The use of surface waves in the determination of ground stiffness profiles

M. C. Matthews, V. S. Hope and C. R. I. Clayton

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■ During the past decade two important parallel discoveries have taken place in geotechnical engineering which have resulted in geophysical measurements being used increasingly to provide design parameters.

- **Methods of measuring strain locally on laboratory samples have shown that the stress–strain behaviour of many soils and weak rocks is significantly non-linear, with very high stiffness being observed at small strains.**
- **Field observations of ground deformations around full-scale structures, which could not be modelled using linear elastic theory, can be predicted satisfactorily if non-linear formulations (incorporating very high initial stiffness) are used.**

These developments have closed the gap which had been thought to exist between static and (very small strain) dynamic measurements of stiffness and have enabled meaningful stiffness parameters to be determined from seismic velocity measurements. As a result, increasing use is being made of seismic velocity measurements to ascertain the variation of stiffness with depth in the design of engineering structures. These methods require the use of one or more boreholes, which adds to the cost and time for each stiffness measurement. A relatively little used but promising method which permits the determination of a modulus–depth profile without the aid of boreholes is surface-wave geophysics. This paper describes surface-wave geophysics in terms of field techniques and, equipment and briefly reviews the range of data interpretation techniques that is currently available.

Introduction

Geotechnical design routinely requires that the in situ strength, stiffness and permeability of the ground be determined. In order to ensure that an adequate margin of safety is maintained in the design of constructions such as buildings, excavations and tunnels, measurements of the stiffness are required so that ground movements, both during and after construction, can be calculated. Traditionally these measurements have been made using in situ loading tests and laboratory tests. However, during the past decade important developments have taken place in geotechnical engineering resulting in geophysical measurements being used to provide design parameters. In particular there is a growing appreciation of the value of measuring the shear modulus, G, using seismic methods[1–3] as part of a site investigation. These measurements are often carried out in addition to in situ and laboratory tests. Unlike these more conventional methods, seismic methods have the advantage of not being affected by sampling disturbance and insertion effects.

2. Resonant column tests together with local strain measurements in the triaxial apparatus have indicated that for most soils the stiffness–strain relationship is of the form shown in Fig. 1. It is believed that most soils behave elastically at very small strains (i.e. < 0·001%) giving rise to a constant stiffness. Although the strain induced by the propagation of seismic energy (e.g. body and surface waves) has not been measured directly it is considered to be very small and hence provides an upper bound measurement of stiffness (G_{max}). At small strains (0·001–0·1%) the stiffness becomes sensitive to the magnitude of strain and significant reduction in stiffness has been observed with increasing strain. At intermediate and large strains (> 0·1%) plastic behaviour dominates and the stiffness becomes less sensitive to strain and approaches a minimum value as the

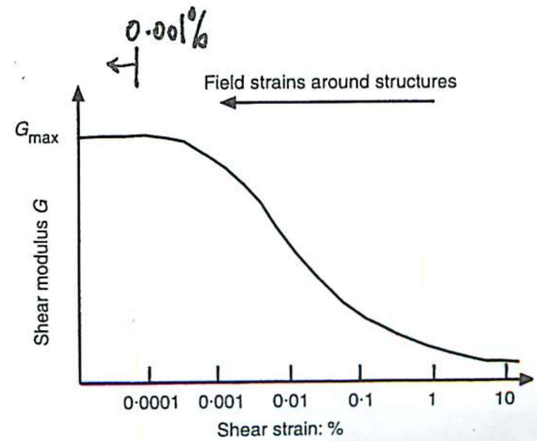


Fig. 1. Idealized stiffness–strain behaviour exhibited by most soils

M. C. Matthews, Department of Civil Engineering, University of Surrey

V. S. Hope, Department of Civil Engineering, University of Surrey

C. R. I. Clayton, Department of Civil Engineering,

erial is brought to failure. Weak rocks display a more linear stress–strain behaviour and hence the loss of stiffness with increasing strain is much less than for most soils.

3. The measurement of stiffness using seismic methods relies on the propagation of elastic waves through the ground. When a hammer strikes the ground, two types of elastic wave are generated; body waves and surface waves. Body waves comprise compressional (or P) waves and slower shear (or S) waves, both of which propagate into the ground. Surface waves cause deformations near the ground surface. Approximately two-thirds of the impact energy of a hammer blow propagates away in the form of surface waves of the kind first described by Rayleigh in 1885.[4] Exploration geophysicists have traditionally regarded Rayleigh waves, or 'ground roll', as a nuisance. However, Rayleigh waves travel at speeds governed by the stiffness–depth profile of the near-surface material: geotechnical engineers have long recognized that Rayleigh waves might offer a useful non-invasive method of investigating the ground in situ.[2,5–7]

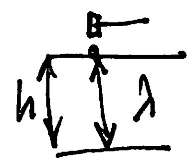
4. As early as the 1940s, Terzaghi[8] and Hvorslev[9] presented reviews of the then state-of-the-art of surface-wave testing. They described work, carried out principally in Germany in the 1930s, in which a mechanical source of continuous vibratory motion was placed on the ground surface. Measurements of the signal frequency, phase and amplitude at various distances from the source were used to deduce the thickness, dip and seismic propagation properties of layered soils, as well as the natural frequency of the ground. The latter was used in the design of foundations for machinery. Both reviews indicate that dispersion (described later) was found to make data processing difficult: nowadays, it is the dispersion of Rayleigh waves which is exploited. Hvorslev[9] described the surface-wave method as being *'under development'*.

5. Despite intermittent renewal of interest in the technique, the surface-wave method has not become widely established in the UK. This is surprising because it is potentially a very attractive addition to the repertoire of site investigation tools. The surface-wave method enables stiffness–depth profiles to be determined rapidly on site, to depths of up to 10 m in soils, and deeper in rock. Typically, the data for a profile comprising more than 20 stiffness values can be acquired within two hours. The technique makes measurements in situ, and so is unaffected by the problems of testing localized, disturbed or non-representative samples; for the same reason it takes into account the effects of fissures and fracturing on mass compressibility. The tests are non-invasive, with at most the upper few centimetres of topsoil or rubble needing to be cleared from a very small area of the test site. On the basis of cost-per-data point, surface-wave testing is by far the cheapest of all the direct methods of stiffness measurement. It allows ground variability to be assessed swiftly in terms of stiffness, and permits ready appraisal of ground improvement schemes such as dynamic compaction. Additionally, unlike other surface-based seismic methods, surface-wave testing can reveal a low-stiffness layer sandwiched between relatively high-stiffness layers.[7,10]

6. The paper describes the principles of the surface-wave technique, and describes how the method is implemented in the field. The ways in which measurements of the dispersion of Rayleigh waves can be used to derive stiffness–depth profiles are outlined.

Rayleigh waves, and the principles of the surface-wave method

7. A Rayleigh wave can be visualized as being similar to a wave on the surface of water; the particle motion is in a vertical ellipse, parallel to the direction of propagation which is along the ground surface. The absolute magnitude of the shear strains induced by Rayleigh waves is thought to be very small, less than 0·001%. The amplitude of the particle motion in a Rayleigh wave diminishes exponentially with distance from the free surface. In practice, the majority of the wave energy is contained within a zone that extends to a depth of approximately one wavelength. Thus, the velocity with which a Rayleigh wavefront propagates away from an impact point is influenced by the properties of the ground to a depth equivalent to about one wavelength.



8. Consider the case of a soil layer of thickness h overlying a much thicker stratum. A surface wave of wavelength less than h would propagate mainly within the upper layer, and so would travel at a speed dependent on the soil properties in that layer. A surface wave of wavelength significantly greater than h would be affected principally by the lower layer. There will also be a range of wavelengths for which both layers have an influence on the wave.

9. A Rayleigh wave propagating along the surface of a uniform, isotropic elastic half-space will travel at a speed that is independent of its wavelength. If, however, there is a variation of stiffness, Poisson's ratio or density with depth, then the speed of the Rayleigh wave will depend on its wavelength. This is because a low-frequency (long-wavelength) Rayleigh wave will extend into and be influenced by deeper material than would a higher-frequency (shorter) wave. When the velocity and frequency (or wavelength) of a wave are not independent, the wave is said to be *dispersive*. It is the dispersive behaviour exhibited by Rayleigh waves in non-

uniform materials that can be exploited by geotechnical engineers. Through field measurements, the velocity of Rayleigh waves of various frequencies, termed phase velocities, can be determined. An estimate of the Rayleigh wave velocity–depth profile that gave rise to the observed dispersion can then be deduced. According to elastic theory, the velocity of a Rayleigh wave is a function of, *inter alia*, the shear modulus of the host medium. Thus a Rayleigh-wave velocity profile can be converted to a stiffness–depth profile.

Test method

10. The use of Rayleigh waves to derive stiffness profiles has been studied by several investigators.[2,6,7,11,12] The basic procedures that have been adopted involve the following

(*a*) the generation of predominantly vertical ground motions using a point source of energy, as either a transient impulse or a continuous wave
(*b*) measurement of ground surface motions using geophones or other sensors placed in a line which is co-linear with the source
(*c*) the recording of ground surface motions with an oscilloscope, spectrum analyzer or a conventional seismograph
(*d*) use of spectral analysis of the data to produce the dispersion curve, showing the variation of Rayleigh wave velocity with wavelength
(*e*) the determination of the Rayleigh wave velocity–depth profile, based on the inversion of the measured dispersion curve
(*f*) application of elastic theory to derive a stiffness–depth from the Rayleigh wave velocity profile.

Surface wave generation

11. There are two forms of surface-wave source in use: impact sources, such as a hammer or a drop weight, which produce a transient impulse, and vibrators which produce continuous waves. The choice of source (transient or continuous) affects the details of the way in which the field data are acquired and subsequently processed. Impact sources have been frequently used in North America,[13,14] with the data being processed using the Spectral Analysis of Surface Waves (SASW) method described by Stokoe and Nazarian.[13] Vibrator sources have been widely adopted in the UK[2,15,16] and Japan,[12] with the Continuous Surface Wave (CSW) method.

12. A typical survey will require the generation of Rayleigh waves of frequencies in the range 3 Hz to 200 Hz. The lower frequencies correspond to long wavelength Rayleigh waves, and it is these waves which provide information about the ground at depth. Therefore, it is essential that the chosen source of Rayleigh waves can produce low frequency energy.

13. The frequency content of a surface-wave signal generated by a hammer or dropped-weight impulse source will depend upon its shape and mass. All such sources produce a signal made up of a spectrum of frequencies. In general, heavier weights generate signals predominated by lower frequencies. To produce Rayleigh waves having the range of wavelengths needed to 'sample' the zone of interest, it may be necessary to use a suite of different impact sources. The lack of precise control over the wave frequency generated when using an impact-type source is a serious disadvantage.

14. Vibration sources generate a continuous sinusoidal signal dominated by a single frequency which, within limits, can be varied in a controlled and systematic manner. The simplest form of vibrator uses a pair of counter-rotating discs on parallel horizontal shafts. The discs are eccentrically weighted or mounted in such a manner as to produce a sinusoidal, vertical force. The earliest mechanical vibrator of this type was developed by Degebo[17] (Deutsche Gesellschaft für Bodenmechanik) in the 1930s. This vibrator had a base area of approximately 1 m^2, weighed over 2000 kg and produced frequencies between 5 and 60 Hz.[8,9] In practice, mechanical vibrators are little used nowadays.

15. Greater frequency ranges and portability can be achieved using electromechanical vibrators, which can weigh less than 15 kg. The frequency output of these devices can be controlled to within 0·1 Hz using a signal generator. However, such vibrators do not provide good-quality sinusoidal waveforms at frequencies below about 7 Hz. The problems of poor signal quality at low frequencies can be overcome by using a much heavier device which can accommodate a greater stroke. For example, the Norwegian Geotechnical Institute source is based on a servo-hydraulic actuator, working against a dead weight.[18] The frequency range of this device is 3–50 Hz but its minimum mass is 200 kg, which rather limits its portability. Another approach is to use ground-borne vibrations from site machinery. Surface waves of adequate quality with frequencies less than 3 Hz have been obtained from earthmoving and drilling equipment.[19]

Measurements of ground motion

16. Surface waves are detected using sensors planted in the ground surface at known distances in one or more lines which are co-linear with the source. Geophones (velocity transducers) are the most widely used sensors; accelerometers have rarely been used to measure ground vibration in Rayleigh wave surveys.

17. Modern geophones are almost entirely of the moving-coil electromagnetic type

(Fig. 2(a)). The output voltage of a vertically mounted geophone is a function of the vertical velocity component of ground motion which causes a relative movement between the coil and magnet. A typical frequency response curve for a geophone is shown in Fig. 2(b). At frequencies greater than the natural frequency, the output is approximately constant; the natural frequency of the geophone should be less than the smallest input frequency. The best geophones used in most surface-based geophysics, such as seismic refraction surveys, typically have a natural frequency of approximately 4·5 Hz, which limits their usefulness in surface-wave surveys. Some authors[12] report the use of geophones with a natural frequency of 1 Hz, termed seismometers. These are extremely delicate instruments which require careful installation in the field. A possible drawback is their price: a seismometer costs about 100 times more than a conventional geophone.

18. The geophone sensors are arranged as shown in Fig. 3. At least two sensors are needed, although as many as 24 are sometimes used. Figures 4 and 5 show typical acquisition geometries for the SASW and the CSW methods. The spacing of the geophones is important. When using only two sensors the distances d, between the sensors, and L, between the source and the mid-point of the sensors (Fig. 3), are key factors in the survey design. Heisey *et al.*[20] suggested that, due to limitations of recording equipment and the attenuative properties of the ground, d should be $\lambda/3 < d < 2\lambda$ where λ is the wavelength of the surface wave under consideration. Based on a more comprehensive study of Rayleigh wave propagation and particle orbits Tokimatsu *et al.*[12] recommended the following empirical rules

$$\lambda/4 \leqslant L$$

and

$$\lambda/16 \leqslant d < \lambda$$

For a typical range of wavelengths of, for example, 0·3–30 m, L would range from 75 mm to 7·5 m and the minimum inter-sensor spacing d would range from 19 mm to 1·9 m. In practice, the recommended lower values of L and d are impractical since the dimensions of the base plate of the energy source would exceed L, and

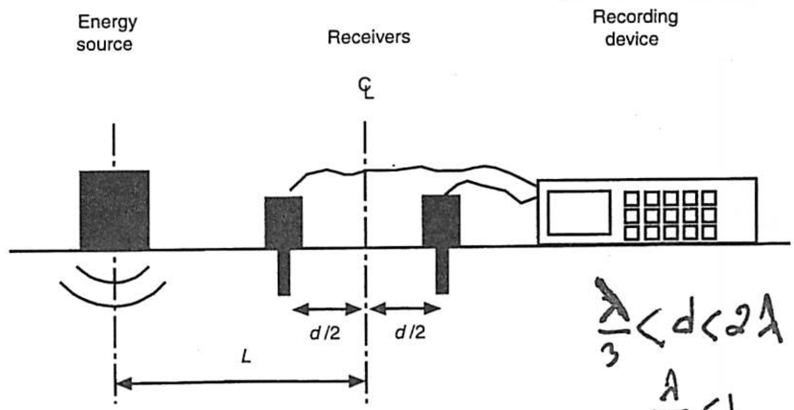


Fig. 3. Basic equipment required for surface-wave surveys

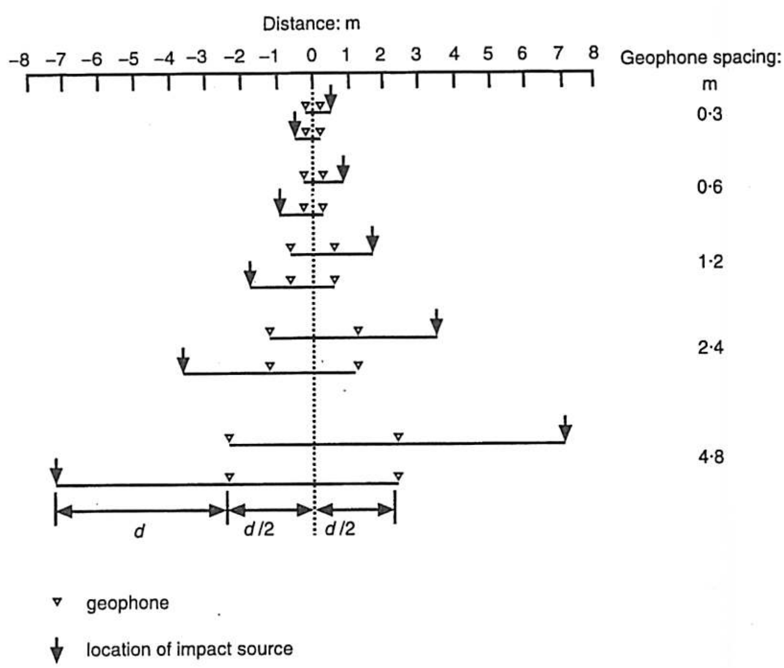


Fig. 4. Common receivers midpoint geometry used with the SASW method

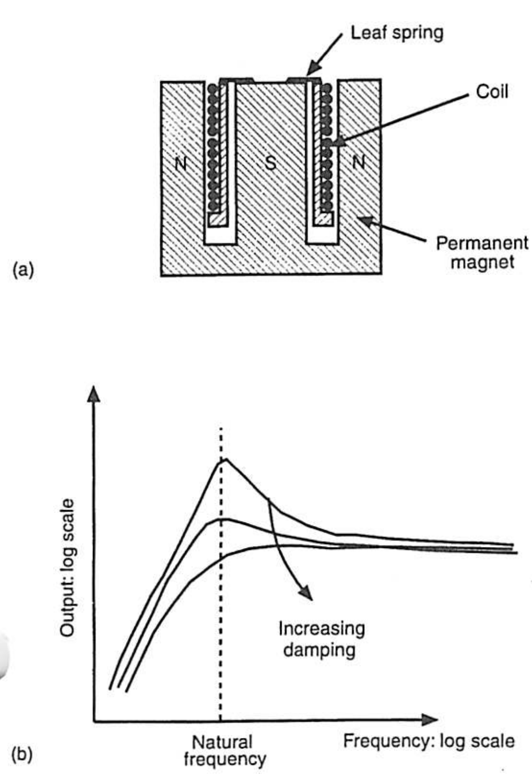


Fig. 2. (a) Schematic diagram of a typical geophone; and (b) typical frequency response for a geophone

geophone housings are greater than 20 mm in diameter.

Recording devices

19. The most common recording system used for surface-wave surveys is the spectrum analyser. A spectrum analyser captures signals from the ground motion sensors, usually a pair, in the time domain. The signals are passed through a high-grain amplifier, digitized and saved. The time-domain data are transformed into the frequency domain. From these spectral data, the phase difference between the signals at each geophone and the coherence of the cross-correlated signals can be determined. In practice, if the coherence drops below 0·9, the phase information should be considered unreliable. Analysers are expensive, but hire costs are low. A key advantage of these devices is that they can provide dispersion data whilst on-site, and so allow an immediate, preliminary assessment of the stiffness–depth profile.

20. Seismographs are also used in surface-wave surveys. Seismographs are multi-channel digital recorders, and most allow at least twelve geophones to be used simultaneously. The data collected in the field are in the time domain, and must be transferred to a computer for transformation into the frequency domain in preparation for the determination of phase shifts between signals. In general, this step precludes on-site data processing. This is a disadvantage, since the quality and range of data acquired cannot be assessed in detail before leaving the site.

21. Some workers[14] record surface-wave field data using a microcomputer equipped with an analogue-to-digital converter and a direct memory access card. A low-pass filter is provided to eliminate aliasing. Appropriate Fourier transform firmware or software may be mounted on the microcomputer so that the Rayleigh dispersion curve can be derived while the survey is in progress.

Spectral analysis of field data

22. From the raw ground motion data, it is necessary to derive the Rayleigh wave dispersion curve of wavelength against phase velocity. This is the dispersion curve from which the stiffness–depth profile can be deduced. The following description of the derivation of a dispersion curve refers to field data acquired using a seismograph. The processes are similar when a spectrum analyser or a computer and A to D converter are used but, with these devices, many of the steps taken are hidden in a 'black box'.

23. Consider the simple case in which a continuous vibratory source of surface waves is placed on the ground and driven at a known frequency, f. Two geophones are positioned as

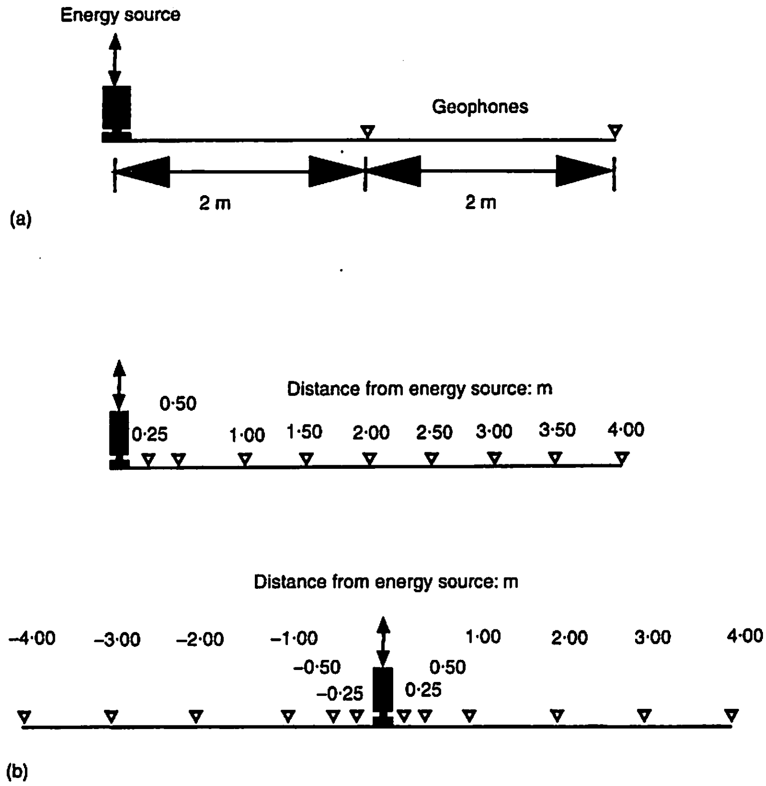


Fig. 5. Typical geophone layouts used with the CSW method for (a) spectrum analyser; and (b) a seismograph

shown in Fig. 4, at a distance d from each other. The phase difference, ϕ in radians, between the steady-state signals received at each geophone is measured. If d is less than the wavelength, λ, of the Rayleigh wave, then by proportions

$$\lambda = \frac{2\pi d}{\phi} \tag{1}$$

If d is greater than λ, then

$$\lambda = \frac{2\pi d}{(2\pi n + \phi)} \tag{2}$$

where n is an integer. The velocity of the Rayleigh wave, V_R, of frequency f at the site is given by the familiar relationship

$$V_R = f\lambda \tag{3}$$

The plot of V_R against λ, for various frequencies, is the site dispersion curve.

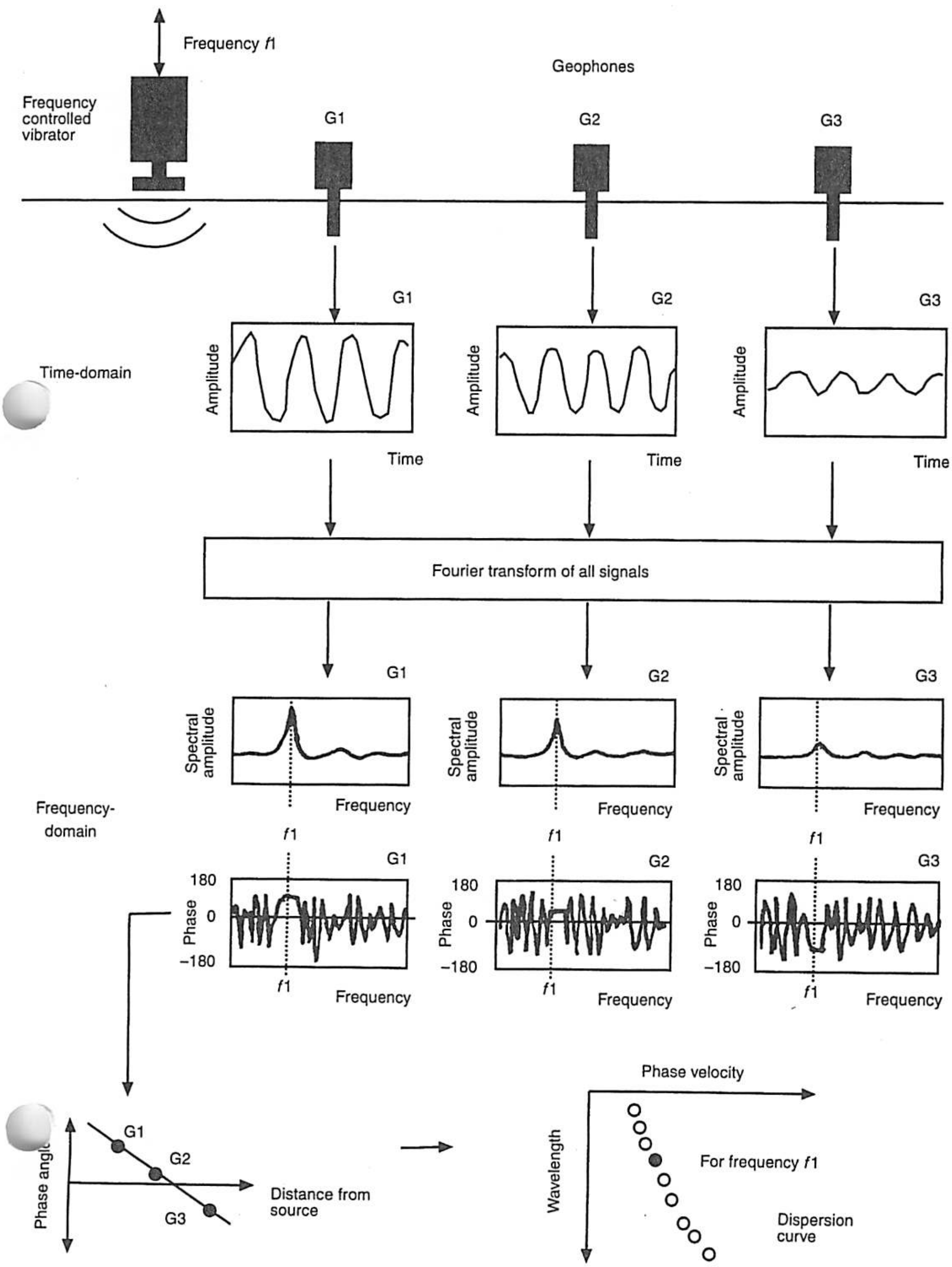
24. So, a question remains: how is ϕ, the phase difference between the ground motions at each geophone, obtained? If the motions at the geophones were pure, mono-frequency sinusoids, then a crude and laborious estimate of the phase difference between the signals could be made using a light table by shifting paper traces of the recordings over each other to find a match between the shapes of the waveforms. In practice, this approach is unusable because the received signals will be slightly corrupted by noise and, in the case of an impact source (SASW), the wave will be transient and exhibit a broad frequency spectrum.

25. It will be recalled that any continuous signal can be decomposed into an equivalent

summation of an infinite series of harmonics, using the Fourier transform. If the signal was sampled at intervals of Δt seconds, as with a digital seismograph, then these time-domain data can be transformed into a finite series of harmonics ranging from 0 to the Nyquist frequency $1/(2\Delta t)$ Hz. Each data point in the frequency domain comprises a complex number (a_f, b_f). Its magnitude $(a_f^2 + b_f^2)^{1/2}$ is the spectral amplitude of that frequency. This indicates how much of the recorded signal was 'made up' of that frequency. The angle $\tan^{-1}(b_f/a_f)$ is the phase of the harmonic, at time zero. For example, for a pure sinusoidal wave of frequency f_1 which was at a peak when recording began, the phase angle would be 90° and a plot of spectral amplitude against frequency would show a spike at f_1.

26. Figures 6 and 7 show schematically the stages by which a dispersion curve is drawn up for the CSW and SASW methods, respectively. With reference to Fig. 6, ground motion data

Fig. 6. Schematic diagram showing the steps followed in the determination of a dispersion curve using the CSW method



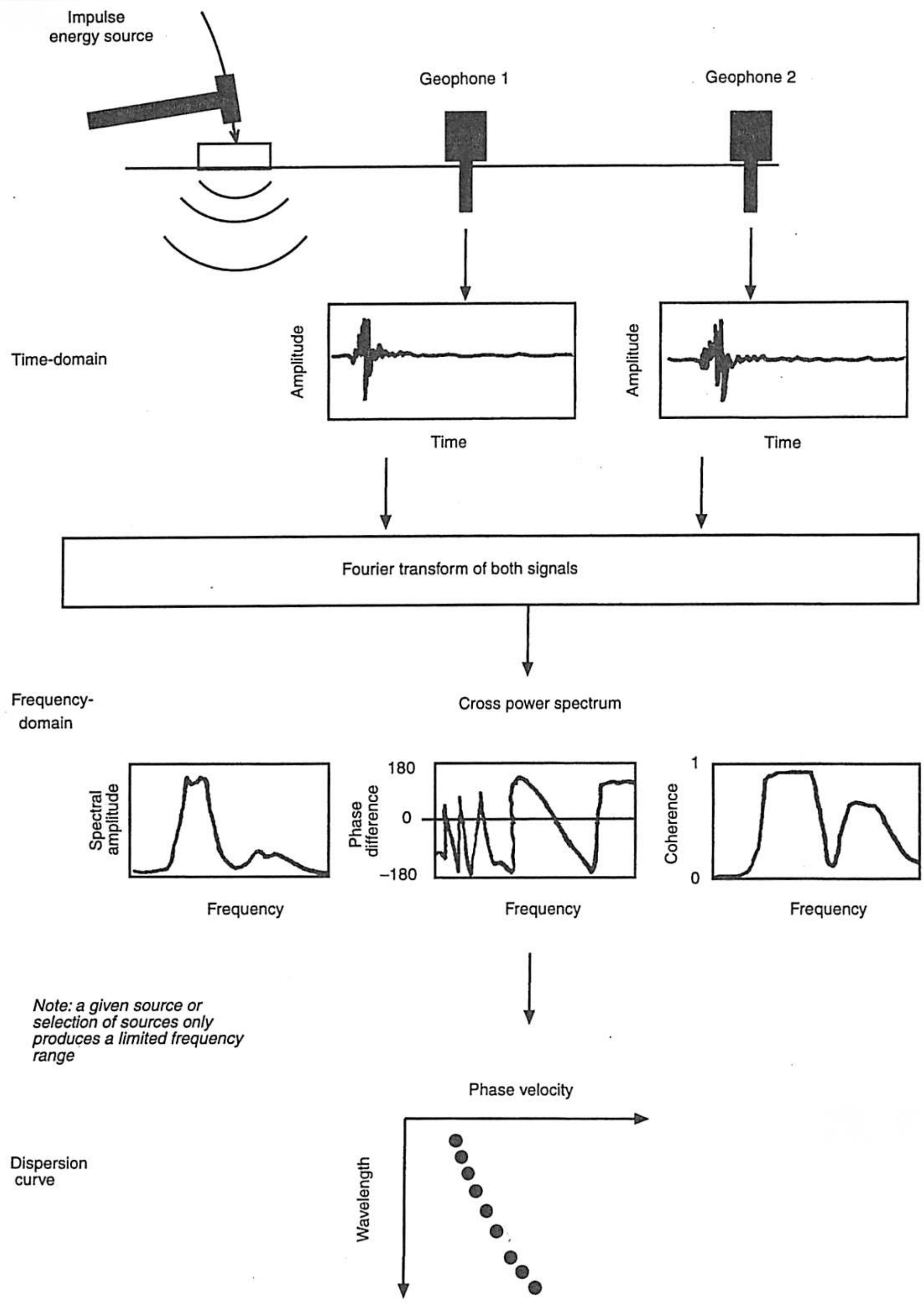


Fig. 7. Schematic diagram showing the steps followed in the determination of a dispersion curve using the SASW method

recorded in the time domain are transformed, using a Fourier algorithm, to the frequency domain. The spectral amplitude curves can be used to assess the quality of the signals: a sharp peak should be seen at the driving frequency of the vibrator. The phase angle at that frequency can be determined. From equation (1), the gradient of a plot of phase angle against distance from the source will yield the wavelength of the Rayleigh wave of that frequency. Then, with equation (3), a new point can be added to the dispersion curve. An advantage of using several geophones is that a best fit line can be drawn through the phase angle–distance plot, minimizing the influence of variations in the data. It is for this reason that the arrays of geophones shown in Fig. 5(b) are used. The calculated phase angle is necessarily limited, for example to the range $-180°$ to $+180°$: if n, in equation (2), is not zero, then it may be necessary to add or subtract multiples of 180° to the calculated phase angle for a particular geophone, in order to determine the phase angle–distance gradient. In practice, this additional step does not pose any problems. Figure 7 shows the comparable processing stages for SASW. In this case, the coherence of the cross-power spectrum of the signals from the two geophones is used to assess the quality of the data and to identify the range of frequencies for which phase difference information can be utilized reliably.

27. Examples of some field dispersion curves are shown in Fig. 8. The dispersion curves for London Clay and Chalk shown in Fig. 8 represent profiles of increasing Rayleigh wave velocity, and hence stiffness, with depth. The curve for the Oxford Clay site shows a different pattern, one typical of a layer of high velocity sandwiched between layers of lower velocity.

Inversion of the field dispersion curve

28. The process of converting a field dispersion curve to a Rayleigh velocity–depth relationship is known as inversion. There are three principal inversion methods

- the wavelength–depth method
- Haskell–Thomson matrix techniques
- finite element approaches.

29. The wavelength–depth method is the simplest, but least exact, of the methods. It is of value because it offers a relatively quick way of processing data while on-site, for preliminary assessment. If using either of the other techniques, then the wavelength–depth method can provide a useful initial estimate of the velocity–depth profile to input to the other algorithms. To establish the depth profile, it is necessary to determine at what depth, z, is the calculated phase velocity representative of the propagation properties of the ground. Recalling that the amplitude of a Rayleigh wave diminishes with depth. In the wavelength–depth method the representative depth is taken to be a fraction of the wavelength, λ. That is, (λ/z) is assumed to be a constant. A ratio of 2 is commonly, but arbitrarily, used.[1,2,6,7] Gazetas[21] recommended that 4 is used at sites where the stiffness increases significantly with depth, and that 2 is suitable at more homogeneous sites. He suggested that $(\lambda/z) = 3$ is a reasonable compromise.

30. Haskell[22] described a method of calculating the dispersion of Rayleigh waves in multi-layered media, based on a matrix approach due to Thomson.[23] The method was intended for global seismologists interested in using earthquake-induced Rayleigh wave data to delineate the structure of the mantle. The use of Haskell–Thomson in surface-wave ground investigations was popularized by Stokoe and his co-workers. In their approach, the Haskell–Thomson algorithm is used to determine a synthetic dispersion curve for an initial estimate of the soil profile. This is compared with the field dispersion curve. Through a repetitive, trial-and-error process, the estimate of the velocity–depth profile is adjusted until there is close agreement between the two curves.

31. Finite element techniques are utilized in a similar way to the Haskell–Thomson method. From an initial estimate of the stiffness distribution, a synthetic dispersion curve is generated using dynamic finite elements, and the stiffness distribution is progressively adjusted

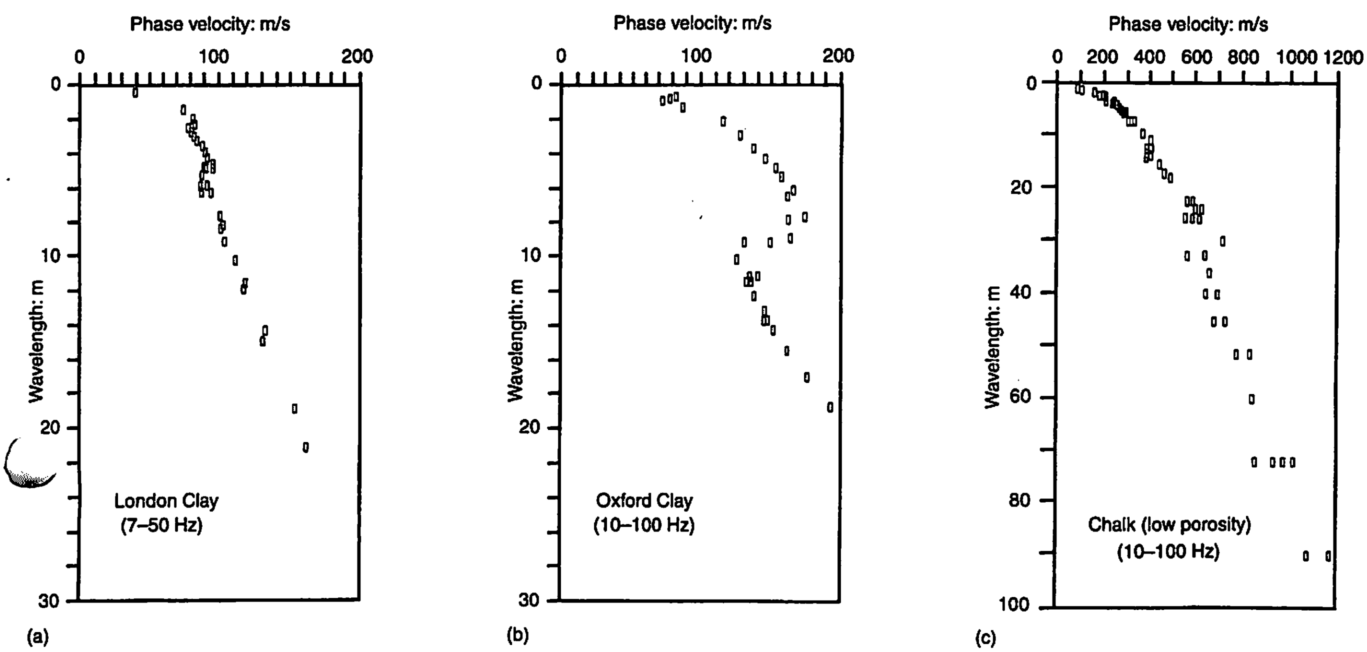


Fig. 8. Typical dispersion curves determined using the CSW method

until the synthetic dispersion curve matches the curve obtained in the field.[24] The ground is divided into layers of constant stiffness. For simple sub-surface geometries a two-dimensional (axi-symmetric) idealization of surface-wave tests can be made. The equations of motion are integrated with respect to time to model the ground motion at the actual geophone locations used in the field. These data are used to determine the synthetic dispersion curve. Care should be taken in the selection of suitable mesh size and time steps to avoid aliasing.[25,26] For complex sub-surface geometries a three-dimensional analysis may be necessary in order to yield a more accurate dispersion curve. However, such an approach is time-consuming in terms of computer time and hence expensive. The principal advantage the finite element method has over the Haskell–Thomson method is its ability to model the near field and complex sub-surface geometries.

Calculation of the stiffness–depth profile

32. According to elastic theory, the velocity of shear wave propagation, V_s, is related to V_R by

$$V_s = p V_R \tag{4}$$

where p is a rather complicated function of Poisson's ratio, ν. For $\nu = 0{\cdot}25$, $p = 1{\cdot}088$ and for $\nu = 0{\cdot}5$, $p = 1{\cdot}047$; mis-estimating ν has little effect on V_s. The shear modulus, G, is related to the shear wave velocity by

$$G = \rho V_s^2 \tag{5}$$

where ρ is the bulk density of the soil. Hence

$$G = \rho p^2 V_R^2 \tag{6}$$

which permits a straightforward conversion from a Rayleigh velocity–depth profile to a stiffness–depth profile. Some field results for different materials are shown in Fig. 9.

Discussion

33. Stiffness values derived from seismic tests, including the Rayleigh method, are usually denoted G_{max}. They represent the very small strain stiffness, which is considered to be the maximum shear modulus exhibited by a material. G_{max} can be used in engineering calculations with only minor modifications. For soils, which show markedly non-linear stress–strain behaviour, the operational stiffness is likely to be less than G_{max} at the strain levels normally associated with ground deformations around structures (0·01–0·1%). For clays the ratio $G_{0{\cdot}01}/G_{max}$ is generally between 0·5 and 0·8[3,27–30] and for sands it approaches 1.[31] Weak rocks, such as chalk, show approximately linear stress–strain behaviour, and $G_{0{\cdot}01}/G_{max}$ may be taken as unity.[15,32] Rayleigh wave testing identifies the *shape* as well as the magnitude of the G_{max} profile. A possible extension of the technique at a particular site would be to use (a small number) of laboratory tests to 'calibrate' the ratio of G_{max} and the shear modulus at the strain of interest. This would yield a stiffness–depth profile of direct use in, for example, finite element analyses.

34. G_{max} values derived using other seismic methods, such as cross-hole surveying, sometimes differ from those obtained using the

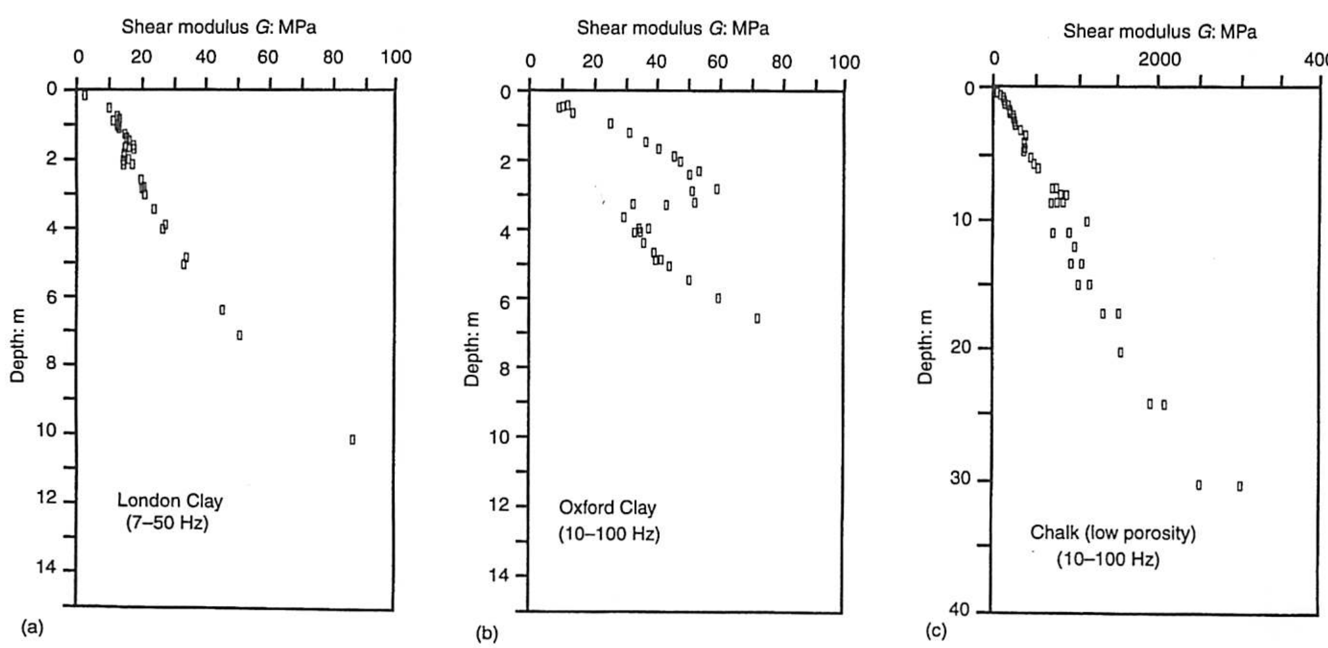


Fig. 9. Shear modulus–depth profiles for sites underlain by London Clay, Oxford Cl

Rayleigh technique. The various modes of propagation of seismic energy (shear, compressional, Rayleigh, etc.) induce different deformations in the ground. In a material that exhibits stress–strain anisotropy, the elastic parameters that would be derived from seismic data would depend on the wave mode used and its direction of propagation. For example, in a homogeneous, transversely isotropic soil, the same speed of propagation would be expected for a horizontally travelling vertically polarized shear wave, a vertically travelling vertically polarized shear wave and a vertically travelling horizontally polarized shear wave. Their speed would be a function of the shear modulus in vertical planes, G_v. In contrast, a horizontally travelling horizontally polarized shear wave would travel at a speed governed by the modulus in the horizontal plane, G_h. Seismic techniques offer a means of determining the degree of anisotropy of the ground.[33]

35. The analysis of surface waves is complicated by the different ways in which soil layers of different stiffness can vibrate relative to each other.[34] Just as a weighted cantilever will vibrate in preferential harmonic modes, so too do Rayleigh waves show modal behaviour when propagating in stratified ground. A system of soil layers will vibrate in different modes, depending on the signal frequency as well as the thickness and stiffness of each layer. Higher-mode Rayleigh waves have been observed by many authors, both intentionally[12,35,36] and accidentally.[6] To resolve the contribution of higher modes and that of the fundamental component, it is necessary to use a large number of geophone sensors in order to avoid spatial aliasing.[35]

36. In the immediate vicinity, termed the *near field*, of a source, the elastic deformations induced in the ground are not yet resolved into distinct propagation modes. Well-developed Rayleigh waves come into being at a distance of about two wavelengths from a point source.[34] In surface-wave surveys, it is common for geophone sensors to be rather closer to the source than this. It is problematic that the Haskell–Thomson method models far-field Rayleigh waves only, since this is not necessarily the form of surface-borne wave that is encountered in practice. Near-field effects around the source can be incorporated in dynamic finite element analyses.

37. The Haskell–Thomson method of inversion treats only laterally invariant, horizontally layered geologies and a flat ground surface. On site, one is likely to encounter hills, ditches and even cliffs. The focusing and distorting effects that such topographical features can have on surface waves are well known to seismologists. The finite element method has the advantage over Haskell–Thomson in that there is no restriction that the ground surface should be flat. Furthermore, there is the potential to treat lateral soil variations, and there is no requirement that interfaces between strata should be parallel.

38. The maximum depth of penetration that can be achieved with the Rayleigh method depends on the stiffness of the ground and the lowest frequency for which reliable data may be obtained. The authors have carried out a number of surface-wave tests which have provided stiffness measurements within the weathering profile of heavily overconsolidated soils and weak rocks. In general, reliable data were obtained to a depth of 8 m in the heavily overconsolidated soils and to 20 m in weak rock (see Fig. 9), using frequencies greater than 6 Hz. The achievable depth of investigation may be increased by using lower frequencies, although this depth may be limited at sites where there are discrete layers of strongly contrasting stiffness.

39. Of the two approaches to surface-wave testing, SASW may at first sight appear to be more attractive due to the simplicity and cheapness of the sources used. However, the restricted range of and the lack of control over the frequencies generated by impact-type sources impose a serious limitation on the SASW method. The use of a vibrator source with the CSW method overcomes these problems. Moreover, since a vibration source produces, in essence, mono-frequency signals, unwanted noise is easily filtered from the data, giving more accurate stiffness–depth profiles. Our experience to date has been that the CSW method gives better quality data.

Conclusions

40. The surface-wave method provides a rapid means of determining stiffness–depth profiles in near-surface soil and rock without the need for boreholes. The equipment required for surface-wave tests includes an energy source (hammer or vibrator), two or more receivers (usually geophones), a recording device (typically a spectrum analyser or a seismograph) and a portable computer for data processing.

41. There are two methods of surface-wave surveying; the Spectral Analysis of Surface Waves (SASW) method and the Continuous Surface Wave (CSW) method. SASW uses a hammer blow as an energy source. The major limitation of this technique is a lack of frequency control and resolution. CSW uses a steady-state vibrator as an energy source. Such sources are relatively costly, but have the advantage of frequency control and good frequency resolution.

42. The maximum depth of investigation of a Rayleigh survey depends on the lowest frequency that can be generated by the energy source and the stiffness of the ground. Using available CSW sources, the maximum depth of

investigation is about 8 m and 20 m in cohesive soils and weak rock, respectively. SASW hammer sources can permit greater depths of investigation.

43. Both SASW and CSW require about two hours to complete a test at a given location. A test will usually give between 20 and 50 stiffness measurements at different depths. On this basis surface-wave testing must be considered to be very cost effective for relatively shallow investigations, when compared with other site investigation techniques.

44. A preliminary stiffness–depth profile can be obtained rapidly while on site using the empirical wavelength–depth method for inverting the field dispersion curve. More sophisticated inversion methods can provide improved profiles, but these usually take more time and so are better suited to the office.

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