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Nonlinear Soil-Site Effects in Probabilistic Seismic-Hazard Analysis

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Abstract This study presents effective probabilistic procedures for evaluating ground-motion hazard at the free-field surface of a nonlinear soil deposit located at a specific site. Ground motion at the surface, or at any depth of interest within the soil formation (e.g., at the structure foundation level), is defined here in terms either of a suite of oscillator-frequency-dependent hazard curves for spectral acceleration, $S_a^s(f)$, or of one or more spectral acceleration uniform-hazard spectra, each associated with a given mean return period. It is presumed that similar information is available for the rock-outcrop input. The effects of uncertainty in soil properties are directly included.

This methodology incorporates the amplification of the local soil deposit into the framework of probabilistic seismic hazard analysis (PSHA). The soil amplification is characterized by a frequency-dependent amplification function, $AF(f)$, where f is a generic oscillator frequency. $AF(f)$ is defined as the ratio of $S_a^s(f)$ to the spectral acceleration at the bedrock level, $S_a^r(f)$. The estimates of the statistics of the amplification function are obtained by a limited number of nonlinear dynamic analyses of the soil column with uncertain properties, as discussed in a companion article in this issue (Bazzurro and Cornell, 2004). The hazard at the soil surface (or at any desired depth) is computed by convolving the site-specific hazard curve at the bedrock level with the probability distribution of the amplification function.

The approach presented here provides more precise surface ground-motion-hazard estimates than those found by means of standard attenuation laws for generic soil conditions. The use of generic ground-motion predictive equations may in fact lead to inaccurate results especially for soft-clay-soil sites, where considerable amplification is expected at long periods, and for saturated sandy sites, where high-intensity ground shaking may cause loss of shear strength owing to liquefaction or to cyclic mobility. Both such cases are considered in this article.

In addition to the proposed procedure, two alternative, easier-to-implement but approximate techniques for obtaining hazard estimates at the soil surface are also briefly discussed. One is based on running a conventional PSHA with a rock-attenuation relationship modified to include the soil response, whereas the other consists of using a simple, analytical, closed-form solution that appropriately modifies the hazard results at the rock level.

Introduction

In probabilistic seismic-hazard analysis (PSHA) the effects of local soil deposits on the seismic hazard at the surface are often treated with less rigor than this critical aspect deserves.

A simple and widely used approach for noncritical facilities assumes that the soil conditions at the site resemble those at the stations in the database considered for the development of one of the many soil-site ground-motion predictive equations available in the literature (e.g., see the many references in *Seismological Research Letters*, 1997). This approach ignores virtually all site-specific information and, therefore, produces only a broad, generic assessment of the hazard.

Because of soil nonlinearity in the soil response for severe bedrock motions, the use of a generic soil-attenuation relation may, however, yield inaccurate results even when the assumption of generic soil conditions is appropriate. Most empirical predictive equations do not explicitly incorporate the effects of site response beyond a simple term that includes a macro-parameter of the soil deposit. For example,

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Campbell (1997) uses the depth to basement rock, whereas Boore *et al.* (1997) adopt the average shear-wave velocity in the first 30 m of soil, $\overline{V}_{30}$. These two models predict the same amplification for mild and severe bedrock shaking; that is, they do not include any nonlinearity in the site response. To our knowledge, the attenuation equation by Abrahamson and Silva (1997) is the only one published to date that explicitly allows for the nonlinear response of local soil. To accommodate soil nonlinearity, however, this equation uses the expected peak bedrock ground acceleration, $\overline{PGA}_{\text{rock}}$, which in our companion article in this issue (Bazzurro and Cornell, 2004) has been shown to be a rather poor predictor of soil amplification at low frequencies.

Another method (used almost exclusively for important facilities) consists of explicitly characterizing the soil response via a deterministic amplification function (usually the *mean* or the *median*) computed for a generally small suite of appropriate seismograms driven through a computer model of the soil column with best estimates of the soil parameters. The equivalent-linear SHAKE program (Schnabel *et al.*, 1972) is used for such purposes in the majority of cases. The seismograms are either real recordings from events consistent with those dominating the site hazard or synthetic time histories that match a predefined target-response spectrum. In most such applications the desired hazard at the soil surface is obtained by multiplying the bedrock hazard (in the form either of uniform hazard spectra or of hazard curves) by this deterministic amplification function. This approach, which is the hybrid method in Cramer (2003) and is similar to approach 1 in U.S. Nuclear Regulatory Commission (2001), does not explicitly account for the amplification function record-to-record variability. It produces surface ground-motion levels whose exceedance rates are unknown, non-uniform, inconsistent across frequency, and generally nonconservative. Similarly, soil surface ground motions with unknown exceedance rates are in general obtained if one multiplies the bedrock hazard by the average National Earthquake Hazards Reduction Program (NEHRP) amplification factors (e.g., see Tables 4.1.2a and 4.1.2b in Federal Emergency Management Agency, 2001).

In more refined studies (e.g., Cramer, 2003) the uncertainty of the site response is indeed considered but often is not properly coupled with the ground-motion variability in the PSHA computations that lead to the hazard estimates at the bedrock. The result of this operation, as pointed out in Silva *et al.* (2000), is an artificially inflated hazard at the soil surface. This subject will be clarified below.

The approach presented here, which is heavily based on the findings presented in our companion article, overcomes the shortcomings previously outlined. Applications are shown for two very different offshore soil sites, one sandy and one clayey. For illustration purposes, the hazard results from this method are compared with those obtained by using an attenuation equation for generic soil. We also discuss the use of two alternative, efficient but approximate techniques: the first requires modifying a rock ground-motion-attenuation relation to attain a site-specific soil-surface predictive equation to be used in PSHA; the second modifies the hazard at the rock level by means of a generalization of the so-called risk equation (e.g., Cornell, 1994, 1996).

Methodology

The methodology that follows is a fully probabilistic procedure to account for nonlinear soil response in PSHA. Preliminary versions were presented in Cornell and Bazzurro (unpublished manuscript, 1997), Bazzurro (1998), and Bazzurro *et al.* (1998). This body of work has also inspired, directly or indirectly, similar studies on this subject by Lee *et al.* (1998, 1999), Lee (2000), and, more recently, Cramer (2003). Tsai (2000) also recently investigated a similar issue. The methodology presented by the authors in their early work is an integral part of the procedure prepared for the U.S. Nuclear Regulatory Commission (USNRC) for nuclear-facility sites (e.g., see section 6 and appendix I in USNRC, 2001; and applications in USNRC, 2002) for developing hazard-consistent spectra on soil.

The objective is to estimate the soil-surface-hazard curve for the spectral acceleration, $S_a^s(f, \xi)$, at a generic oscillator frequency, f, and damping, ξ. This is simply a relationship between $S_a^s(f, \xi)$ and the annual mean rate of exceedance (MRE) or alternatively versus the mean return period (MRP). The MRP is defined as the reciprocal of the annual MRE (e.g., an MRE of 4×10^{-4} implies an MRP of 2500 years). For small values of engineering interest the rare-event assumption holds. This assumption implies that the likelihood of two or more events occurring in the period of interest is small in comparison to the likelihood that one event happens. The rare-event assumption implies that the MRE of a value s is numerically equal to the probability that the annual maximum spectral acceleration exceeds s. Throughout this article we make this assumption, and therefore we are permitted to use the notation of $G_S(s)$ for this complementary cumulative distribution function (CCDF), understanding that it is numerically identical to the hazard or MRE curve.

The uniform hazard spectrum (UHS) at the soil surface for any desired MRP can be obtained by interpolating hazard curves, if available, for a suitably dense range of frequencies. Throughout this study the damping is assumed equal to 5% of critical, and therefore ξ is dropped hereafter from the notation. Note that the following procedure can be applied to obtain the hazard at any depth (e.g., at the structure-foundation level), provided that the soil response is computed accordingly.

Consistently with our companion article (Bazzurro and Cornell, 2004), the effect of the soil layers on the intensity of the ground motion at the surface is incorporated via a site-specific, frequency-dependent amplification function, $AF(f)$:

$$AF(f) = \frac{S_a^s(f)}{S_a^r(f)}, \tag{1}$$

where f is a generic oscillator frequency, $S_a^s(f)$ and $S_a^r(f)$ are the 5%-damped spectral-acceleration values at the soil surface and at the bedrock, respectively.

The prediction of $AF(f)$, given the knowledge of bedrock-ground-motion parameters, was studied in our companion article by running multiple recordings through different realizations of the soil column with uncertain properties. That study has shown that $S_a^r(f)$ is the most effective predictor variable for estimating $AF(f)$ (at the same frequency f) among different bedrock-ground-motion parameters such as magnitude, M, of the causative event; source-to-site distance, R; PGA_r; and spectral-acceleration values, $S_a^r(f_{sc})$, at the initial resonant frequency, f_{sc}, of the soil column.

Furthermore, results showed that once the $S_a^r(f)$ value of a record at the bedrock is known, the additional knowledge of M and R, which implicitly define its average response spectrum shape, do not appreciably improve the estimation of $AF(f)$ at the same frequency f. In other words, $AF(f)$, conditioned on $S_a^r(f)$, is virtually independent on M and R. The interested reader is referred to our companion article for a quantification of such a statement. $S_a^r(f)$ can be said to be, therefore, a sufficient estimator of $AF(f)$ (Luco, 2001). The procedure for estimating surface hazard in this case is given in the subsection that follows.

Strictly speaking, this lack of M and R conditional dependence does not hold for frequencies f below f_{sc}, where a mild dependence on M—or, more significantly, on $S_a^r(f_{sc})$—was found. We will treat this latter, more complicated case in a subsequent section. Finally, if a particular application warrants keeping the dependence of $AF(f)$ on M and R, besides on $S_a^r(f)$, then the procedure to estimate $S_a^s(f)$ versus MRE requires knowledge of the hazard curve at bedrock plus results that can be extracted from standard hazard disaggregation. The interested reader can find relevant details in Bazzurro (1998).

Convolution: $AF(f)$ Dependent on $S_a^r(f)$

The proposed method for computing surface-hazard curves for $Z = S_a^s(f)$ convolves the site-specific rock-hazard curves for $X = S_a^r(f)$, which may be exogenously provided (e.g., by a site-specific PSHA analysis or by the hazard maps at the U.S. Geological Survey's Web site (http://geohazards.cr.usgs.gov/eq/), with the probability distribution of $Y = AF(f)$, an estimate of which is obtained through nonlinear dynamic analyses of the soil. The change in notation is adopted to make the following equations less cumbersome. Note that $Z = X \cdot Y$. Based on the elementary probability theory (e.g., Benjamin and Cornell, 1970, pp. 112–124), the convolution goes as follows:

$$G_Z(z) = \int_0^\infty P\left[Y \geq \frac{z}{x}\middle| x\right] f_X(x)dx = \int_0^\infty G_{Y|X}\left(\frac{z}{x}\middle| x\right) f_X(x)dx, \quad (2)$$

where $G_W(w)$ is the CCDF of any random variable (RV), W—for example, $G_Z(z)$ is the sought hazard curve for $S_a^s(f)$, that is, the annual probability of exceeding level z, and $G_{Y|X}$ is the CCDF of $Y = AF(f)$, conditional on a rock-level amplitude x—and $f_X(x)$ is the probability density function (pdf) of $S_a^r(f)$. Note that $f_X(x)$ is the absolute value of the derivative of $G_X(x)$ with respect to x, and recall that $G_X(x)$ is numerically equal to MRE and $f_X(x)$ is referred to as the mean rate density. Because in practice $G_Z(z)$ is always numerically computed, this equation is more readily useful in discretized form:

$$G_Z(z) = \sum_{\text{all } x_j} P\left[Y \geq \frac{z}{x}\middle| x_j\right] p_X(x_j) = \sum_{\text{all } x_j} G_{Y|X}\left(\frac{z}{x}\middle| x_j\right) p_X(x_j). \quad (3)$$

The term $p_X(x_j)$ represents the probability that the rock-input level is equal to (or better, in the neighborhood of) x_j. This term can be approximately derived by differentiating the rock-hazard curve in discrete or numerical form. An easy way of interpreting this equation is considering that if the random variable X takes on some value x (say, 0.25 g) then Z will exceed the value z (say, 0.5 g) if and only if Y takes on some value $y \geq z/x = 2$. We consider the probability of this event by the product $P[Y \geq z/x|x_j]p_X(x_j)$, and then we sum over all possible values of x_j.

Assuming the lognormality of Y given X, when all the RVs are expressed in continuous rather than discrete form the $G_{Y|X}$ is given by:

$$G_{Y|X}\left(\frac{z}{x}\middle| x\right) = \hat{\Phi}\left(\frac{\ln\left[\frac{z}{x}\right] - \ln[\hat{m}_{Y|X}(x)]}{\sigma_{\ln Y|X}}\right) \quad (4)$$

in which $\hat{\Phi}(\cdot) = 1 - \Phi(\cdot)$ is the widely tabulated complementary standard Gaussian CDF. Estimates of the distribution parameters of Y (i.e., the conditional median of Y, $\hat{m}_{Y|X}$, and the conditional standard deviation of the natural logarithm of Y, $\sigma_{\ln Y|X}$) can be found by driving a suite of n rock-ground-motion records through a sample of soil-column representations (recall that the soil properties are uncertain) and then regressing, for each frequency f, the values of ln Y on ln X. These regressions, based on a large suite of records, were shown for two soil sites in figure 8 of our companion article. We emphasize again, however, that in real applications, given the relative small values of $\sigma_{\ln Y|X}$, only 10 soil response analyses are sufficient to achieve a level of accuracy of $\pm 10\%$ in estimating the median $AF(f)$. The large database was used only to validate the procedure.

From the computational standpoint, implementing the convolution described by equations (2) (or 3) and (4) is an easy task. A word of caution, however, is in order regarding the wide range of $X = S_a^r(f)$ values to be considered in achieving accuracy in the computation of $G_Z(z)$. In extreme cases, the exceedance of a particular soil-ground-motion level equal to z may in fact occur both for a very small bedrock ground motion significantly amplified by the nearly linear soil response or for a large bedrock ground motion greatly deamplified by the nonlinear behavior of the soil column. Both the amplification function and the rock seismic-hazard curves need to span over a domain of X that is considerably larger than the domain of Z for which the results are desired.

In the development of equation (2) we have used the rare-event assumption. Relaxing this assumption implies permitting the occurrence of more than one event in the period of interest. In this case, one would have to carefully distinguish the contributions to the random $AF(f)$ that are the same for every event (e.g., the soil properties) from those that vary randomly from event to event (e.g., the ground-motion details). The occurrence of two or more events may become important for MRE values greater than, say, 0.1, and this case is seldom of earthquake-engineering significance.

Convolution: $AF(f)$ Dependent on $S_a^r(f)$ and $S_a^r(f_{sc})$

As pointed out in our companion article, in the frequency range below f_{sc} the $AF(f)$ is jointly negatively correlated with $S_a^r(f)$ and positively correlated with M. M seems to explain part of the variability in $AF(f)$ because it carries information about the spectral shape and therefore about $S_a^r(f_{sc})$, the bedrock spectral acceleration at the initial resonance frequency of the soil. The higher $S_a^r(f_{sc})$ is, the more significant the shift of the peak of the $AF(f)$ function is toward lower f values, a shift that in turn tends to increase the value of $AF(f)$ at $f < f_{sc}$. Hence, in this frequency range $S_a^r(f)$ alone is somewhat an insufficient parameter for the estimation of $AF(f)$ (Luco, 2001).

The consequence of this insufficiency is that the estimates of $\hat{m}_{AF(f)S_a^r(f)}$ and $\sigma_{\ln AF(f)S_a^r(f)}$ that are obtained using a limited dataset of real recordings are not necessarily accurate, and this may potentially lead to imprecise estimates of the hazard curves for $S_a^s(f)$ (Bazzurro and Cornell, 2002). The inaccuracy may arise because the record set does not necessarily have the right, hazard-consistent spectral shape for a site as determined by the combinations of M–R events that nearby faults are more likely to generate. The inaccuracy can be reduced if the records are selected with care to ensure that the M reflects the controlling scenario for the site. Even with an infinitely large database of seismograms, however, a perfect record selection for a site may prove to be an impossible task, because the scenario that dominates the hazard generally changes with hazard level. Note that this potential source of inaccuracy is a limitation of any statistical approach that provides estimates based on limited (and potentially biased) samples of data.

On the basis of the experience gained so far, we believe that, given $S_a^r(f)$, the conditional dependence of $AF(f)$ on $S_a^r(f_{sc})$ in the frequency range below f_{sc} is not strong enough to generate large errors in the estimate of the soil-hazard curves, if neglected. Potential errors can be reduced or possibly avoided altogether by keeping the soil response jointly dependent on $S_a^r(f)$ and $S_a^r(f_{sc})$ and coupling it with the site-specific joint hazard of $S_a^r(f)$ and $S_a^r(f_{sc})$ rather than with the rock-hazard curves for $S_a^r(f)$ from conventional scalar PSHA. The site-specific joint hazard can be computed via vector-valued probabilistic seismic-hazard analysis (VPSHA) (Bazzurro and Cornell, 2002).

In this framework the convolution method for computing surface-hazard curves for $Z = S_a^s(f)$, based on $X_1 = S_a^r(f)$ and $X_2 = S_a^r(f_{sc})$, can be described by the following equation:

$$G_Z(z) = \int_0^\infty \int_0^\infty G_{Y|X_1;X_2}\left(\frac{z}{x_1}\bigg| x_1;x_2\right) f_{X_1;X_2}(x_1;x_2)dx_1dx_2, \quad (5)$$

or in discretized form:

$$G_Z(z) = \sum_{\text{all}\, x_{1,j}} \sum_{\text{all}\, x_{2,i}} G_{Y|X_1;X_2}\left(\frac{z}{x_{1,j}}\bigg| x_{1,j};x_{2,i}\right) p_{X_1;X_2}[x_{1,j};x_{2,i}]. \quad (6)$$

The $f_{X_1;X_2}(x_1;x_2)$ term in equation (5) is (numerically equal to) the site-specific mean rate density of $\ln S_a^r(f) = \ln X_1$ and $\ln S_a^r(f_{sc}) = \ln X_2$ for a site obtained via VPSHA (Bazzurro and Cornell, 2002). In equation (6) the $G_{Y|X_1;X_2}$ represents the CCDF of Y conditional on X_1 and X_2, and $p_{X_1;X_2}[x_{1,j};x_{2,i}]$ denotes the probability that the rock spectral-acceleration values at frequencies f and f_{sc} are in the neighborhood of $x_{1,j}$ and $x_{2,i}$, respectively. As before, assuming lognormality of Y, given X_1 and X_2, the $G_{Y|X_1;X_2}$ in continuous form is given by:

$$G_{Y|X_1;X_2}\left(\frac{z}{x_1}\bigg| x_1;x_2\right) = \hat{\Phi}\left(\frac{\ln\left[\frac{z}{x_1}\right] - \ln[\hat{m}_{Y|X_1;X_2}(x_1;x_2)]}{\sigma_{\ln Y|x_1;x_2}}\right) \quad (7)$$

where the estimates of the distribution parameters of Y, $\hat{m}_{Y|X_1;X_2}(x_1;x_2)$, and $\sigma_{\ln Y|x_1;x_2}$, are obtained through the multiple regression of $\ln AF(f) = \ln Y$ on $\ln S_a^r(f) = \ln X_1$ and $\ln S_a^r(f_{sc}) = \ln X_2$ (see companion article).

PSHA with Soil-Specific Attenuation Equation

The most straightforward approach for estimating seismic-hazard curves at the surface of a soil deposit is to develop an attenuation relationship for that soil condition and use it during conventional PSHA calculations (W. J. Silva, personal comm., 1996). Developing a robust soil-specific attenuation law requires that a significant number of rock-ground-motion accelerograms (more than 100) be propagated (and perhaps deconvolved first through a generic rock profile; see, e.g., Kramer, 1996) through a numerical model of the sediment deposit. The computed $S_a^s(f)$ data could then be processed by using, for example, advanced regression-analysis techniques (e.g., Abrahamson and

Youngs, 1992). These techniques account for correlations in the data, owing to multiple recordings of a single earthquake, for obtaining the desired attenuation equation for $\widehat{S_a^s(f)}$, that is, the median of $S_a^s(f)$, given M, R, and $\boldsymbol{\theta}$, where the vector $\boldsymbol{\theta}$ represents source mechanism, wave travel path, and local site conditions, characteristics that are commonly included in modern attenuation relations.

This direct method, although feasible, is rather impractical. It produces, however, hazard curves at the soil surface that may be considered "exact." The soil-hazard curves provided by this exact method can be used as a benchmark for testing the validity of the proposed convolution approach mentioned in the two previous subsections. Such a validation was conducted in Bazzurro (1998) and will not be repeated in this article. In contrast we explore here the use of existing rock-attenuation laws amended with an additional term to account for local soil conditions with uncertain properties. As pointed out earlier, this frequency-specific correction factor, which is simply the amplification factor, $AF(f)$, can be obtained without driving an extensive dataset of records through the soil column (Bazzurro and Cornell, 2004).

If a linear predictive model for (log) $AF(f)$ in terms of (log) $S_a^r(f)$ is appropriate, then closed-form equations can be derived to integrate $AF(f)$ directly into an existing rock-attenuation equation for $S_a^r(f)$, transforming it into a site-specific soil-attenuation equation. If the linearity condition does not hold, difficulties arise in finding a closed-form relationship when coupling the attenuation error term for $S_a^r(f)$, given M, R, and $\boldsymbol{\theta}$, with the error term of the regression of (log) $AF(f)$ on (log) $S_a^r(f)$. It is emphasized however, that the equations that follow can be applied also to piecewise–linear models to approximate, for example, a quadratic behavior such as that shown in figure 8 in our companion article (see also Fig. 2, to come).

In logarithmic terms, equation (1) can be written as:

$$\ln S_a^s(f) = \ln S_a^r(f) + \ln AF(f). \tag{8}$$

Analytical expressions for $\ln S_a^r(f)$ in terms of M, R, and $\boldsymbol{\theta}$ are numerous in the literature (for a review, see Abrahamson and Shedlock, 1997). Such attenuation functions for a specific frequency value f have typically the form:

$$\begin{aligned}\ln S_a^r(f) &= g_1(M, R, \boldsymbol{\theta}) + \varepsilon_{\ln S_a^r(f)}\sigma_{\ln S_a^r(f)} \\ &= \ln \widehat{S_a^r(f)} + \varepsilon_{\ln S_a^r(f)}\sigma_{\ln S_a^r(f)},\end{aligned} \tag{9}$$

where $\ln \widehat{S_a^r(f)}$ is the median of $\ln S_a^r(f)$, given M, R, and $\boldsymbol{\theta}$. The remaining variation unexplained by the nonlinear regression is captured by a (standardized Gaussian) random variable, $\varepsilon_{\ln S_a^r(f)}$, which can be defined as the number of (logarithmic) standard deviations by which the random (logarithmic) spectral acceleration deviates from its median value as predicted by an attenuation equation in terms of M, R, and $\boldsymbol{\theta}$. Values of $\sigma_{\ln S_a^r(f)}$ are available for standard attenuation relationships.

Our companion article has shown that $AF(f)$, conditional on the spectral value at the rock level, $S_a^r(f)$, is virtually independent of M and R, and other ground-motion parameters such as $S_a^r(f_{sc})$ or PGA. Assuming that a linear regression in logarithmic space is appropriate to describe the dependence of $\widehat{AF(f)}$ on $S_a^r(f)$, at least within a limited range of $S_a^r(f)$ values, then:

$$\begin{aligned}\ln AF(f) &= \ln \widehat{AF(f)} + \varepsilon_{\ln AF(f)}\sigma_{\ln AF(f)} \\ &\approx c_0 + c_1 \ln S_a^r(f) + \varepsilon_{\ln AF(f)}\sigma_{\ln AF(f)},\end{aligned} \tag{10}$$

where c_0 and c_1 are coefficients of the linear regression in logarithmic space of $AF(f)$ on $S_a^r(f)$, $\varepsilon_{\ln AF(f)}$ is a standard normal variable, and $\sigma_{\ln AF(f)}$ represents the standard error of estimation from the regression that has been found in work to date (Bazzurro and Cornell, 2004; U.S. Nuclear Regulatory Commission, 2001) to be not significantly functionally dependent on $S_a^r(f)$ (or M and R).

From these equations it follows that:

$$\begin{aligned}\ln S_a^s(f) = {} & \ln \widehat{S_a^r(f)} + \varepsilon_{\ln S_a^r(f)}\sigma_{\ln S_a^r(f)} + \ln \widehat{AF(f)} \\ & + \varepsilon_{\ln AF(f)}\sigma_{\ln AF(f)}.\end{aligned} \tag{11}$$

However from equation (10):

$$\begin{aligned}\ln \widehat{AF(f)} &\approx c_0 + c_1 \ln S_a^r(f) \\ &= c_0 + c_1[\ln \widehat{S_a^r(f)} + \varepsilon_{\ln S_a^r(f)}\sigma_{\ln S_a^r(f)}].\end{aligned} \tag{12}$$

Substituting equation (12) into equation (11):

$$\begin{aligned}\ln S_a^s(f) \approx {} & c_0 + (c_1 + 1) \ln \widehat{S_a^r(f)} \\ & + (c_1 + 1)\varepsilon_{\ln S_a^r(f)}\sigma_{\ln S_a^r(f)} + \varepsilon_{\ln AF(f)}\sigma_{\ln AF(f)}.\end{aligned} \tag{13}$$

Equation (13) states that the median $S_a^s(f)$ can be predicted from the median of $S_a^r(f)$ as:

$$\ln \widehat{S_a^s(f)} \approx c_0 + (c_1 + 1) \ln \widehat{S_a^r(f)}. \tag{14}$$

R. K. McGuire, (personal comm., 2002) found that $\varepsilon_{\ln AF(f)}$ and $\varepsilon_{\ln S_a^r(f)}$ are mildly negatively correlated, mostly for large values of $\ln S_a^r(f)$. However, if we (conservatively) neglect this negative correlation, then the dispersion measure for $\ln S_a^s(f)$ becomes:

$$\sigma_{\ln S_a^s(f)} \approx \sqrt{(c_1 + 1)^2\sigma^2_{\ln S_a^r(f)} + \sigma^2_{\ln AF(f)}}. \tag{15}$$

Equations (14) and (15) permit the assembly of an approximate attenuation relationship for the soil-surface-motion parameter, $S_a^s(f)$, by coupling the available attenuation equation for the rock-motion parameter, $S_a^r(f)$ (equation 14), with the site-specific regression for $AF(f)$ on $S_a^s(f)$ (equation 10).

It is interesting to note that because, in general, $AF(f)$ is negatively correlated with the amplitude of $S_a^r(f)$—that is, the higher the value of $S_a^r(f)$, the lower the value of $AF(f)$, at least at frequencies f higher than 0.5 Hz (see companion article), the coefficient c_1 is negative. Hence, from equation (15) it follows that, depending on how large $\sigma^2_{\ln AF(f)}$ is, the variability in $S_a^s(f)$ may be less than the variability in $S_a^r(f)$. This phenomenon, which is presumably due to the nonlinearity in the soil response during severe ground shaking, has in fact been observed from real recordings on rock and soil conditions (see Abrahamson and Sykora, 1993; Toro *et al.*, 1997). As mentioned before, accounting for the negative correlation between $AF(f)$ and $S_a^r(f)$ also avoids the concern pointed out by Silva *et al.* (2000) that artificially inflated soil-hazard curves may be obtained owing to overestimated uncertainty on $S_a^s(f)$ if one simply adds the amplification uncertainty to that already contained in the rock-hazard curve without recognition of this negative correlation.

An application of this method will be shown later.

An Analytical Estimate of the Soil Hazard

Under the same assumptions of a previous subsection, namely, that the dependence of $AF(f)$ on M and R, given $S_a^r(f)$, is weak, the convolution in equation (2) can be rewritten without the explicit use of $AF(f)$ as follows:

$$G_Z(z) = \int_0^{\infty} P[Z > z|x] f_X(x)dx = \int_0^{\infty} G_Z(z|x) f_X(x)dx. \tag{16}$$

Under the mild assumptions (e.g., Cornell, 1994, 1996; Bazzurro *et al.*, 1998) that (1) $Z = S_a^s(f)$, given $X = S_a^r(f)$ follows a lognormal distribution as before for $AF(f)$, given $S_a^r(f)$; (2) a linear (or piecewise–linear) model in log-space for the dependence of $\widehat{S_a^s(f)}$ on $\widehat{S_a^r(f)}$ (equation 14) is appropriate and homoscedastic, or that the standard error of estimation, $\sigma_{\ln S_a^s(f)}$, is constant with respect to $S_a^r(f)$; and (3) the hazard curve for $S_a^r(f)$ in log–log space can be locally replaced by its linear tangent, resulting in the equation $H[S_a^r(f)] \approx k_0 S_a^r(f)^{-k_1}$; then the convolution integral in equation (16) can be solved in closed form. This is a generalization of the so-called risk equation (see also U.S. Nuclear Regulatory Commission, 2001, for a similar derivation):

$$G_Z(z) = H(\hat{S}^r_{a,z}) e^{\frac{1}{2}\frac{k_1^2\sigma^2}{(c_1+1)^2}}. \tag{17}$$

$H(\hat{S}^r_{a,z})$ is the value of the site's bedrock-hazard curve, corresponding to the median level, $\hat{S}^r_{a,z}$, of $S_a^r(f)$ capable of inducing the specified $S_a^s(f)$ level z as predicted by the fitted linear regression model, that is, $\hat{S}_{a,z} = (z/c_0)^{1/(c_1+1)}$ (equation 14). The value of σ is, of course, numerically equivalent to $\sigma_{\ln AF(f)}$, the dispersion measure of the linear regression model of ln $AF(f)$ on ln $S_a^r(f)$ (equation 10). The constant k_1 is the slope (in log–log scale) of the straight-line tangent to the hazard curve at the point $\hat{S}^r_{a,z}(f)$ (or, alternatively, the slope of the secant straight line that approximates the hazard curve in the neighborhood of the tangent point). Such a linearization is usually an adequate approximation over an order-of-magnitude probability range around the tangency point. To quantify this term, note that $k_1 = 1/\log a_{10}$, where a_{10} is the factor by which one must increase $S_a^r(f)$ in order to increase the hazard (or reduce the mean return period) by one order of magnitude (U.S. Department of Energy, 1993).

Hence, loosely speaking, the annual probability that $S_a^r(f)$ exceeds a given level of z is equal to the value of the bedrock-hazard curve at a bedrock value x that induces a soil ground motion in exceedance of z (50% of the time) multiplied by an exponential correction factor. Given the typical values of $\sigma = \sigma_{\ln AF(f)}$ in the order of 0.3, and of the slopes of the hazard curves for most sites commonly varying from 2.5 to 6 and for c_1 ranging from 0 to approximately -0.8 (values observed for the two soil sites in our companion article), such a correction factor is often not larger than 10. However, for some combinations of these parameter values—for example, large negative c_1 and large $\sigma = \sigma_{\ln AF(f)}$—it may become very large. We do not advise the use of this method when the correction factor exceeds 10.

Applications

The two soil deposits, one sandy and one clayey, considered in our companion article, were assumed to be in the Santa Barbara Channel (SBC) (Fig. 1), southern California, for which a seismotectonic model was readily available (N. A. Abrahamson, personal comm., 1996). Considering a single location permits a direct comparison of the effects of the different amplification of the two soil sediments, given the same seismic hazard at the bedrock.

The site hazard was computed by a conventional PSHA approach with the Abrahamson and Silva (1997) ground-motion-attenuation law (both for rock and, for comparison purposes, for generic soil conditions), and by the proposed site-specific convolution method (equation 2). Again, the latter method makes use of the rock-hazard curves found by means of the aforementioned ground-motion predictive equation and the site-specific statistics of $AF(f)$ conditional on $S_a^r(f)$ only. We consider both a quadratic model and a piecewise–linear model of (log) $AF(f)$ on (log) $S_a^r(f)$. Figure 2 shows the fitted models for selected frequency values for both soil columns (see also fig. 8 in our companion article). The consideration of a piecewise–linear model is necessary for applying the two approximate methods proposed earlier. The piecewise–linear model is also used here to investigate the effect on the soil hazard of using alternative regression models for $AF(f)$.

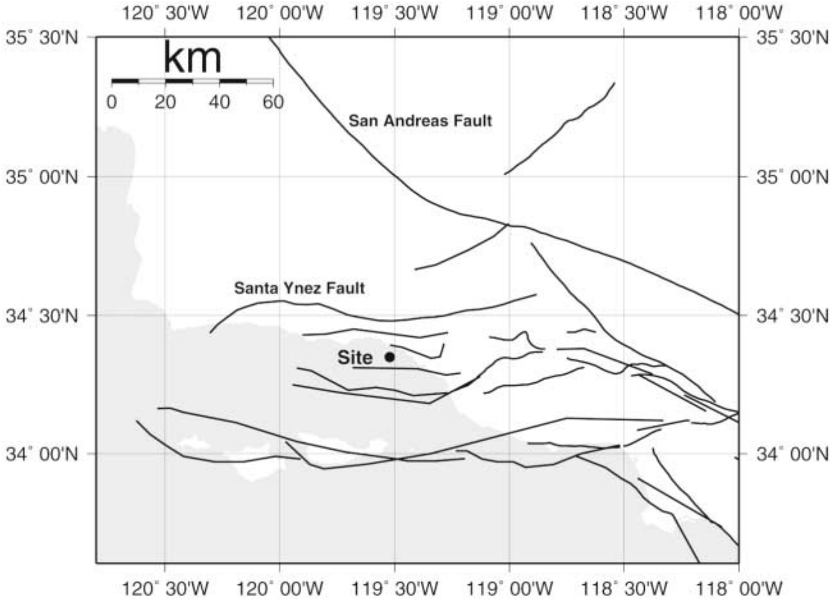

Figure 1. Location of the hypothetical site in the Santa Barbara Channel.

Hazard Estimates from the Numerical Convolution Method

The soil UHS with probability of exceedance (PE) of 50%, 10%, 5%, and 2% in 50 years are shown in Figure 3 for both soil conditions. The convolution results in Figure 3 were obtained by using the quadratic model of $AF(f)$ on $S_a^r(f)$. The rock UHS are also included for comparison. As reported in our companion article, the sandy site can be classified as NEHRP Type D soil ($\overline{V}_{30} \approx 260$ m/sec), whereas the clayey site is a Type E soil ($\overline{V}_{30} \approx 160$ m/sec).

It is apparent that using a generic soil attenuation law in this case may lead to severe underestimation of the hazard for $S_a^s(f)$ below approximately $f = 2$ Hz at low MRP values. The hazard at high frequencies (here above 2 Hz) is overestimated by the predictive equation for generic soil conditions especially at high MRP values. The gap at high frequencies between the UHS found by convolution and by conventional PSHA (using generic soil conditions of Abrahamson and Silva, 1997), however, may be partly due to the application of rock-outcrop motions directly to the column base (Bazzurro and Cornell, 2004). For the most part, however, these differences in hazard prediction are due to the significant nonlinear response (Fig. 2) of the two soil columns considered in this study. Even higher differences between soil-specific and soil-generic hazard estimates are expected when empirical attenuation laws that, in contrast to the Abrahamson and Silva (1997) relation used here, do not account for nonlinear soil behavior are adopted in the PSHA.

The soil-specific UHS for the four MRP values considered in Figure 3 show much less relative difference from one MRP value to another than their counterparts found by using the generic soil-attenuation relation. For example, at 5 Hz the amplitudes of the 500-year and 2500-year UHS differ by only 20% in the sandy soil-specific case and by a difference that goes up to 80% for bedrock. This "saturation" or relative insensitivity to MRP of the UHS ordinates is due, again, to the extremely nonlinear responses of both these example soil columns, which, in general, amplify low bedrock motions and de-amplify large bedrock motions much more significantly than the empirical attenuation.

It is also worth noting that the UHS specifically developed for a given soil deposit are not as smooth as those customarily obtained via PSHA with a soil-generic ground-motion prediction equation. For example, the large peak at 1.5 Hz in the UHS for the clayey site is due to the relatively large value of $AF(f)$ that is present at the second harmonic frequency, despite the high nonlinearity of the soil response (fig. 5b in our companion article). This higher harmonic effect is less pronounced in the sandy-soil deposit (fig. 5a in our companion article).

Figure 4 shows the variation in the soil hazard at the sandy site resulting from the use of two alternative models for $AF(f)$ on $S_a^r(f)$, the quadratic model and the piecewise–linear model displayed in Figure 2. The hazard curves for $S_a^s(f)$ at frequencies of 0.33 Hz, 1 Hz, and 5 Hz and for PGA reveal that the discrepancies arise mostly in the high acceleration range, where the estimates of $AF(f)$, based on the quadratic model, are lower. The hazard curves, in fact, tend to separate in the same neighborhood of $S_a^r(f)$ where the two fitted curves for $AF(f)$ begin diverging. In this high-frequency interval the quadratic model for $AF(f)$ falls off more sharply than the linear model fitted to the upper portion of the data (see Fig. 2). Note that to produce the hazard curves in Figure 4 the range of $S_a^r(f)$ used in the numerical convolution process (equation 3) was within the range of applicability of the fitted models for $AF(f)$, namely 0.01–0.5 g for $f = 0.33$ Hz, 0.01–2.0 g for $f = 1.0$ Hz, 0.01–3.0 g for

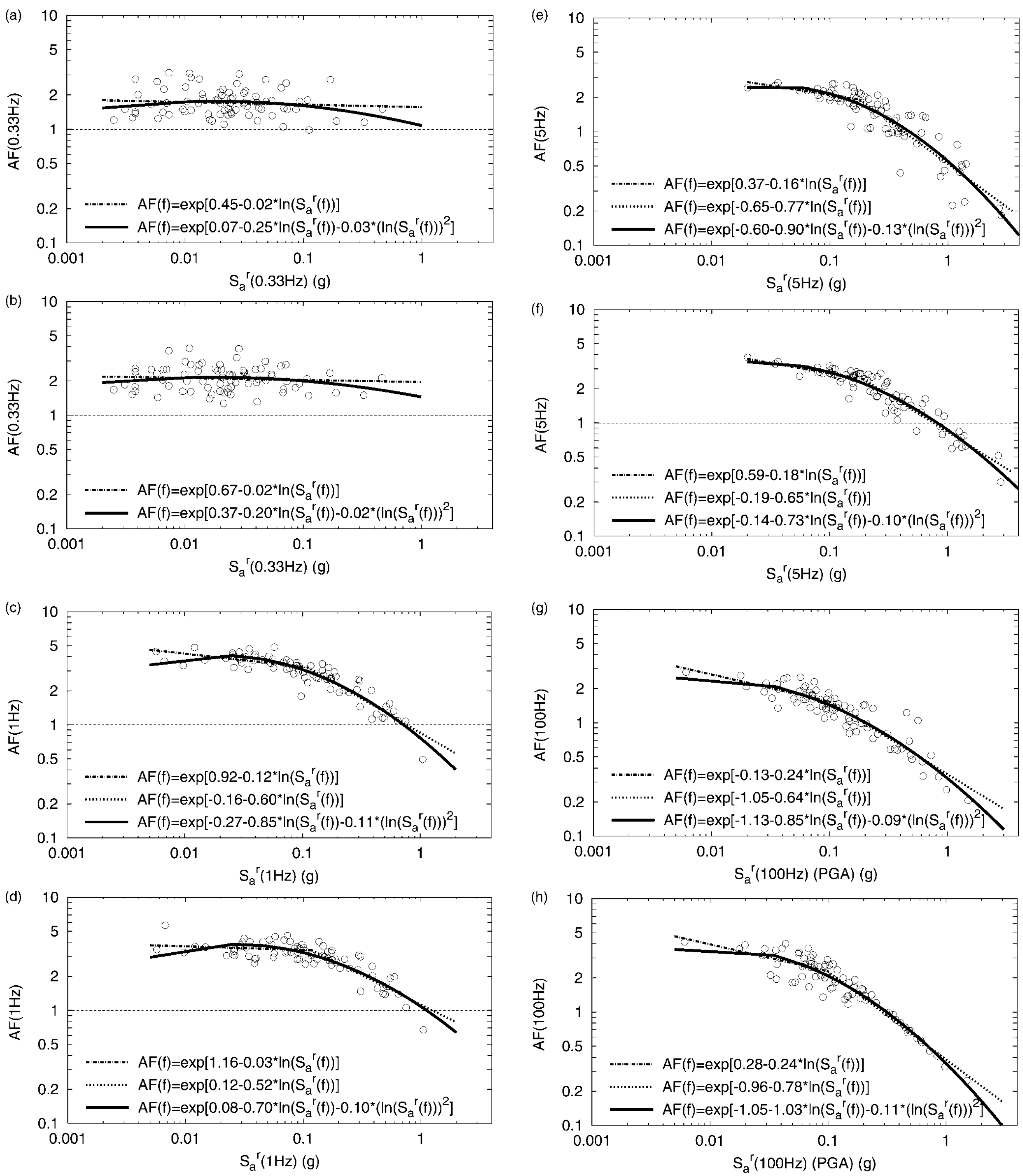

Figure 2. Quadratic and piecewise–linear regression models of $AF(f)$ on $S_a^r(f)$ at different f values for the two soil colums, sand (panels a, c, e, and g) and clay (panels b, d, f, and h).

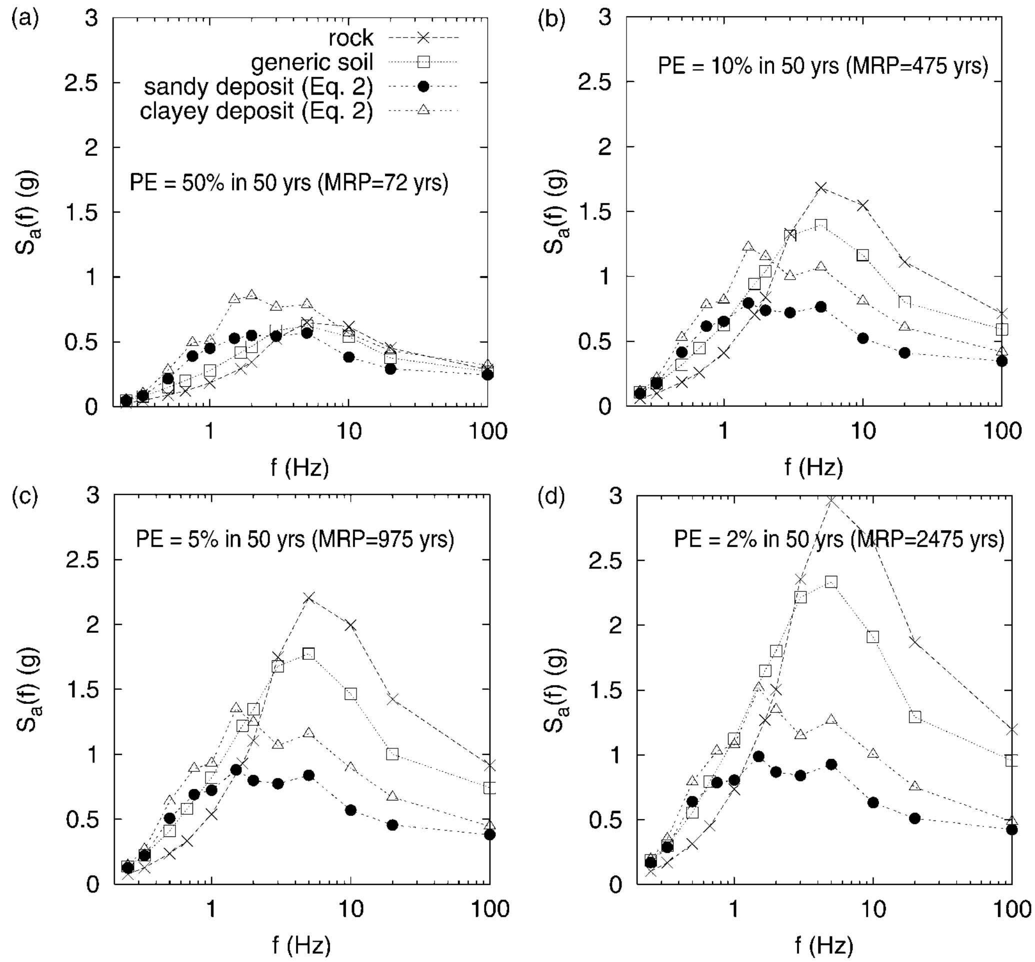

Figure 3. Uniform hazard spectra (UHS) for the hypothetical SBC site. (PE = annual probability of exceedance; MRP = mean return period.)

$f = 5.0$ Hz, and 0.01–2.0 g for $f = 100.0$ Hz (i.e., PGA). To achieve acceptable accuracy in the estimate of the surface-hazard curve, we used 100 values of $S_a^r(f)$ (called x_js in equation 3) during the numerical convolution.

Finally, before moving further it is interesting to compare surface UHS obtained from convolution (i.e., equation 2) to those obtained by using a traditional engineering approach for computing surface-design ground motion. Borrowing from Cramer (2003), we will call it a "hybrid" method here because it is a mix of probabilistic and deterministic thinking. To obtain the surface UHS, this method, which as mentioned before is similar to approach 1 in U.S. Nuclear Regulatory Commission (2001), multipies the ordinates of the rock UHS by the values of the median (or, more often, the mean) $AF(f)$ at each frequency. Figure 5 shows the comparison of hybrid-based and convolution-based surface UHS. The hybrid method tends to be nonconservative at all frequencies and at all MRPs. The underestimation of the surface ground motion, which is not constant across frequency, is more prominent for longer MRPs and for higher frequencies where the (neglected) variability in $AF(f)$ is larger.

Hazard Estimates from PSHA with Soil-Specific Attenuation Law

The soil-specific prediction equations for $S_a^s(f)$ at f of 0.33 Hz, 1 Hz, and 5 Hz and for PGA were constructed for the sandy site according to equations 14 and 15. The correction factor was introduced in the existing rock-ground-motion predictive equation from Abrahamson and Silva (1997). We used the values of c_0 and c_1 of the piecewise–linear models shown in Figure 2 and the values of $\sigma_{\ln AF(f)}$ in Table 1. This table also includes the values of the threshold spectral acceleration, $S_{a,\mathrm{tr}}^r(f)$, that separate the upper from the lower range of the data.

In the case examined here, this procedure leads to two soil-specific attenuation equations for each frequency, f', one valid in the lower interval of $S_a^r(f')$ and one valid in the higher interval of $S_a^r(f')$. These two equations were used in two separate runs of a PSHA code (N. A. Abrahamson personal comm., 1996) to estimate the soil-hazard curve for $S_a^s(f')$. Of course, the equation that uses the $AF(f)$ model fitted to the lower portion of the data will greatly overestimate the amplification beyond the $S_{a,\mathrm{tr}}^r(f)$ threshold value

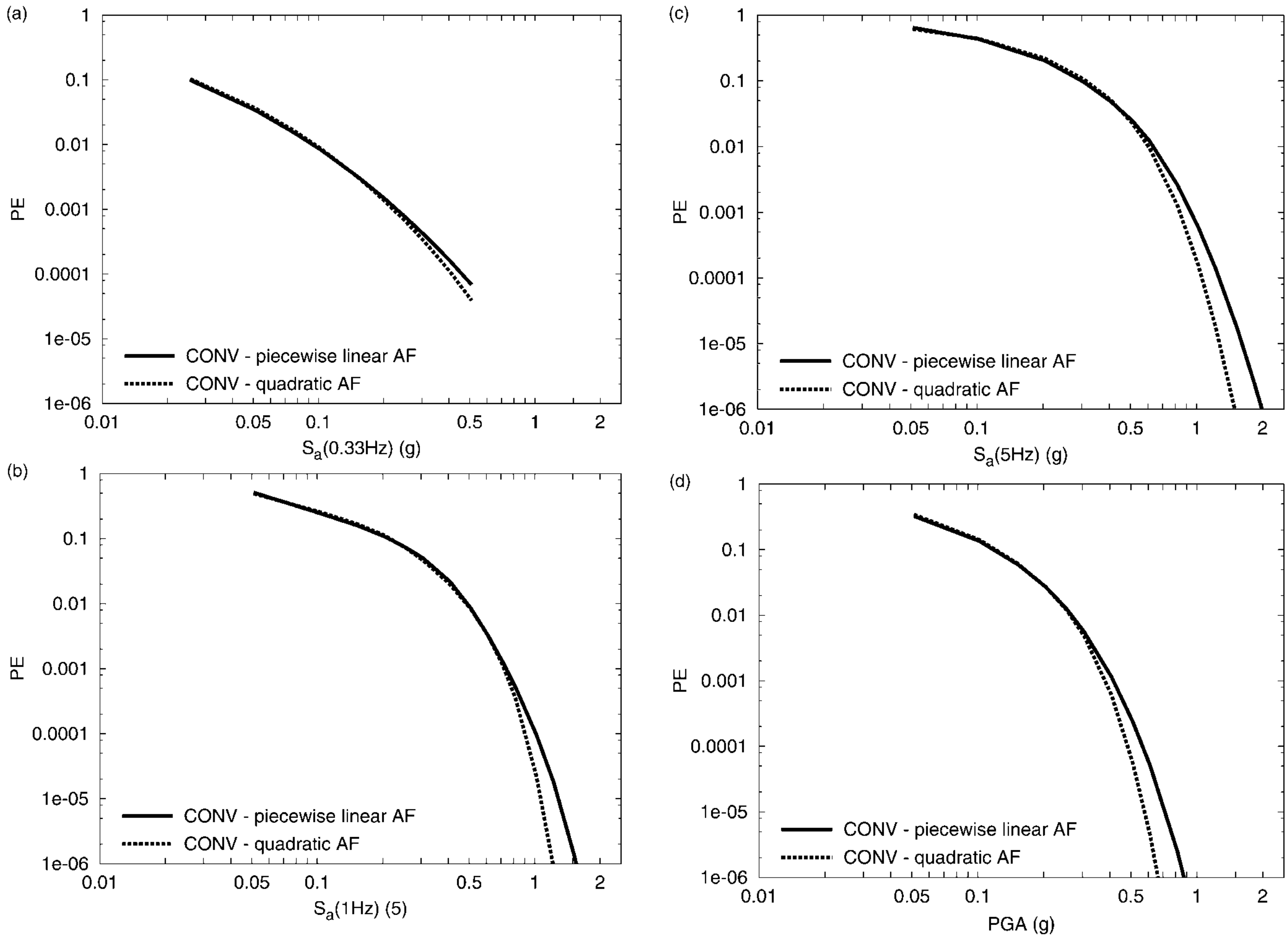

Figure 4. Soil-hazard curves for the sandy site obtained by convolving rock-hazard curves with the quadratic and the piecewise–linear models of $AF(f)$ on $S_a^r(f)$.

(e.g., 0.11 g for $S_a^r(f)$ at 1 Hz) used during the regression. In turn, this overestimation will lead to a significant overestimation of the $S_a^s(f)$ in the high-acceleration range. The converse is true for the $AF(f)$ model fitted to the higher portion of the data.

An approximate estimate of the desired soil-hazard curve can be obtained simply by combining the two resulting hazard curves as shown in bold dashed lines in Figure 6. A comparison with the benchmark hazard curves from the convolution method with piecewise $AF(f)$ versus $S_a^r(f)$ relationship shows that the approximation error implicit in this procedure is not significant across the entire acceleration range. However, in the neighborhood of the crossing point of the two hazard curves some smoothing is necessary to locally improve the accuracy of the estimates. Of course, in the case of $S_a^s(f)$ at 0.33 Hz (Fig. 6a), where only one linear model of $AF(f)$ was fitted for the entire $S_a^r(f)$ range, no crossing point and therefore no "kink" is present in the hazard curves.

Running the PSHA code twice with two different additional terms included into the rock-attenuation law is a fast and efficient way of acquiring an approximate estimate of the desired site-specific soil hazard. The necessary PSHA code modifications are trivial. A more elegant and accurate, but disproportionately more complicated, code change would be required to incorporate in the same attenuation relation the two terms that characterize the soil amplification for the lower and higher $S_a^r(f)$ ranges. It is in fact not correct to use one model in the PSHA calculations when the median estimate, $\widehat{S_a^r(f)}$, of $S_a^r(f)$ at the site for a scenario event (i.e., for a given M, R, and $\boldsymbol{\theta}$) is below $S_{a,tr}^r(f)$ (in the example above, 0.11 g), and the other when it is above. For each scenario event the model of $AF(f)$, developed for the lower $S_a^r(f)$ range, should be used when $S_a^r(f) < S_{a,tr}^r(f)$ rather than when $\widehat{S_a^r(f)} < S_{a,\mathrm{tr}}^r(f)$. In the PSHA calculations for each scenario, the quantity $\widehat{S_a^r(f)}$ is readily available, whereas $S_a^r(f)$, which is an RV, takes on a distribution of possible values. If implemented, however, this more complicated code change would allow one to run only a single PSHA, which would yield a smooth $S_a^s(f)$ curve.

As before, similar results can be obtained for the clayey site.

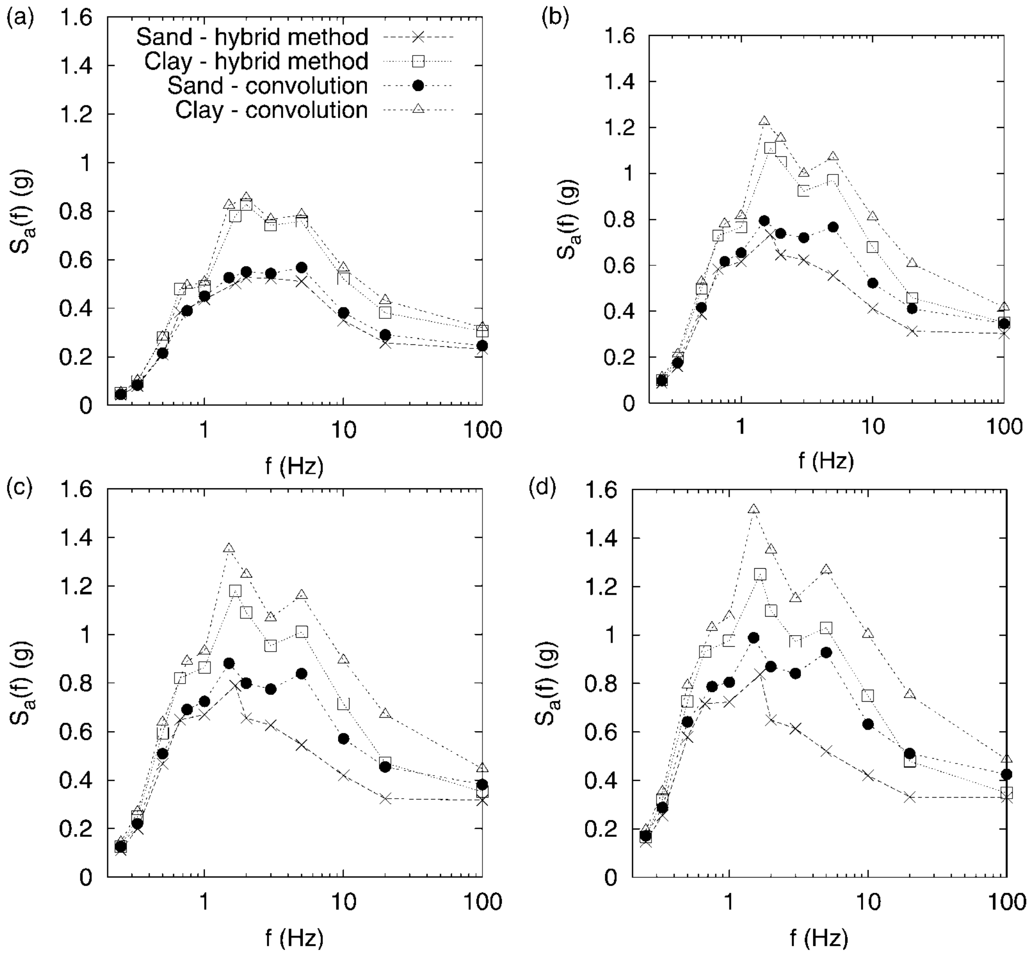

Figure 5. Uniform hazard spectra (UHS) for the hypothetical SBC site.

Hazard Estimates from the Analytical Method

The accuracy of the soil-hazard estimates obtained via the analytical method (equation 17) is shown here (Fig. 7) only for $S_a^s(f)$ at 1 Hz for the sandy site. The hazard curve, computed by using the numerical convolution approach (equations 3 and 4) with a piecewise–linear relationship between (log) $AF(f)$ and (log) $S_a^r(f)$, is used as a benchmark. The numerical convolution with the piecewise–linear model is used instead of the quadratic one for consistency with the analytical approximation, which requires a piecewise–linear model in log space between $S_a^s(f)$ and $S_a^r(f)$. The agreement of the two soil-hazard curves is excellent. Similar results are expected to hold for many frequencies and for both soil columns. However, this method may not yield accurate results for those cases (e.g., f of 5 Hz for the sandy column) where large negative values of c_1, coupled with the value of $\sigma = \sigma_{\ln AF(f)}$ greater than 0.3, make the correction factor very large.

To facilitate the duplication of the results in Figure 7, we have listed the values of the parameters of equation (17) and the resulting hazard curve for $S_a^s(f)$ in Table 2. Note that instead of computing a different value of k_1 at each $\hat{S}_a^r(f)$ point, to simplify the computations we held k_1 constant for all the $\hat{S}_{a,z}^r(f)$ values in the PE ranges of 0.1 to 0.01, 0.01 to 0.001, 0.001 to 1E-4, 1E-4 to 1E-5. The value of k_1 in each interval, computed as described in the methodology section, is the slope of the secant straight line (in log space) that approximates the $S_a^r(f)$ hazard curve in that interval. To ensure accuracy in the results, it is emphasized that the $H(\hat{S}_{a,z}^r)$ values should be computed by interpolating the existing $S_a^r(f)$ hazard curve in log space.

Table 1
Parameter Values of the Piecewise–Linear Model of $AF(f)$ on $S_a^r(f)$ for the Sandy Site

		$\sigma_{\ln AF(f)}$	
Freq. (Hz)	$S_{a,tr}^r$ (g)	$S_a^r < S_{a,tr}^r$	$S_a^r \geq S_{a,tr}^r$
0.33	–	0.28	0.28
1	0.11	0.16	0.19
5	0.18	0.16	0.33
100	0.10	0.18	0.26

Frequency of 100 Hz corresponds to PGA.

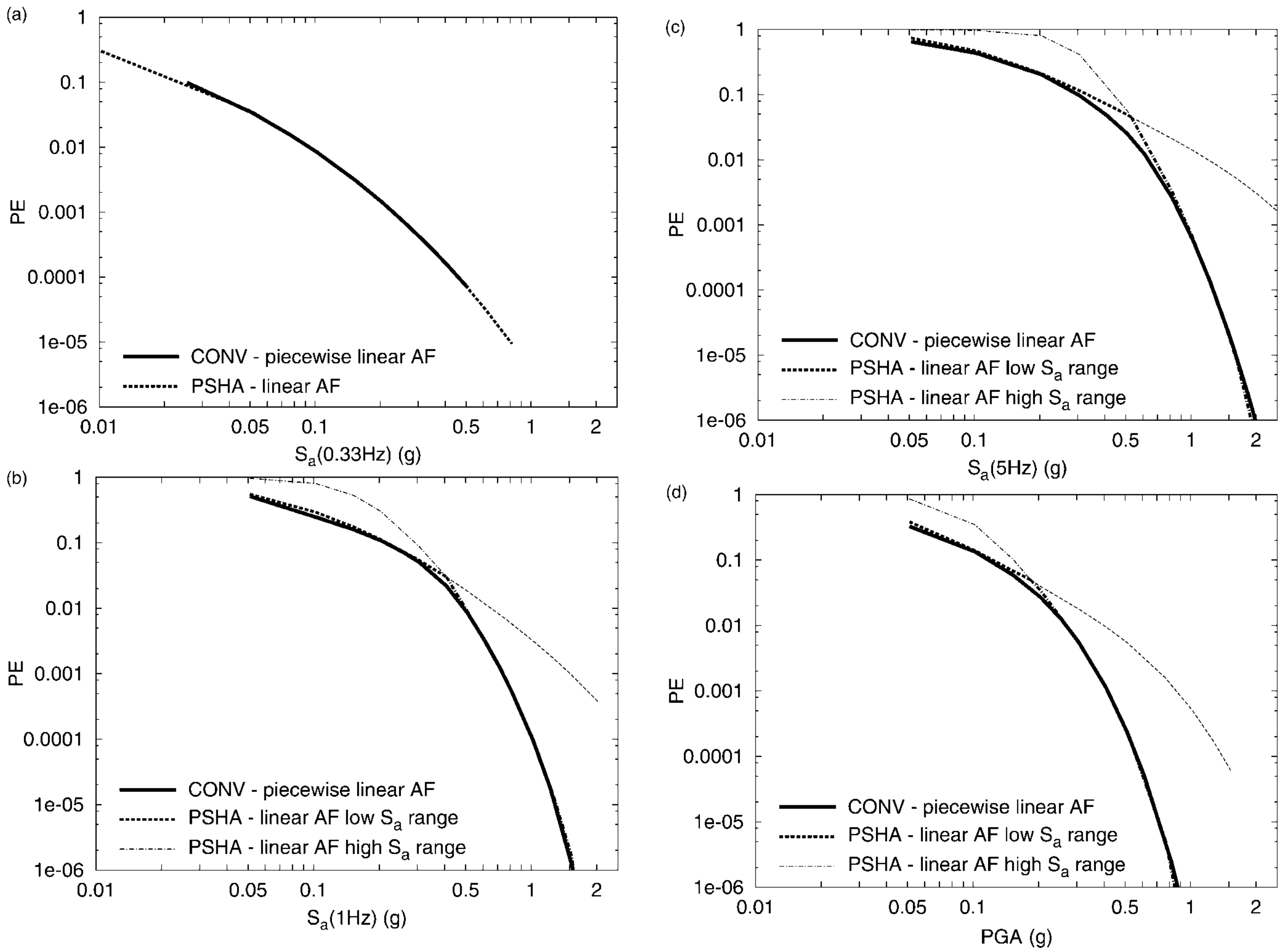

Figure 6. Soil-hazard curves from PSHA with rock-attenuation relations modified for the site-specific sandy-soil conditions. Results are compared with those from the numerical convolution method with a piecewise–linear model (Fig. 4).

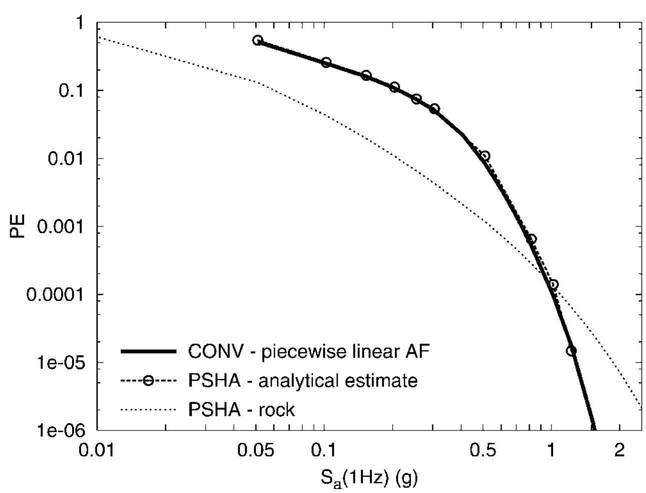

Figure 7. Estimates by numerical convolution (equations 3 and 4) and by analytical approach (equation 17) of the $S_a^s(f)$ hazard curve at 1 Hz for the sandy site. The hazard curve for $S_a^r(f)$ at 1 Hz, on which both the numerical-convolution and the analytical-method results are based, was found by PSHA.

Summary and Conclusions

This paper has presented a methodology for computing the seismic hazard at the soil surface and its application to two soil deposits with uncertain properties, one sandy and one clayey. This procedure accounts for nonlinearity in the soil response, an aspect which is, at best, only marginally considered in the attenuation relationships for generic soil conditions routinely used in PSHA calculations. This methodology is particularly useful for those site conditions for which the use of standard soil-attenuation relationships is questionable. Two examples include soft-soil sites, which are known to greatly amplify the motion at low frequencies, and saturated sandy sites, which are prone to liquefaction and cyclic-mobility phenomena under severe levels of ground shaking. Both cases were considered here. When the soil properties are adequately known, explicit site-specific consideration of the nonlinear soil response in the PSHA framework provides soil-surface-hazard estimates that, in our opinion, are more precise than those from attenuation equations for generic soil conditions.

Table 2
Parameter Values Used for the Computation of the Hazard Curve for $S_a^s(f)$ at 1 Hz for the Sandy Site

$S_a^s = z\ (g)$	$\hat{S}_{a,z}^r\ (g)$	k_1	$\sigma_{\ln AF(f)}$	$c_1 + 1$	$H(\hat{S}_{a,z}^r)$	EXP*	$G_Z(z)$
0.05	0.01	1.79	0.16	0.88	5.17E-01	1.05	5.45E-01
0.10	0.03	1.79	0.16	0.88	2.44E-01	1.05	2.57E-01
0.15	0.04	1.79	0.16	0.88	1.57E-01	1.05	1.66E-01
0.20	0.06	1.79	0.16	0.88	5.06E-01	1.05	1.12E-01
0.25	0.07	1.79	0.16	0.88	7.07E-02	1.05	7.46E-02
0.31	0.09	1.79	0.16	0.88	5.07E-02	1.05	5.35E-02
0.51	0.28	2.50	0.19	0.40	5.40E-03	2.03	1.09E-02
0.82	0.90	3.20	0.19	0.40	2.05E-04	3.18	6.53E-04
1.02	1.57	4.07	0.19	0.40	2.20E-05	6.46	1.42E-04
1.22	2.47	4.07	0.19	0.40	2.30E-06	6.46	1.48E-05

*EXP represents the exponential correction factor in equation (17).

The proposed approach convolves (numerically) the hazard at the bedrock with the nonlinear response of the soil column computed via dynamic analyses. The necessary statistics of the soil-amplification function can be estimated to a sufficient level of accuracy with as few as 10 ground motions (see our companion article for details). This makes the methodology presented here highly suitable for practical application, in particular for those critical facilities that require accurate site-hazard estimates. We also showed that the widely used approach of multiplying the rock UHS by the median amplification function leads to generally nonconservative estimates of the surface UHS by an amount that is larger for longer mean return periods and for higher frequencies.

Two alternative but approximate methods have also been described. The first condenses the soil response into an additional term to be included in existing attenuation laws for rock ground motion. The resulting soil-specific attenuation relationship can then be used with any PSHA software. The second replaces, under certain assumptions, the numerical convolution by a closed-form analytical approximation, which includes two terms: the rock hazard and a correction factor. The correction factor accounts for the uncertainty in the amplification of the soil at an appropriate level of bedrock ground shaking. The two approximate methods have shown a good accuracy, at least for the two test cases investigated in this study. The latter one, however, should be used with caution when the correction factor takes on values greater than 10.

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