Transportation Economics and Decision Making
Arithmetic Gradient Series

Amount increases by “G” each period

This is equivalent to
Arithmetic Gradient Series

Present worth of base amount + Present worth of gradient amount
Arithmetic Gradient

Arithmetic Gradient Uniform Series:

To Find $A$

Given $G$

\[
(A/G, i, n) \quad A = G \left[ \frac{(1+i)^n - in - 1}{i(1+i)^n - i} \right]
\]

or

\[
A = G \left[ \frac{1}{i} - \frac{n}{(1+i)^n - 1} \right]
\]

Arithmetic Gradient Present Worth:

To Find $P$

Given $G$

\[
(P/G, i, n) \quad P = G \left[ \frac{(1+i)^n - in - 1}{i^2(1 + i)^n} \right]
\]
A city department of transportation (DOT) expects cost of maintenance of a midblock to be $5,000 in the first year and increase annually by $500 until year 10. At an interest rate of 10% per year, determine the present worth of maintenance cost.

\[ P = 5000 \times \left( \frac{P}{A} \right)_{10\%,10} + 500 \times \left( \frac{P}{G} \right)_{10\%,10} \]

\[ = 5000 \times 6.1446 + 500 \times 22.8913 \]

\[ = $42,269 \]
The Present Worth of the above Payment Plan is

\[ PW = 1.9[PW]_{n=1}^{i=9} + 1.81[PW]_{n=2}^{i=9} + 1.72[PW]_{n=3}^{i=9} + \ldots + 1.09[PW]_{n=10}^{i=9} \]

\[ = 10K \]
**Geometric Gradient**

- $g$ is the geometric gradient over the time period
  - (time period: Time 0 to Time n, 1st flow at Time 1)
- $P$ is the present value of the flow at Time 0
  - (n periods in the past)
- $i$ is the effective interest rate for each period

Note: cash flow starts with $A_1$ at Time 1, increases by constant $g\%$ per period

\[ P = A_1 \left( \frac{P}{A, g, i, n} \right) \]

\[
(P/A, g, i, n) = \begin{cases} 
1 - \left( \frac{1 + g}{1 + i} \right)^n & \text{when } i \neq g \\
\frac{n}{(1 + i)} & \text{when } i = g 
\end{cases}
\]
A state department of transportation has four toll bridges and combined salaries obtained at the end of year 1 is $250,000. If the toll booths are expected to raise revenue 5% each year, what is the present worth of the revenue in next ten years.

\[
P = 250,000 \left( \frac{P}{A} \right)_{g,i,n10\%,10} \\
P = 250,000 \left( \frac{P}{A} \right)_{5\%,10\%,10} \\
= 250,000 \times 3.94005 \\
= $985,015
\]
Example

- You have just begun your first job as a civil engineer and decide to participate in the company’s retirement plan. You decide to invest the maximum allowed by the plan which is 6% of your salary. Your company has told you that you can expect a minimum 4% increase in salary each year assuming good performance and typical advancement within the company. Choose a realistic starting salary and estimate the following:
  - Assuming you stay with the company, the company matches your 6% investment in the retirement plan, expected minimum salary increases, and an interest rate of 10%, how much will you have in your retirement account after 40 years?

1) Assuming a starting salary of $50,000, \[ A1 = 0.12 \times 50,000 = 6000 \]
\[ P = 6000 \left[ \frac{(1 - (1.04)^{40} \times (1.1)^{40})}{(1 - .04)} \right] = $89,392.18 \]
\[ F = 89,392.18 (1.1^{40}) = $4,045,823.50 \]
Summary of Gradient Growths

- **Arithmetic gradient consists of two parts**
  - A uniform series that has amount equal to the period-1
  - A gradient that has value equal to the difference of cash flow between period 1 and 2
- **Gradient factor is preceded by a + sign for increasing gradient, and –ve sign for decreasing gradient**
- **Geometric gradients are handled just by one equation.**
The six methods of economic analysis (to be discussed) have the **common objective** of comparing the future streams of costs and benefits in such a way that for a specific future period of time the analysis will disclose the probable net return on the proposed investment, or the most economical design required to produce the returns.
Basic Characteristics of Methods of Analysis

- Each method applies the principles and concepts of compound interest in a way to take into the calculations the differences in the worth of money over time.
Traditional Analysis Methods

- 1. Equivalent Uniform Annual Cost Method (EUAC)
- 2. Present Worth of Costs Method (PWOC)
- 3. Equivalent Uniform Annual Net Return Method (EUANR)
- 4. Net Present Value Method (NPV)
- 5. Benefit to Cost Ratio (B/C)
- 6. Rate of Return Method (ROR)
  - * Incremental Benefit-Cost Ratio
  - * Cost/Effectiveness Method
Traditional Analysis Methods

1. Equivalent Uniform Annual Cost Method (EUAC)

Combines all investment costs and all annual expenses into a single annual sum that is equivalent to all disbursements during the analysis period if spread uniformly over the period.

When more than one alternative is being examined, the one with the lowest equivalent uniform annual cost is the more economical.
2. PRESENT WORTH OF COSTS METHOD (PWOC)

Combines all investment costs and all annual expenses into a single present-worth sum, which represents the sum necessary to finance the total disbursements over the analysis period.

Of the alternatives compared, the one with the lowest present worth of cost is the more economical.
3. Equivalent Uniform Annual Net Return Method (EUANR)

Is the equivalent uniform annual cost method plus the inclusion of an income factor or benefit factor.

The answer indicates the amount by which the equivalent uniform annual income exceeds (or is less than) the equivalent uniform annual cost.
4. Net Present Value Method (NPV)

This method gives the algebraic difference in the present worth of both cost and benefits.

The alternative having the greater net present value is the one with greatest economy.
5. Benefit to Cost Ratio (B/C)

This method expresses the ratio of equivalent uniform annual benefit (or its present worth) to the equivalent uniform annual cost (or its present worth).

Any alternative that has a benefit/cost ratio greater than 1.0 is economically feasible and the alternative that has the highest incremental benefit/cost ratio is indicated as the preference.
6. Rate of Return Method (ROR)

This method determines the interest rate, which will equalize the negative costs and the positive rates returns or benefits.

While comparing the alternatives, the higher the rate of return, the greater the economy.
Economic Analysis

- **EUAC** = \(-I(CR)_n + T(SF)_n - K - U\)
  - I = Initial cost
  - T = Terminal value
  - K = Total uniform annual costs
  - U = Uniform annual road user costs

- **PWOC** = \(-I + T(PW)_n - K(SPW)_n - U(SPW)_n\)

- **EUANR** = \(-I(CR)_n + T(SF)_n - K + R\)
  - R = Uniform annual gross benefit
## Economic Analysis Example

<table>
<thead>
<tr>
<th>Cash Flow Item</th>
<th>Alternative 1</th>
<th>Alternative 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Investment</td>
<td>140,000</td>
<td>160,000</td>
</tr>
<tr>
<td>Terminal Value</td>
<td>40,000</td>
<td>50,000</td>
</tr>
<tr>
<td>Total O&amp;M annual cost</td>
<td>7,000</td>
<td>8,000</td>
</tr>
<tr>
<td>Uniform Annual Road User Cost</td>
<td>74,000</td>
<td>70,000</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>8%</td>
<td>8%</td>
</tr>
<tr>
<td>Analysis Period (years)</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>
EUAC\textsubscript{A1} = -140,000 \times (CR\textsubscript{n=10}^{i=8}) + 40,000 \times (SF\textsubscript{n=10}^{i=8}) - 7,000 - 74,000

= -140,000 \times (0.149029) + 40,000 \times (0.0629) - 81,000

EUAC\textsubscript{A1} = -99,103

EUAC\textsubscript{A2} = -160,000 \times (CR\textsubscript{n=10}^{i=8}) + 50,000 \times (SF\textsubscript{n=10}^{i=8}) - 8,000 - 70,000

= -160,000 \times (0.149029) + 50,000 \times (0.069029) - 78,000

EUAC\textsubscript{A2} = -98,394
EUAC Method

\[ \text{EUAC}_{A_1} - \text{EUAC}_{A_2} = +99,103 - 98,394 = 709 \]

\( A_2 \) Alternative has $709 less annual costs

\( A_2 \) is better than \( A_1 \).
PWOC_{A1} = -140,000 + 40,000 \text{ PW}^{8}_{10} - 7000 \text{ SPW}^{8}_{10} - 74,000 \text{ SPW}^{8}_{10} \\
= -140,000 + 40,000 \times (0.46319) - 7,000 \times (6.710081) - 74,000 \times (6.710081) \\
PWOC_{A1} = -664,989
**PWOC Method**

\[
\text{PWOC}_{A2} = - 160,000 + 50,000 (PW_{810}) - 8000 \times (SPW_{810}) - 70,000 \times (SPW_{810})
\]

\[
\text{PWOC}_{A2} = -660,227
\]

\[
\text{PWOC}_{A2} - \text{PWOC}_{A1} = -660,227 - (-664,989)
\]

\[
= $4,762
\]

The cost of Alternative A\(_1\) is $4762 more than A\(_2\).

\[\therefore A_2\] is better.
Alternatively $A_1$ and $A_2$ cannot be examined separately since they do not have the equivalent of a sales income. However, reduction in Road user costs is equivalent to cash income.

$$EUANR_{A2} = -(I_p - I_B) (CR_{10}^8) + (T_p - T_B) \times (SF_{10}^8) - (U_p - U_B) - (K_p - K_B)$$

The subscripts P and B stand for proposed and base alternatives respectively.

$$EUANR_{A2} = - (160K - 140K) (0.149029) + (50K - 40K) (0.069029) - (70K - 74K) - (8K - 7K) = $709$$

$\therefore A_2$ is better.
Net Present Value Method

Similar to previous comparison

\[ NPV_{A2} = -(I_p - I_B) + (T_p - T_B) \times (PW_{10}^{8}) - \\
\quad [ (U_p - U_B) + (K_p - K_B) ] \times (SPW_{10}^{8}) \]

\[ = -20K + 10K \times (0.463193) - \\
\quad (-4K + 1K) \times (6.71008) \]

\[ NPV_{A2} = 4,762 \]

\[ \therefore A_2 \text{ is better.} \]
**B/C Ratio**

\[ B/C = \frac{EUAB}{EUAC} \]

or

\[ B/C = \frac{PWOB}{PWOC} \]

**Benefit**

- Reduction in road user costs and reduction in annual expenses together is the benefit

- Initial cost and Terminal value should be considered as costs
B/C Ratio (Continued)

\[
EUAB = -(U_P - U_B) - (K_P - K_B)
\]

\[
= -(70,000 - 74,000) - (8,000 - 7,000)
\]

EUAB = 3000

\[
EUAC = -(I_P - I_B)(CR_{10}) + (T_P - T_B)(SF_{10})
\]

\[
= -(160K-140K)(0.149029) +
\]

\[
(50K-40K)(.069029)
\]

EUAC = - 2291

B/C = + 3000/-2291 = 1.31

This is the B/C ratio applied to alternatives A₁ and A₂.
Rate of Return Method

Assume, $i = 10\%$ on the basis of present worth

$$ 0 = B - C $$

$$ \therefore 0 = -(160,000 - 140,000) + (50,000 - 40,000)(PW_{10}^{10}) - (70,000 - 74,000) $$

$$ (SPW_{10}^{10}) - (8,000 - 7,000)(SPW_{10}^{10}) $$

$$ = -20,000 + 10,000*(0.385543) + 4,000(6.144567) - 1,000 (6.144567) $$

$$ = $2,288 $$
Rate of Return Method (Continued)

Second Iteration: Assume i = 12%

\[ 0 = -20,000 + 10,000(0.321973) + 4,000(5.65023) - 1,000(5.65023) \]

\[ = 171 \]

By linear extrapolation, Rate of Return = 12.15%

How?
The solutions indicate that the incremental investment of $20,000 in Alternative A₂ over A₁ will produce a ratio of benefits of 1.31 or a rate of return on the $20,000 of 12.1 percent.

None of the solutions tests the base alternative A₁ to see whether it is economically desirable. The solutions, in each case, compare alternative A₂ to alternative A₁ and determine the relative worth of A₂ over A₁.
Comparison of the Methods - Group of Mutually Exclusive Alternatives

- All the methods give identical alternative as the best choice when the proper procedure of application and calculation are followed.
- Thus in this respect there is no best method of analysis so far as the final ranking of alternatives is concerned.
- The basis of choice between methods must be based on
  - The form of available data
  - Whether benefit or income amounts are available
  - The preferences of the analyst and the decision maker.
Traffic signals were redesigned and implemented in the following intersections in the city of Detroit.

The cost of implementation and benefits accrued in terms of accident reductions are shown in the following table. Perform a benefit-cost analysis.
Low Cost Improvement Project Example

**Project Cost**

Location 1 = $34,100  
Location 2 = $35,200  
Location 3 = $29,400  
Total = $98,700

Annual Operation and Maintenance cost = $1000 per Intersection per year

Assume i = 7% and n = 15 years
## Low Cost Improvement Project Example

<table>
<thead>
<tr>
<th>All/ Year</th>
<th>Before</th>
<th>After</th>
<th>Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>PDO</td>
<td>Injury</td>
<td>PDO</td>
<td>Injury</td>
</tr>
<tr>
<td>Location-1</td>
<td>49</td>
<td>18.67</td>
<td>29</td>
</tr>
<tr>
<td>Location-2</td>
<td>35</td>
<td>16.67</td>
<td>24.57</td>
</tr>
<tr>
<td>Location-3</td>
<td>21.67</td>
<td>13.33</td>
<td>12.83</td>
</tr>
<tr>
<td>Total</td>
<td>39.27</td>
<td>30.59</td>
<td></td>
</tr>
</tbody>
</table>

Reduction in injuries and PDO for different locations.
National Safety Council (NSC)

Costs

Injury = $34,100
PDO = $6,400

Annual reduction in road user costs related to traffic crashes

\[ = 39.27(6,400) + 30.59(34,100) \]

\[ = 251,328 + 1,043,119 \]

\[ = $1,294,447 \]
PWOC and PWOB

PWOC = -98,700 − (3000)∗(SPW^{7}_{15})
   = -98,700 − 3000 (9.1079)
   = -126,023.7

PWOB = 1,294,447∗(SPW^{7}_{15})
   = 11,789,693

B/C = 11,789,693/126,023.7
   B/C = 94:1
Incremental B/C Ratio

Assume that you want to pick an alternative amongst 5 solutions.

Say, the alternatives were ranked in increasing cost. The dolor costs and benefits are in thousands.

<table>
<thead>
<tr>
<th></th>
<th>B</th>
<th>C</th>
<th>B-C</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>10</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>A2</td>
<td>20</td>
<td>14</td>
<td>6</td>
</tr>
<tr>
<td>A3</td>
<td>50</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>A4</td>
<td>80</td>
<td>30</td>
<td>50</td>
</tr>
<tr>
<td>A5</td>
<td>90</td>
<td>45</td>
<td>45</td>
</tr>
</tbody>
</table>
The method involves examining the ratio
\[
\frac{B_{i+1} - B_i}{C_{i+1} - C_i}
\]
If it is > 1, then alt. \(A_{i+1}\) is accepted and compared with \(A_{i+2}\)
If it is < 1, then alt. \(A_i\) is accepted and compared with \(A_{i+2}\)
## Incremental B/C Ratio

<table>
<thead>
<tr>
<th>Comparison</th>
<th>Incremental Benefits</th>
<th>Incremental Costs</th>
<th>Inc. Benefits/ Inc. Costs</th>
<th>Decision in Favor of</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1 to A0</td>
<td>10</td>
<td>2</td>
<td>5.00</td>
<td>A1</td>
</tr>
<tr>
<td>A2 to A1</td>
<td>10</td>
<td>12</td>
<td>0.83</td>
<td>A1</td>
</tr>
<tr>
<td>A3 to A1</td>
<td>40</td>
<td>23</td>
<td>1.74</td>
<td>A3</td>
</tr>
<tr>
<td>A4 to A3</td>
<td>30</td>
<td>5</td>
<td>6.00</td>
<td>A4</td>
</tr>
<tr>
<td>A5 to A4</td>
<td>10</td>
<td>15</td>
<td>0.67</td>
<td>A4</td>
</tr>
</tbody>
</table>

Select A4

If benefits and costs are in different units, this method cannot be applied.
Nominal and Effective Rates of Interest

- Nominal and effective interest rates have similar relationship to that of simple and compound interest rates.
- The difference is that nominal and effective interest rates are used when compounding period (or interest period) is less than one year.
- Let $i =$ interest rate per base period conversion; quoted interest rate
  - $r =$ nominal rate per annum
  - $j =$ effective rate per annum
  - $m =$ times per year, or base period, the nominal rate is converted
Effective Rates of Interest

- Let $i =$ interest rate per base period conversion; quoted interest rate
  - $r =$ nominal rate per annum
  - $j =$ effective rate per annum
  - $m =$ times per year, or base period, the nominal rate is converted
Effective Rates of Interest

Since \( i = \frac{r}{m} \)

Effective Interest Rate

\[ j = \left[ 1 + \frac{r}{m} \right]^m - 1 \]

Example:
Find the effective rate of interest for $100 for 1 year at nominal interest of 12% per year, interest payable monthly:

\[ F = 100 \times \left[ 1 + \frac{0.12}{12} \right]^{12} = 100 \times (1.01)^{12} = 112.6825 \]

Effective Interest Rate = \((1 + .01)^{12} - 1 = 0.1268 = 12.68\%\)
Example

- A bank pays 6% nominal interest rate. Calculate the effective interest with:
  - a) monthly, b) daily, c) hourly d) secondly compounding

- $i = (1 + \frac{r}{m})^m - 1$
- $i$ monthly = $(1 + \frac{.06}{12})^{12} - 1 = 6.1678\%$
- $i$ daily = $(1 + \frac{.06}{365})^{365} - 1 = 6.183\%$
- $i$ hourly = $(1 + \frac{.06}{8760})^{8760} - 1 = 6.1836\%$
- $i$ secondly = $(1 + \frac{.06}{31.5M})^{31.5M} - 1 = 6.18365\%$
Engineering Economy in Highways

- Construction should be planned with an eye for the future
- Roads should be built only to the extent and of such types as will pay themselves.
- There must be enough traffic and type of improvement shall be such that the savings in cost of transportation is at least equal to the cost of improvement.
Basic Premise of Transportation Economics

1. Instinctive desire to save
   - Save for future use
   - Save for different use
2. Conservation of commodities
   - Future use
3. Conservation of Labor
   - Alternative use
4. Long range result of conservation of resources
   - Growth with least amount of resources
5. Public versus Private
   - Public viewpoint - Welfare of everyone
   - Private viewpoint - Welfare of one
Principles of Analysis

1. Complete Objectivity
   - Selection of Factors
   - Selection of Cost
   - Selection of Vest Charge

2. Economic analysis is not a management decision

3. “Hunch” has no place in economic analysis

4. Study all possible alternatives
5. Always consider the “Do Nothing” alternative

6. Separate market and non-market factors
   - Factors of general socio-economic consequences are excluded from calculations

7. The analysis is a study of future conditions
   - Careful forecasting is necessary
8. Past events and investments are irrelevant.

9. Use same time periods for all factors

10. Analysis period should not extend beyond the period of reliable forecasts.

11. Same time frame for all factors

12. Differences in alternatives are controlling

13. Common factors of equal magnitude may be omitted
14. Use the net basis for all costs and consequences

15. Analysis for economy is independent of financing

16. Uncertainties need to be acknowledged

17. Separate decisions are made at separate levels of management

18. Viewpoints should be established before final decisions are made
19. Establish criteria for decision making

20. Consider all consequences to whomsoever they may accrue

21. Final decision should also consider market factors
Methods of Economic Analysis

- Economic Analysis is an art and not an exact science.
- Every method cannot be applied to every different type of proposal, an understanding of the characteristics and limitations of each of the methods is essential for the analyst.
- When properly applied in accordance with their limitations, each method will give a reliable result.
Methods of Economic Analysis

- Economic Analysis is more commonly being used as a management tool for decision making.

- Economic analysis is performed for:
  - Project Evaluation
  - Project Formulation
  - Project Priority Selection
  - Project Justification (Relative to other projects)- public financial policy