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Wavelet-based generation of spectrum-compatible time-histories

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Abstract

This paper deals with the well-known problem of generating spectrum-compatible synthetic accelerograms for the linear and non-linear time-history analyses of structural systems. A wavelet-based procedure has been used to decompose a recorded accelerogram into a desired number of time-histories with non-overlapping frequency contents, and then each of the time-histories has been suitably scaled for matching of the response spectrum of the revised accelerogram with a specified design spectrum. The key idea behind this iterative procedure is to modify a recorded accelerogram such that the temporal variations in its frequency content are retained in the synthesized accelerogram. The proposed procedure has been illustrated by modifying five recorded accelerograms of widely different characteristics such that those are compatible with the same USNRC design spectrum.
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Keywords: Strong motion synthesis; Spectrum-compatible accelerograms; Time-history analysis; Wavelet transform

1. Introduction

Characterization of seismic hazard at a site is usually done in terms of design (response) spectra. These spectra are convenient to use by the practising engineers for linear analyses of single-degree-of-freedom and multi-degree-of-freedom systems. For seismic analyses of complex and non-linear structural systems, however, these spectra cannot be directly used. It is therefore considered acceptable: (i) to generate artificial time-histories of specified durations and compatible with these spectra and (ii) to directly integrate the equations of motion with these time-histories as input. The problem of generating spectrum-compatible time-histories is not new and it has been attempted several times in the past. Most available methods do not, however, account for the temporal variations in the frequency composition of an accelerogram that arise due to the arrivals of different types of seismic waves at different time-instants and due to the phenomenon of dispersion in these waves. It has been already established (see, for example Refs. [1,2]) that non-linear systems respond quite differently to the ground motions with time-invariant frequency contents, even if there is no difference in the response of linear systems. Due to this reason, few efforts on generating spectrum-compatible accelerograms have focussed on properly simulating non-stationary characteristics instead of simply using a time-dependent modulating function. Wong and Trifunac [3] initially attempted the use of site-specific group velocity curves of various waves with success in the SYNACC program. Gupta and Joshi [4] and Shrikhande and Gupta [5] used the phase characteristics of recorded accelerograms. Conte and Peng [6] directly modelled the evolutionary power spectral density function of the ground motion process, while Shrikhande and Gupta [7] modelled the phase spectrum to optimize a given objective function.

This paper considers the possibility of: (i) decomposing a recorded accelerogram into a finite number of time-histories with energy in non-overlapping frequency bands and (ii) scaling those up/down iteratively such that the assembled time-history is compatible with a specified design spectrum. It is assumed that a motion recorded under similar seismic environment and site conditions is available from the database of recorded ground motions, and thus, the waves in an arbitrary, small frequency band of the chosen motion exhibit desired temporal characteristics. A wavelet-based approach employing the modified Littlewood–Paley (L–P) basis function [8] is used to decompose the recorded accelerogram. The proposed method is illustrated by modifying five example accelerograms so that those are compatible with a design spectrum proposed by USNRC [9].

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2. Decomposition of recorded accelerogram

If $f(t)$ is a function belonging to $L^2(R)$—a space of all finite energy functions, i.e. if it satisfies the condition

$$\int_{-\infty}^{\infty} |f(t)|^2 \, \mathrm{d}t < \infty \tag{1}$$

it can be decomposed into wavelet coefficients and reconstructed back from those by using the wavelet transformation and the inverse wavelet transformation, respectively. The transient nature and the finite energy content of all recorded accelerograms make it possible to have their wavelet domain representations. Following the work of Basu and Gupta [8], it is possible to decompose $f(t)$ of length T into N number of time-histories, i.e.

$$f(t) = \sum_{j=1}^{N} f_j(t) \tag{2}$$

such that no two time-histories have energy in overlapping frequency bands and that all N time-histories completely span a frequency band from $\omega_{\min}$ to $\omega_{\max}$. Here, $f_j(t)$ may be expressed as

$$f_j(t) = \frac{K\Delta b}{a_j} \sum_i W_\psi f(a_j, b_i)\psi\left(\frac{t - b_i}{a_j}\right) \tag{3}$$

where

$$\psi(t) = \frac{1}{\pi\sqrt{\sigma - 1}} \frac{\sin(\sigma\pi t) - \sin(\pi t)}{t} \tag{4}$$

with $\sigma = 2^{1/4}$ is the mother wavelet or basis function; $a_j = 2^{j/4}$ and $b_i = (i-1)\Delta b$ are the discretized values of scale and shift parameters; and

$$W_\psi f(a_j, b_i) = \frac{1}{\sqrt{a_j}} \int_0^T f(t)\psi\left(\frac{t - b_i}{a_j}\right)\mathrm{d}t \tag{5}$$

is the wavelet coefficient for scale parameter, a_j, and shift parameter, b_i. Further, in Eq. (3)

$$K = \frac{1}{4\pi C_\psi}\left(\sigma - \frac{1}{\sigma}\right) \tag{6}$$

and

$$C_\psi = \int_{-\infty}^{\infty} \frac{|\hat{\psi}(\omega)|^2}{|\omega|} \mathrm{d}\omega \tag{7}$$

are constants, where

$$\hat{\psi}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \psi(t)\mathrm{e}^{-\mathrm{i}\omega t} \, \mathrm{d}t \tag{8}$$

is the Fourier transform of the basis function, $\psi(t)$. Δb is taken as 0.02 s. In Eq. (2), the jth time-history, $f_j(t)$, has energy in the period band, $(2a_j/\sigma, 2a_j)$, and thus, for j chosen from -21 to 6, all 28 decomposed time-histories span over 0.044–0.053, 0.053–0.063, …, 4.76–5.66 s period bands, respectively.

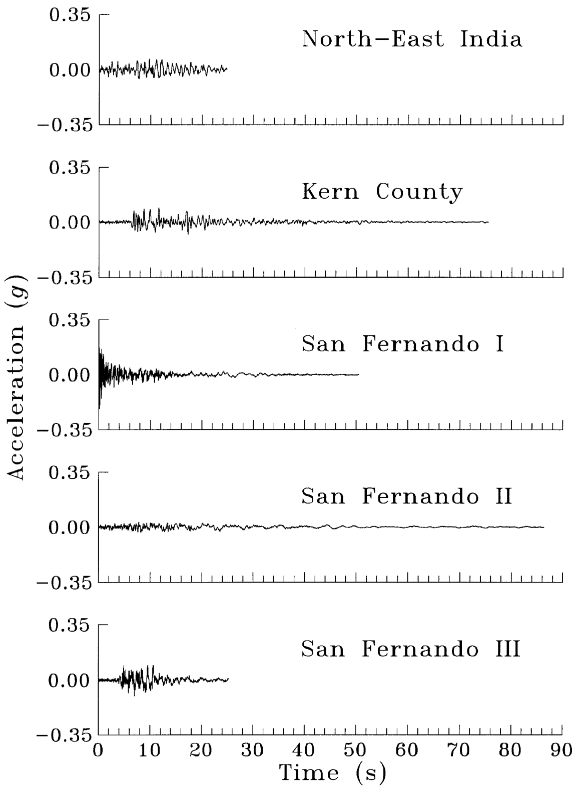

Fig. 1. Recorded (original) acceleration time-histories of the example motions.

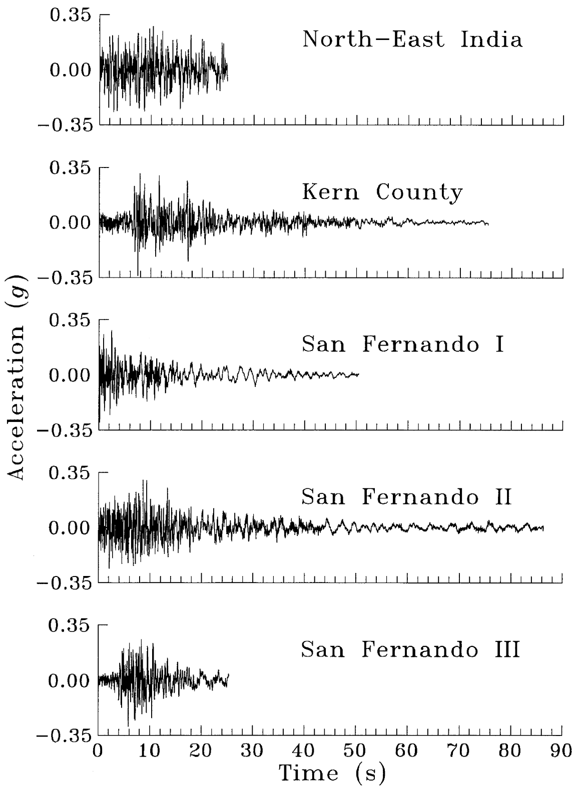

Fig. 2. Modified acceleration time-histories of the example motions.

3. Modification of recorded accelerogram

It has been shown by Basu and Gupta [8] that in case of the modified L–P basis function, the excitation wavelet coefficients corresponding to a particular scale parameter mainly contribute to the responses of those single-degree-of-freedom oscillators which have natural periods falling within the period-band corresponding to that parameter. Thus, the oscillators with periods between 0.053 and 0.063 s will respond primarily to $f_2(t)$, if $f_2(t)$ corresponds to $j = -20$. It appears, thus, reasonable that response spectrum of the recorded accelerogram is matched with the target design spectrum by scaling each of the decomposed time-histories (as in Eq. (2)) up/down based on the amplification/reduction required to reach target spectral ordinates in the period-band corresponding to that time-history. Since modifying a particular time-history also affects the spectral values for the oscillators of other period-bands, an iterative scheme has been considered in which each decomposed time-history is scaled up/down so as to lead to the averaged value of target spectrum over the corresponding period-band. Thus, in the ith iteration, $f_j^{(i)}(t)$ (with $f_j^{(1)}(t) \equiv f_j(t)$) is modified to $f_j^{(i+1)}(t)$ such that

$$f_j^{(i+1)}(t) = f_j^{(i)}(t) \frac{\int_{2a_j/\sigma}^{2a_j} [\mathrm{PSA}(T)]_{\mathrm{target}}\, \mathrm{d}T}{\int_{2a_j/\sigma}^{2a_j} [\mathrm{PSA}^{(i)}(T)]_{\mathrm{calculated}}\, \mathrm{d}T}, \qquad (9)$$

$j = 1, 2, \ldots, N$

where $[\mathrm{PSA}(T)]_{\mathrm{target}}$ is the target pseudo-spectral acceleration (PSA) ordinate at period, T, and $[\mathrm{PSA}^{(i)}(T)]_{\mathrm{calculated}}$ is the PSA calculated from $f^{(i)}(t)(= \sum_{j=1}^{N} f_j^{(i)}(t))$ by integrating the equation of motion for the oscillator of

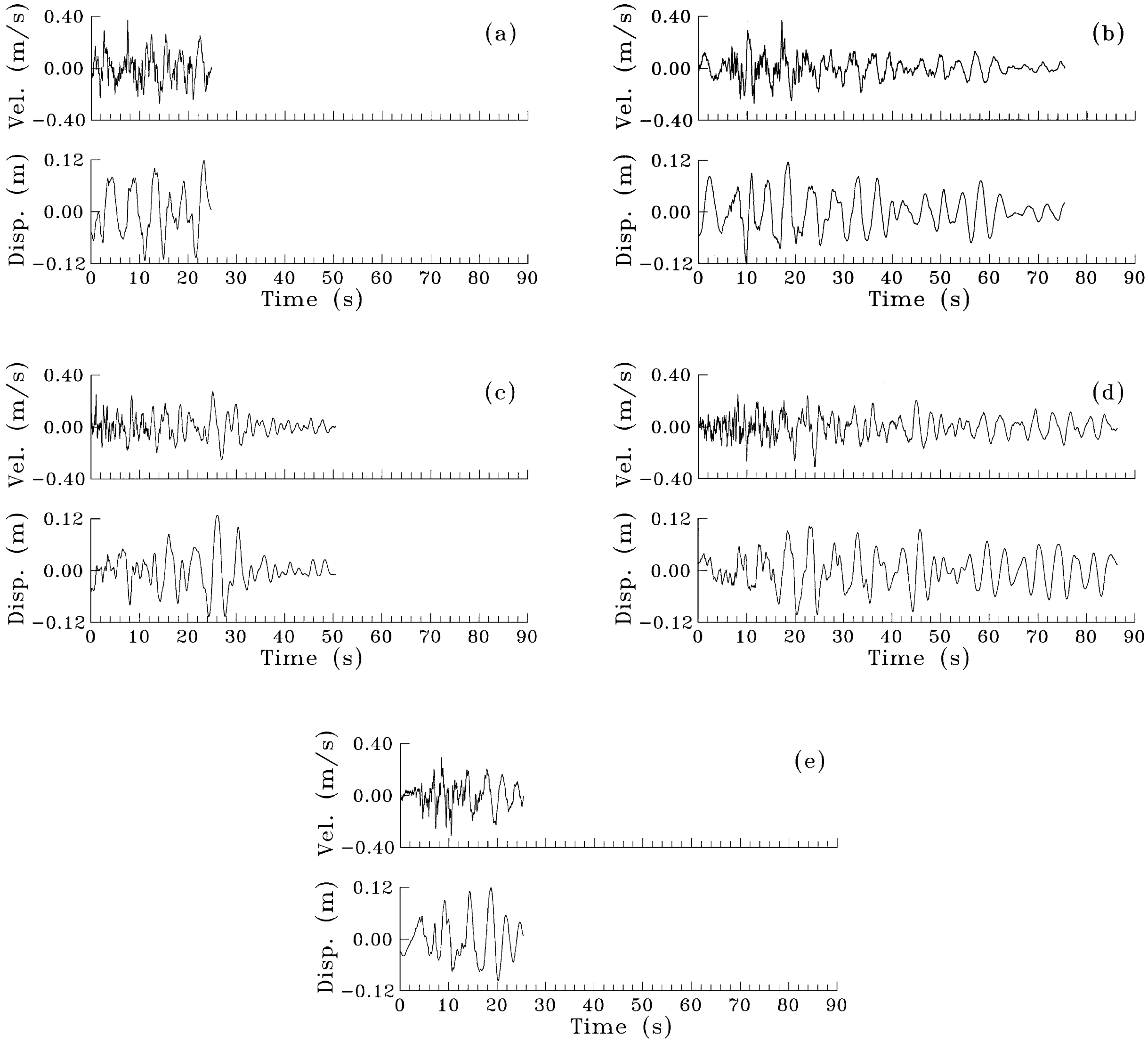

Fig. 3. Velocity and displacement time-histories for the modified: (a) North-east India, (b) Kern County, (c) San Fernando I, (d) San Fernando II, and (e) San Fernando III motions.

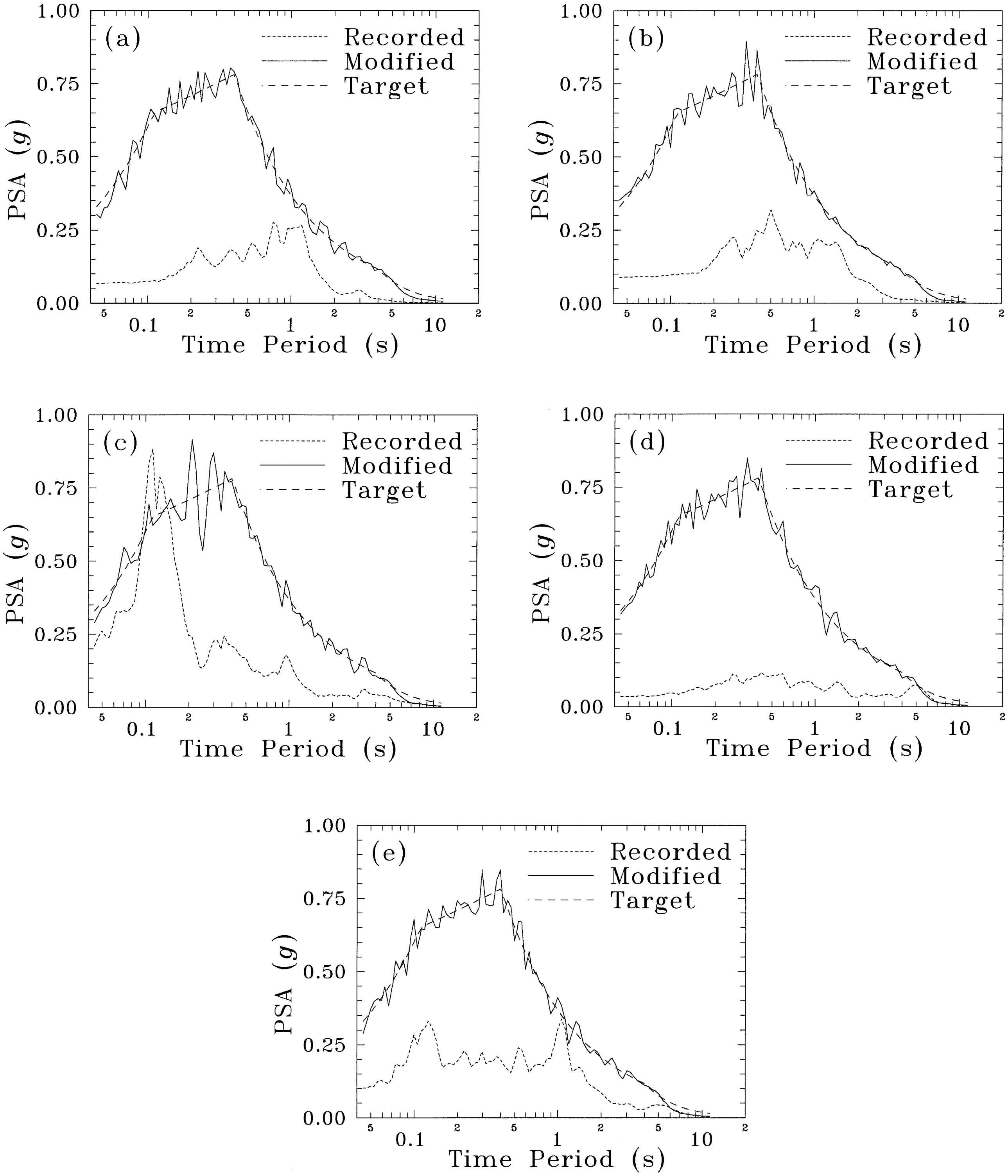

Fig. 4. Comparison of PSA spectra for recorded and modified time-histories with the target spectrum in case of: (a) North-east India, (b) Kern County, (c) San Fernando I, (d) San Fernando II, and (e) San Fernando III motions.

period, T. In the ith iteration, error at time-period, T, is calculated as

$$\mathrm{ERR}^{(i)}(T) = \frac{\left|[\mathrm{PSA}(T)]_{\mathrm{target}} - [\mathrm{PSA}^{(i)}(T)]_{\mathrm{calculated}}\right|}{[\mathrm{PSA}(T)]_{\mathrm{target}}} \qquad (10)$$

This error is averaged over various time periods and further iterations are continued until the average error falls below a tolerance limit or fails to register further reduction. It may be noted that convergence from $f^{(1)}(t)$ to $f^{(2)}(t)$, $f^{(3)}(t)$, $f^{(4)}(t), \ldots$ requires the decomposition of accelerogram into N time-histories only once.

It may be mentioned that the accelerograms generated by the proposed method may need appropriate baseline corrections, as in case of any recorded accelerogram, so that the associated velocity and displacement time-histories do not become unrealistic due to systematic low-frequency errors.

4. Numerical examples

For illustrating the proposed iterative method, five recorded ground motions have been modified so as to be compatible with the 5% USNRC design spectrum for 'mean + SD' confidence level and 0.25g peak ground acceleration. The example motions are: (i) S89E component recorded at Katakhal site during the 1988 North-East India earthquake (to be denoted as 'North-East India motion'), (ii) N42E component recorded at Santa Barbara Courthouse during the 1952 Kern County earthquake (to be denoted as 'Kern County motion'), (iii) S08E component recorded at Santa Felicia Dam outlet works during the 1971 San Fernando earthquake (to be denoted as 'San Fernando I motion'), (iv) S45E component recorded at 8639 Lincoln Avenue Basement, Los Angeles during the 1971 San Fernando earthquake (to be denoted as 'San Fernando II motion'), and (v) N61W component recorded at 2500 Wilshire Boulevard Basement, Los Angeles during the 1971 San Fernando earthquake (to be denoted as 'San Fernando III motion'). Fig. 1 shows all five example motions. These motions are based on 'as available' recorded data, without any truncation in the beginning and/or end for this study. On considering 32 levels from $j = -21$ to 6 with period spanning from 0.044 to 5.66 s, these motions get modified after 11 iterations to the motions shown in Fig. 2. The time-histories in Figs. 1 and 2 are drawn on a common scale to facilitate a convenient comparison. The time-histories in Fig. 2 have been baseline-corrected so as to have zero velocity at the end. Fig. 3 shows the corresponding velocity and displacement time-histories. It may be observed from Figs. 1 and 2 that the modified motions nicely preserve the temporal characteristics of the original motions. Fig. 2 also shows that for the same seismic hazard description, we can obtain significantly different accelerograms with realistic features.

Fig. 4 shows the comparisons of PSA spectra for the original and modified motions, as shown in Figs. 1 and 2, with the target spectrum. It may be observed that despite widely different energy distributions, spectra of modified motions match very well with the target spectrum. Fig. 5 describes the rate of convergence in iterations through plots of average error versus iteration level for all five example motions. Average error in each case has been calculated by considering three periods in each period-band, and thus, by considering a total of 85 period values. It is seen that average error less than 8% is obtained in 6, 4, 10, 6 and 6 iterations, respectively. The results here could have been obtained by taking a larger value of N; however, it was found that taking additional frequency bands did not make substantial difference to the final results.

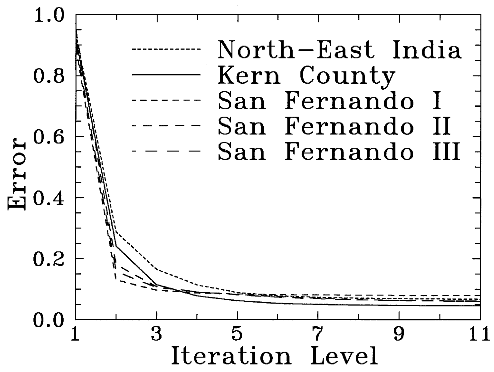

Fig. 5. Variations of average error with iteration level in case of the example motions.

It may be mentioned that the success of the proposed procedure is not dependent on the features of the (target) design spectrum, and therefore, any spectrum (other than the USNRC spectrum) could have been chosen in the above numerical illustration. Further, since the modified accelerograms have the same non-stationary characteristics as the parent accelerograms, a 'complete' recorded accelerogram (with no truncation(s) in the beginning and/or the end) should be chosen as a parent accelerogram in order to obtain a realistic modified accelerogram.

5. Conclusions

An iterative procedure has been proposed to modify a recorded accelerogram so that it becomes compatible with a given design (response) spectrum. The modification has been achieved by decomposing the recorded accelerogram through wavelet analysis into a finite number of time-histories with energy in non-overlapping frequency bands and by scaling those time-histories by suitable amounts. The proposed procedure is computationally efficient as decomposing of the recorded accelerogram is carried out only once and since an acceptable matching with the target spectrum is achieved after about 6–8 iterations. It is also seen that the modified motions nicely preserve the non-stationary characteristics of the parent motions. The proposed procedure is direct and more realistic than other procedures based on using the phase characteristics of a recorded accelerogram, as it does not involve the use of a (time-invariant) Fourier spectrum or power spectral density function.

The success of the proposed procedure in any practical situation will depend much on whether and how judiciously we can choose a recorded accelerogram to simulate the desired non-stationary features. It is expected that this will not remain a serious limitation with more and more recorded data gradually becoming available to the engineering community.

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