

In general for cohesion soil the angle of friction is  $\phi = 0$ .

Frk  
6.6

For normally consolidated clay

$$k_0 = 0.95 - \sin \phi$$

$$k_0 = 0.95 - \sin 0 = 0.95$$

$\sigma'_m$  = mean principal effective stress

$$\sigma'_m = \frac{\sigma'_1 + \sigma'_2 + \sigma'_3}{3} = \frac{1 + 2k_0}{3} \sigma'_v$$

at depth of 20 m from Problem 6.5

$$\sigma'_v = \sigma' - u_m$$

$$\sigma'_v = 14 \times 18.8 + (20 - 14) \times 15.9 = 358.6 \text{ kN / m}^2$$

$$u_m = (20 - 2) \times 9.81 = 176.58 \text{ kN / m}^2$$

$$\sigma'_v = 358.6 - 176.58 = 182.02 \text{ kN / m}^2$$

$\sigma'_m$  = mean principal effective stress

$$\sigma'_m = \frac{\sigma'_1 + \sigma'_2 + \sigma'_3}{3} = \frac{1 + 2k_0}{3} \sigma'_v = \frac{1 + 2 \times 0.95}{3} \times 182.02 = 175.83 \text{ kN / m}^2$$

# 2

## *Weight-Volume Relationships, Plasticity, and Soil Classification*

The preceding chapter presented the geological processes by which soils are formed, the description of the soil-particle size limits, and the mechanical analysis of soils. In natural occurrence, soils are three-phase systems consisting of soil solids, water, and air. This chapter discusses the weight-volume relationships of soil aggregates, their structures and plasticity, and their engineering classification.

### 2.1

### *Weight-Volume Relationships*

Figure 2.1a shows an element of soil of volume  $V$  and weight  $W$  as it would exist in a natural state. To develop the weight-volume relationships, we separate the three phases (that is, solid, water, and air) as shown in Figure 2.1b. Thus, the total volume of a given soil sample can be expressed as

$$V = V_s + V_v = V_s + V_w + V_a \quad (2.1)$$

where  $V_s$  = volume of soil solids  
 $V_v$  = volume of voids  
 $V_w$  = volume of water in the voids  
 $V_a$  = volume of air in the voids

Assuming the weight of the air to be negligible, we can give the total weight of the sample as

$$W = W_s + W_w \quad (2.2)$$

where  $W_s$  = weight of soil solids  
 $W_w$  = weight of water

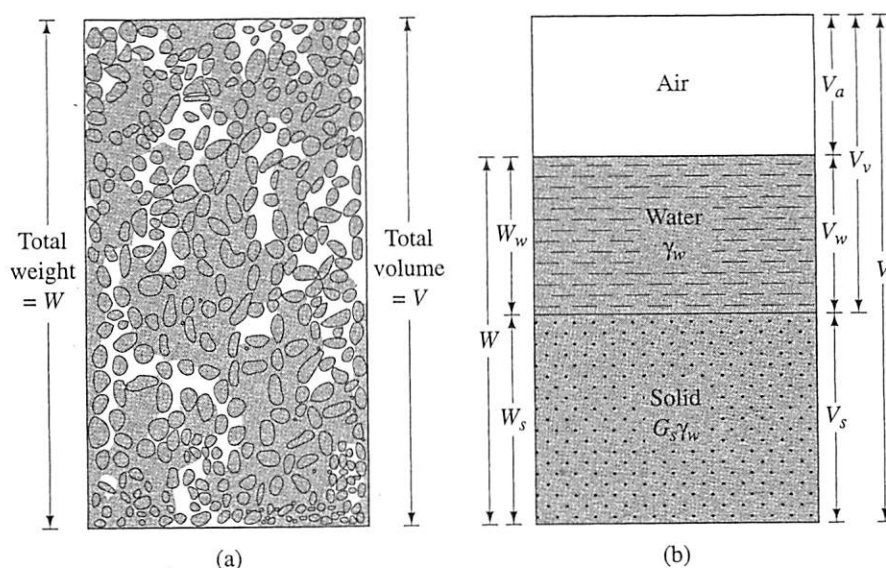


FIGURE 2.1 (a) Soil element in natural state; (b) three phases of the soil element

The volume relationships commonly used for the three phases in a soil element are *void ratio*, *porosity*, and *degree of saturation*. *Void ratio* ( $e$ ) is defined as the ratio of the volume of voids to the volume of solids, or

$$e = \frac{V_v}{V_s} \quad (2.3)$$

*Porosity* ( $n$ ) is defined as the ratio of the volume of voids to the total volume, or

$$n = \frac{V_v}{V} \quad (2.4)$$

*Degree of saturation* ( $S$ ) is defined as the ratio of the volume of water to the volume of voids, or

$$S = \frac{V_w}{V_v} \quad (2.5)$$

The degree of saturation is commonly expressed as a percentage.

The relationship between void ratio and porosity can be derived from Eqs. (2.1), (2.3), and (2.4), as follows:

$$e = \frac{V_v}{V_s} = \frac{V_v}{V - V_v} = \frac{\left(\frac{V_v}{V}\right)}{1 - \left(\frac{V_v}{V}\right)} = \frac{n}{1 - n} \quad (2.6)$$

Also, from Eq. (2.6), we have

$$n = \frac{e}{1 + e} \quad (2.7)$$

The common weight relationships are *moisture content* and *unit weight*. *Moisture content* ( $w$ ) is also referred to as *water content* and is defined as the ratio of the weight of water to the weight of solids in a given volume of soil, or

$$w = \frac{W_w}{W_s} \quad (2.8)$$

*Unit weight* ( $\gamma$ ) is the weight of soil per unit volume:

$$\gamma = \frac{W}{V} \quad (2.9)$$

The unit weight can also be expressed in terms of weight of soil solids, moisture content, and total volume. From Eqs. (2.2), (2.8), and (2.9), we have

$$\gamma = \frac{W}{V} = \frac{W_s + W_w}{V} = \frac{W_s \left[ 1 + \left( \frac{W_w}{W_s} \right) \right]}{V} = \frac{W_s (1 + w)}{V} \quad (2.10)$$

Soils engineers sometimes refer to the unit weight defined by Eq. (2.9) as the *moist unit weight*.

It is sometimes necessary to know the weight per unit volume of soil excluding water. This is referred to as the *dry unit weight*,  $\gamma_d$ . Thus,

$$\gamma_d = \frac{W_s}{V} \quad (2.11)$$

From Eqs. (2.10) and (2.11), we can give the relationship among unit weight, dry unit weight, and moisture content as

$$\gamma_d = \frac{\gamma}{1 + w} \quad (2.12)$$

Unit weight is expressed in kilonewtons per cubic meter ( $\text{kN/m}^3$ ). Since the newton is a derived unit, it may sometimes be convenient to work with densities

( $\rho$ ) of soil. The SI unit of density is kilograms per cubic meter ( $\text{kg/m}^3$ ). We can write the density equations [similar to Eqs. (2.9) and (2.11)] as

$$\rho = \frac{m}{V} \quad (2.13a)$$

and

$$\rho_d = \frac{m_s}{V} \quad (2.13b)$$

where  $\rho$  = density of soil ( $\text{kg/m}^3$ )

$\rho_d$  = dry density of soil ( $\text{kg/m}^3$ )

$m$  = total mass of the soil sample (kg)

$m_s$  = mass of soil solids in the sample (kg)

The unit of total volume,  $V$ , is  $\text{m}^3$ .

The unit weights of soil in  $\text{N/m}^3$  can be obtained from densities in  $\text{kg/m}^3$  as

$$\gamma = \rho \cdot g = 9.81\rho \quad (2.14a)$$

and

$$\gamma_d = \rho_d \cdot g = 9.81\rho_d \quad (2.14b)$$

where  $g$  = acceleration due to gravity =  $9.81 \text{ m/sec}^2$ .

## 2.2

### **Relationships Among Unit Weight, Void Ratio, Moisture Content, and Specific Gravity**

To obtain a relationship among unit weight (or density), void ratio, and moisture content, consider a volume of soil in which the volume of the soil solids is 1, as shown in Figure 2.2. If the volume of the soil solids is 1, then the volume of voids is numerically equal to the void ratio,  $e$  [from Eq. (2.3)]. The weights of soil solids and water can be given as

$$W_s = G_s \gamma_w$$

$$W_w = wW_s = wG_s \gamma_w$$

where  $G_s$  = specific gravity of soil solids

$w$  = moisture content

$\gamma_w$  = unit weight of water



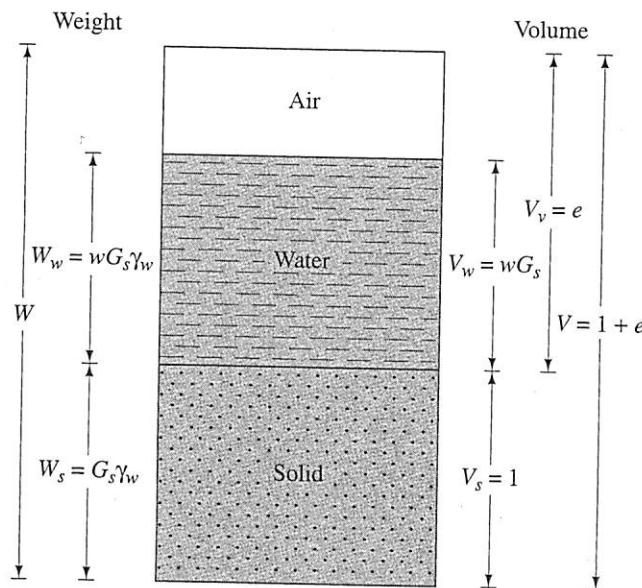


FIGURE 2.2 Three separate phases of a soil element with volume of soil solids equal to 1

The unit weight of water is  $9.81 \text{ kN/m}^3$ . Now, using the definitions of unit weight and dry unit weight [Eqs. (2.9) and (2.11)], we can write

$$\gamma = \frac{W}{V} = \frac{W_s + W_w}{V} = \frac{G_s\gamma_w + wG_s\gamma_w}{1 + e} = \frac{(1 + w) G_s\gamma_w}{1 + e} \quad (2.15)$$

and

$$\gamma_d = \frac{W_s}{V} = \frac{G_s\gamma_w}{1 + e} \quad (2.16)$$

Since the weight of water in the soil element under consideration is  $wG_s\gamma_w$ , the volume occupied by it is

$$V_w = \frac{W_w}{\gamma_w} = \frac{wG_s\gamma_w}{\gamma_w} = wG_s$$

Hence, from the definition of degree of saturation [Eq. (2.5)], we have

$$S = \frac{V_w}{V_v} = \frac{wG_s}{e}$$

or

$$Se = wG_s \quad (2.17)$$

This is a very useful equation for solving problems involving three-phase relationships.

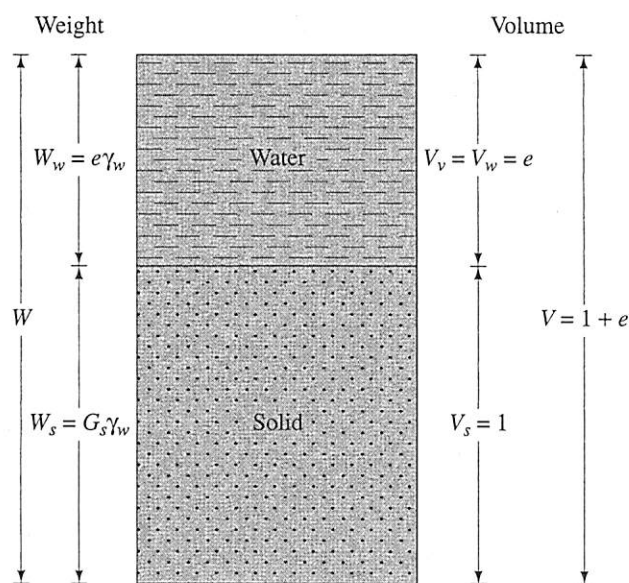
If the soil sample is *saturated*—that is, the void spaces are completely filled with water (Figure 2.3)—the relationship for saturated unit weight can be derived in a similar manner:

$$\gamma_{\text{sat}} = \frac{W}{V} = \frac{W_s + W_w}{V} = \frac{G_s \gamma_w + e \gamma_w}{1 + e} = \frac{(G_s + e) \gamma_w}{1 + e} \quad (2.18)$$

where  $\gamma_{\text{sat}}$  = saturated unit weight of soil.

As mentioned before, because it is convenient to work with densities, the following equations [similar to the unit-weight relationships given in Eqs. (2.15), (2.16), and (2.18)] are useful:

$$\text{Density} = \rho = \frac{(1 + w)G_s \rho_w}{1 + e} \quad (2.19a)$$



**FIGURE 2.3** Saturated soil element with volume of soil solids equal to 1

$$\text{Dry density} = \rho_d = \frac{G_s \rho_w}{1 + e} \quad (2.19b)$$

$$\text{Saturated density} = \rho_{\text{sat}} = \frac{(G_s + e) \rho_w}{1 + e} \quad (2.19c)$$

where  $\rho_w$  = density of water = 1000 kg/m<sup>3</sup>.

The relationships among unit weight, porosity, and moisture content can also be developed by considering a soil specimen that has a total volume equal to 1.

## 2.3 Relative Density

The term *relative density* is commonly used to indicate the *in situ* denseness or looseness of granular soil. It is defined as

$$D_r = \frac{e_{\text{max}} - e}{e_{\text{max}} - e_{\text{min}}} \quad (2.20)$$

where  $D_r$  = relative density, usually given as a percentage

$e$  = *in situ* void ratio of the soil

$e_{\text{max}}$  = void ratio of the soil in the loosest condition

$e_{\text{min}}$  = void ratio of the soil in the densest condition

The values of  $D_r$  may vary from a minimum of 0 for very loose soil to a maximum of 1 for very dense soil. Soils engineers qualitatively describe the granular soil deposits according to their relative densities, as shown in Table 2.1. Some typical values of void ratio, moisture content in a saturated condition, and dry unit weight as encountered in a natural state are given in Table 2.2.

By using the definition of dry unit weight given in Eq. (2.16), we can also express relative density in terms of maximum and minimum possible dry unit weights. Thus,

**Table 2.1** Qualitative description of granular soil deposits

Relative density (%)	Description of soil deposit
0–15	Very loose
15–50	Loose
50–70	Medium
70–85	Dense
85–100	Very dense



**Table 2.2** Void ratio, moisture content, and dry unit weight for some typical soils in a natural state

Type of soil	Void ratio, $e$	Natural Moisture content in a saturated state (%)	Dry unit weight, $\gamma_d$ (kN/m <sup>3</sup> )
Loose uniform sand	0.8	30	14.5
Dense uniform sand	0.45	16	18
Loose angular-grained silty sand	0.65	25	16
Dense angular-grained silty sand	0.4	15	19
Stiff clay	0.6	21	17
Soft clay	0.9–1.4	30–50	11.5–14.5
Loess	0.9	25	13.5
Soft organic clay	2.5–3.2	90–120	6–8
Glacial till	0.3	10	21

$$D_r = \frac{\left[ \frac{1}{\gamma_{d(\min)}} \right] - \left[ \frac{1}{\gamma_d} \right]}{\left[ \frac{1}{\gamma_{d(\min)}} \right] - \left[ \frac{1}{\gamma_{d(\max)}} \right]} = \left[ \frac{\gamma_d - \gamma_{d(\min)}}{\gamma_{d(\max)} - \gamma_{d(\min)}} \right] \left[ \frac{\gamma_{d(\max)}}{\gamma_d} \right] \quad (2.21)$$

where  $\gamma_{d(\min)}$  = dry unit weight in the loosest condition (at a void ratio of  $e_{\max}$ )

$\gamma_d$  = *in situ* dry unit weight (at a void ratio of  $e$ )

$\gamma_{d(\max)}$  = dry unit weight in the densest condition (at a void ratio of  $e_{\min}$ )

### EXAMPLE 2.1

In the natural state, a moist soil has a volume of 0.0093 m<sup>3</sup> and weighs 177.6 N. The oven dry weight of the soil is 153.6 N. If  $G_s = 2.71$ , calculate the moisture content, moist unit weight, dry unit weight, void ratio, porosity, and degree of saturation.

**Solution** Refer to Figure 2.4. The moisture content [Eq. (2.8)] is

$$w = \frac{W_w}{W_s} = \frac{W - W_s}{W_s} = \frac{177.6 - 153.6}{153.6} = \frac{24}{153.6} \times 100 = 15.6\%$$

The moist unit weight [Eq. (2.9)] is

$$\gamma = \frac{W}{V} = \frac{177.6}{0.0093} = 19,906 \text{ N/m}^3 \approx 19.1 \text{ kN/m}^3$$

For dry unit weight [Eq. (2.11)], we have

$$\gamma_d = \frac{W_s}{V} = \frac{153.6}{0.0093} = 16,516 \text{ N/m}^3 \approx 16.52 \text{ kN/m}^3$$

The void ratio [Eq. (2.3)] is found as follows:

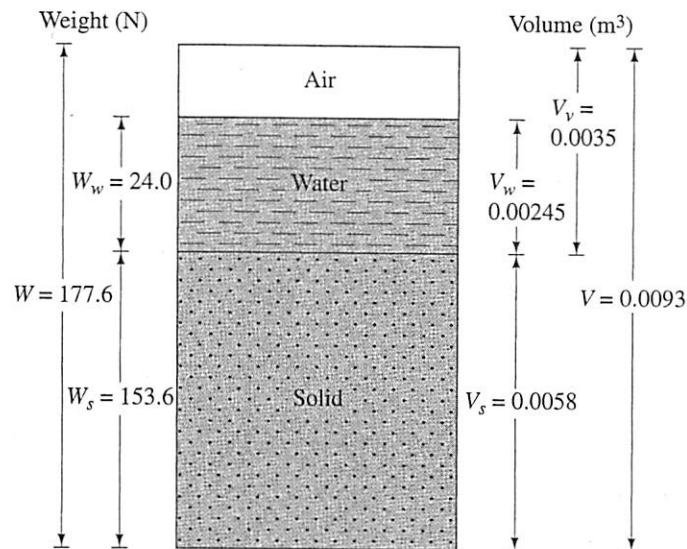


FIGURE 2.4

$$e = \frac{V_v}{V_s}$$

$$V_s = \frac{W_s}{G_s \gamma_w} = \frac{0.1536}{2.71 \times 9.81} = 0.0058 \text{ m}^3$$

$$V_v = V - V_s = 0.0093 - 0.0058 = 0.0035 \text{ m}^3$$

so

$$e = \frac{0.0035}{0.0058} \approx 0.60$$

For porosity [Eq. (2.7)], we have

$$n = \frac{e}{1 + e} = \frac{0.60}{1 + 0.60} = 0.375$$

We find the degree of saturation [Eq. (2.5)] as follows:

$$S = \frac{V_w}{V_v}$$

$$V_w = \frac{W_w}{\gamma_w} = \frac{0.024}{9.81} = 0.00245 \text{ m}^3$$

so

$$S = \frac{0.00245}{0.0035} \times 100 = 70\%$$

**EXAMPLE  
2.2**

For a given soil,  $e = 0.75$ ,  $w = 22\%$ , and  $G_s = 2.66$ . Calculate the porosity, moist unit weight, dry unit weight, and degree of saturation.

**Solution** The porosity [Eq. (2.7)] is

$$n = \frac{e}{1 + e} = \frac{0.75}{1 + 0.75} = 0.43$$

To find the moist unit weight, we use Eq. (2.19a) to calculate the moist density:

$$\rho = \frac{(1 + w)G_s\rho_w}{1 + e}$$

$$\rho_w = 1000 \text{ kg/m}^3$$

$$\rho = \frac{(1 + 0.22)2.66 \times 1000}{1 + 0.75} = 1854.4 \text{ kg/m}^3$$

$$\text{Hence, the moist unit weight is } \gamma \text{ (kN/m}^3\text{)} = \rho \cdot g = \frac{9.81 \times 1854.4}{1000} = 18.19 \text{ kN/m}^3$$

To find the dry unit weight, we use Eq. (2.19b):

$$\rho_d = \frac{G_s\rho_w}{1 + e} = \frac{2.66 \times 1000}{1 + 0.75} = 1520 \text{ kg/m}^3$$

so

$$\gamma_d = \frac{9.81 \times 1520}{1000} = 14.91 \text{ kN/m}^3$$

The degree of saturation [Eq. (2.17)] is

$$S(\%) = \frac{wG_s}{e} \times 100 = \frac{0.22 \times 2.66}{0.75} \times 100 = 78\%$$

**EXAMPLE  
2.3**

The following data are given for a soil: porosity = 0.45, specific gravity of the soil solids = 2.68, and moisture content = 10%. Determine the mass of water to be added to 10 m<sup>3</sup> of soil for full saturation.

**Solution** From Eq. (2.6), we have

$$e = \frac{n}{1 - n} = \frac{0.45}{1 - 0.45} = 0.82$$

The moist density of soil [Eq. (2.19a)] is

$$\rho = \frac{(1 + w)G_s\rho_w}{1 + e} = \frac{(1 + 0.1)2.68 \times 1000}{1 + 0.82} = 1619.8 \text{ kg/m}^3$$