Transportation Economics and Decision Making

Lecture-4

Example of Mutually Exclusive Alternatives

- Problem definition
  - A certain section of highway is now in location A. Number of perspective designs are proposed. A total of 13 alternatives are proposed.
  - Current location
    - A-1 (do nothing), A-2, A-3, A-4
  - New location-B
    - B-1, B-2, B-3, B-4, B-5
  - New location-C
    - C-1, C-2, C-3, and C-4

Example with Multiple Alternatives

Alternative	Capital Cost	Annualized Maintenance Cost	Annual Road User Cost
A-1	-	60	2,200
A-2	1,500	35	1,920
A-3	2,000	30	1,860
A-4	3,500	40	1,810
B-1	3,000	30	1,790
B-2	4,000	20	1,690
B-3	5,000	30	1,580
B-4	6,000	40	1,510
B-5	7,000	45	1,480
C-1	5,500	40	1,620
C-2	8,000	30	1,470
C-3	9,000	40	1,400
C-4	11,000	50	1,340

Other Assumptions

- Project life: 30 years
- Minimum attractive rate of return: 7%
- A fixed road user cost throughout the lifetime (simplified for the example problem)
- No terminal or salvage value

Observations

- Alternative A-1 is do nothing, no capital costs involved but has highest road user cost
- Alternative C-4 is involved with highest cost but least road user cost
- All other alternatives has intermediate capital cost

Analysis Based on Total Cost

Alternative	Capital Cost	Annualized Cost	Annualized Maintenance Cost	Annual Highway Cost	Annual Road User Cost	Total Annual Cost
A-1	-	-	60	60	2,200	2,260
A-2	1,500	121	35	156	1,920	2,076
A-3	2,000	161	30	191	1,860	2,051
A-4	3,500	282	40	322	1,810	2,132
B-1	3,000	242	30	272	1,790	2,062
B-2	4,000	322	20	342	1,690	2,032
B-3	***5,000***	***403***	***30***	***433***	***1,580***	***2,013***
B-4	6,000	484	40	524	1,510	2,034
B-5	7,000	564	45	609	1,480	2,089
C-1	5,500	443	40	483	1,620	2,103
C-2	8,000	645	30	675	1,470	2,145
C-3	9,000	725	40	765	1,400	2,165
C-4	11,000	886	50	936	1,340	2,276

Analysis Based on Benefit Cost Ratio (Compare do nothing with all alternatives)

Alternative	Capital Cost	Annualized Cost	Annualized Maintenance Cost	Annual Highway Cost	Annual Road User Cost	Total Annual Cost	User Benefits Compared to A-1	Highway Costs Compared to A-1	B/C Ratio
A-1	-	-	60	60	2,200	2,260			
A-2	1,500	121	35	156	1,920	2,076	280	96	2.92
A-3	2,000	161	30	191	1,860	2,051	340	131	2.59
A-4	3,500	282	40	322	1,810	2,132	390	262	1.49
B-1	3,000	242	30	272	1,790	2,062	410	212	1.94
B-2	4,000	322	20	342	1,690	2,032	510	282	1.81
B-3	5,000	403	30	433	1,580	2,013	620	373	1.66
B-4	6,000	484	40	524	1,510	2,034	690	464	1.49
B-5	7,000	564	45	609	1,480	2,089	720	549	1.31
C-1	5,500	443	40	483	1,620	2,103	580	423	1.37
C-2	8,000	645	30	675	1,470	2,145	730	615	1.19
C-3	9,000	725	40	765	1,400	2,165	800	705	1.13
C-4	11,000	886	50	936	1,340	2,276	860	876	0.98

Does not mean A-2 is the most preferred alternative

Incremental B/C Ratio

Alternative	Annual Highway Cost	Annual Road User Cost	Comparison	Incremental Benefit	Incremental Cost	Incremental B/C	Decision
A-1	60	2,200					
A-2	156	1,920	A-1	280	96	2.92	**A-2**
A-3	191	1,860	A-2	60	35	1.70	**A-3**
A-4	322	1,810	A-3	50	131	0.38	**A-3**
B-1	272	1,790	A-3	70	81	0.87	**A-3**
B-2	342	1,690	A-3	170	151	1.12	**B-2**
B-3	433	1,580	B-2	110	91	1.21	**B-3**
B-4	524	1,510	B-3	70	91	0.77	**B-3**
B-5	609	1,480	B-3	100	176	0.57	**B-3**
C-1	483	1,620	B-3	(40)	50	-0.80	**B-3**
C-2	675	1,470	B-3	110	242	0.45	**B-3**
C-3	765	1,400	B-3	180	332	0.54	**B-3**
C-4	936	1,340	B-3	240	504	0.48	**B-3**

Transportation Demand

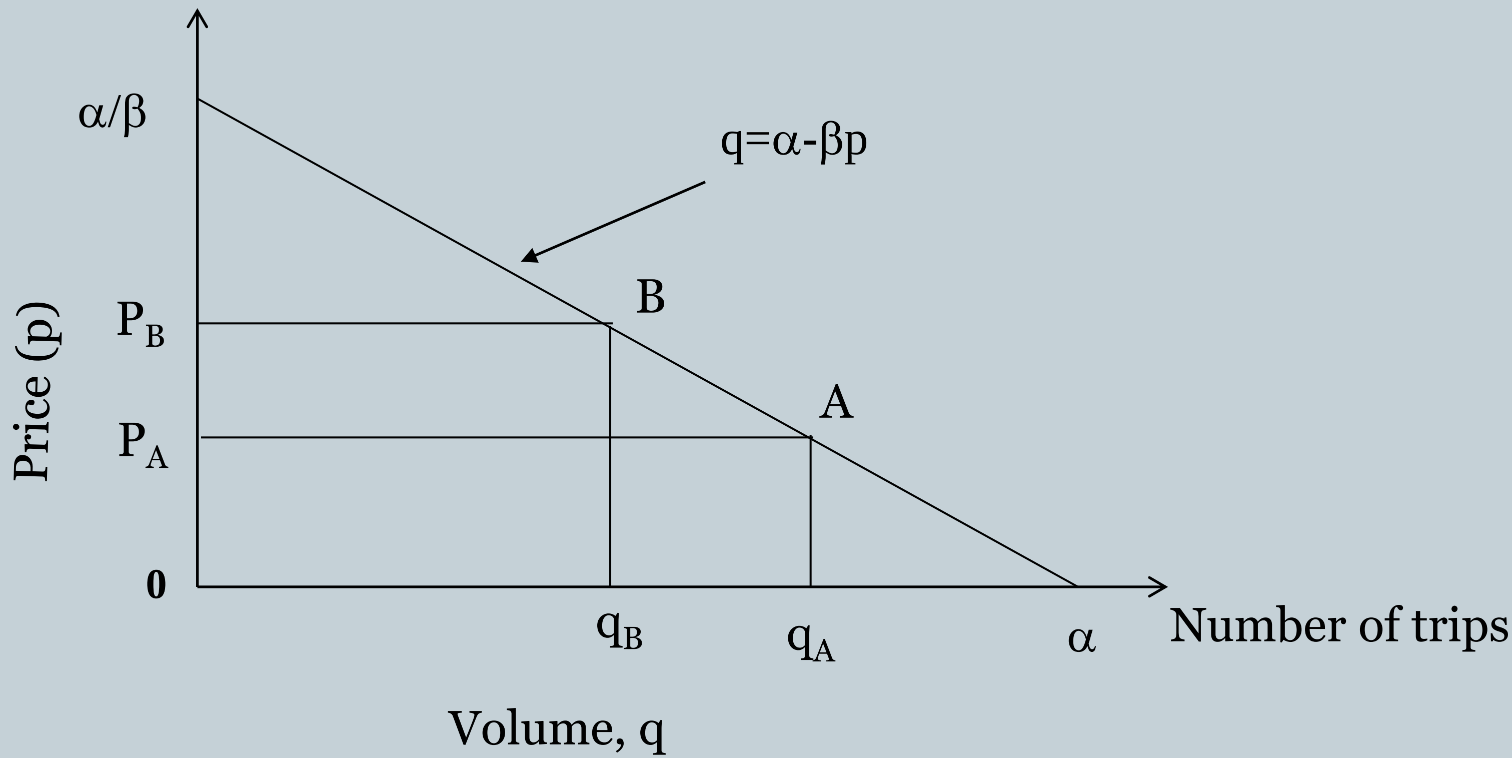
- The demand for goods and services in general depends largely on consumer's income and price of a particular good relative to other prices.
- Example-1: Demand for travel depends on income of the traveler
- Example-2: Choice of travel mode depends on several factors such as
  - Trip purpose
  - Distance travelled
  - Income of the traveller

Transportation Demand

- A demand function for a particular product represents the willingness of consumers to purchase the product at alternative price.
- Example: A number of passengers willing to use a commuter train at different price levels between O-D pair during a given time period.
- Willingness to pay depends on
  - Out of pocket cost
  - Waiting time
  - In-vehicle time
  - Comfort, convenience
  - Safety,
  - Reliability
  - In combination the total price is called as "generalized cost"

Linear Demand Curve

- A linear transportation demand can be represented as



Linear Demand Curve

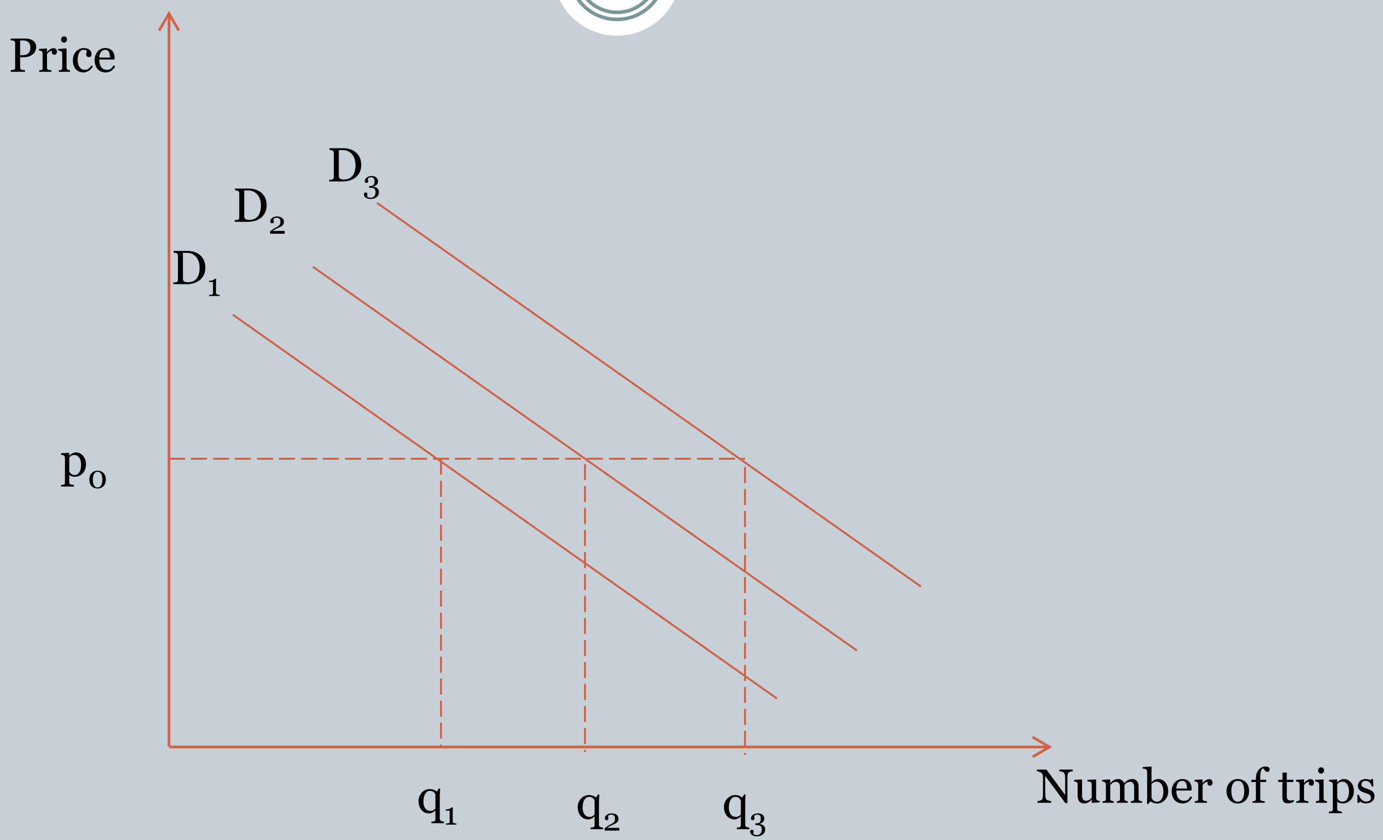
- A linear demand curve represents volume of trips demanded at different prices by a group of travellers

$$q = \alpha - \beta p$$

Where,
q-> quantity of trips demanded
p-> price
α, β->demand parameters

- The demand curve take a negative slope representing decrease in perceived price will result in increased travel (*may not be always true for transportation!*)

Shifted Demand Curve



A scenario represents increase in demand because of variables other than perceived price

Typical and Shifted Demand Curve

- It is crucial to distinguish short-run changes in the quantity of travel because of price changes
  - Relationship represented by a single demand curve
- Long-run changes are because of activity and behavioral changes
  - Relationship represented in a shifted demand curve

Demand Supply Equilibrium

- Equilibrium is said to be attained when factors that affect the quantity demanded and those that determine the quantity supplied are equal (or converging towards equilibrium)
- If the demand and supply functions for a transportation facility are known, then it is possible to deal with the concept of equilibrium

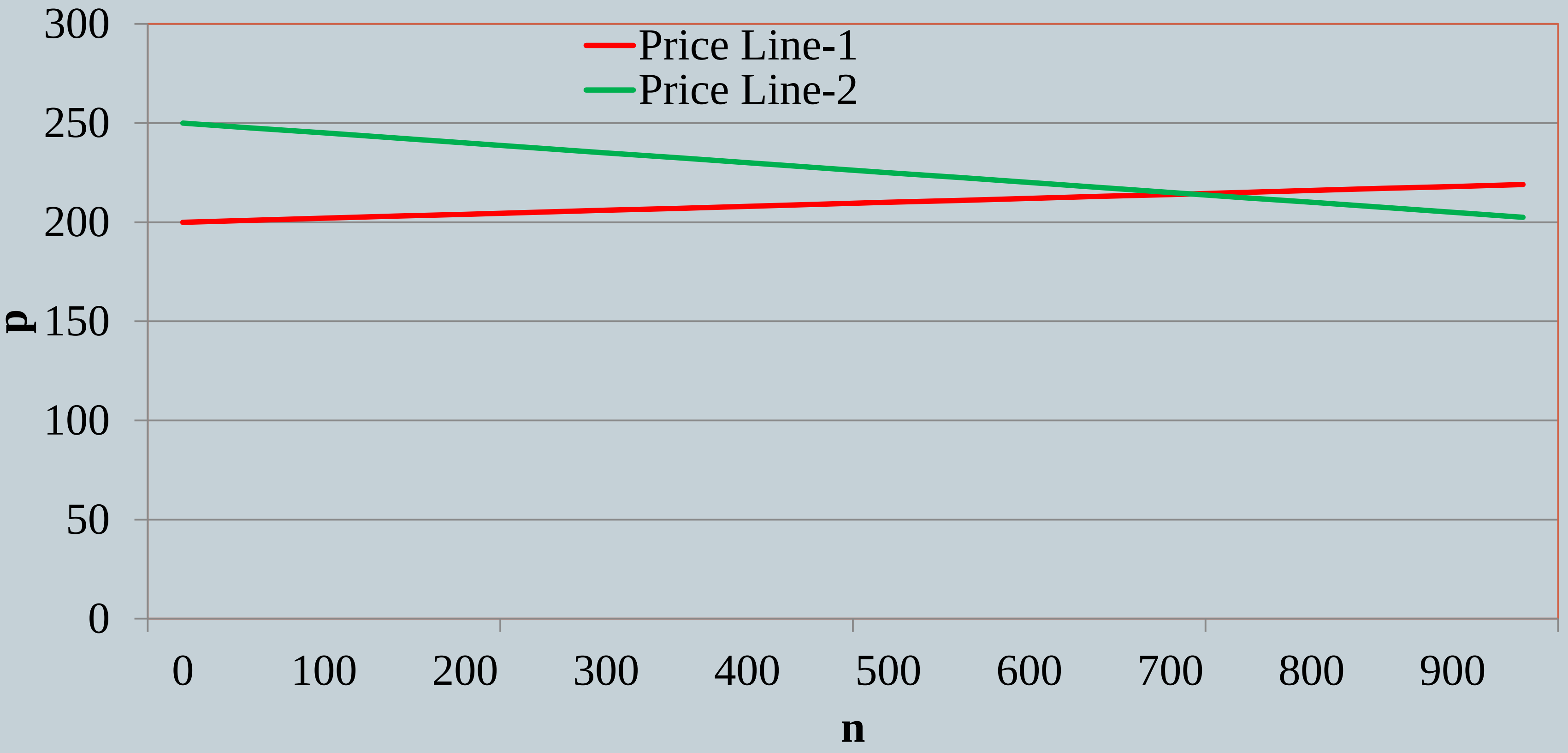
Example-1

- An airline company has determined the price of a ticket on a particular route to be $p=200+0.02n$. The demand for this route by air has been found to be $n=5000-20p$. Where,
  - p-> price in dollars
  - n->number of tickets sold per day
- Determine the equilibrium price charged and the number of seats sold per day

Example-1

- Solving two equations:
- $p=200+0.02n$ and $p = (5000-n)/20$;
- $p = \$214.28$; and $n = 714$ tickets
- The logic of two equations appears reasonable. If the price of airline ticket goes up, then demand would fall.

Example-1



Price Line-1
Price Line-2
300
250
200
150
100
50
0
p
0
100
200
300
400
500
600
700
800
900
n

Sensitivity of Travel Demand

- A useful descriptor to explain degree of sensitivity to the change in price is the elasticity of demand *(e_p)*
- e_p is the percentage in quantity of trips demanded that accompanies a 1% change in price

$$q = \alpha - \beta p$$

$$e_p = \frac{\delta q / q}{\delta p / p} = \frac{\delta q}{\delta p} \; x \, \frac{p}{q}$$

- Where δq is the change in number of trips that accompanies change in price δp

Price Elasticity

$$e_p = \frac{\delta q}{\delta p} \frac{p}{q}$$

$$= \frac{Q_1 - Q_0}{P_1 - P_0} \frac{(P_1 + P_0)/2}{(Q_1 + Q_0)/2}$$

Where, Q_1, Q_0 represent the quantity of travel demanded corresponding to P_1, P_0 prices respectively

After derivation elasticity may take the following forms

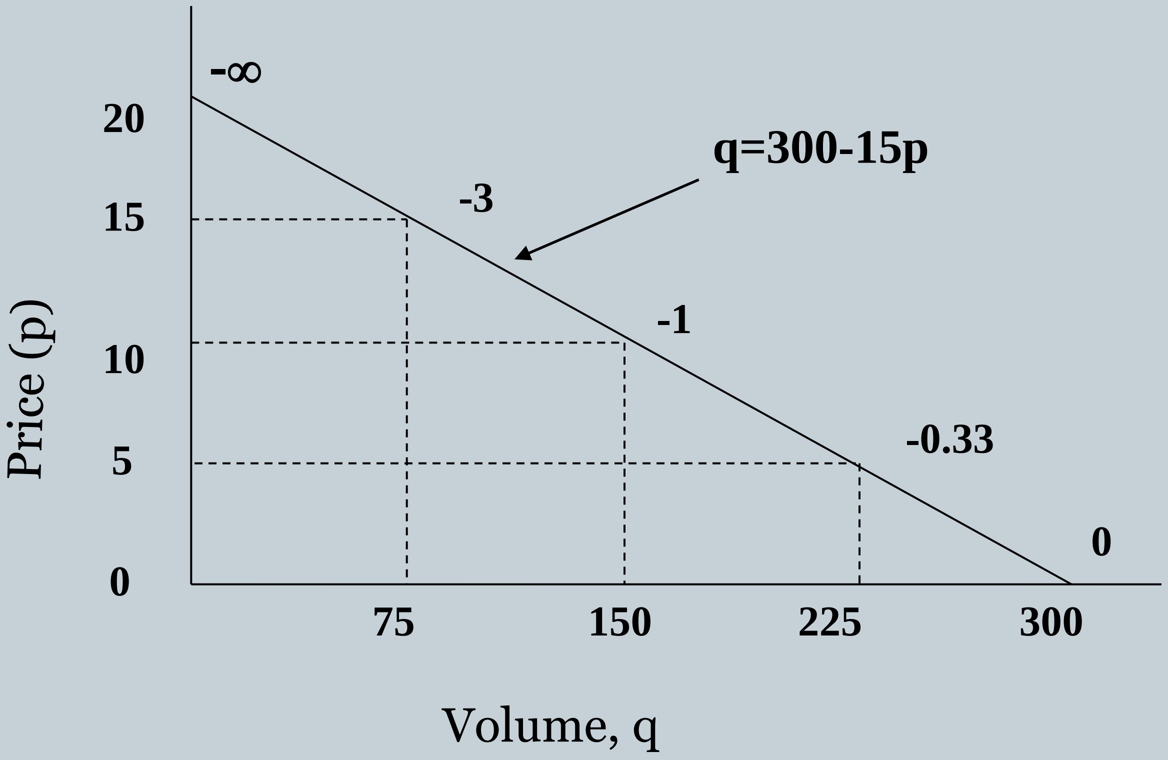
$$e_p = \frac{-\beta p}{q} = 1 - \frac{\alpha}{q}$$

Example: Elasticity

- An aggregate demand is represented by the equation
- q= 200-10p where q is the number of trips made and p is the price per trip
- Find the price elasticity of demand for the following conditions

Demand (q)	Price (p)
0	20
50	15
100	10
150	5
200	0

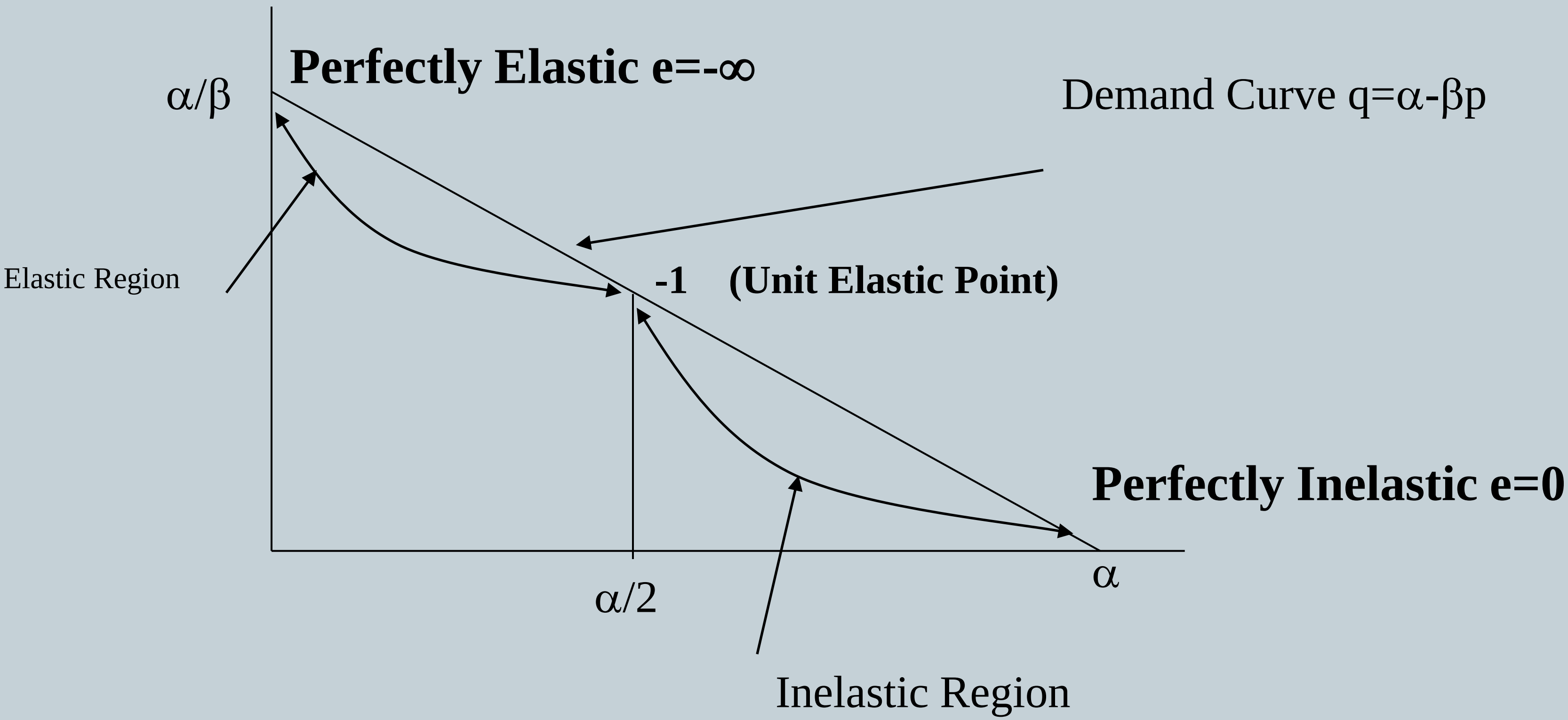
Demand Function and Elasticities



Elasticity: Discussion

- When elasticity is less than -1: demand is described as elastic
  - i.e. the resulting percentage change in quantity of trip making is larger than price
  - In this case demand is relatively sensitive to price change
- When the elasticity is between -1 and 0, demand is described to be inelastic or relatively insensitive.

Elasticity: General Case



Elasticity Observations

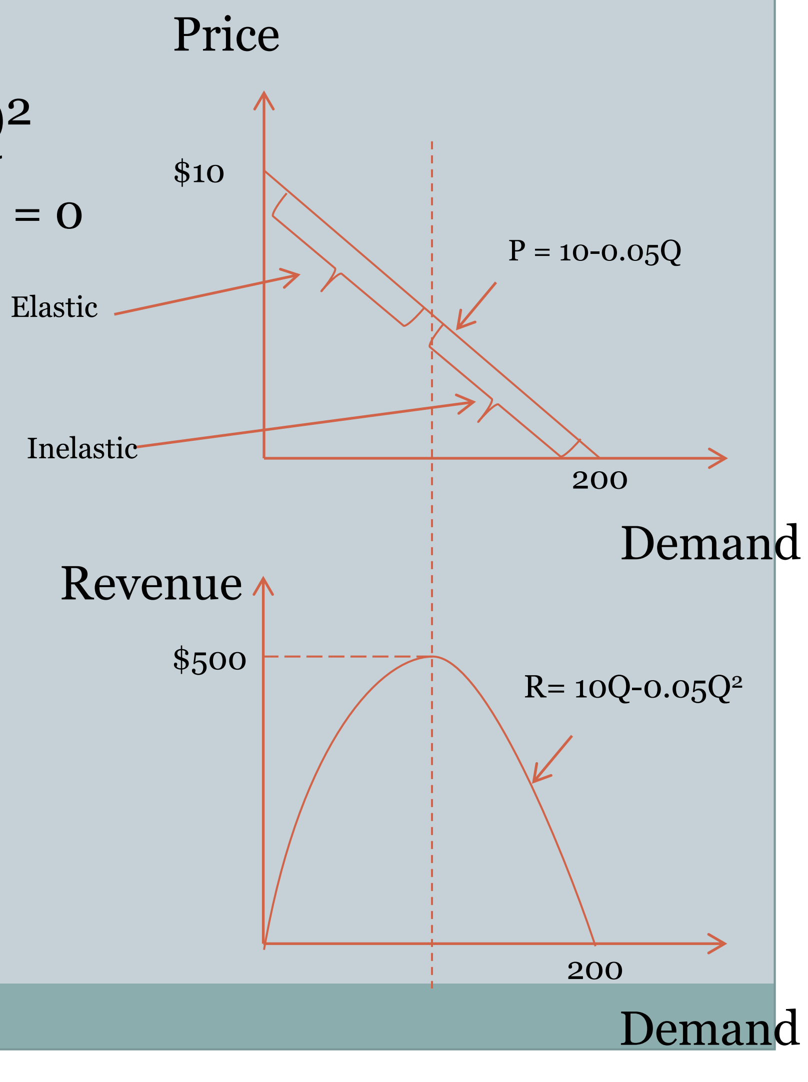
- The linear demand curve has several interesting properties
  - As one moves down the demand curve, the price elasticity becomes smaller (i.e. more inelastic)
- Slope of the line is constant
- Elasticity changes from ∞ to 0

Example-2

- A bus company's linear demand curve is $P=10-0.05Q$. Where P is the price of one way ticket, and Q is number of tickets sold per hour. Determine the maximum revenue ?

Example-2: Demonstration

- $P = 10-0.05Q$
- $R = Q(10-0.05Q) = 10Q-0.05Q^2$
- Maximum revenue will occur when $dR/dQ = 0$
- i.e. $Q= 100$, $R = 500$



Kraft Demand Model

- We occasionally come across a demand function where the elasticity of demand of travel with respect to price is constant. The demand function is represented as

$$q = \alpha(p)^{\beta}$$

$$e_p = \frac{\delta q}{\delta p}\,\frac{p}{q}$$

$$= \alpha\beta p^{\beta-1}\,\frac{p}{q}$$

$$= \alpha\beta p^{\beta-1}\,\frac{p}{\alpha(p)^{\beta}}$$

$$= \beta$$

Example

- The elasticity of transit demand with respect to price has been found to be -2.75. A transit line on this system carries 12,500 passengers per day with a flat fare of 50 cents/ride. The management would like to rise the fare to 70 cents/ride. Will this be a prudent decision?

$q = \alpha(p)^{\beta}$
$12{,}500 = \alpha(50)^{-2.75}$
$\alpha = 5.876 \times 10^{8}$

Hence q= $5.876 \times 10^{8} (50)^{-2.75}$

Fare 70 cents would result in demand = $5.876 \times 10^{8} (70)^{-2.75}$ = 4995 passengers

Revenue @ 50cents/ride = 50 * 12, 50 = $6,250

Revenue @ 70cents/ride = 70 * 4,995 = $3,486.50

It would not be prudent to increase the fare.

Example

- The demand function from suburbs to university of Memphis is given by

$$Q = T^{-0.3}\, C^{-0.2} A^{-0.1} I^{-0.25}$$

Where
Q-> number of transit trips
T-> travel time on transit (hours)
C-> Fare on transit (dollars)
A-> Average cost of automobile trip (dollars)
I-> Average income (dollars)

- There are currently 10,000 persons per hour riding the transit system, at a flat fare of $1 per ride. What would be the change in ridership with a 90 cent fare?
- By auto the trip costs $3 (including parking). If the parking fees are raised by 30 cents, how would it affect the transit ridership?

Solution

- This is essentially a modified kraft demand model. The price elasticity of demand for transit trips is $\frac{\delta Q/Q}{\delta C/C} = 0.2$
- This means 1% reduction in fare would lead to a 0.2% increase in transit ridership.
- Because the fare reduction is (100-90)/100 = 10%, one would expect 2% increase in ridership.
- New ridership will be 10,000 * 1.02 = 10,200
- Revenue @$1/ride = 10,000 * 1 = $10,000
- Revenue @$0.9/ride = 10,200*0.9 = $9,180
- The company will loose $820

Solution

- The automobile elasticity of demand is 0.1, i.e. $\frac{\delta Q/Q}{\delta C/C} = 0.1$,
- 1% rise in auto costs will lead to a 0.1% rise in transit trips,
- 10% rise in auto cost (0.3 is 10% of $3) would result in 1% increase in transit ridership, i.e. 1.1*10,000 = 10,100

Direct and Cross Elasticity

- Direct elasticity
  - The effect of change in the price of a good on the demand for the same good is referred as direct elasticity
- Cross elasticity
  - The measure of responsiveness of the demand for a good to the price of another good is referred as cross elasticity