PROBABILISTIC SEISMIC HAZARD ANALYSIS

The following is a simple example to show how the probabilistic seismic hazard analysis procedure works. Assume that there is only one seismic source zone around the study site (Figure 1). The seismic parameters of this source zone are:

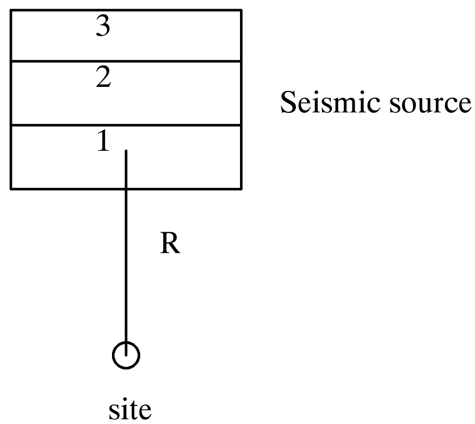

Figure 1. Seismic Source Zone

A. Minimum Magnitude m_0 = 5

B. Maximum Magnitude m_u = 7.4

C. Frequency–Magnitude Relationship

$$\ln N(m) = 2.55 - 0.88m \tag{1}$$

where m is earthquake magnitude, $N(m)$ is the number of earthquakes with magnitude equal or greater than m in a given period, and b describes the relative rate of occurrence of earthquakes with different magnitudes. In seismic hazard analysis, we are only interested in the earthquakes with magnitude larger than the minimum magnitude, therefore Equation (1) is expanded to include a lower bound magnitude as well as an upper bound magnitude, the relationship becomes nonlinear at large magnitudes. The probability density function also can be derived from the recurrence relationship. Incorporating the magnitude range, this is the truncated exponential distribution of earthquake magnitudes:

$$f_M(m) = k\beta \exp(-\beta(m - m_0)) \quad m_0 < m < m_u \tag{2}$$

where $\beta = b \ln(10)$, m_0 and m_u are the minimum and maximum magnitude respectively, and, k is a normalizing constant

$$k = [\, 1 - e^{-\beta(m_u - m_o)}\,]^{-1} \tag{3}$$

D. Assume for the study area, the acceleration attenuation function is

$$\ln y = 1.31 + 1.15m - 0.83 \ln R - 0.0028R \qquad (unit : cm/sec^2) \tag{4}$$

where m is the magnitude of earthquake and R is the epicentral distance of the study site.

The recurrence relationship is combined with statistical model of the ground acceleration attenuation relationship and earthquake occurrence (Poisson model) to relate the intensity of expected strong ground motion to its annual probability or occurrence rate of being exceeded. This can be calculated as

$$P[\, y \geq Y\,] = v(m \geq m_0) \int_R \int_{m_0}^{m_u} P[\, y \geq Y/m, R]\, f(m)\, f(R)\, dmdR \tag{5}$$

where Y is the ground motion level we interest, $v(m \geq m_0)$ is the occurrence rate of earthquakes with magnitude $m \geq m_0$, $P[\, y \geq Y/m, R\,]$ is the conditional probability of $y \geq Y$ specified earthquake magnitude m and epicentral distance R, which is determined by the strong ground motion attenuation function, $f(m)$ is probability density function of earthquake magnitude m of seismic source zone, and, $f(R)$ is probability density function of distance which can be computed from the geometrical position of site respect to the seismic source zone. Because it is difficult to do the integration of equation (5), we usually divide magnitude range into M subrange, and the seismic source zone into N subzones and assume the distance from each subzone to the site is the same, then do discrete integration. Equation (5) is revised as following:

$$P[\, y \geq Y\,] = \frac{v(m \geq m_0)}{S} \sum_{i=1}^{N} S_i \sum_{j=1}^{M} P[\, y \geq Y/m_j, R_i]\, f(m_j) \tag{6}$$

where S_i is area of subzone i, and S is total area of seismic source zone. Therefore, to compute the seismic hazard curve, we have to do the following work:

A. Divide the seismic source zone into a set of subsonic zone (Figure 1). For this example, the area and distance of each subzone are as table 1

Table 1. Parameters of Subzone

No.	*1*	*2*	*3*	$\sum$
Area (km^2)	*2000*	*1500*	*2500*	*6000*
R_i *(km)*	*20*	*30*	*40*	

B. Calculate the occurrence rate of earthquakes with magnitude larger than minimum magnitude from equation (1). For the example, the minimum magnitude is 5. Therefore the occurrence rate can be computed as

$$v(m \geq 5) = N(m = 5) = e^{2.55 - 0.88*5} = 0.1572 \tag{7}$$

C. Divide the magnitude range into a set subrange, then compute the probability density function of central magnitude of each subrange from equation (2). Here, we use the magnitude increment 0.4, so there are six subranges for $5 \leq m \leq 7.4$.

Table 2. Probability Density Function

m	5–5.4	5.4–5.8	5.8–6.2	6.2–6.6	6.6–7.0	7.0–7.4
m_{midj}	5.2	5.6	6.0	6.4	6.8	7.2
$f(m_{midj})$	1.362	0.605	0.269	0.120	0.053	0.024

D. Makeup an acceleration attenuation table according to magnitude of table 2 and the attenuation function of equation (4)

Table 3. Acceleration Attenuation Table

Distance	m = 5.2	m = 5.6	m = 6.0	m = 6.4	m = 6.8	m = 7.2
R = 20 km	0.115g	0.182	0.289	0.458	0.726	1.150
R = 30 km	0.080	0.126	0.201	0.318	0.504	0.799
R = 40 km	0.061	0.097	0.154	0.244	0.386	0.612

D. Specified a set of acceleration values (Table 4)

Table 4. Specified Acceleration Values

0.10 g	0.20 g	0.30 g	0.40 g

Now we compute the occurrence rate of specified acceleration values when earthquakes occur in subzone 1 (R = 20 km).

A. m = 5.2. According to Table 3, the acceleration value is A = 0.115 g. Therefore

P_{11}[A $\geq$0.10/ m=5.2, R = 20] = 1.0

P_{11}[A $\geq$0.20/ m=5.2, R = 20] = 0.0

P_{11}[A $\geq$0.30/ m=5.2, R = 20] = 0.0

P_{11}[A $\geq$0.40/ m=5.2, R = 20] = 0.0

The first subscript in P_{11} [ ] means order of subzone, the second one stands for the magnitude order.

B. m = 5.6. According to Table 3, A = 0.182 g. Therefore

P_{12}[A $\geq$0.10/ m=5.6, R = 20] = 1.0

P_{12}[A $\geq$0.20/ m=5.6, R = 20] = 0.0

P_{12}[A $\geq$0.30/ m=5.6, R = 20] = 0.0

P_{12}[A $\geq$0.40/ m=5.6, R = 20] = 0.0

C. m = 6.0. According to Table 3, A = 0.289 g. Therefore

P_{13}[A $\geq$0.10/ m=6.0, R = 20] = 1.0

P_{13}[A $\geq$0.20/ m=6.0, R = 20] = 1.0

P_{13}[A $\geq$0.30/ m=6.0, R = 20] = 0.0

P_{13}[A $\geq$0.40/ m=6.0, R = 20] = 0.0

D. m = 6.4. According to Table 3, A = 0.458 g. Therefore

P_{14}[A $\geq$0.10/ m=6.4, R = 20] = 1.0

P_{14}[A $\geq$0.20/ m=6.4, R = 20] = 1.0

P_{14}[A $\geq$0.30/ m=6.4, R = 20] = 1.0

$P_{14}[\ A \geq 0.40/\ m=6.4,\ R = 20\ ] = 1.0$

E. m = 6.8. According to Table 3, A = 0.726 g. Therefore

$P_{15}[\ A \geq 0.10/\ m=6.8,\ R = 20\ ] = 1.0$

$P_{15}[\ A \geq 0.20/\ m=6.8,\ R = 20\ ] = 1.0$

$P_{15}[\ A \geq 0.30/\ m=6.8,\ R = 20\ ] = 1.0$

$P_{15}[\ A \geq 0.40/\ m=6.8,\ R = 20\ ] = 1.0$

F. m = 7.2. According to Table 3, A = 1.150 g. Therefore

$P_{16}[\ A \geq 0.10/\ m=7.2,\ R = 20\ ] = 1.0$

$P_{16}[\ A \geq 0.20/\ m=7.2,\ R = 20\ ] = 1.0$

$P_{16}[\ A \geq 0.30/\ m=7.2,\ R = 20\ ] = 1.0$

$P_{16}[\ A \geq 0.40/\ m=7.2,\ R = 20\ ] = 1.0$

Now, we can obtain the occurrence rate specified acceleration values when earthquakes occur in subzone 1 by summation the multiple of contribution of each magnitude range and its probability density function *f(m$_j$)*

$$P_i[\ y \geq Y/R_i\ ] = \sum_{j=1}^{6} P_{1j}[\ y \geq Y/m_j, R_i\ ]f(m_j) \tag{8}$$

$P_1[\ A \geq 0.10/\ R = 20\ ] = 2.433$

$P_1[\ A \geq 0.20/\ R = 20\ ] = 0.466$

$P_1[\ A \geq 0.30/\ R = 20\ ] = 0.197$

$P_1[\ A \geq 0.40/\ R = 20\ ] = 0.197$

We repeat the computation when earthquakes occur in subzone 2. Now the epicentral distance R = 30 km.

A. m = 5.2. According to Table 3, A = 0.080 g. Therefore

$P_{21}[\ A \geq 0.10/\ m=5.2,\ R = 30\ ] = 0.0$

$P_{21}[\ A \geq 0.20/\ m=5.2,\ R = 30\ ] = 0.0$

$P_{21}[\ A \geq 0.30/\ m=5.2,\ R = 30\ ] = 0.0$

$P_{21}[\ A \geq 0.40/\ m=5.2,\ R = 30\ ] = 0.0$

B. m = 5.6. According to Table 3, A = 0.126g. Therefore

$P_{22}[\ A \geq 0.10/\ m=5.6,\ R = 30\ ] = 1.0$

$P_{22}[\ A \geq 0.20/\ m=5.6,\ R = 30\ ] = 0.0$

$P_{22}[\ A \geq 0.30/\ m=5.6,\ R = 30\ ] = 0.0$

$P_{22}[\ A \geq 0.40/\ m=5.6,\ R = 30\ ] = 0.0$

C. m = 6.0. According to Table 3, A = 0.201 g. Therefore

$P_{23}[\ A \geq 0.10/\ m=6.0,\ R = 30\ ] = 1.0$

$P_{23}[\ A \geq 0.20/\ m=6.0,\ R = 30\ ] = 1.0$

$P_{23}[\ A \geq 0.30/\ m=6.0,\ R = 30\ ] = 0.0$

$P_{23}[\ A \geq 0.40/\ m=6.0,\ R = 30\ ] = 0.0$

D. m = 6.4. According to Table 3, A = 0.318 g. Therefore

$P_{24}[\ A \geq 0.10/\ m=6.4,\ R = 30\ ] = 1.0$

$P_{24}[\ A \geq 0.20/\ m=6.4,\ R = 30\ ] = 1.0$

$P_{24}[\ A \geq 0.30/\ m=6.4,\ R = 30\ ] = 1.0$

$P_{24}[\ A \geq 0.40/\ m=6.4,\ R = 30\ ] = 0.0$

E. m = 6.8. According to Table 3, A = 0.504 g. Therefore

$P_{25}[\ A \geq 0.10/\ m=6.8,\ R = 30\ ] = 1.0$

$P_{25}[\ A \geq 0.20/\ m=6.8,\ R = 30\ ] = 1.0$

$P_{25}[\ A \geq 0.30/\ m=6.8,\ R = 30\ ] = 1.0$

$P_{25}[\ A \geq 0.40/\ m=6.8,\ R = 30\ ] = 1.0$

F. m = 7.2. According to Table 3, A = 0.799 g. Therefore

$P_{26}[\ A \geq 0.10/\ m=7.2,\ R = 30\ ] = 1.0$

$P_{26}[\ A \geq 0.20/\ m=7.2,\ R = 30\ ] = 1.0$

$P_{26}[\ A \geq 0.30/\ m=7.2,\ R = 30\ ] = 1.0$

$P_{26}[\ A \geq 0.40/\ m=7.2,\ R = 30\ ] = 1.0$

We can obtain the occurrence rate specified acceleration values when earthquakes occur in subzone 2 according to equation (8).

$P_2[\ A \geq 0.10/\ R = 30\ ] = 1.071$

$P_2[\ A \geq 0.20/\ R = 30\ ] = 0.466$

$P_2[\ A \geq 0.30/\ R = 30\ ] = 0.197$

$P_2[\ A \geq 0.40/\ R = 30\ ] = 0.077$

When earthquakes occur in subzone 3, R = 40 km.

A. m = 5.2. According to Table 3, A = 0.061 g. Therefore

$P_{31}[\ A \geq 0.10/\ m=5.2,\ R = 40\ ] = 0.0$

$P_{31}[\ A \geq 0.20/\ m=5.2,\ R = 40\ ] = 0.0$

$P_{31}[\ A \geq 0.30/\ m=5.2,\ R = 40\ ] = 0.0$

$P_{31}[\ A \geq 0.40/\ m=5.2,\ R = 40\ ] = 0.0$

B. m = 5.6. According to Table 3, A = 0.097 g. Therefore

$P_{32}[\ A \geq 0.10/\ m=5.6,\ R = 40\ ] = 0.0$

$P_{32}[\ A \geq 0.20/\ m=5.6,\ R = 40\ ] = 0.0$

$P_{32}[\ A \geq 0.30/\ m=5.6,\ R = 40\ ] = 0.0$

$P_{32}[\ A \geq 0.40/\ m=5.6,\ R = 40\ ] = 0.0$

C. m = 6.0. According to Table 3, A = 0.154 g. Therefore

$P_{33}[\ A \geq 0.10/\ m=6.0,\ R = 40\ ] = 1.0$

$P_{33}[\ A \geq 0.20/\ m=6.0,\ R = 40\ ] = 1.0$

$P_{33}[\ A \geq 0.30/\ m=6.0,\ R = 40\ ] = 0.0$

$P_{33}[\ A \geq 0.40/\ m=6.0,\ R = 40\ ] = 0.0$

D. m = 6.4. According to Table 3, A = 0.244 g. Therefore

$P_{34}[\ A \geq 0.10/\ m=6.4,\ R = 40\ ] = 1.0$

$P_{34}[\ A \geq 0.20/\ m=6.4,\ R = 40\ ] = 1.0$

$P_{34}[\ A \geq 0.30/\ m=6.4,\ R = 40\ ] = 0.0$

$P_{34}[\ A \geq 0.40/\ m=6.4,\ R = 40\ ] = 0.0$

E. m = 6.8. According to Table 3, A = 0.386g. Therefore

$P_{35}[\ A \geq 0.10/\ m=6.8,\ R = 40\ ] = 1.0$

$P_{35}[\ A \geq 0.20/\ m=6.8,\ R = 40\ ] = 1.0$

$P_{35}[\ A \geq 0.30/\ m=6.8,\ R = 40\ ] = 1.0$

$P_{35}[\ A \geq 0.40/\ m=6.8,\ R = 40\ ] = 0.0$

F. m = 7.2. According to Table 3, A = 0.612 g. Therefore

$P_{36}[\ A \geq 0.10/\ m=7.2,\ R = 40\ ] = 1.0$

$P_{36}[\ A \geq 0.20/\ m=7.2,\ R = 40\ ] = 1.0$

$P_{36}[\ A \geq 0.30/\ m=7.2,\ R = 40\ ] = 1.0$

$P_{36}[\ A \geq 0.40/\ m=7.2,\ R = 40\ ] = 1.0$

We can obtain the occurrence rate specified acceleration values when earthquakes occur in subzone 3 from equation (8)

$P_3[\ A \geq 0.10/\ R = 40\ ] = 0.466$

$P_3[\ A \geq 0.20/\ R = 40\ ] = 0.466$

$P_3[\ A \geq 0.30/\ R = 40\ ] = 0.077$

$P_3[\ A \geq 0.40/\ R = 40\ ] = 0.024$

Now we calculate the total occurrence rate specified acceleration values when earthquakes occur in this seismic zone as following

$$P[\,y \geq Y\,] = \frac{v(m \geq 5)}{S} \sum_{i=1}^{3} S_i P_i [\,y \geq Y/ R_i\,] \tag{9}$$

The results of this example are

$$P[A \geq 0.1] = 0.1572\left[\frac{2000}{6000} \times 2.433 + \frac{1500}{6000} \times 1.070 + \frac{2500}{6000} \times 0.466\right] = 0.200$$

$P[A \geq 0.10] = 0.200$

$P[A \geq 0.20] = 0.056$

$P[A \geq 0.30] = 0.023$

$P[A \geq 0.40] = 0.015$

In equation (9), the value of $P[y \geq Y]$ is the annual occurrence rate of the event of $y \geq Y$. The occurrence model of large earthquakes in a region is often described by a stationary Poisson random process, hence the probability of $y \geq Y$ occurs at least one time in one year can be computed as:

$$P_{1yr}[y \geq Y] = 1 - e^{-P[y \geq Y]} \tag{10}$$

For this example, we have

$P_{1yr}[A \geq 0.10] = 1. - e^{-0.200} = 0.181$

$P_{1yr}[A \geq 0.20] = 1. - e^{-0.073} = 0.070$

$P_{1yr}[A \geq 0.30] = 1. - e^{-0.023} = 0.023$

$P_{1yr}[A \geq 0.40] = 1. - e^{-0.009} = 0.009$

It is clear when the annul occurrence rate of $y \geq Y$ is a very small value, it equals to the probability of the same event occurring in one year . Therefore the calculation of equation (10) can be approximated by:

$$P_{1yr}[y \geq Y] = P[y \geq Y] \tag{11}$$

From the probabilities specified acceleration values, we can draw the seismic hazard curve for this site (Figure 2.). Figure 2. includes two more points we do not calculate in the example. From this hazard curve, we can obtain the acceleration level of any specified return period.

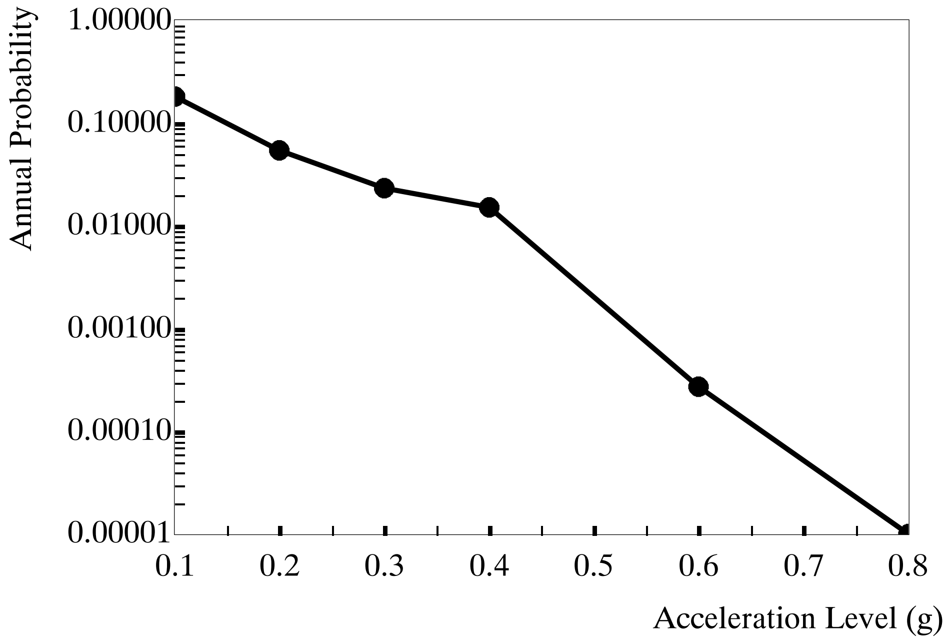

Figure 2. The Seismic Hazard Curve of Example