Transportation Economics and Decision Making

Lecture-12

Risk and Uncertainty

- Risk and uncertainty are very different looking animals but they are of the same species
  - The line of demarcation is often blurred
- A distinction is critical
- A story on sky diving

Difference example

- Example
  - Suppose you play a coin toss game
  - If heads come up you win $1, otherwise you loose everything
  - The uncertainty is that tails may appear; the risk is that you may loose everything
  - Uncertainty is the possibility of occurrence of an event; and risk is the ramification of loosing everything
  - Types
    - Unknown
    - Unknowable

Risk is the outcome of uncertainty

- The concept of risk and uncertainty are related but different.
- Uncertainty involves variables that are known and changing, but its uncertainty will become known and resolved through passage of time, events and action.
- Risk is something one bears and is the outcome of uncertainty

Why Risk is Important

- Example: safety alternatives

Project Name	Cost	Returns	Risk
X	50	50	25
Y	250	200	200
Z	100	100	10

Why Risk is Important (2)

- With a $1,00 budget

Project X:20 Project Xs returning $1,000, with $500 risk
Project Y: 4 Project Xs returning $800, with $800 risk
Project Z: 10 Project Xs returning $1,000, with $100 risk

Project X: For each $1 return, $0.5 risk is taken
Project Y: For each $1 return, $1.0 risk is taken
Project Z: For each $1 return, $0.1 risk is taken

Project X: For each $1 of risk taken, $2 return is obtained
Project Y: For each $1 of risk taken, $1 return is obtained
Project Z: For each $1 of risk taken, $10 return is obtained

Conclusion: Risk is important. Ignoring risks results in making the wrong decision.

Certainty, Risk, and Uncertainty

- Certainty – Everything know for sure; not present in the real world of estimation, but can be 'assumed'
- Risk – a decision making situation where all of the outcomes are know and the associated probabilities are defined
- Uncertainty – One has two or more observable values but the probabilities associated with the values are unknown
  - Observable values – states of nature

Types of Decision Making

- Decision Making under Certainty
  - Process of making a decision where all of the input parameters are known or assumed to be known
  - Outcomes – known
  - Termed a *deterministic* analysis
  - Parameters are estimated with certainty
- Decision Making under Risk
  - Inputs are viewed as uncertain, and element of chance is considered
  - Variation is present and must be accounted for
  - Probabilities are assigned or estimated
  - Involves the notion of *random variables*

Analyzing Risk

- Risks have
  - a time horizon
  - Exists in the future and will evolve over time
  - Changing scenario’s effect on the system can be measured
  - Measurement has to be set against a benchmark

Measuring Risk

- Since in risk variables can have ranges, we deal with probability distributions.
  - Measuring center of distribution- The first moment
  - Measuring spread of distribution – The second moment
  - Measuring skew of the distribution- The third moment
  - Measuring catastrophic tail events of the distribution – The fourth moment

Two Ways to Consider Risk

- ✓ Expected Value (EV) analysis
  - Applies the notion of expected value
  - Calculation of EV of a given outcome
  - Selection of the outcome with the most advantageous outcome
- ✓ Simulation Analysis
  - Form of generating artificial data from assumed probability distributions
  - Relies on the use of random variables and the laws associated with the algebra of random variables

Quantifying Risk

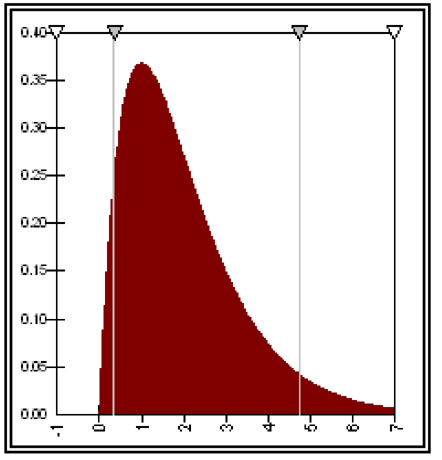
- Probability of occurrence
- Standard deviation and variance
- Semi-standard deviation
- Volatility
- Beta
- Coefficient of Variation
- Value at Risk
- Worst case scenario and Regret
- Risk adjusted return on capital

Elements Important to Decision Making Under Risk

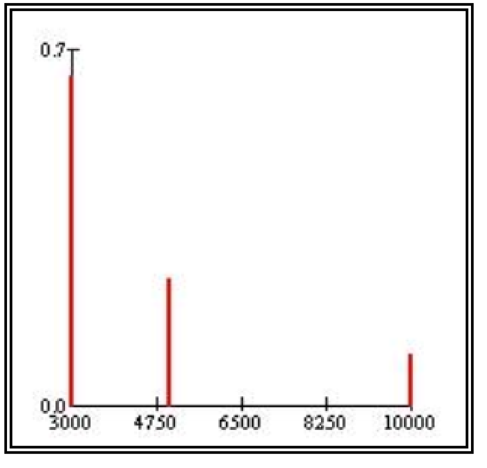
- The concept of a random variable
  - A decision rule that assigns an outcome to a sample space
  - Discrete variable or Continuous variable
    - Discrete variable – finite number of outcomes possible
    - Continuous variable – infinite number of outcomes
- Probability
  - Number between 0 and 1
  - Expresses the “chance” in decimal form that a random variable will take on any specific value

Types of Random Variables

- Continuous

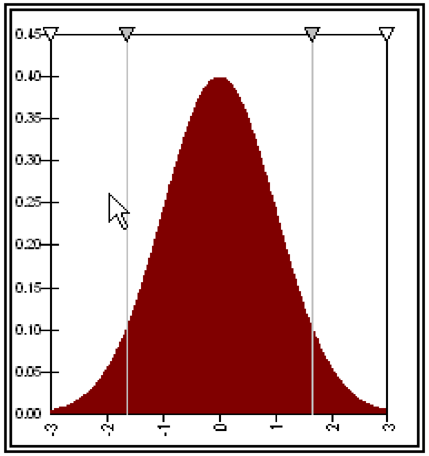


- Discrete



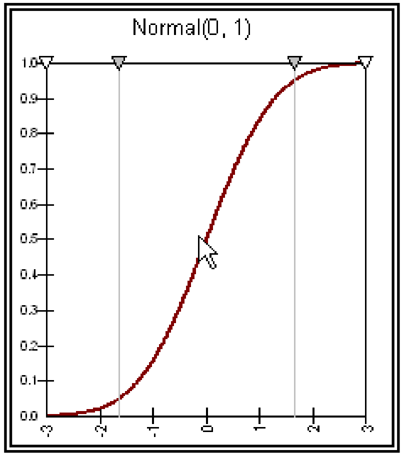
Distributions - Continuous Variables

- Probability Distribution (pdf)
  - A function that describes how probability is distributed over the different values of a variable
  - $P(X_i)$ = probability that $X = X_i$



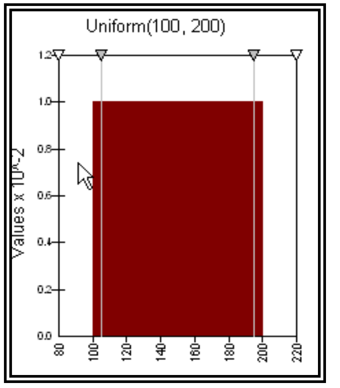
- Cumulative Distribution (cdf)
  - Accumulation of probability over all values of a variable up to and including a specified value
  - $F(X_i)$ = sum of all probabilities through the value X_i

    $= P(X \leq X_i)$

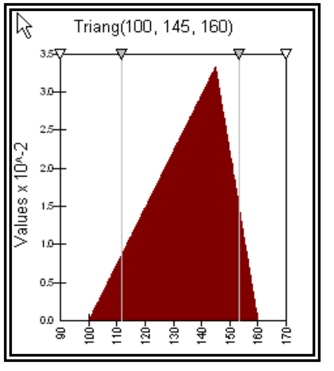


Three Common Random Variables

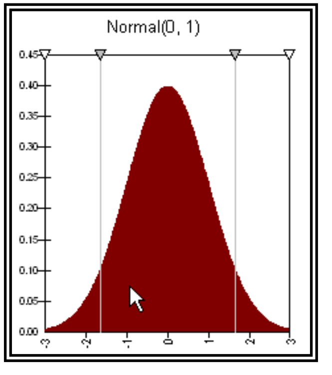
- Uniform – equally likely outcomes



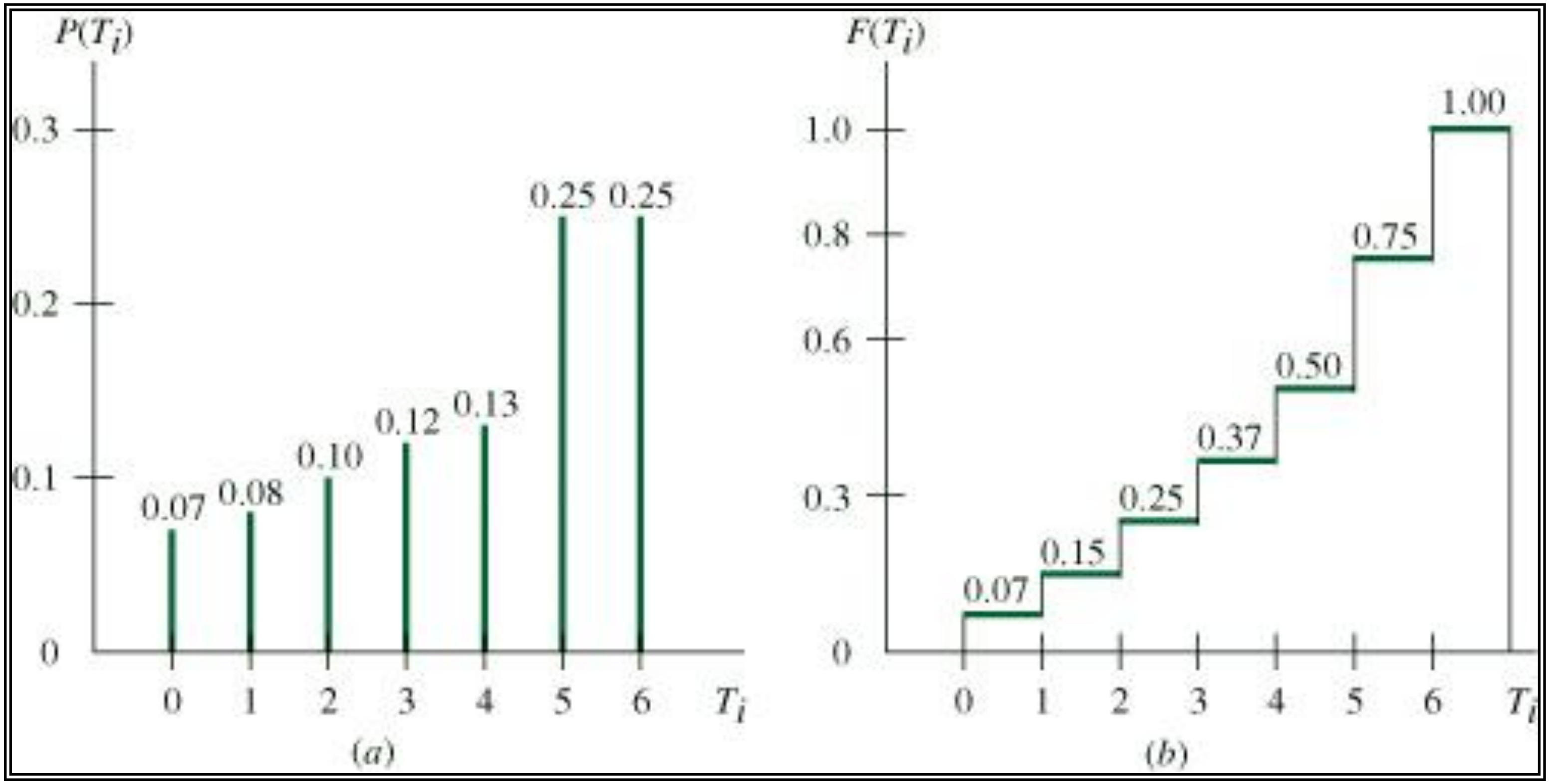
- Triangular



- Normal



Discrete Density and Cumulative Example



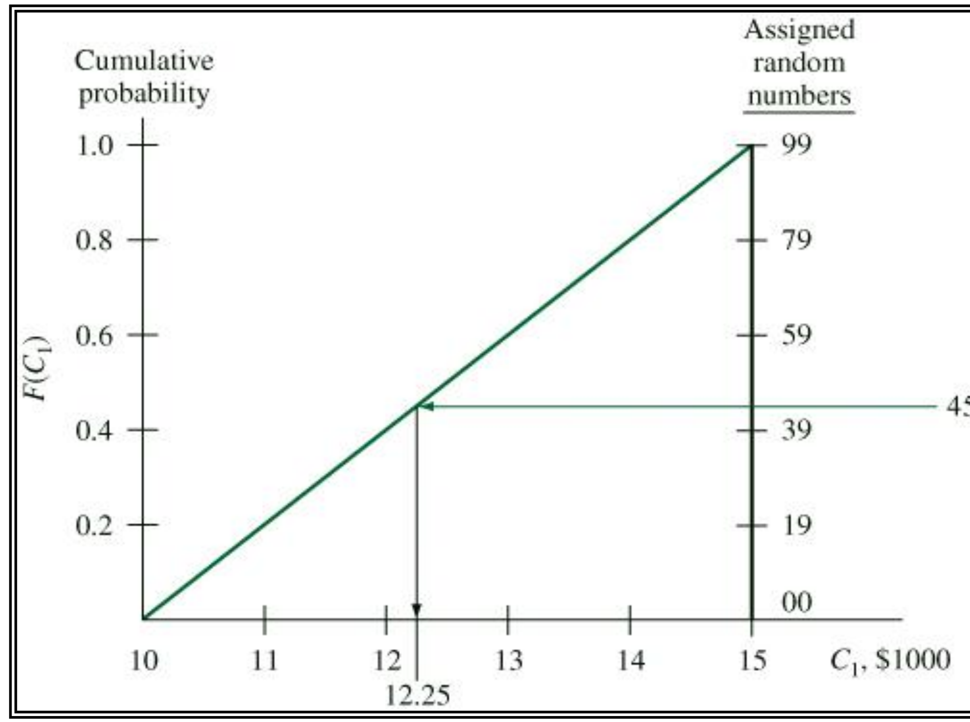
pdf cdf

Random Samples

- Random Sample
  - A random sample of size n is the selection in a random fashion with an assumed or known probability distribution such that the values of the variable have the same chance of occurring in the sample as they appear in the population
  - Basis for Monte Carlo Simulation
- Can sample from:
  - Discrete distributions … or
  - Continuous distributions

Sampling from a Continuous Distribution

- Form the cumulative distribution in closed form from the pdf
- Generate a uniform random number on the interval {0 – 1}, called U(0,1)
- Locate U(0,1) point on y-axis
- Map across to intersect the cdf function
- Map down to read the outcome (variable value) on x-axis



Expected Value and Standard Deviation

- Two important parameters of a given random variable:
  - Mean - μ
    - Measure of central tendency
  - Standard Deviation - σ
    - Measure of variability or spread
- Two Concepts to work within
  - Population
  - Sample from a population

Population vs Sample

Population

- μ - population mean
- σ^2 - population variance
- σ - population standard deviation
- Often sample from a population in order to make estimates

Sample

$\bar{X}$	Sample mean
S^2	Sample variance
S	Sample standard deviation

These values, properly sampled, attempt to estimate their population counterparts

Important Relationships

- Population Mean μ
- Distribution
  - E(x) = $\sum X_i P(X_i)$
- Sample
- $$\frac{\sum X_i}{n} = \frac{\sum f_i X_i}{n}$$
- Measure of the central tendency of the population
- If one samples from a population the hope is that sample mean is an unbiased estimator of the true, but unknown, population mean

Variance and Standard Deviation

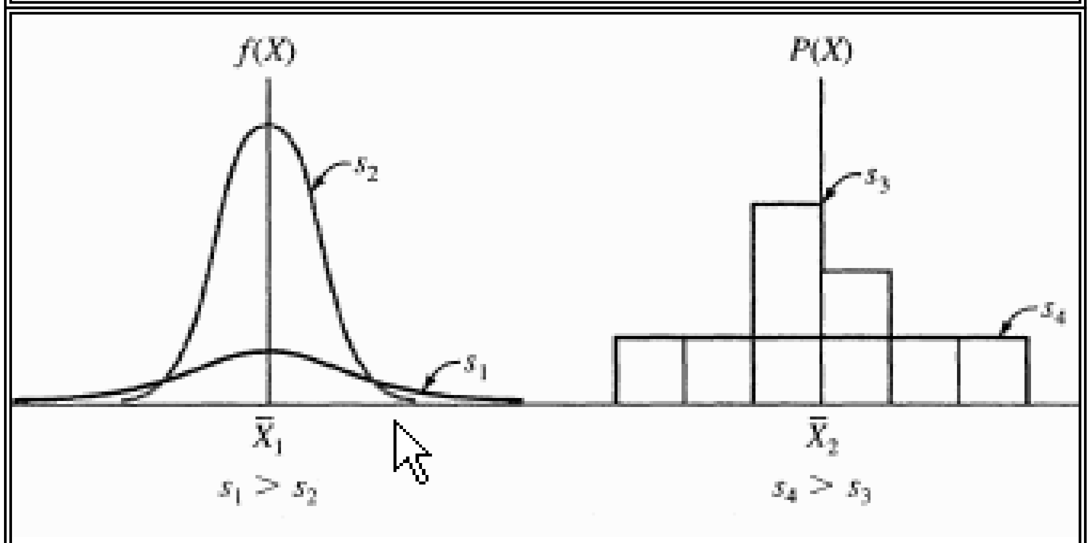
Population: $\sigma^2 = \mathrm{Var}(X)$ and $\sigma = \sqrt{\sigma^2} = \sqrt{\mathrm{Var}(X)}$

Probability distribution: $\mathrm{Var}(X) = \sum [X_i - E(X)]^2 P(X_i)$

Sample: $$s^2 = \frac{\text{sum of (sample value} - \text{sample average)}^2}{\text{sample size} - 1}$$

$$= \frac{\Sigma (X_i - \bar{X})^2}{n - 1}$$

$$s = \sqrt{s^2}$$



Notes relating to variance and standard deviation properties

Illustration of variances for discrete and continuous distributions

Population vs. Sample

- Variance of a population

$$\sigma^2 = \frac{\sum_{i=1}^{N}(x_i - \mu)^2}{N}$$

- Standard deviation of a population:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \mu)^2}{N}}$$

$$\sigma_X = \sqrt{\sigma_X^2} = \sqrt{\mathrm{Var(X)}}$$

- **Variance of a sample**

$$s^2 = \frac{\sum (X_i - \bar{X})}{n-1}$$

$$s^2 = \frac{\sum X_i^2}{n-1} - \frac{n}{n-1}\bar{X}^2$$

- **Standard deviation of a sample**

$$s = \sqrt{s^2}$$

S is termed an unbiased estimator of the population standard deviation

Combining the Average and Standard Deviation

- Determine the percentage or fraction of the sample that is within ±1, ±2, ±3 standard deviations of the sample mean . . .

$$\bar{X} \pm ts, \text{ t} = 1,2,3$$

- In terms of probability…

$$P(\bar{X} - ts \le X \le \bar{X} + ts)$$

- Virtually all of the sample values will fall within the ±3s range of the sample mean

Example

- Project-A has following returns
  - 40,66,75,92, 107,159, 275
- Project B has following returns
  - 84, 90, 104, 187, 190
- What are the ranges of returns of the following two projects

Continuous Random Variables

- Expected value:

$$E(X) = \int_R X f(x) dx$$

R **represents the defined range of the variable in question.**

- Variance:

$$Var(x) = \int_R X^2 f(X) dX - [E(X)]^2$$

- For uniform pdf

$$f(x) = \frac{1}{5} = 0.2 \quad \$10 \leq X \leq \$15$$

$$\text{E(X)} = \int_R (0.2)\text{Xdx} = 0.1\text{X}^2 \Big|_{10}^{15} = 0.1(225 - 100) = \$12.50$$

$$Var(X) = \int_R X^2 (0.2) dX - (12.5)^2 = \frac{0.2}{3} X^3 \Big|_{10}^{15} - (12.5)^2$$

$$= 0.06667(3375\text{-}1000)\text{-}156.25 = 2.08$$

$$\sigma_X = \sqrt{2.08} = \$1.44$$

Monte Carlo Sampling and Simulation Analysis

- Simulation involves the generation of artificial data from a modeled system
- Monte Carlo Sampling
  - The generation of samples of size n for selected parameters of formulated alternatives
  - The sampled parameters are expected to vary according to a stated probability distribution (assumed)

Key Assumption - Independence

- For a given problem:
  - All parameters are assumed to be <u>independent</u>
  - One variable’s distribution in no way impacts any other variable’s distribution
  - Termed:
    - Property of independent random variables

Steps

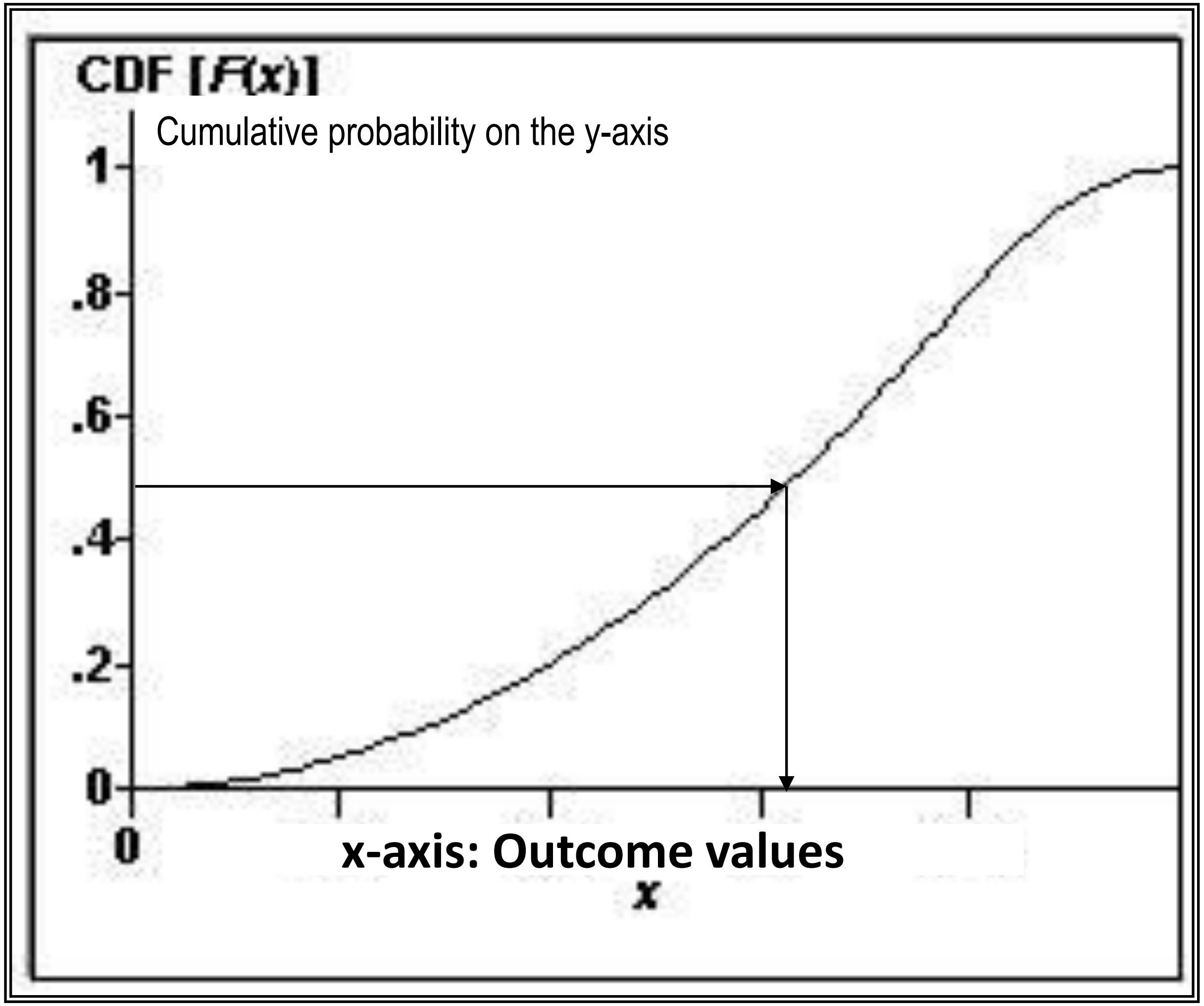
- Formulate alternatives
- Parameters with variation
- Determine probability distribution
- Random sampling
- Measure of worth calculation
- Measure of worth description (confidence intervals)
- Conclusion

Sampling Process

- Requires:
  - The cdf of the assumed pdf;
  - A uniform random number generator;
  - Application of the inverse transform approach.
- Why require the cdf?
  - The y-axis of a cdf is scaled from 0 to 1.
  - That is the same as the range of U(0,1).
  - Facilitates mapping a RN to achieve the outcome value on the x-axis.
  - The U(0,1) selects a X-value from the cdf.

The Need for the cdf

Generate a U(0,1) random number: Locate that value on the y-axis: Map across to the cdf then map down to the x-axis to obtain the outcome



Summary of the Modeling Steps

- Formulate the economic analysis:
  - The alternatives – if more than one;
  - Define which parameters are "constants" and which are to be viewed as random variables.
  - For the random variables, assign the appropriate pdf:
    - Discrete and or continuous.
  - Apply Monte Carlo sampling – a sample size of "n" where it is suggested that n = 30.
  - Compute the measure of worth (PW, AW, . .)
  - Evaluate and draw conclusions.

Notes

- To perform decision making under risk implies that some parameters of an engineering alternative are treated as random variables.
- Assumptions about the shape of the variable's probability distribution are used to explain how the estimates of parameter values may vary.
- Additionally, measures such as the expected value and standard deviation describe the characteristic shape of the distribution.

Notes

- We discussed several of the simple, but useful, discrete and continuous population distributions used in engineering economy -uniform and triangular - as well as specifying our own distribution or assuming the normal distribution.
- It is important to note that a sound background in applied statistics is vital to the complete understanding of the simulation process

Value at Risk

- *VaR is the maximum loss over a target horizon within a confidence interval (or, under normal market conditions)*

- In other words, if none of the "extreme events" (i.e., low-probability events) occurs, what is my maximum loss over a given time period?

Another Definition of VAR

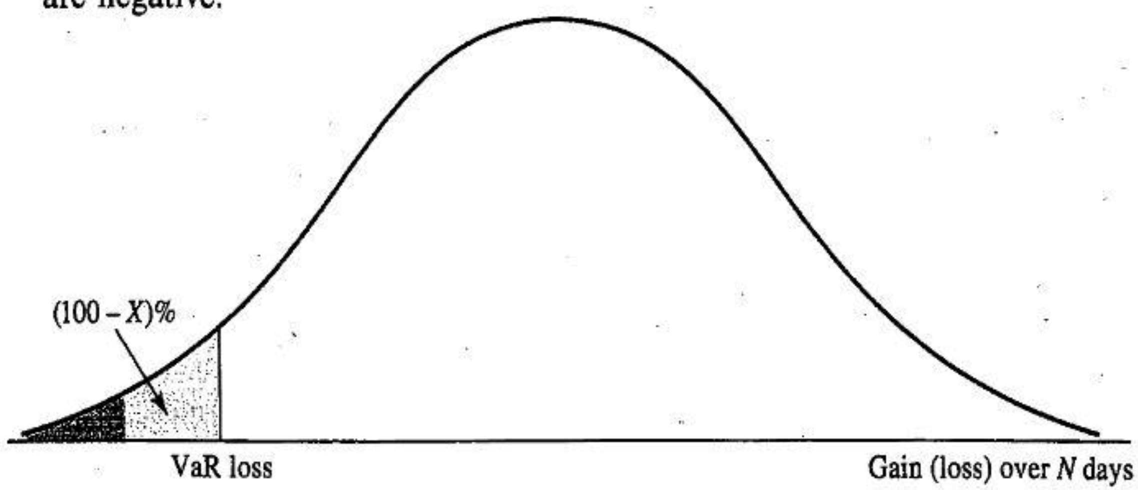
- *A forecast of a given percentile, usually in the lower tail, of the distribution of returns on a portfolio over some period; similar in principle to an estimate of the expected return on a portfolio, which is a forecast of the 50th percentile.*

  Ex: *95% one-tail normal distribution is 1.645 sigma ($Pr(x<=X)=0.05$, $X=-1.645$) while 99% normal distribution is 2.326 sigma*

Value at Risk (VaR)

- "We are *X* percent certain that we will not lose more than *V* dollars in time *T*."
- Function of confidence level *X* and time *T*

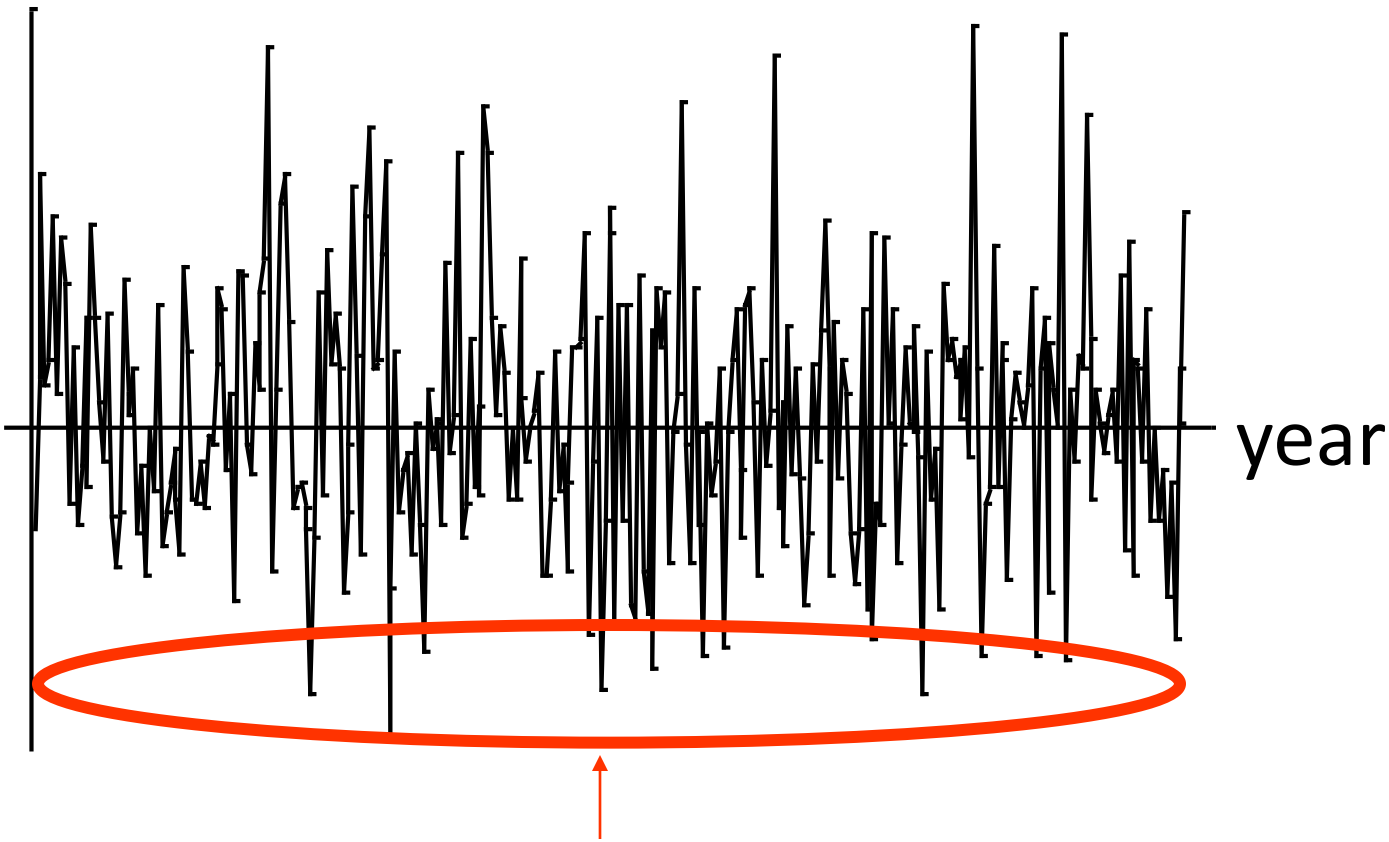
Figure 20.1 Calculation of VaR from the probability distribution of the change in the portfolio value; confidence level is $X\%$. Gains in portfolio value are positive; losses are negative.



Brief History

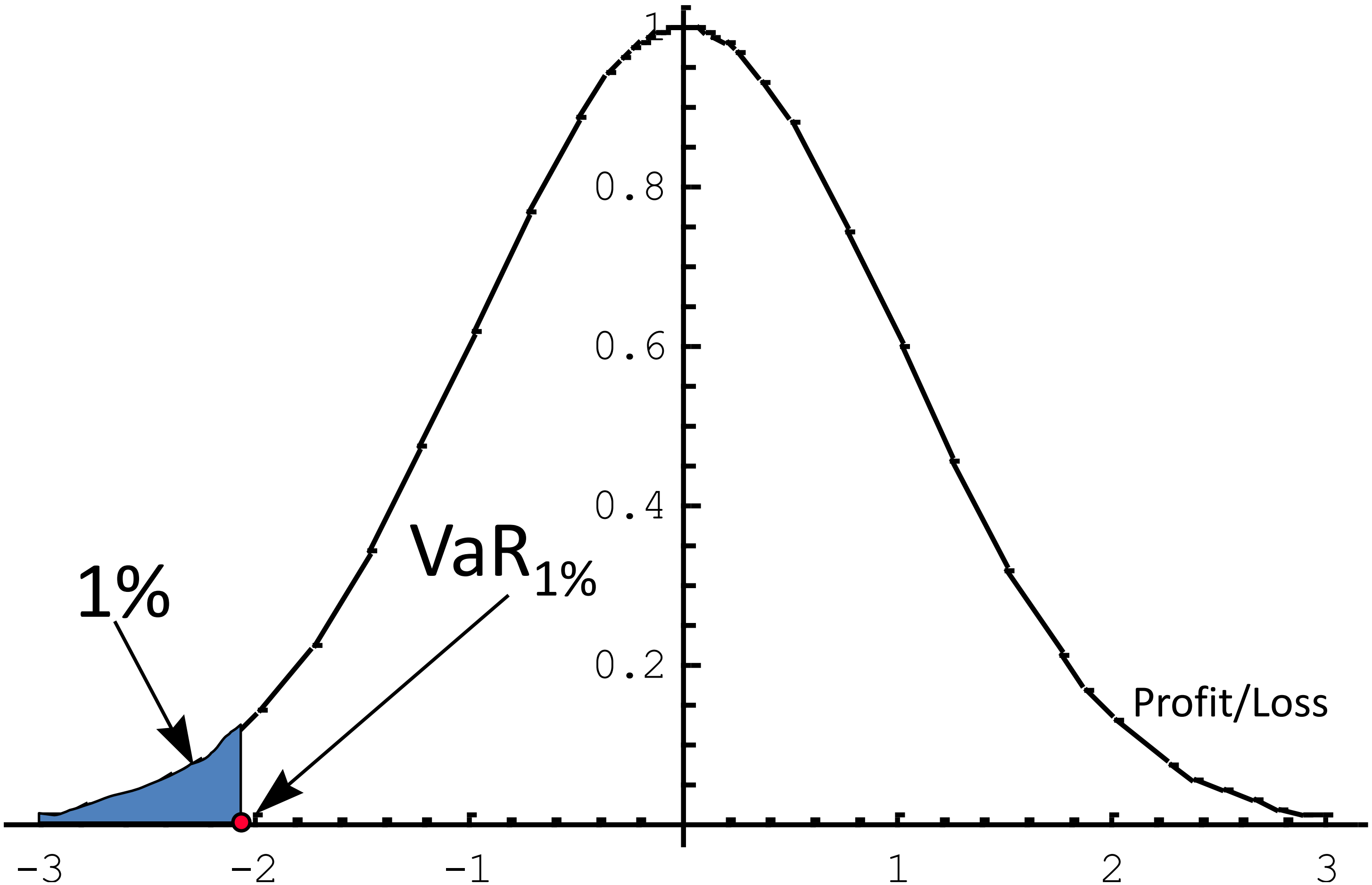
- Increasing need for risk management after the 1987 market crash
- J.P. Morgan employees credited for developing VaR
- Known as the 4:15pm report
- RiskMetrics spinoff in 1994
- CreditMetrics and CorporateMetrics

Returns



1% of worst cases

VaR

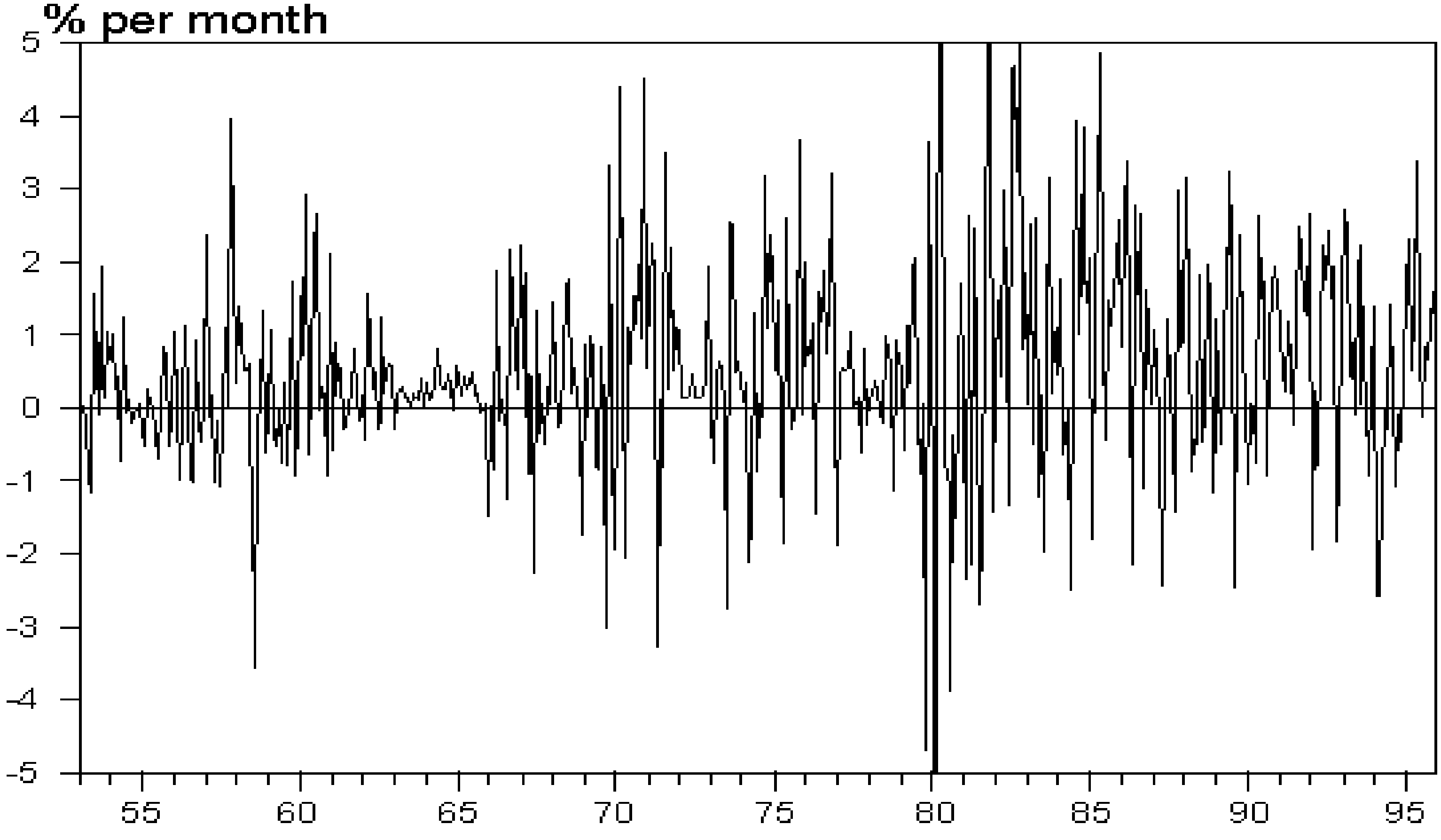


VAR: Example

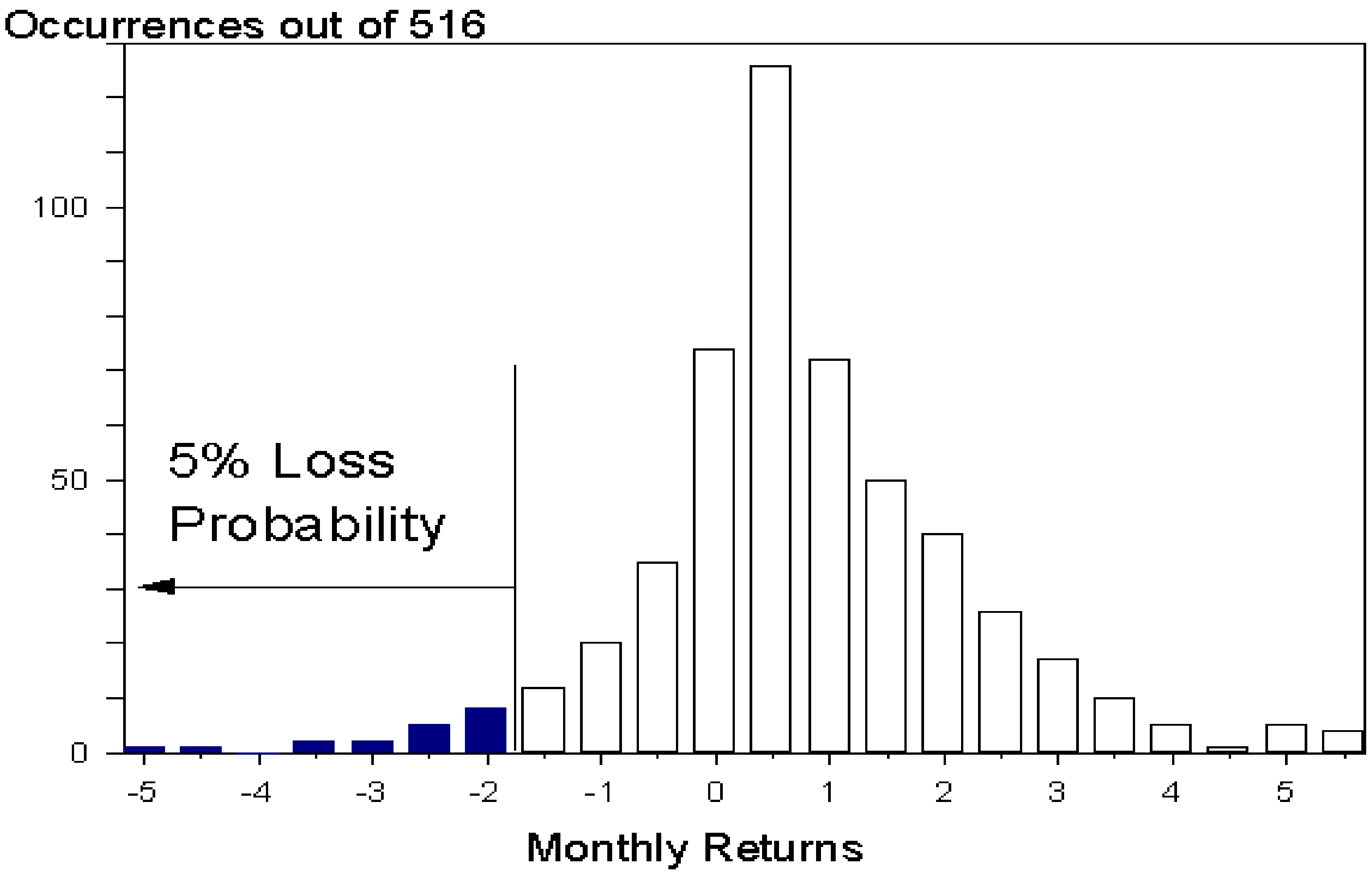
- **Consider a $100 million return. Suppose my confidence interval is 95% (i.e., 95% of possible market events is defined as "normal".) Then, what is the maximum monthly loss under normal markets over any month?**

- **To answer this question, let's look at the monthly returns from 1953 to 1995:**
- ***Lowest: -6.5% vs. Highest: 12%***

History of Returns



Distribution of Returns



Calculating VAR at 95% Confidence

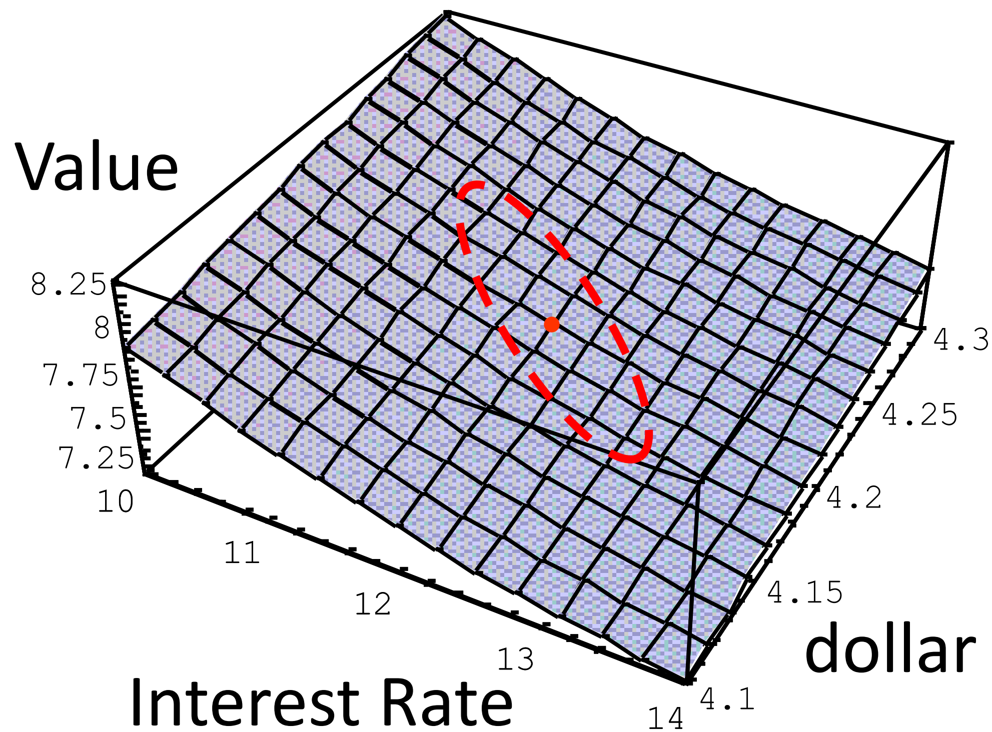
- At the 95% confidence interval, the lowest monthly return is -1.7%. *(I.e., there is a 5% chance that the monthly return is lower than -1.7%)*

  That is, there are 26 months out of the 516 for which the monthly returns were lower than -1.7%.

- **VAR = 100 million X 1.7% = $1.7 million**
- (95% of the time, the portfolio's loss will be no more than $1.7 million!)

Another way

- Take standard deviation of the returns (-1.03%)
- 95% of the standard normal distribution is 1.645
- VaR = 1.645*1.03 = 1.70 million
- Under normal market conditions, the project can loose at most 1.7 million at 95% level of confidence.



interest rates and dollar are NOT independent

How to calculate VaR

- Historical Simulation
- Variance-Covariance Method
- Monte-Carlo Simulation

Monte Carlo simulation

- Monte Carlo simulation randomly generates values for uncertain variables over and over to simulate a model.
- It's used with the variables that have a known range of values but an uncertain value for any particular time or event.
- For each uncertain variable, you define the possible values with a probability distribution.
- Distribution types include:

- A simulation calculates multiple scenarios of a model by repeatedly sampling values from the probability distributions
- Computer software tools can perform as many trials (or scenarios) as you want and allow to select the optimal strategy

Issues to Ponder

- What horizon is appropriate?

  A day, a month, or a year?

  the holding period should correspond to the longest period needed for an orderly portfolio liquidation.

-

- What confidence level to consider?

  * *Are you risk averse?*

  The more risk averse => (1) the higher confidence level necessary & (2) the lower VAR desired.

VaR Computation-continued

- Historical simulation

  going back in time, e.g. over the last 5 years, and applying current weights to a time-series of historical asset returns. This return does not represent an actual portfolio but rather reconstructs the history of a hypothetical portfolio using the current position
- *(1) for each risk factor, a time-series of actual movements, and*
- *(2) positions on risk factors.*

VaR Computation-continued

Monte Carlo Simulation

two steps:

- *Specifies a stochastic process for financial variables as well as process parameters; the choice of distributions and parameters such as risk and correlations can be derived from historical data.*
- *Fictitious price paths are simulated for all variables of interest. At each horizon considered, one day to many months ahead, the portfolio is marked-to-market using full valuation. Each of these ``pseudo'' realizations is then used to compile a distribution of returns, from which a VAR figure can be measured.*

Required:

- *for each risk factor, specification of a stochastic process (i.e., distribution and parameters),*
- *valuation models for all assets in the portfolio, and*
- *positions on various securities.*

VaR in practice

- J.P.Morgan Riskmetrics

  allows users to compute a portfolio VAR using the Delta-Normal method based on a 95% confidence level over a daily or monthly horizon

- Deutsche Bank, RAROC 2020 system

  provides VaR estimates at the 99% level of confidence over an annual horizon, using the Monte Carlo method.

Weaknesses

- *VaR does not measure "event" (e.g., market crash) risk. That is why portfolio stress tests are recommended to supplement VaR.*
- *VaR does not readily capture liquidity differences among instruments. That is why limits on both tenors and option greeks are still useful.*
- *VaR doesn't readily capture model risks, which is why model reserves are also necessary.*