and opened for usage at end of year 0, \(q_{A,0}\) is the diverted traffic.

\(q_{B,0} - q_{A,0}\) is the induced traffic.

The additional increase in traffic flow from year to year can be regarded as normal traffic growth for facility B.

- During the first full year, the normal traffic growth for B would be \(q_{B,1} - q_{A,1}\).
- During year 0 to 5, increased growth is \(q_{B,5} - q_{A,5}\).
Consumer Surplus

- If we would like to measure changes in CS over five years then
  - We should measure changes year by year and then accumulate over the planning horizon.
The total cost of owning and operating a facility is broken into:
- Fixed cost
- Variable cost
- Total cost = Fixed Cost + Variable Cost

Fixed cost does not depend on production levels or degree of utilization:
- Purchase price

Variable cost depends on degree of production or utilization:
- Depends on use (increased wear and tear)
Fixed and Variable Cost

Case-A: When production is doubled unit cost drops from $0.10 to $0.09 (10% reduction)
Case-B: When production is doubled unit cost drops from $0.20 to $0.11 (45% reduction)
Economy of scale is defined as the decrease in average cost as the output increases.
<table>
<thead>
<tr>
<th>Number of Wagons</th>
<th>Fixed Cost</th>
<th>Variable Cost</th>
</tr>
</thead>
<tbody>
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<td>0</td>
</tr>
<tr>
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<tr>
<td>Number of Wagons</td>
<td>Fixed Cost</td>
<td>Variable Cost</td>
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<td>555</td>
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<tr>
<td>10</td>
<td>55</td>
<td>705</td>
</tr>
</tbody>
</table>
Example-1

Total Cost Curve

Average and Marginal Cost Curve
A city is considering building one of the following two types of transit systems, Type A and Type B. Type A is a conventional high-speed bus system on a freeway network, and Type B is an advanced, energy-efficient, light-trail transit system on a fully controlled-access network.

<table>
<thead>
<tr>
<th></th>
<th>Type A</th>
<th>Type B</th>
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<tbody>
<tr>
<td>Initial Cost</td>
<td>$45 \times 10^6$</td>
<td>$80 \times 10^6$</td>
</tr>
<tr>
<td>Project Life</td>
<td>20 years</td>
<td>20 years</td>
</tr>
<tr>
<td>Operating Cost</td>
<td>$0.25$/pass. mile</td>
<td>$0.18$/pass. mile</td>
</tr>
<tr>
<td>Ridership</td>
<td>180,000 Pass. Miles/day</td>
<td>216,000 Pass. Miles/day</td>
</tr>
</tbody>
</table>

(a) Using a discount rate of 6% per year, compute the fixed cost, variable cost and total cost of the two systems on an annualized basis, as well as the unit cost/pass. mile. Which system should be built?

(b) Assuming an average 15-mile trip length per passenger, and a 30% subsidy, what should be the fare/passenger for the recommended system? Also compute the minimum demand for the Type B system in order for it to be more cost efficient than Type A. Assume demand for Type B to be 20% higher than that for Type A.