The Present Worth of the above Payment Plan is

\[ PW = 0.9 [SPW]_{n=10}^i + 10 [PW]_{n=10}^i \]

\[ = 0.9 \times 5.995 + 10 \times 0.461 \]

\[ = 10 \text{ K} \]
The Present Worth of the above Payment Plan is

\[PW = 1.9[PW]_{n=1}^{i=9} + 1.81[PW]_{n=2}^{i=9} + 1.72[PW]_{n=3}^{i=9} +
\]

\[+ \ldots \ldots + 1.09[PW]_{n=10}^{i=9}\]

\[= 10K\]
Estimate Present Worth

The Present Worth of the above Payment Plan is

\[
PW = 1558.2 \left[ SPW \right]_{i=9}^{n=10} \\
= 1558.2 \times 6.418 \\
= 10K
\]
Estimate Present Worth

\[ PW = 23675 \left[ PW \right]_{i=9}^{n=10} \]
\[ = 23675 \times 0.4224 \]
\[ = 10K \]
The Present Worth of the above Payment Plan is

\[ PW = 3881 [PW]_{i=9}^{n=2} + 3504 [PW]_{i=9}^{n=4} + 3129 [PW]_{i=9}^{n=6} + \]
\[ + \ldots + 2376 [PW]_{i=9}^{n=10} \]
\[ = 3881 \times 0.8417 + 3504 \times 0.7084 + 3129 \times 0.5963 + \]
\[ + \ldots + 2376 \times 0.4224 \]
\[ = 10K \]
Estimate Future Value

Let \( i = 10\% \)

\[
F = 10\left[CA\right]_{n=20}^{i=10} + R_1\left[SCA\right]_{n=8}^{i=10} \times \left[CA\right]_{n=10}^{i=10} + \\
R_2\left[SCA\right]_{n=5}^{i=10} \times \left[CA\right]_{n=5}^{i=10}
\]

\[
= 10 \times 6.728 + 2 \times 11.436 \times 2.594 + 3 \times 6.105 \times 1.611
\]

\[
= 156.11 \text{ K}
\]
Cash Flow Diagrams

Estimate Present Worth

Let \( i = 10\% \)

\[
PW = 10 + R_1 \left[ SPW \right]_{i=10}^{n=8} \times \left[ PW \right]_{i=10}^{n=2} + R_2 \left[ SPW \right]_{i=10}^{n=5} \times \left[ PW \right]_{i=10}^{n=10}
\]

\[
= 10 + 2 \times 5.3349 \times 0.8265 + 3 \times 3.7908 \times 0.3856
\]

\[
= 23.204 \text{ K}
\]
Let \( i = 10\% \)

\[
PW = 23.204K \text{ (As calculated before)}
\]

This present value is equivalent to annuity of

\[
A = P\left[CR\right]_{i=10}^{n=20} = 23.204 \times 0.1175
\]

\[
= 2.72647 \text{ K}
\]

This is also similar to Equivalent Uniform Annual Cost (EUAC)
Arithmetic Gradient Series

Amount increases by “G” each period

This is equivalent to

\[ A + (n-1)G \]
Arithmetic Gradient Series

Present worth of base amount + Present worth of gradient amount

\[ P' + (n-1)G + 2G + 3G \]
Arithmetic Gradient Uniform Series:

To Find $A$

Given $G$

$$\left(\frac{A}{G},i,n\right) \quad A = G \left[ \frac{(1+i)^n - in - 1}{i(1+i)^n - i} \right]$$

or

$$A = G \left[ \frac{1}{i} - \frac{n}{(1+i)^n - 1} \right]$$

Arithmetic Gradient Present Worth:

To Find $P$

Given $G$

$$\left(\frac{P}{G},i,n\right) \quad P = G \left[ \frac{(1+i)^n - in - 1}{i^2(1+i)^n} \right]$$
A city department of transportation (DOT) expects cost of maintenance of a midblock to be $5,000 in the first year and increase annually by $500 until year 10. At an interest rate of 10% per year, determine the present worth of maintenance cost.

\[
P = 5000 \times \left( \frac{P}{A} \right)_{10\%,10} + 500 \times \left( \frac{P}{G} \right)_{10\%,10}
= 5000 \times 6.1446 + 500 \times 22.8913
= $42,269
\]
Geometric Gradient

- **g** is the geometric gradient over the time period
  - (time period: Time 0 to Time n, 1st flow at Time 1)
- **P** is the present value of the flow at Time 0
  - (n periods in the past)
- **i** is the effective interest rate for each period

Note: cash flow starts with $A_1$ at Time 1, increases by constant g% per period

\[
P = A_1 \cdot (P/A, g, i, n)
\]

\[
(P/A, g, i, n) = \begin{cases} 
\frac{1 - \left(\frac{1 + g}{1 + i}\right)^n}{i - g} & \text{when } i \neq g \\
\frac{n}{(1+i)} & \text{when } i = g
\end{cases}
\]
A state department of transportation has four toll bridges and combined salaries obtained at the end of year 1 is $250,000. If the toll booths are expected to raise revenue 5% each year, what is the present worth of the revenue in next five years.

\[
P = 250,000 \times \left( \frac{P}{A} \right)_{g,i,n10\%,10}
\]

\[
P = 250,000 \times \left( \frac{P}{A} \right)_{5\%,10\%,10}
\]

\[
= 250,000 \times 3.94005
\]

\[
= $985,015
\]
You have just begun your first job as a civil engineer and decide to participate in the company’s retirement plan. You decide to invest the maximum allowed by the plan which is 6% of your salary. Your company has told you that you can expect a minimum 4% increase in salary each year assuming good performance and typical advancement within the company. Choose a realistic starting salary and estimate the following:

Assuming you stay with the company, the company matches your 6% investment in the retirement plan, expected minimum salary increases, and an interest rate of 10%, how much will you have in your retirement account after 40 years?

1) Assuming a starting salary of $50,000, \( A1 = 0.12 \times 50,000 = 6000 \)
\[ P = 6000 \left[ (1 -(1.04)^{40} \times (1.1)^{-40}) / (0.1 - 0.04) \right] = \$89,392.18 \]
\[ F = 89,392.18 \times (1.1)^{40} = \$4,045,823.50 \]
Summary of Gradient Growths

- **Arithmetic gradient consists of two parts**
  - A uniform series that has amount equal to the period-1
  - A gradient that has value equal to the difference of cash flow between period 1 and 2

- **Gradient factor is preceded by a + sign for increasing gradient, and –ve sign for decreasing gradient**

- **Geometric gradients are handled just by one equation.**
Nominal and Effective Rates of Interest

- Interest rates are nominally quoted on bank and other loans on the basis of interest per one year, even though the interest may be paid monthly, quarterly or twice yearly.

- The interest rate per annum is the nominal rate; the effective rate is that rate corresponding to compounding the interest for the conversion periods less than a year.
Effective Rates of Interest

- When the interest is paid more frequently than indicated by the time period attached to the quoted interest rate, the effective rate will be higher than the nominal (quoted) rate.

- Let $i =$ interest rate per base period conversion; quoted interest rate
  $r =$ nominal rate per annum
  $j =$ effective rate per annum
  $m =$ times per year, or base period, the nominal rate is converted
Effective Rates of Interest

Since \( i = \frac{r}{m} \)

Effective Interest Rate \( j = \left[ 1 + \frac{r}{m} \right]^m - 1 \)

Example:
Find the effective rate of interest for $100 for 1 year at nominal interest of 12% per year, interest payable monthly:

\[
F = 100 \times \left[ 1 + \frac{0.12}{12} \right]^{12} = 100 \times (1.01)^{12} = 112.6825
\]

Effective Interest Rate = \((1 + .01)^{12} - 1 = 0.1268 = 12.68\%\)
Example

- A bank pays 6% nominal interest rate. Calculate the effective interest with
- a) monthly, b) daily, c) hourly d) secondly compounding

- $i = (1 + i)^m - 1$
- $i$ monthly = $(1 + .06/12)^{12} - 1 = 6.1678\%$
- $i$ daily = $(1 + .06/365)^{365} - 1 = 6.183\%$
- $i$ hourly = $(1 + .06/8760)^{8760} - 1 = 6.1836\%$
- $i$ secondly = $(1 + .06/31.5M)^{31.5M} - 1 = 6.18365\%$
Construction should be planned with an eye for the future.

Roads should be built only to the extent and of such types as will pay themselves.

There must be enough traffic and type of improvement shall be such that the savings in cost of transportation is at least equal to the cost of improvement.
Basic Premise of Transportation Economics

1. Instinctive desire to save
   - Save for future use
   - Save for different use

2. Conservation of commodities
   - Future use

3. Conservation of Labor
   - Alternative use
4. Long range result of conservation of resources
   - Growth with least amount of resources
5. Public versus Private
   - Public viewpoint - Welfare of everyone
   - Private viewpoint - Welfare of one
Principles of Analysis

1. Complete Objectivity
   - Selection of Factors
   - Selection of Cost
   - Selection of Vest Charge

2. Economic analysis is not a management decision

3. “Hunch” has no place in economic analysis

4. Study all possible alternatives
5. Always consider the “Do Nothing” alternative

6. Separate market and non-market factors
   - Factors of general socio-economic consequences are excluded from calculations

7. The analysis is a study of future conditions
   - Careful forecasting is necessary
8. Past events and investments are irrelevant.

9. Use same time periods for all factors

10. Analysis period should not extend beyond the period of reliable forecasts.

11. Same time frame for all factors

12. Differences in alternatives are controlling

13. Common factors of equal magnitude may be omitted
14. Use the net basis for all costs and consequences

15. Analysis for economy is independent of financing

16. Uncertainties need to be acknowledged

17. Separate decisions are made at separate levels of management

18. Viewpoints should be established before final decisions are made
19. Establish criteria for decision making

20. Consider all consequences to whomsoever they may accrue

21. Final decision should also consider market factors