Inversion of seismic reflection traveltimes using a nonlinear optimization scheme

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Abstract

We present the use of a nonlinear optimization scheme called generalized simulated annealing to invert seismic reflection times for velocities, reflector depths, and lengths. A finite-difference solution of the eikonal equation computes reflection traveltimes through the velocity model and avoids raytracing. We test the optimization scheme on synthetic models and compare it with results from a linearized inversion. The synthetic tests illustrate that, unlike linear inversion schemes, the results obtained by the optimization scheme are independent of the initial model. The annealing method has the ability to produce a suite of models which satisfy the data equally well. We make use of this property to determine the uncertainties associated with the obtained model parameters. Synthetic examples demonstrate that allowing the reflector length to vary, along with its position, helps the optimization process obtain a better solution. This we put to use in imaging the Garlock fault, whose geometry at depth is poorly known. We use reflection times picked from shot gathers recorded along COCORP Mojave Line 5 to invert for the Garlock fault and velocities within the Cantil Basin below Fremont Valley, California. The velocities within the basin obtained by our optimization scheme are consistent with earlier studies, though our results suggest that the basin might extend 1 - 2 km further south. The reconstructed reflector seem to suggest shallowing of the dip of the Garlock fault at depth.

Introduction

Determination of subsurface velocities is a difficult proposition in areas where lithology and structure undergo significant lateral changes. Further complications arise in reflection problems where the velocity as well as the reflector depth is not known. The last decade has seen the development of numerous inversion schemes to obtain velocity and reflector depth from reflection traveltimes. They have evolved from the use of straight rays and fixed reflector depths (Kjartansson, 1980) to more complicated raytracing and simultaneous velocity-depth determination.

We can achieve simultaneous inversion in two ways. The first is to parametrize both velocity and reflector depth and perform a joint inversion (e.g., Bishop et al., 1985; Stork and Clayton, 1986a, b; Farra and Madariaga; 1988, Williamson, 1990). The second approach is to use a migration technique to
image the reflector with the existing velocity field, then update the velocity keeping the reflector fixed (Bording et al., 1987; Stork and Clayton, 1987). The main advantage of the second technique is that it incorporates multiple arrivals from a reflector, while the joint inversions use only first arrival reflections. However, the second technique is computationally more expensive.

The objective of joint inversion is to minimize the least-square difference between the observed and calculated times through a model. This is cast as a generalized nonlinear least-squares problem (Tarantola and Valette, 1982; Tarantola, 1987). Farra and Madariaga (1988) solve this problem iteratively using the Gauss-Newton method while Williamson (1990) uses an iterative subspace search method to do the same. van Trier (1990a, b) simultaneously inverts for the reflector geometry and interval velocities using a Gauss-Newton method. Reflection times depend on both the velocity and the depth of the reflectors, making traveltime inversion a nonlinear process. All the above methods involve local linearization of the problem, and require the starting model to be close to the desired final solution. We demonstrate this requirement later using synthetic models.

In this paper we examine the use of a nonlinear optimization scheme, namely simulated annealing, to invert reflection traveltimes. We must rapidly compute traveltimes through models by a method avoiding raytracing. It employs a fast finite-difference scheme based on a solution to the eikonal equation (Vidale, 1988). This accounts for curved rays and all types of primary arrivals, i.e., the fastest arrival, be it a direct arrival or a diffraction. We test the optimization process on synthetic models and compare its performance with linear inversion schemes that use curved rays. Synthetic examples and optimizations of data demonstrate that the method is not dependent on the choice of the initial model. Another advantage of the simulated-annealing algorithm is that it produces a suite of final models with comparable least-square error. This enables us to choose the model most likely to represent the geology of the region. We also use this property to determine the uncertainties associated with the model parameters obtained.

Lastly, we use the scheme to invert reflection times to image the Garlock fault and the velocity within the Cantil Basin below Fremont Valley, California. The traveltime picks are made from the shot records of COCORP Mojave Line 5. Strong lateral velocity variations across the basin make it difficult to image the Garlock fault by standard seismic velocity analysis techniques. The use of prestack Kirchhoff migration (Louie and Qin, 1991), which accounts for curved rays and lateral velocity variations, requires prior knowledge of the velocities. This makes it important to use an inversion scheme that will image the velocities with minimum a priori constraints. In addition, the geometry of the Garlock fault at depth is not unequivocally known. The nonlinear optimization scheme we use can reconstruct velocities, as well as recover estimates of dip or length of the fault. We need not specify the length or dip prior to the inversion.

**Methods**

**Traveltime calculation**
Any inversion scheme requires the solution of the forward problem. In our case, the computation of reflection times through a known velocity field given the reflector location constitutes the forward problem. We do this by a method avoiding raytracing. First, assume each point along the discretized reflector to be a source and compute the traveltimes from these points using a finite-difference method (Vidale, 1988). The next step is to determine the reflection depth point location for each source-receiver pair. For each source-receiver pair, we calculate the total traveltime by adding appropriate times, corresponding to all reflector points (each one acting as a source). Invoking Fermat's principle, the minimum total traveltime will correspond to the time of primary reflection arrival. Thus we can rapidly compute reflection times through any model given the velocity and reflector location. Figure 1 shows a schematic representation of this process. It also shows ray tracing through a test model using the above method, though during the optimization we do not have to find any raypaths.
Figure 1: (a) Schematic illustration of the use of Fermat's principle and seismic reciprocity to calculate reflection traveltimes. We use a finite-difference solution of the eikonal equation (Vidale, 1988) to calculate the direct arrivals (each ray). (b) Rays traced through a test model using the above method. (Click on image for printable Adobe Acrobat PDF file.)

Optimization by simulated annealing

Kirkpatrick et al. (1983) were the first to introduce the simulated-annealing method of optimization, and its first use in geophysics was by Rothman (1985) for the estimation of residual statics corrections. Subsequently, the method has been put to use for a variety of purposes like coherency optimization for reflection problems (Landa et al., 1989), transmission tomography (Ammon and Vidale, 1990) and seismic waveform inversion (Sen and Stoffa, 1991). Simulated annealing is a Monte Carlo optimization technique that mimics the physical process by which a crystal grows from a melt. We can relate crystallization to optimization by characterizing the nonlinear inversion as a transformation from disorder (initial model) to order (the solution). This method avoids local linearization and does not require the calculation of partial derivatives. Ideally, it has the ability to test a series of local minima in search of the global minimum.

We make use of a variation of this optimization process called generalized simulated annealing (Bohachevsky et al., 1986; Landa et al., 1989). Its basis is a Monte Carlo technique due to Metropolis et al. (1953). The algorithm essentially comprises the following steps:

1) Compute traveltimes through an initial model. Determine the least-square error, $E_0$. For any iteration $i$, we can define the least-square error (which is the objective function):

$$E_i = (1/n)(\Sigma(t^{obs} - t^{cal})^2), \quad (1)$$

where $n$ is the number of observations, $j$ denotes each observation and $t^{obs}$ and $t^{cal}$ are the observed (data) and calculated traveltimes respectively. The summation goes from $j=1$ to $j=n$. 

2) The next step is to perturb the model. To preserve the nonlinearity of the problem, we carry out simultaneous perturbations of the medium velocity and the reflector depth. We perturb the velocity by adding random sized boxes, followed by smoothing with averaging over four adjacent cells. The boxes can vary between one cell size and the entire model size. To perturb the reflector we add random length lines that are smoothed by averaging adjacent nodes. Again, the added lines can be as small as one grid spacing or as long as the length of the model. Following the perturbation, we compute the new least-square error $E_1$.

3) The following criteria are put to use to determine the probability $P$ of accepting the new model:

$$P = 1 ; E_1 \leq E_0$$  \hspace{1cm} (2) $$

$$P = P_c = \exp\{(E_{\text{min}} - E_1) q \Delta E / T\}; E_1 > E_0$$ \hspace{1cm} (3) 

where $P_c$ is the probability of conditional acceptance, $T$ is called the *temperature*, $\Delta E = E_0 - E_1$, $q$ is an even integer (which we determine empirically) and $E_{\text{min}}$ is the value of the objective function at the global minimum.

Ideally, the value of $E_{\text{min}}$ is zero. Equation (2) means that we always accept the new model when the least-square error is less than that of the previous iteration. Equation (3) provides for the conditional acceptance of models with a larger least-square error. This gives the inversion the ability to escape out of local minima in its search for the global minimum. The factor $(E_{\text{min}} - E_1)^q$ makes the probability of accepting a detrimental step towards local minima tend to zero as the inversion approaches the global minimum. This is because, when $E_i$ tends to $E_{\text{min}}$ this factor tends to zero. This makes $P_c$ approach one, or in other words the probability of taking a step away from $E_{\text{min}}$ (equal to $1 - P_c$) approaches zero. The temperature is chosen by a process which we will explain in the next section.

4) Repeat steps 3 and 4 till the annealing converges, where the difference in least-square error between successive models and the probability of accepting new models becomes very small. We discern the latter condition when there are a large number of iterations (50,000 or more) over which no model is accepted.

**Determination of the annealing parameters**

The convergence of the inversion scheme is sensitive to the value of $q$ and the rate of *cooling*. The rate of *cooling* refers to the variation of parameter $T$ with iteration to aid convergence. One iteration includes one perturbation of both velocity and reflector depth. Rothman (1986) found that simulated annealing finds the global minimum when one starts at a high temperature, rapidly cools to a lower temperature...
and then performs a number of iterations at that temperature. This low temperature, also referred to as the critical temperature ($T_c$), must be high enough to allow the inversion to escape local minima, but low enough to ensure that it settles in the global minimum.

We use a procedure based on the one developed by Basu and Frazer (1990) to determine the critical temperature. We determine the parameter before performing the inversion, and it may not be the same for a different inversion problem. The objective is to perform a series of short runs for a range of $T$ values. In our case, 1000 iterations comprise a short run (a full run would be between 100,000 and 300,000 iterations).

1) First, for a fixed value of $T$, we compute the average least-square error, $E_{av}$, as shown below:

$$E_{av}(T) = \frac{1}{n} \left( \sum E_k \right)$$

where $E_k$ is the least-square error of the $k$th accepted model and $n$ is the number of accepted models. The summation is from $k=1$ to $k=n$.

2) Next, repeat the above step for a range of $T$ values. Typically, we vary $T$ from $10^{-6}$ to $10^3$ by factors of 10 (thus resulting in 10 values).

3) Then make a plot of $E_{av}$ that we obtain for each $T$, versus $\log T$.

4) The value of $T$ which corresponds to the minimum $E_{av}$ is the critical temperature, $T_c$ (Figure 2a).

Having found $T_c$ we can proceed to fix the cooling rate. The initial temperature is high ($T = 10$ in our inversions). This destroys any order in the model, so as not to bias the inversion. Within a few thousand iterations we rapidly decrease $T$ to the critical temperature. On reaching $T_c$, we slow the rate of decrease of $T$, enabling the optimization process to settle in the deepest minimum. Figure 2b shows a typical cooling rate curve. Here, we keep the temperature constant at $T_c$ from the 2000th to the 50,000th iteration, and then decrease the rate. After some trial runs we found this to work best, though it is still a guess.

To determine the other empirical parameter, $q$ we do the following:

5) At the critical temperature, $T_c$, compute the average least-square error, $E_{av}$ for a short run of the optimization process.

6) Repeat the above step for a suite of $q$ values.
7) The value of \( q \) which corresponds to the minimum \( E_{av} \) will give the optimum convergence (Figure 2c).
Results

In this section we present the results of our tests of the annealing optimization scheme on synthetic models. In addition we also compare its performance with linear inversion schemes that employ curved rays.

We use two synthetic models in our study, namely the three box model (Figure 3a) and the lateral velocity gradient model (Figure 5a). The models are 30 km long and 8 km deep. There are sharp velocity contrasts in the three box model, with slowness ranging from 0.75 s/km to 1.25 s/km. In contrast, the velocity variation is gradual in the lateral gradient model (patterned after Williamson, 1990). Here the slowness varies between 0.382 s/km and 0.418 s/km. In both cases the true reflector is fault-shaped, ranging between depths of 7 km and 5 km. There are 30 sources each with 30 receivers along the top of the model, thus resulting in 900 traveltime observations.

We first determine the critical temperature $T_c$. For the two synthetic cases $T_c$ was equal to 0.1. Accordingly we fix the cooling rate. Finally, before performing the inversion, we determine the value of $q$. In our case $q$ was equal to 2 for optimum convergence. Figure 2 illustrates the above steps.

Optimization results of the three box model

The starting model has a slowness of 0.6 s/km and a flat reflector at a depth of 6 km. We allow the slowness to vary between 0.6 s/km and 1.5 s/km while the reflector can assume depths between 4 km and 7 km. Figures 3b and 3d show the results of the optimization process. It reconstructs the high velocity region and the background velocity well. But it fails to recover the low velocity boxes. We attribute this to poor ray coverage in those regions. The rays bend around them and sample the faster regions. As regards the reflector, the scheme recovers the sharp features and the depth very well. The least-square error reduces from a value of 24.41 s$^2$ for the initial constant velocity and flat reflector model to 0.0578 s$^2$ for the final model (Figure 4). From the figure we see that during the initial stages of the optimization, when $T$ is high, the least-square error varies rapidly. This indicates that it tests numerous models and can easily escape from a local minimum. As the optimization progresses, $T$ becomes smaller, the probability of escaping local minima also decreases. The optimization settles into the deepest minimum. This shows that the choice of $T_c$ and the cooling rate affect the convergence of the annealing process.
Figure 3: Nonlinear optimization and linearized inversion results of the three box model. (a) The true model, showing slowness and the reflector depth. The initial slowness for the optimization is 0.6 s/km. (b) The slowness model obtained after annealing. (c) Slowness model obtained from a linearized
inversion scheme using SVD. (d) The reconstructed reflector depth after annealing and a linearized inversion. It also shows the true depth and initial depth.

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**Figure 4**: Least square error variation during the optimization of the three box model. (a) The initial stages of the process (note the rapid variation in the least square error). (b) The final stages. The gaps in the curve are regions of no accepted models.

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**Optimization results of the lateral gradient model**
The initial model is one of constant slowness, with a value of 0.3 s/km, and has a flat reflector 6 km deep. During the inversion, the slowness can assume values between 0.3 s/km and 0.5 s/km. Figures 5b and 5d show the annealing results. The optimization scheme does a poor job of recovering the small lateral variations; it reconstructs only the average velocity. On the other hand, it images the reflector depth and shape very well. The recovered reflector is a smoothed version of the true reflector, whereas the linearized scheme recovers a heavily averaged shape. The least-square error decreases from 2.97 s$^2$ for the initial model to 0.008543 s$^2$ for the final model.

**Comparison with a linearized inversion scheme**

We compare the performance of annealing with a linearized inversion scheme. In the linearized inversion, we linearize the nonlinear relationship between traveltime and slowness and the reflector depth. Generally, this linear scheme is cast in a matrix form and the solution obtained using a standard least-squares technique (Jackson, 1972; Wiggins, 1972). We use a singular value decomposition method to do this. Instead of solving for the perturbations to the model parameters, we compute a new model during each iteration (Shaw and Orcutt, 1985). We compute the traveltimes by the same method we used in the annealing. To speed up convergence and stabilize the inversion we smooth the velocity perturbations using a Laplacian operator (Lees and Crosson, 1989) and the reflector depth by a second-difference operator (Ammon et al., 1990).

Figures 3 and 5 compare the results we obtain from the linearized inversion with those from the nonlinear optimization scheme. Neither method recovers the velocity well when there is a gradual lateral variation in the true model (Figure 5). The range of the reflection traveltime through the lateral gradient model is only about 8 seconds while it is 23 seconds through the three box model. Hence, the survey aperture might be too small to recover the slowness variations in the former case. Annealing does better in recovering the sharp features of the model - both in the case of velocity and of reflector shape. The smoothing applied during the linearized inversion scheme helps convergence, but smears the velocities and averages out the reflector depth. Both methods reconstruct the reflector depth and shape fairly well close to the edges of the model. It is noteworthy that the velocity above these regions are not well recovered. This can be attributed to the inherent ambiguity in the simultaneous reconstruction of velocity and reflector characteristics. It illustrates that velocity-depth trade-offs near the reflector are hard to resolve. In our tests the reflector depth is better resolved probably because it is described by fewer parameters (30 as compared to 240 velocity parameters). The second-difference smoothing applied during the linearized inversions and the averaging during annealing also helps in reconstructing the reflector shape better.
Figure 5: Nonlinear optimization and linearized inversion results of the lateral gradient model, patterned after Williamson (1990). (a) The true model, showing slowness and the reflector depth. The
starting slowness for the optimization is 0.3 s/km. (b) The slowness model obtained after annealing. (c)
Slowness model obtained from a linearized inversion scheme using SVD. (d) The reconstructed reflector
depth after annealing and after linearized inversion. It also shows the true depth and the initial depth.

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We also note that the convergence of the linearized inversions is initial model dependent (in the
synthetic tests the starting model is close the true solution for the linearized inversions). To demonstrate
this we do another synthetic test. We use the slowness model used by Louie and Qin (1991) for their
prestack migrations (which will be put to further use later on) as the true model. The model has
dimensions of 26 km by 7 km with a fault shaped reflector along the bottom as shown in the top panel
of Figure 6. The slowness varies between 0.2 s/km and 0.45 s/km. We assume the reflector position and
shape are known, and invert for only the velocities. Twenty-six receivers along the surface record
traveltimes from twenty-six sources. Figure 6 shows inversion results of the linearized scheme using two
different starting models. The solutions are different, though the least-square errors are comparable in
both cases. The inversion with an initial model of 0.2 s/km reconstructs the model better (a global
minimum) than the one with a starting slowness of 0.4 s/km (a local minimum).
Figure 6: A true slowness model is shown at the top, from Louie and Qin (1991). We invert the reflection times for only the velocity, assuming the reflector depth is known a priori, to demonstrate that the scheme is initial-model dependent. The bottom two panels show the inversion results obtained using a linearized scheme, starting out with two different initial constant slowness models - 0.2 s/km (middle panel) and 0.40 s/km (bottom).
Figure 7 shows the optimization results by annealing. The final models we obtain by starting with two different models look similar. The random perturbations used during the annealing scheme make it independent of the initial model. It destroys any order in the starting model within the first few iterations. The Laplacian smoothing applied during the inversion might explain the better reconstruction of the low velocity (high slowness) basin in the linearized scheme. Note that in both the methods, the region below the reflector is unconstrained due to absence of any ray coverage. Thus, even when we eliminate the nonlinearity due to simultaneous dependence of reflection traveltimes on the velocity field and reflector depth, by keeping the reflector position fixed, the linearized inversion scheme shows dependence on the initial model. Though the linearized inversion takes fewer iterations to converge, more time is spent in the matrix operations, making the total computation time comparable in both cases.
Figure 7: Results of the nonlinear optimization scheme for the slowness (assuming the reflector depth is known) using two different starting models (0.2 s/km and 0.40 s/km, middle and bottom frames). The true slowness model is shown at the top.

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For the nonlinear optimization scheme, the computation time is proportional to the size of the model and the number of reflector nodes. Models with 1500 cells and a reflector along the whole length took about 60 hours on a SPARCstation 2. With a grid spacing of 250 m and for a survey line 15 km long, this would mean we can invert for velocities and reflector depths down to 7 km in the above mentioned time.

Imaging the Garlock Fault, Cantil Basin

The left-lateral Garlock fault branches into two strands in the Fremont Valley, California, region. The eastern branch strikes almost east-west while the western branch strikes southwest-northeast. Cantil Basin itself is a pull-apart basin, and Aydin and Nur (1982) show that it is one of the largest of its kind in the San Andreas fault system.

The Consortium for Continental Reflection Profiling (COCORP) Mojave Line 5 collected seismic reflection data across the Garlock fault in Fremont Valley. Stacked sections of Line 5 displayed by Cheadle et al. (1986) show portions of the basin floor and the southern wall. But strong lateral velocity variations across the basin made the imaging inadequate. Louie and Qin (1991) used a prestack Kirchhoff migration, which accounts for curved rays and lateral velocity variations, on the Line 5 data. They infer that Cantil Basin was either a negative flower structure, or a detachment headwall with the Garlock fault flattening at depth. They prefer the detachment model since it allows a simple mechanism to explain the observed broadening of Cantil basin with fault movement over time.

Kirchhoff migration assumes prior knowledge about the velocity. Hence, an iterative application of this method would demand the initial model be close to the solution to keep it computationally inexpensive. Use of the nonlinear optimization will not require any constraints on the model. Since one of the goals is to determine the nature of the fault at depth, we do not put any constraints either on the location or the shape of the fault (the reflector). Resolving the structure of the Garlock fault and Cantil Basin would help constrain regional tectonics, mainly the configuration of the Mojave block.
Results of the traveltime inversion

The data we use are reflection times picked from shot gathers (Figure 8). Louie and Qin (1991) compute synthetic seismograms to show that these are reflections off the east branch of the Garlock fault. The total number of observations is 621 reflection times, picked from 28 shot gathers. In order to determine an optimum grid spacing to use in the inversions, we do a convergence test (not shown). We compute traveltimes through models with the same dimensions, but different grid spacings. For a range of grid spacings the optimum spacing is that for which smaller spacings did not appreciably alter the computed times (i.e.: less than the error in the data). The dimensions of the model were 14.4 km by 4 km with a grid spacing of 277.85 m. We incorporate elevation corrections (C. Ammon pers. comm.) by extrapolating the times to sources and receivers not on the grid, assuming plane wave incidence at the receivers.

Figure 8: Sample shot gather from COCORP Mojave Line 5 showing example reflection time picks from VP 361, used to invert for velocities in Cantil Basin and image the Garlock fault. We used a total of 621 reflection times picked from 28 shot gathers.

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Before embarking on the optimization process, it is essential to determine whether letting the reflector length vary hinders the inversion from reaching the solution. For this study our true velocity model is the one used by Louie and Qin (1991) for the Cantil Valley region. The source-receiver configuration is same as the actual survey. The shape of the reflector corresponds to the dip of the east branch of the Garlock fault shallowing at depth. It has a
dip of about 50 degrees near the surface, changing to about 30 degrees at 2.5 km depth (Figure 9a). We do two optimization runs - one keeping the reflector length fixed and equal to the true length, and the other letting the length vary. From Figure 9b we see that keeping the reflector length fixed hinders the optimization process. The velocity and reflector are reconstructed poorly. This is an example of a bad choice of the objective function. The annealing process does a better job of reconstructing the velocities and reflector shape when the reflector length is allowed to vary (Figure 9c), suggesting that it is a more useful objective function. The optimization reconstructs only the lower part of the reflector because the picked reflections see only this segment of the true reflector. We demonstrate this in a later section.
Figure 9: Results of the synthetic study using a velocity model for the region from Louie and Qin (1991) and keeping the source-receiver configuration the same as the actual survey. (a) The true model, with the part of the reflector seen by picked reflection rays delineated by the square. (b) Optimization result when the reflector length is kept fixed and equal to the true length. (c) The reconstructed velocity and reflector (broken line) when the reflector length is allowed to vary during the optimizations. The true reflector (solid line) is shown for comparison.

Hence during the data optimizations, we let the reflector length vary along with its position. The length can be as short as 277.85 m (the grid spacing) or as long as 14.4km (the length of the model). The reflector position can vary from near surface to 4 km depth. Hence, the reflector can assume any dip and the dip can change along its length. Each cell in the model can assume any velocity between 1.5 km/s and 6.3 km/s. This is the range of velocities expected in this region. We perform the optimization with three different constant velocity starting models - 2.5 km/s, 4.0 km/s and 8.3 km/s. We see that (Figure 10) the final model in all three cases are similar, even if we start out with the unreasonably high velocity of 8.3 km/s. The features that are common to all three results would be the most reliable.
Figure 10: Results of the nonlinear optimization on reflection time picks from COCORP Mojave Line 5 data. The models are the final accepted models after the optimization, each with a different starting velocity. The initial velocity is indicated on left side of each figure. (Click on image for printable Adobe Acrobat PDF file.)
The inversion reconstructs a prominent low velocity basin (2.4 - 2.6 km/s) whose location corresponds to that of Cantil Basin. The basin seems to extend 1 - 2 km further south than suggested by Louie and Qin (1991) and may be 2 to 2.5 km deep. The reflector is about a kilometer long and extends from a depth of 3 km to about 3.5 km. The dip of this segment of the reflector is between 20 to 30 degrees and it lies beneath the low velocity basin. If we interpret this as part of the Garlock fault, it indicates that the fault shallows at depth, since diffraction studies (Louie and Qin, 1991) indicate that the fault dips about 45 degrees near the surface. The least-square error decreases from 3.477 s\(^2\) for the constant velocity starting model to 0.0044465 s\(^2\) for the final model. The traveltime residual between the final model and the data is less than 0.1 seconds for all the observations. The range of the observed traveltimes was between 2.25 and 3.65 seconds.

By using three different initial models, we allow the optimization process to sample different parts of the model space. It approaches the minimum least-square error from different directions. Hence, we can use the final models, with comparable least-square error, from each of the three runs to estimate the uncertainty associated with the obtained model parameters. In our analysis, we use about 15 models from each run and thus a total of about 40 models. Figure 11a shows the reflector points sampled by the rays for these models along with the recovered reflector of the final model. Most of the midpoints cluster around the recovered reflector. The standard deviation of the velocities (Figure 11b) range from 0.46 km/s to 0.85 km/s. We note here that these may not represent the actual deviations in the model parameters because of the nonlinear nature of the problem. It is difficult to separate out the contributions due to mislocation of the reflector and the errors in the velocities. But it does provide an idea about well- and poorly-resolved regions in the model. Our statistical analysis is over accepted models only. So there might be unconstrained regions in the model which are not perturbed, and hence have a small deviation. But, in general, the regions with higher standard deviation are parts of the model that have poor ray coverage. Hence, changing velocities in these regions do not affect the calculated traveltimes much and thus contribute minimally to the least-square error.

Our synthetic study using the velocity model constructed by Louie and Qin (1991) for the region and the actual survey's source-receiver configuration (Figure 9c), indicate that the scheme recovers velocities in and around the low velocity basin to within 0.5 km/s to 1.0 km/s. The resolution decreases with distance from and depth below the basin. This approximately corresponds to the deviations we obtain from our statistical analysis on the models obtained using the real data. This study also shows that the optimization images only the lower part of the reflector, suggesting that the small segment of the reflector imaged during the data optimization is fairly robust.

We attribute the poor imaging of the reflector to the source-receiver configuration of the survey, that is, all the sources are towards the north side of the model and most of the receivers to the south. To confirm this, we determine the depth points along the reflector for every source-receiver pair in a manner similar to how we compute the traveltimes. We plot these along the true reflector in Figure 9a (delineated by a square). This shows that the picked reflection rays see only the lower part of the reflector. Thus poor ray coverage causes the steeply-dipping part not to be reconstructed and might also explain the poor reconstruction of velocities away from the
Doing this study suggests what features of the model are robust and which are not. One has to note that we use only the primary reflected arrivals and a small subset of the data (only 28 shot gathers along Line 5) in our optimizations. In contrast, the prestack Kirchhoff migration (Louie and Qin, 1991) uses all types of arrivals and more data (228 shot gathers), to image the Garlock fault. So including more data in our optimizations could enable us to image more of the fault.

![Figure 11](image)

**Figure 11:** (a) Reflection depth points for the models used to determine the deviations in the velocities. The recovered reflector for one of the final models is shown by the solid line. (b) Uncertainties associated with the velocities obtained after the nonlinear optimization scheme.

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**Conclusions**
We have presented a nonlinear optimization scheme to invert seismic reflection traveltimes, tested it on synthetic data and used it to image the Garlock fault and velocities within Cantil Basin.

Our synthetic study using the three box model and lateral gradient model demonstrate the inherent ambiguity in recovering the velocity and reflector depth from reflection traveltimes. But the nonlinear optimization scheme avoids local linearization of the problem, allowing it to settle into a global (or deepest) minimum. The random perturbations that go into the optimization process make it independent of the initial model.

The method provides for easy implementation of constraints. It facilitates inversion for the reflector depth, its length and shape. The method does not involve matrix inversion; hence it is computationally tractable. The need to perform numerous iterations make the scheme computationally intensive when the depth of interest becomes greater than 10 km, for grid sizes we use in the above studies.

The velocities obtained by our optimization scheme within Cantil basin (2.4 - 2.6 km/s) are consistent with earlier studies using gravity data, refraction data and Kirchhoff migration. The depth of the basin is about 2.5 km and seems to extend further south than suggested by previous studies. The synthetic study shows that the small segment of the reflector recovered by the optimization process is robust. This indicates that the dip of the Garlock fault shallows at 3.4 to 4 km depth. This is consistent with the detachment model suggested for the Mojave region by Louie and Qin (1991). Thus, using only the primary reflected arrivals and a small data set we were able to obtain velocities within the basin and image a segment of the fault.

Successful application of the nonlinear optimization scheme to image the Garlock fault and Cantil basin suggests that the method is very useful in regions where there are strong lateral velocity variations and where neither the fault shape nor depth are known. We demonstrate that inverting for the reflector length is crucial for the optimization to find the best solution. This feature makes the scheme more attractive, since generally we do not know the extent of subsurface structures. Though the recovered reflector is only a collection of depth points, it gives us an idea of the location and nature of the reflector. This information, along with the reconstructed velocity model, can be used in a prestack migration (which uses all arrivals) to recover finer details along the reflector.

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