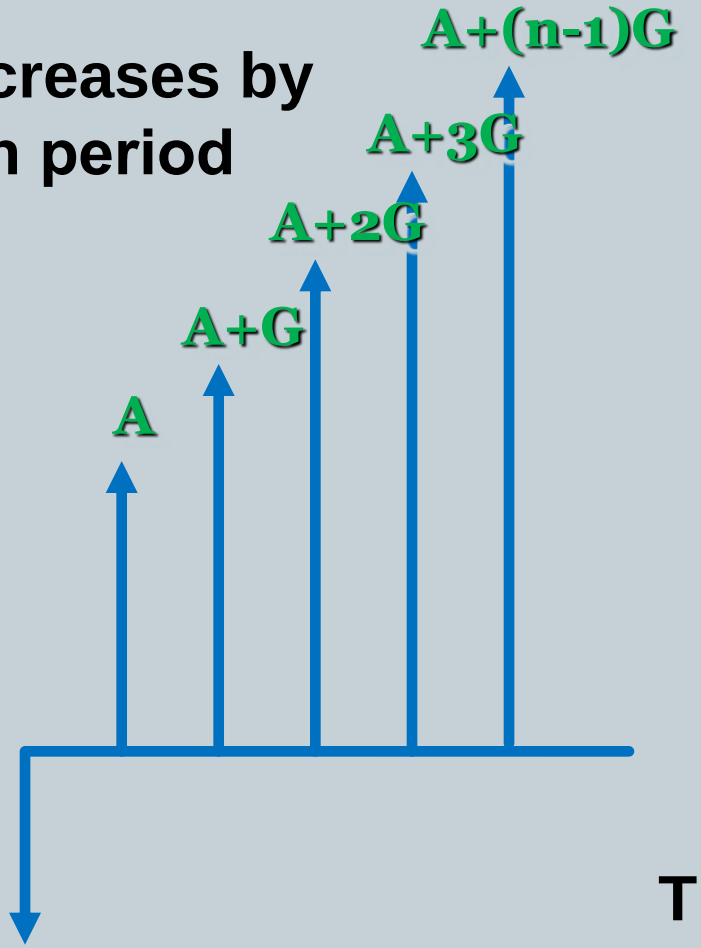
Transportation Economics and Decision Making

Lecture-3

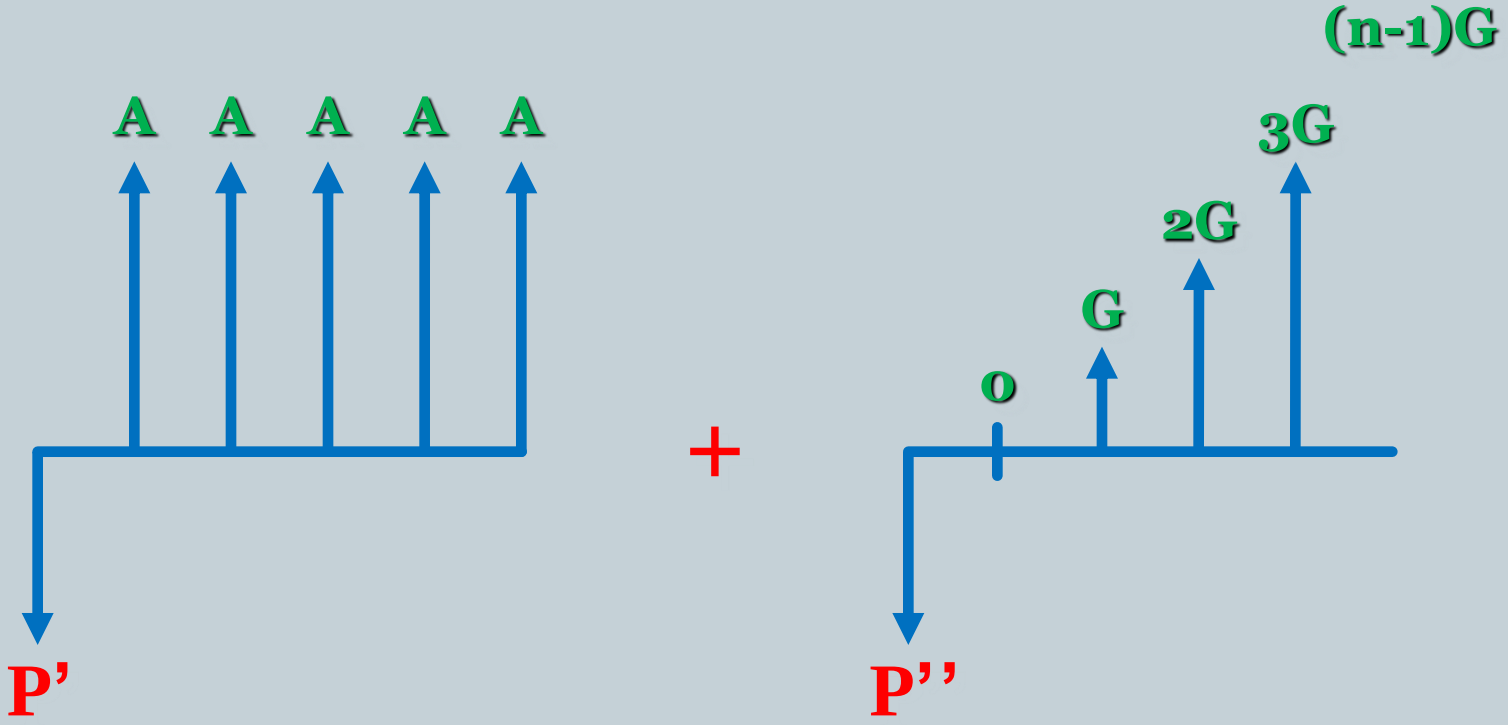
Arithmetic Gradient Series

Amount increases by "G" each period



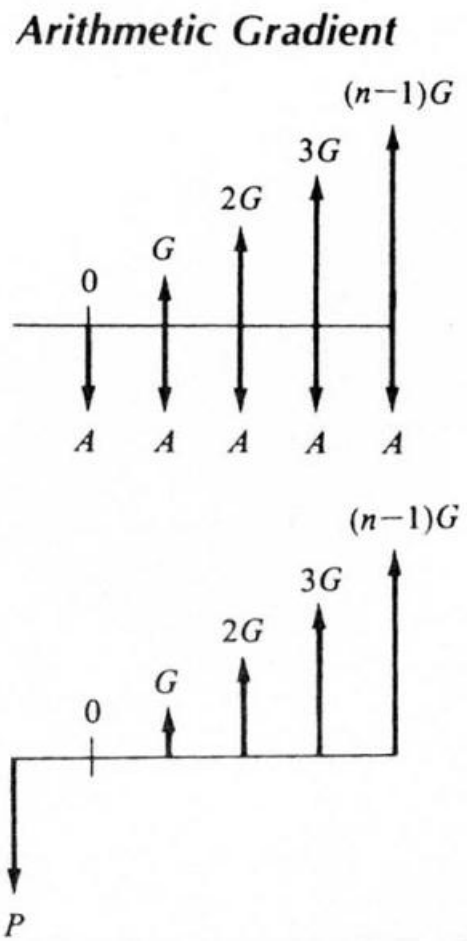
This is equivalent to

Arithmetic Gradient Series



Present worth of base amount + Present worth of gradient amount

Arithmetic Gradient



Arithmetic Gradient Uniform Series:

To Find A
Given G

$$(A/G,i,n) \qquad A = G\left[\frac{(1+i)^n - in - 1}{i(1+i)^n - i}\right]$$

or

$$A = G\left[\frac{1}{i} - \frac{n}{(1+i)^n - 1}\right]$$

Arithmetic Gradient Present Worth:

To Find P
Given G

$$(P/G,i,n) \qquad P = G\left[\frac{(1+i)^n - in - 1}{i^2(1+i)^n}\right]$$

Example

- A city department of transportation (DOT) expects cost of maintenance of a midblock to be $5,000 in the first year and increase annually by $500 until year 10. At an interest rate of 10% per year, determine the present worth of maintenance cost.

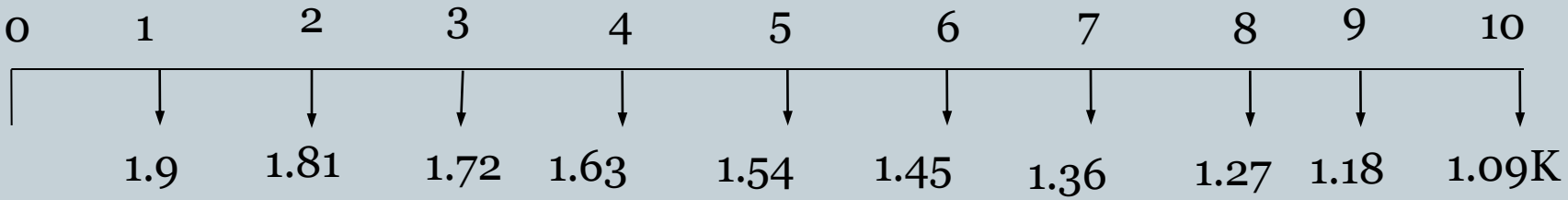
$$P = 5000 * \left(\frac{P}{A}\right)_{10\%,10} + 500 * \left(\frac{P}{G}\right)_{10\%,10}$$

= 5000*6.1446 + 500*22.8913

= $42,269

Cash Flow Diagrams

Estimate Present Worth



The Present Worth of the above Payment Plan is

$$PW = 1.9\left[PW\right]_{n=1}^{i=9} + 1.81\left[PW\right]_{n=2}^{i=9} + 1.72\left[PW\right]_{n=3}^{i=9} +$$

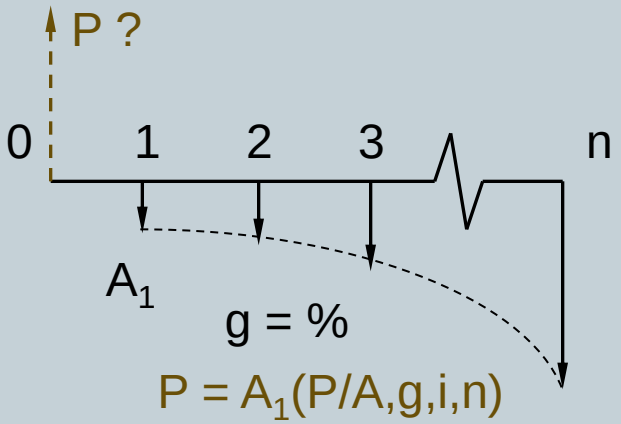
$$+ \ldots\ldots\ldots\ldots\ldots\ldots + 1.09\left[PW\right]_{n=10}^{i=9}$$

$$= 10K$$

Geometric Gradient

- ***g*** is the geometric gradient over the time period
  - (time period: Time 0 to Time n, 1st flow at Time 1)
- ***P*** is the present value of the flow at Time 0
  - (n periods in the past)
- ***i*** is the effective interest rate for each period

Note: cash flow starts with A_1 at Time 1, increases by constant g% per period



$$(P/A,g,i,n) = \begin{cases} \left[\dfrac{1-\left(\dfrac{(1+g)}{(1+i)}\right)^{n}}{i-g}\right] & when \quad i \neq g \\ \dfrac{n}{(1+i)} & when \quad i = g \end{cases}$$

Example

- A state department of transportation has four toll bridges and combined salaries obtained at the end of year 1 is $250,000. If the toll booths are expected to raise revenue 5% each year, what is the present worth of the revenue in next ten years.

$$P = 250{,}000 * \left(\frac{P}{A}\right)_{g,i,n10\%,10}$$

$$P = 250{,}000 * \left(\frac{P}{A}\right)_{5\%,10\%,10}$$

$$= 250{,}000 * 3.94005$$

$$= \$985{,}015$$

Example

- You have just begun you first job as a civil engineer and decide to participate in the company's retirement plan. You decide to invest the maximum allowed by the plan which is 6% of your salary. Your company has told you that you can expect a minimum 4% increase in salary each year assuming good performance and typical advancement within the company. Choose a realistic starting salary and estimate the following:
  - Assuming you stay with the company, the company matches your 6% investment in the retirement plan, expected minimum salary increases, and an interest rate of 10%, how much will you have in your retirement account after 40 years?

1) Assuming a starting salary of $50,000, A1 = .12 x 50,000 = 6000
*P = 6000 [(1 – (1.04)^40 * (1.1)^(-40)) / (.1 - .04)] = $89,392.18*
F = 89,392.18 (1.1^40) = $4,045,823.50

Summary of Gradient Growths

- Arithmetic gradient consists of two parts
  - A uniform series that has amount equal to the period-1
  - A gradient that has value equal to the difference of cash flow between period 1 and 2
- Gradient factor is preceded by a + sign for increasing gradient, and –ve sign for decreasing gradient
- Geometric gradients are handled just by one equation.

Basic Characteristics of Methods of Analysis

- The six methods of economic analysis (to be discussed) have the **common objective** of comparing the future streams of costs and benefits in such a way that for a specific future period of time the analysis will disclose the probable net return on the proposed investment, or the most economical design required to produce the returns.

Basic Characteristics of Methods of Analysis

- Each method applies the principles and concepts of compound interest in a way to take into the calculations the differences in the worth of money over time.

Traditional Analysis Methods

- 1. Equivalent Uniform Annual Cost Method (EUAC)
- 2. Present Worth of Costs Method (PWOC)
- 3. Equivalent Uniform Annual Net Return Method (EUANR)
- 4. Net Present Value Method (NPV)
- 5. Benefit to Cost Ratio (B/C)
- 6. Rate of Return Method (ROR)
- * Incremental Benefit-Cost Ratio
- * Cost/Effectiveness Method

Traditional Analysis Methods

1. Equivalent Uniform Annual Cost Method (EUAC)

Combines all investment costs and all annual expenses into a single annual sum that is equivalent to all disbursements during the analysis period if spread uniformly over the period

When more than one alternative is being examined, the one with the lowest equivalent uniform annual cost is the more economical

Traditional Analysis Methods

2. PRESENT WORTH OF COSTS METHOD (PWOC)

Combines all investment costs and all annual expenses into a single present-worth sum, which represents the sum necessary to finance the total disbursements over the analysis period.

Of the alternatives compared, the one with the lowest present worth of cost is the more economical.

Traditional Analysis Methods

3. Equivalent Uniform Annual Net Return Method (EUANR)

Is the equivalent uniform annual cost method plus the inclusion of an income factor or benefit factor.

The answer indicates the amount by which the equivalent uniform annual income exceeds (or is less than) the equivalent uniform annual cost.

Traditional Analysis Methods

4. Net Present Value Method (NPV)

This method gives the algebraic difference in the present worth of both cost and benefits.

The alternative having the greater net present value is the one with greatest economy.

Traditional Analysis Methods

5. Benefit to Cost Ratio (B/C)

This method expresses the ratio of equivalent uniform annual benefit (or its present worth) to the equivalent uniform annual cost (or its present worth).

Any alternative that has a benefit/cost ratio greater than 1.0 is economically feasible and the alternative that has the highest incremental benefit/cost ratio is indicated as the preference

Traditional Analysis Methods

6. Rate of Return Method (ROR)

This method determines the interest rate, which will equalize the negative costs and the positive rates returns or benefits.

While comparing the alternatives, the higher the rate of return, the greater the economy

Economic Analysis

- $EUAC = -I(CR)^{i}_{n} + T(SF)^{i}_{n} - K - U$

  I = Initial cost

  T = Terminal value

  K = Total uniform annual costs

  U = Uniform annual road user costs
- $PWOC = -I + T(PW)^{i}_{n} - K(SPW)^{i}_{n} - U(SPW)^{i}_{n}$
- $EUANR = -I(CR)^{i}_{n} + T(SF)^{i}_{n} - K + R$

  R = Uniform annual gross benefit

Economic Analysis Example

Cash Flow Item	Alternative 1	Alternative 2
Initial Investment	140,000	160,000
Terminal Value	40,000	50,000
Total O&M annual cost	7,000	8,000
Uniform Annual Road User Cost	74,000	70,000
Interest Rate	8%	8%
Analysis Period (years)	10	10

EUAC Method

$EUAC_{A1}$= -140,000 * ($CR^{i=8}_{n=10}$)+ 40,000 *
($SF^{i=8}_{n=10}$)- 7,000 - 74,000
= - 140,000 *(0.149029) + 40,000*
(0.0629) - 81,000
$EUAC_{A1}$= - 99,103

$EUAC_{A2}$= -160,000 * ($CR^{i=8}_{n=10}$)+ 50,000 *
($SF^{i=8}_{n=10}$)-8,000 - 70,000
= -160,000*(0.149029) + 50,000*
(0.069029) -78,000
$EUAC_{A2}$= - 98,394

EUAC Method

$EUAC_{A1} - EUAC_{A2} = +99{,}103 - 98{,}394$

$= 709$

A_2 Alternative has $709 less annual costs

A_2 is better than A_1.

PWOC Method

$$PWOC_{A1} = -140{,}000 + 40{,}000\,PW^{8}_{10} - 7000 * SPW^{8}_{10} - 74{,}000 * SPW^{8}_{10}$$

$$= -140{,}000 + 40{,}000*(0.46319) - 7{,}000 * (6.710081) - 74{,}000*(6.710081)$$

$$PWOC_{A1} = -664{,}989$$

PWOC Method

$PWOC_{A2} = -160{,}000 + 50{,}000\,(PW^{8}_{10}) - 8000 * (SPW^{8}_{10}) - 70{,}000 * (SPW^{8}_{10})$

$PWOC_{A2} = -660{,}227$

$PWOC_{A2} - PWOC_{A1} = -660{,}227 - (-664{,}989)$

$= \$4{,}762$

The cost of Alternative A_1 is \$4762 more than A_2

$\therefore$ A_2 is better.

Equivalent Uniform Annual Net Return Method

Alternatively A_1 and A_2 cannot be examined separately since they do not have the equivalent of a sales income.

However, reduction in Road user costs is equivalent to cash income.

$EUANR_{A2} = -(I_p - I_B)(CR^8_{10}) + (T_p - T_B) * (SF^8_{10}) - (U_p - U_B) \quad -(K_p - K_B)$

The subscripts P and B stand for proposed and base alternatives respectively.

$EUANR_{A2} = -(160K - 140K)(0.149029) +$

$(50K - 40K)(0.069029) -$

$(70\,K - 74\,K) - (8K - 7K)$

$= \$709$

$\therefore A_2$ is better.

Net Present Value Method

Similar to previous comparison

$NPV_{A2} = -(I_p - I_B) + (T_p - T_B) * (PW^8_{10}) -$

$[(U_p - U_B) + (K_p - K_B)] * (SPW^8_{10})$

$= -20\,K + 10K\,(0.463193) -$

$(-4K+1K)*(6.71008)$

$NPV_{A2} = 4{,}762$

$\therefore A_2$ is better.

B/C Ratio

$$B/C = EUAB/EUAC$$

or

$$B/C = PWOB/PWOC$$

Benefit

- Reduction in road user costs and reduction in annual expenses together is the benefit
- Initial cost and Terminal value should be considered as costs

B/C Ratio (Continued)

$EUAB = -(U_P - U_B) - (K_P - K_B)$

$= -(70{,}000 - 74{,}000) - (8{,}000 - 7{,}000)$

$EUAB = 3000$

$EUAC = -(I_p - I_B)*(CR^8_{10}) + (T_p - T_B)*(SF^8_{10})$

$= -(160K\text{-}140K)(0.149029) +$

$(50K\text{-} \quad 40K)(.069029)$

$EUAC = -2291$

$B/C = +3000/-2291 = 1.31$

This is the B/C ratio applied to alternatives A_1 and A_2.

Rate of Return Method

Assume, i = 10% on the basis of present worth

$0 = B - C$

$\therefore\ 0 = -(160{,}000 - 140{,}000) + (50{,}000 - 40{,}000)*(PW^{10}_{10}) - (70{,}000 - 74{,}000)$

$(SPW^{10}_{10}) - (8{,}000 - 7{,}000)\ (SPW^{10}_{10})$

$= -20{,}000 + 10{,}000*(0.385543) + 4{,}000(6.144567) - 1{,}000\ (6.144567)$

$= \$2{,}288$

Rate of Return Method (Continued)

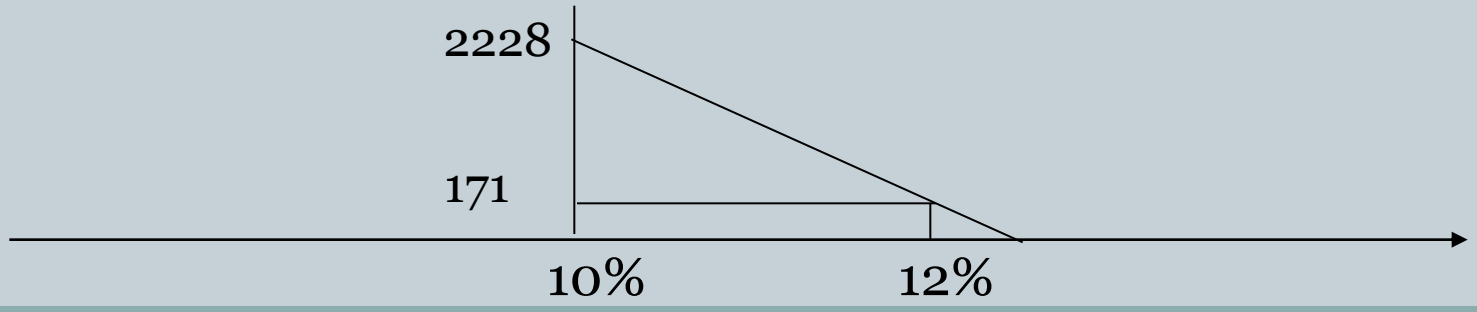
Second Iteration: Assume i = 12%

0 = -20,000 + 10,000(0.321973) +
4,000(5.65023) – 1,000 (5.65023)

= 171

By linear extrapolation, Rate of Return = 12.15%

How?



General discussion on the Solutions

The solutions indicate that the incremental investment of $20,000 in Alternative A_2 over A_1 will produce a ratio of benefits of 1.31 or a rate of return on the $20,000 of 12.1 percent

None of the solutions tests the base alternative A1 to see whether it is economically desirable. The solutions, in each case, compare alternative A_2 to alternative A_1 and determine the relative worth of A_2 over A_1.

Comparison of the Methods - Group of Mutually Exclusive Alternatives

- All the method give identical alternative as the best choice when the proper procedure of application and calculation are followed.
- Thus in this respect there is no best method of analysis so far as the final ranking of alternatives is concerned.
- The basis of choice between methods must be based on
  - The form of available data
  - Whether benefit or income amounts are available
  - The preferences of the analyst and the decision maker.

Low Cost Improvement Project Example

Traffic signals were redesigned and implemented in the following intersections in the city of Detroit.

The cost of implementation and benefits accrued in terms of accident reductions are shown in the following table. Perform a benefit-cost analysis.

Low Cost Improvement Project Example

Project Cost

Location 1 = $ 34,100

Location 2 = $ 35,200

Location 3 = $ 29,400

Total = $ 98,700

Annual Operation and Maintenance cost
= $ 1000 per Intersection per year

Assume i = 7% and n = 15 years

Low Cost Improvement Project Example

All/ Year	Before		After		Reduction	
	PDO	Injury	PDO	Injury	PDO	Injury
Location-1	49	18.67	29	6.5	20	12.17
Location-2	35	16.67	24.57	4.57	10.43	12.1
Location-3	21.67	13.33	12.83	7.01	8.84	6.32
Total					39.27	30.59

National Safety Council (NSC)

Costs

Injury = $ 34,100

PDO = $ 6,400

Annual reduction in road user costs related to traffic crashes

= 39.27(6,400) + 30.59 (34,100)

= 251,328 + 1043119

= $ 1,294,447

PWOC and PWOB

PWOC = -98,700 – (3000)*(SPW^{7}_{15})

= -98,700 – 3000 (9.1079)

= -126,023.7

PWOB = 1,294,447*(SPW^{7}_{15})

= 11,789,693

B/C = 11 ,789,693/126,023.7

B/C = 94:1

Incremental B/C Ratio

Assume that you want to pick an alternative amongst 5 solutions.

Say, the alternatives were ranked in increasing cost. The dolor costs and benefits are in thousands.

	B	C	B-C
A_1	10	2	8
A_2	20	14	6
A_3	50	25	25
A_4	80	30	50
A_5	90	45	45

Incremental B/C Ratio

The method involves examining the ratio

$$\frac{B_{i+1} - B_i}{C_{i+1} - C_i}$$

If it is > 1, then alt. A_{i+1} is accepted and compared with A_{i+2}

If it is < 1, then alt. A_i is accepted and compared with A_{i+2}

Incremental B/C Ratio

Comparison	Incremental Benefits	Incremental Costs	Inc. Benefits/ Inc. Costs	Decision in Favor of
A1 to A0	10	2	5.00	A1
A2 to A1	10	12	0.83	A1
A3 to A1	40	23	1.74	A3
A4 to A3	30	5	6.00	A4
A5 to A4	10	15	0.67	A4

Select A4

If benefits and costs are in different units, this method cannot be applied.

Nominal and Effective Rates of Interest

- Nominal and effective interest rates have similar relationship to that of simple and compound interest rates.
- The difference is that nominal and effective interest rates are used when compounding period (or interest period) is less than one year.
- Let i= interest rate per base period conversion; quoted interest rate

  r = nominal rate per annum

  j = effective rate per annum

  m= times per year, or base period, the nominal rate is converted

Effective Rates of Interest

- Let i= interest rate per base period conversion; quoted interest rate

  r = nominal rate per annum

  j = effective rate per annum

  m= times per year, or base period, the nominal rate is converted

Effective Rates of Interest

Since $i = \frac{r}{m}$

Effective Interest Rate $j = \left[1 + \frac{r}{m}\right]^m - 1$

Example:

Find the effective rate of interest for \$100 for 1 year at nominal interest of 12% per year, interest payable monthly:

$$F = 100 * \left[1 + \frac{0.12}{12}\right]^{12} = 100 * (1.01)^{12} = 112.6825$$

$$\text{Effective Interest Rate} = (1 + .01)^{12} - 1 = 0.1268 = 12.68\%$$

Example

- A bank pays 6% nominal interest rate. Calculate the effective interest with
- a) monthly, b) daily, c) hourly d) secondly compounding
  - $i = (1 + r/m)^m - 1$
  - i monthly $= (1 + .06/12)^{12} - 1 = 6.1678\ \%$
  - i daily $= (1 + .06/365)^{365} - 1 = 6.183\ \%$
  - i hourly $= (1 + .06/8760)^{8760} - 1 = 6.1836\ \%$
  - i secondly $= (1 + .06/31.5M)^{31.5M} - 1 = 6.18365\ \%$

Engineering Economy in Highways

- Construction should be planned with an eye for the future
- Roads should be built only to the extent and of such types as will pay themselves.
- There must be enough traffic and type of improvement shall be such that the savings in cost of transportation is at least equal to the cost of improvement.

Basic Premise of Transportation Economics

1. Instinctive desire to save
    - Save for future use
    - Save for different use
2. Conservation of commodities
    - Future use
3. Conservation of Labor
    - Alternative use

Basic Premise of Engineering (Continued)

4. Long range result of conservation of resources
    - Growth with least amount of resources
5. Public versus Private
    - Public viewpoint - Welfare of everyone
    - Private viewpoint - Welfare of one

Principles of Analysis

1. Complete Objectivity
    - Selection of Factors
    - Selection of Cost
    - Selection of Vest Charge

2. Economic analysis is not a management decision

3. “Hunch” has no place in economic analysis

4. Study all possible alternatives

Principles of Analysis (Cont..)

5. Always consider the “Do Nothing” alternative

6. Separate market and non-market factors

   - Factors of general socio-economic consequences are excluded from calculations

7. The analysis is a study of future conditions

   - Careful forecasting is necessary

Principles of Analysis (Cont..)

8. Past events and investments are irrelevant.

9. Use same time periods for all factors

10. Analysis period should not extend beyond the period of reliable forecasts.

11. Same time frame for all factors

12. Differences in alternatives are controlling

13. Common factors of equal magnitude may be omitted

Principles of Analysis (Cont..)

14. Use the net basis for all costs and consequences

15. Analysis for economy is independent of financing

16. Uncertainties need to be acknowledged

17. Separate decisions are made at separate levels of management

18. Viewpoints should be established before final decisions are made

Principles of Analysis (Cont..)

19. Establish criteria for decision making

20. Consider all consequences to whomsoever they may accrue

21. Final decision should also consider market factors

Methods of Economic Analysis

- Economic Analysis is an art and not an exact science
- Every method can not be applied to every different types of proposal, an understanding of the characteristics and limitations of each of the methods is essential for the analyst.
- When properly applied in accordance with their limitations, each method will give a reliable result.

Methods of Economic Analysis

- Economic Analysis is more commonly being used as a management tool for decision making

- Economic analysis is performed for
  - Project Evaluation
  - Project Formulation
  - Project Priority Selection
  - Project Justification (Relative to other projects)- public financial policy