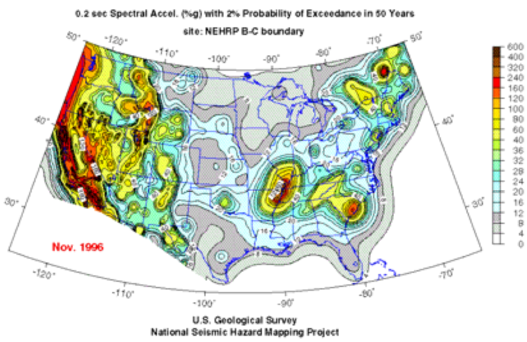
Probabilistic Seismic Hazard Analysis

- Cornell BSSA Paper (1969)



Probabilistic Seismic Hazard Analysis

- Deterministic (DSHA)
  - Assume a single "Scenario"
    - Select a Single Magnitude, M
    - Select a Single Distance, R
    - Select effects due to M and R

→ Ground Motion Parameters

- Probabilistic (PSHA)
  - Assume Many "Scenarios"
    - Consider all Magnitudes
    - Consider all Distances
    - Consider all Effects

→ Ground Motion Parameters

Probabilistic Seismic Hazard Analysis

•Why?
- •we don't know when earthquake will occur,
- •We don't know where they will occur
- •We don't know how big they will be

- Probabilistic (PSHA)
  - Assume Many "Scenarios"
    - Consider all Magnitudes
    - Consider all Distances
    - Consider all Effects

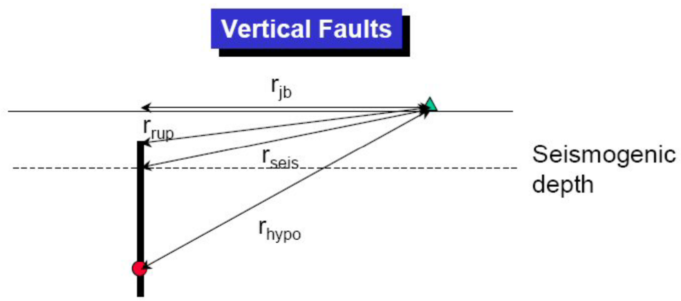
→ Ground Motion Parameters

Probabilistic Seismic Hazard Analysis

- Consists of four primary steps:
  - Identification and Characterization of all Sources
  - Characterization of Seismicity of Each Zone
  - Determination of Motions from Each Source
  - Probabilistic Calculations
- PSHA Characterizes Uncertainty in
  - Location
  - Frequency
  - Effect of earthquakes
  - Then, combine all of them to compute probabilities of different levels of ground shaking

Uncertainty in Source-Site Distance

- Need to specify distance measure based on distance measure in attenuation relationship



Uncertainty in Source-Site Distance

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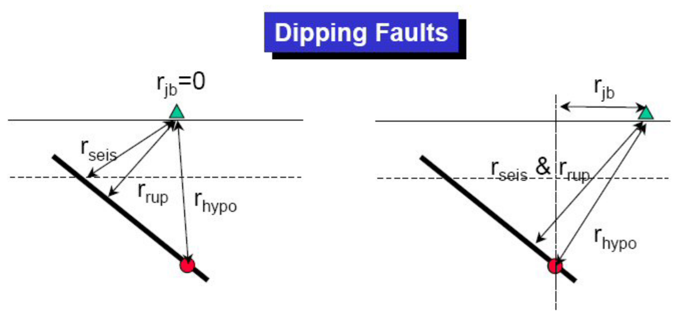
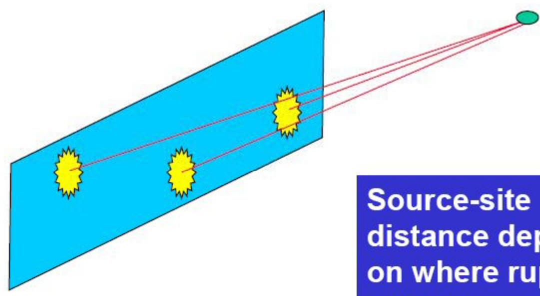


Uncertainty in Source-Site Distance

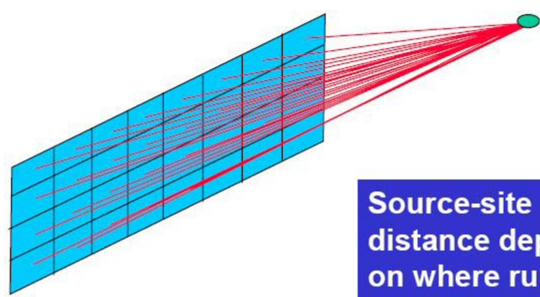
- Where on fault is rupture most likely to occur



Source-site distance depends on where rupture occurs

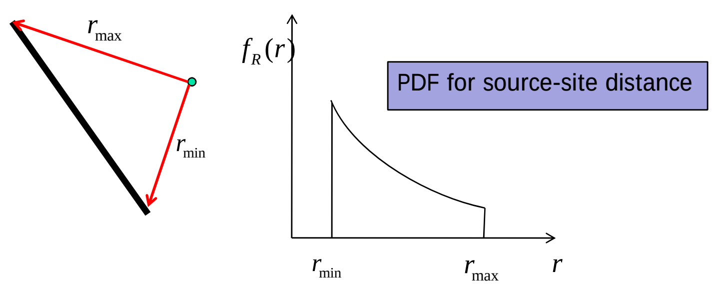
Uncertainty in Source-Site Distance

- Where is rupture most likely to occur? **We don't know**



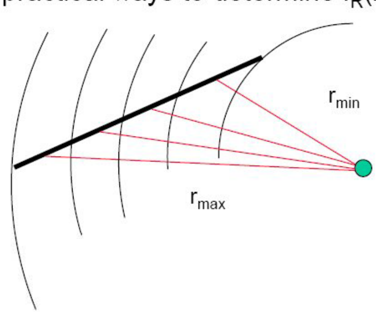
Source-site distance depends on where rupture occurs

Uncertainty in Source-Site Distance



Uncertainty in Source-Site Distance

Two practical ways to determine $f_R(r)$

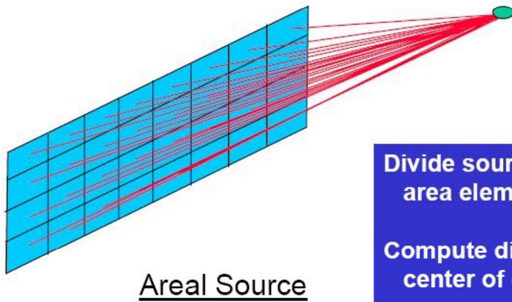


Draw series of concentric circles with equal radius increment

Measure length of fault, L_i, between each pair of adjacent circles

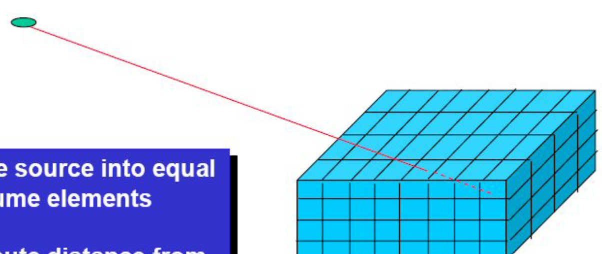
Assign weight equal to L_i/L to each corresponding distance

Uncertainty in Source-Site Distance



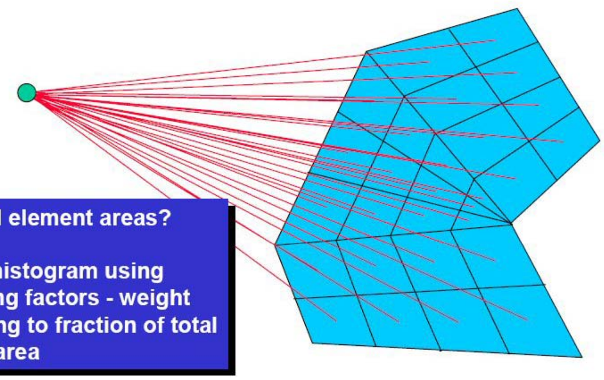
- Divide source into equal area elements
- Compute distance from center of each element
- Create histogram of source-site distance

Uncertainty in Source-Site Distance



- Divide source into equal volume elements
- Compute distance from center of each element
- Create histogram of source-site distance

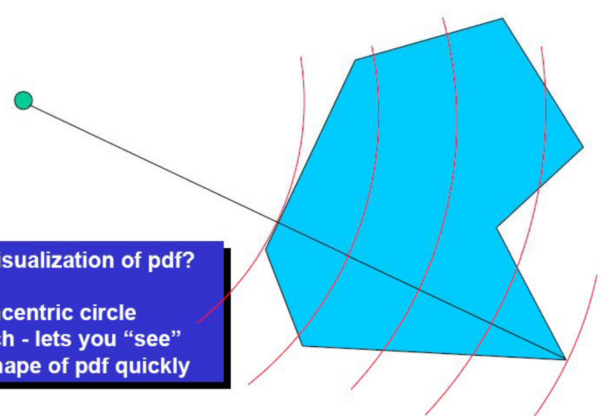
Uncertainty in Source-Site Distance



Unequal element areas?

Create histogram using weighting factors - weight according to fraction of total source area

Uncertainty in Source-Site Distance



Quick visualization of pdf?

Use concentric circle approach - lets you “see” basic shape of pdf quickly

Characterization of Maximum Magnitude

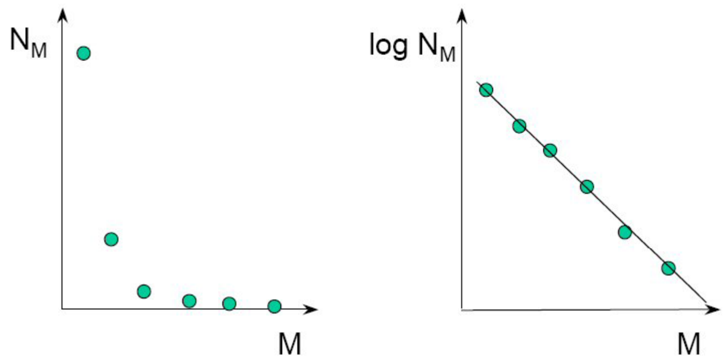
- Determination of M_{max} - Same as DSHA
  - Empirical Correlations
    - Rupture Length Correlations
    - Rupture Area Correlations
    - Maximum Surface Dispalcement Correlations
  - Theoretical Determination
    - Slip Rate Correlations

Need to Know
Distribution of Magnitudes

Distribution of Earthquake Magnitudes

- Given source can produce different earthquakes
  - Low Magnitude – Often
  - Large Magnitude – Rare
- Gutenberg-Richter
  - Southern Carifornia Data – Many Faults
  - Counted number of earthquakes exceeding different magnitude levels over period of many years

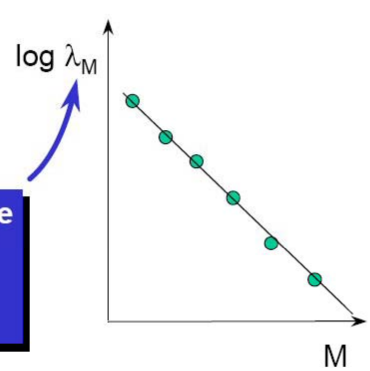
Distribution of Earthquake Magnitudes



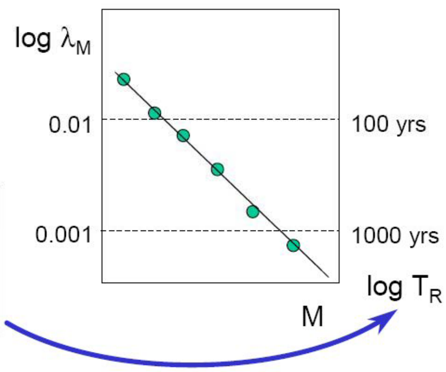
Distribution of Earthquake Magnitudes

Mean annual rate of exceedance

$\lambda_M = N_M / T$



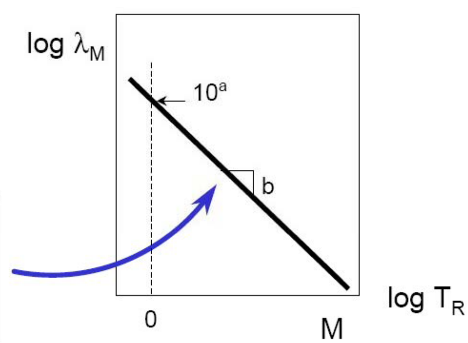
Distribution of Earthquake Magnitudes



Return period (recurrence interval)

$T_R = 1 / \lambda_M$

Distribution of Earthquake Magnitudes



Gutenberg-Richter Recurrence Law

$\log \lambda_M = a - bM$

Frequency of Occurrence

- The equation most commonly used to describe the occurrence of earthquakes is the well-known Gutenberg-Richter relationship:

$$\log(N_c) = a - bM$$

where *Nc* = is the number of events greater than or equal to magnitude *M*; *a* and *b* are constants.

Frequency of Occurrence

$$\log(N_c) = a - bM$$

$$\log(\lambda_m) = a - bM \qquad \ln(\lambda_m) = \alpha - \beta M$$

- The constant *a* , the activity parameter, provides a measure of the overall occurrence rate of earthquakes in the zone considered and is the zero magnitude intercept on a semi-log plot.
- The slope *b*, or *b* value, is controlled by the distribution of events between the higher- and lower-magnitude ranges.

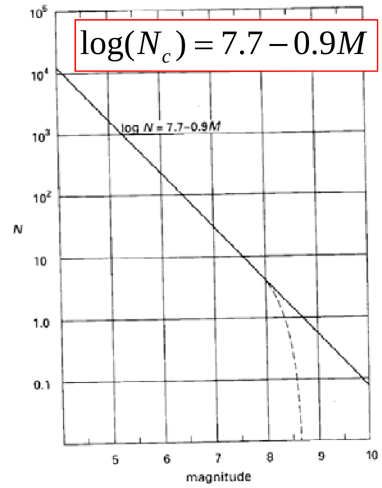
Frequency of Occurrence

For the entire world, the approximate relationship (up to approximately M = 8.2) the approximate number of earthquakes, *N*, of a given magnitude *M* is:

Approximate Expected Frequency of Occurrence of Earthquakes (per 100 years)

Magnitude	Number
4.75-5.25	250
5.25-5.75	140
5.75-6.25	78
6.25-6.75	40
6.75-7.25	19
7.25-7.75	7.6
7.75-8.25	2.1
8.25-8.75	0.6

$$\log(N_c) = 7.7 - 0.9M$$



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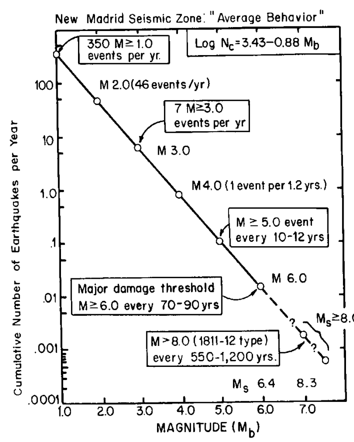
Frequency of Occurrence

- The frequency of occurrence in the New Madrid seismic zone (NMSZ) according to Johnston and Nava (1985) can be estimated by mean recurrence rates for the NMSZ using the historical seismicity and the instrumental records (1974-1983).
- They used both linear regression and maximum likelihood techniques to determine the Gutenberg-Richter constants *a* and *b* for the best-fit line throughout the data.
- For the NMSZ (35°-37°N, 89°-90.5°W), they obtained

$$\log(N_c) = 3.43 - 0.88(M_b)$$



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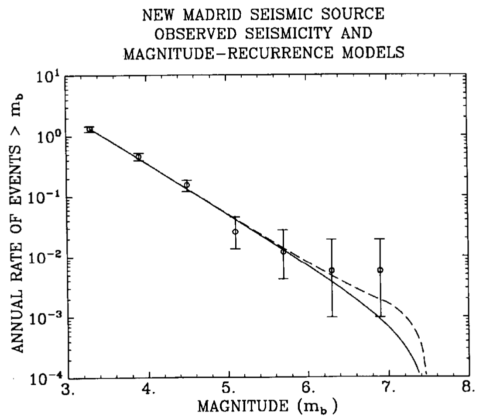
Frequency of Occurrence



- The solid line indicates the exponential magnitude-recurrence model; the dashed line indicates the characteristic model (Toro et al. 1992).

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Frequency of Occurrence

$$\log(\lambda_m) = a - bM \qquad \ln(\lambda_m) = \alpha - \beta M$$

$$\alpha = 2.303a$$
$$\beta = 2.303b$$

For an exponential distribution

$$f_M(m) = \beta e^{-\beta m}$$

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Distribution of Earthquake Magnitudes

$$\lambda_m = \nu \exp\left[\alpha - \beta(m - m_0)\right] \qquad m > m_0$$

$$\upsilon = \exp\left[\alpha - \beta m_0\right]$$

Then,

$$f_M(m) = \beta e^{-\beta(m-m_0)}$$

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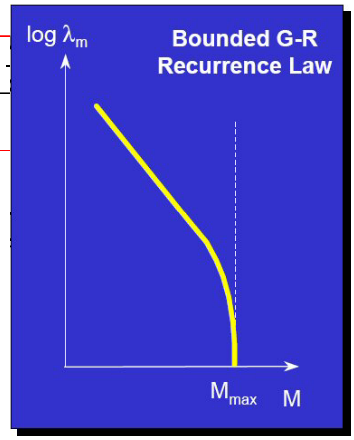
Distribution of Earthquake Magnitudes

- Every source has a maximum magnitude
- Distribution must be modified to account for M_{max}
- Bounded G-R recurrence law

$$\lambda_m = \nu \frac{\exp\left[-\beta\left(m - m_0\right)\right] - \exp\left[-\beta\left(m_{\max} - m_0\right)\right]}{1 - \exp\left[-\beta\left(m_{\max} - m_0\right)\right]}$$

Distribution of Earthquake Magnitudes

$$\lambda_m = \nu \frac{\exp\left[-\beta\left(m - m_0\right)\right] - \exp\left[-\beta\left(m_{max} \cdots \right.\right.}{1 - \exp\left[-\beta\left(m_{max} - m_0\right)\right]}$$

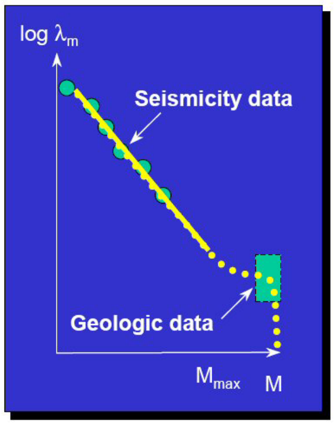


Distribution of Earthquake Magnitudes

- Characteristic Earthquake Recurrence Law
  - Paleoseismic Investigations
    - Show similar displacements in each earthquake
    - Individual faults produce characteristic earthquakes
    - Characteristic earthquake occure at or near M_{max}
    - Could be caused by geologic constraints
    - More research, field observations needed

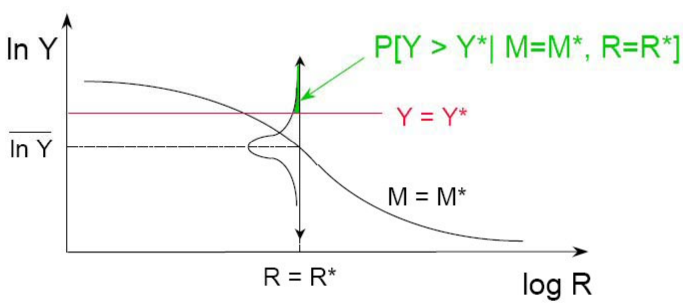
Distribution of Earthquake Magnitudes

- Characteristic Earthquake Recurrence Law



Predictive Relationships

- Standard Error
  - Use to evaluate conditional probability



Predictive Relationships

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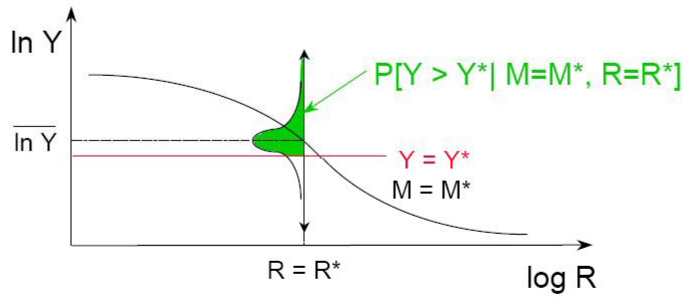


Example 4.4 of text

Temporal Uncertainty

- Poisson process
  - Describe number of occurrences of an event during a given time interval or spatial region
  - The number of occurrences in one time interval are independent of the number that occur in any other time interval
  - Probability of occurrence in a very short time interval is proportional to length of interval
  - Probability of more than one occurrence in a very short time interval is negligible

Temporal Uncertainty

- Poisson process

$$P[N = n] = \frac{\mu^n e^{-\mu}}{n!}$$

- Where *n* is the number of occurrences and μ is the average number of occurrences in the time interval of interest.

Temporal Uncertainty

- Poisson process
  - Letting $\mu = \lambda t$

$$P[N=n] = \frac{(\lambda t)^n e^{-\lambda t}}{n!}$$

- Then

$$P[N \geq 0] = P[N=1] + P[N=2] + \ldots + P[N=\infty]$$

$$P[N \geq 0] = 1 - P[N=0]$$
$$P[N \geq 0] = 1 - e^{-\lambda t}$$

Temporal Uncertainty

- Poisson process

$$P[N \geq 0] = 1 - e^{-\lambda t}$$

  - Consider an event that occurs, on average, every 1,000 years. What is the probability it will occur at least once in a 100 year period?

$$\lambda = \frac{1}{1000} = 0.001$$
$$P[N \geq 0] = 1 - \exp[-(0.001)(100)] = 0.0952$$

Temporal Uncertainty

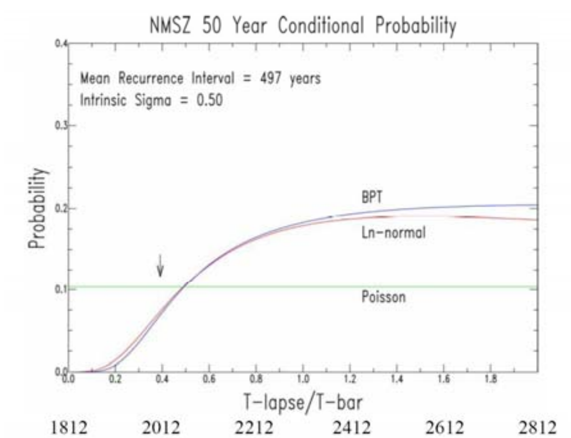
- Poisson process
  - What is the probability it will occur at least once in a 1000 year period?

$$P\left[N \geq 0\right] = 1 - \exp\left[-\left(0.001\right)(1000)\right] = 0.632$$

  - Solving for λ

$$\lambda = -\frac{\ln\left(1-p\right)}{t}$$

Various Probability Models



Temporal Uncertainty

- The annual rate of exceedance for an event with 10% probability of exceedance in 50 years is

$$\lambda = -\frac{\ln(1-p)}{t}$$

$$\lambda = -\frac{\ln(1-0.1)}{50} = 0.0021$$

- The corresponding return period is $T_R = 1/\lambda = 475$yrs

Temporal Uncertainty

- The annual rate of exceedance for an event with 2% probability of exceedance in 50 years is

$$\lambda = -\frac{\ln(1-p)}{t}$$

$$\lambda = -\frac{\ln(1-0.02)}{50} = 0.000404$$

- The corresponding return period is $T_R = 1/\lambda = 2475$ yrs

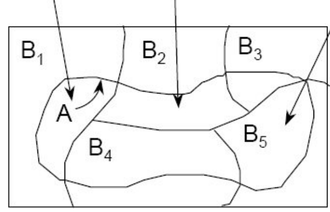
Summary of Uncertainties

Location	$f_R(r)$	← **Source-site distance pdf**
Size	$f_M(m)$	← **Magnitude pdf**
Effects	P[Y > Y*| M=M*, R=R*]	← **Attenuation relationship including standard error**
Timing	$P = 1 - e^{-\lambda t}$	← **Poisson model**

Combining Uncertainties – Probability Computations

$$P[A] = P[A \cap B_1] + P[A \cap B_2] + \ldots + P[A \cap B_N]$$

$$P[A] = P[A|B_1]P[B_1] + P[A|B_2]P[B_2] + \ldots + P[A|B_N]P[B_N]$$



Total Probability Theorem

Combining Uncertainties – Probability Computations

- Applying total probability theorem:

$$p\left[Y>y^*\right]=p\left[Y>y^*\mid X\right]p\left[X\right]=\int p\left[Y>y^*\mid X\right]f_X(X)dx$$

  - Where X is a vector of parameters
  - We Assume that M and R are the most important parameters and that they are independent. Then,

$$p\left[Y>y^*\right]=\iint P\left[Y>y^*\mid m,r\right]f_M(m)\,f_R(r)dmdr$$

Combining Uncertainties – Probability Computations

$$p\left[Y>y^*\right]=p\left[Y>y^*\mid X\right]p\left[X\right]=\int p\left[Y>y^*\mid X\right]f_X(X)dx$$

  - Above equation gives the probability that Y* will be exceeded if an earthquake occurs
  - Can covert probability to annual rate of exceedance by multiplying probability by annual rate of occurrence of earthquakes

$$\lambda_{y^*}=\upsilon\iint P\left[Y>y^*\mid m,r\right]f_M(m)\,f_R(r)dmdr$$

  - where

$$\upsilon=\exp\left[\alpha-\beta m_0\right]$$

Combining Uncertainties – Probability Computations

- If the site of interest is subjected to shaking from more than one site (N_S sites), then

$$\lambda_{y^*} = \sum_{i=1}^{N_s} \upsilon_i \iint P\left[Y > y^* \mid m, r\right] f_{Mi}(m) f_{Ri}(r) dm dr$$

- For realistic cased, PDFs for M and R are too complicated to integrate analytically. Therefore, it is done numerically

Combining Uncertainties – Probability Computations

- Dividing the range of possible magnitudes and distances into N_M and N_R increments, repectively

$$\lambda_{y^*} = \sum_{i=1}^{N_s} \sum_{j=1}^{N_M} \sum_{k=1}^{N_R} \upsilon_i P\left[Y > y^* \mid m_j, r_k\right] f_{Mi}(m) f_{Ri}(r) dm dr$$

- This expression can be written, equivalently as

$$\lambda_{y^*} = \sum_{i=1}^{N_s} \sum_{j=1}^{N_M} \sum_{k=1}^{N_R} \upsilon_i P\left[Y > y^* \mid m_j, r_k\right] P\left[M = m_j\right] P\left[R = r_k\right]$$

Combining Uncertainties – Probability Computations

What does it mean?

$$\lambda_{y^*} = \sum_{i=1}^{N_s}\sum_{j=1}^{N_M}\sum_{k=1}^{N_R} \upsilon_i P\left[Y > y^* \mid m_j, r_k\right] P\left[M = m_j\right] P\left[R = r_k\right]$$

All possible distances are considered - contribution of each is weighted by its probability of occurrence

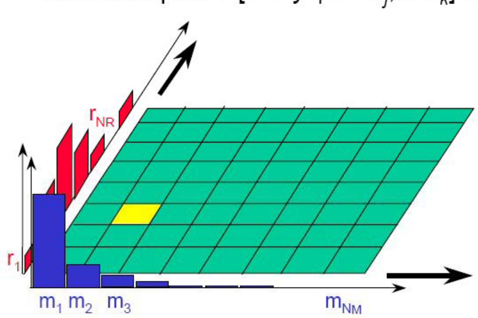
All sites are considered

All possible effects are considered - each weighted by its conditional probability of occurrence

All possible magnitudes are considered - contribution of each is weighted by its probability of occurrence

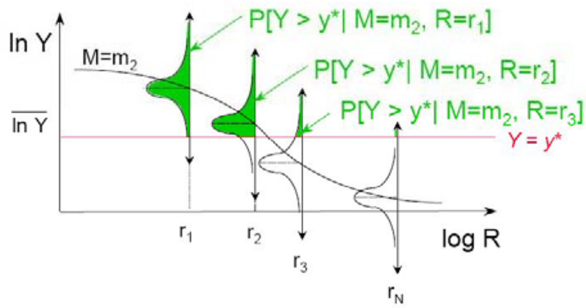
Combining Uncertainties – Probability Computations

N_M x N_R possible combinations
Each produces some probability of exceeding y^*
Must compute P[$Y > y^* | M=m_j, R=r_k$] for all m_j, r_k



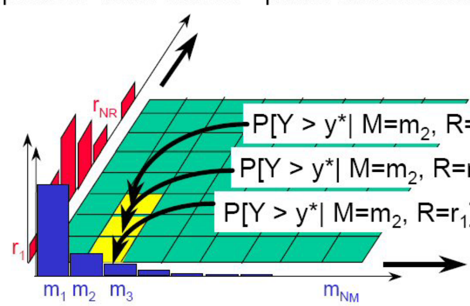
Combining Uncertainties – Probability Computations

- Compute Conditional probability for each element on grid
- Enter in matrix (spreadsheet cell)



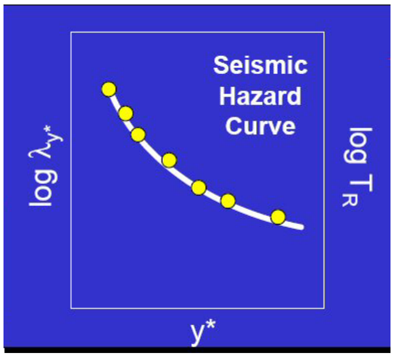
Combining Uncertainties – Probability Computations

"Build" hazard by:
  computing conditional probability for each element
  multiplying conditional probability by $P[m_j]$, $P[r_k]$, ν_i
Repeat for each source - place values in same cells



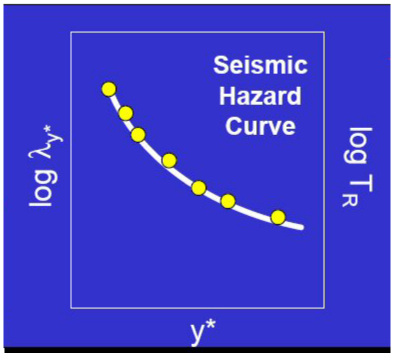
Combining Uncertainties – Probability Computations

- Choose new value of y*
- Repeat entire process
- Develop paris of (y*,λ_{y^*}) points

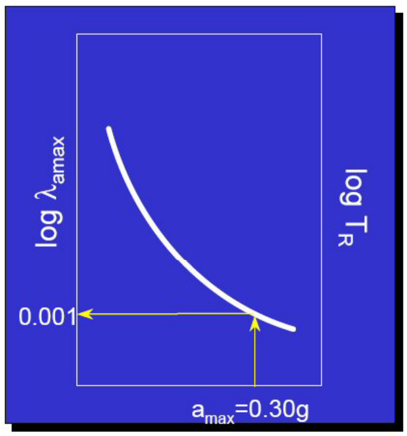


Combining Uncertainties – Probability Computations

- Seismic hazard curve shows the mean annual rate of exceedance of a particular ground motion parameter.
- A seismic hazard curve is the ultimate result of a PSHA.



Using Seismic Hazard Curves



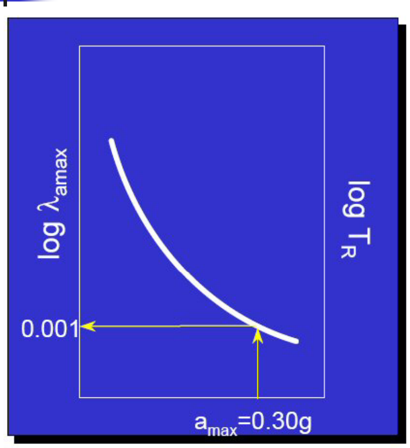
- Probability of exceeding a_{max} = 0.30g in a 50 yr period?

$$p = 1 - e^{-\lambda t}$$
$$= 1 - \exp\left[-(0.001)(50)\right]$$
$$= 0.049 = 4.9\%$$

- In 500 yr period?

$$p = 0.393$$
$$= 39.3\%$$

Using Seismic Hazard Curves

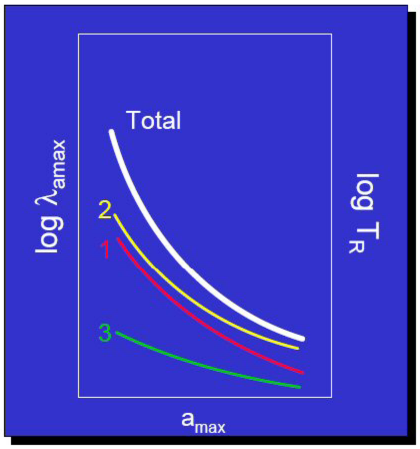


- What peak acceleration has a 10% probability of being exceeded in 50 yr period

$$10\% \quad in \quad 50 yrs \Rightarrow \lambda = 0.0021$$
$$or$$
$$T_R = 475\ yrs$$

- Use seismic hazard curve to find a_{max} value corresponding to l = 0.0021?

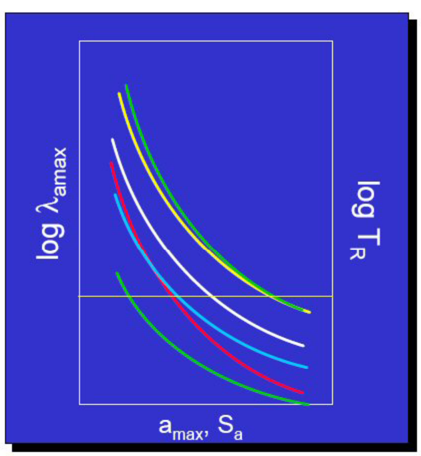
Using Seismic Hazard Curves



Contribution of sources

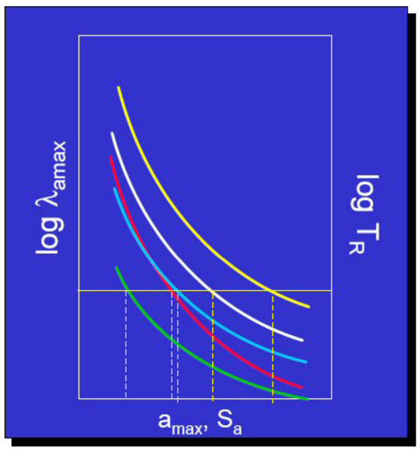
- Can break λ-values down into contributions from each source
- Plot seismic hazard curves for each source and total seismic hazard curve (equal to sum of source curves)
- Curves may not be parallel, may cross
- Shows which source(s) most important

Using Seismic Hazard Curves



- Can develop seismic hazard curves for different ground motion parameters
  - Peak acceleration
  - Spectral accelerations
  - Other
- Choose desired λ-value
- Read corresponding parameter values from seismic hazard curves

Using Seismic Hazard Curves



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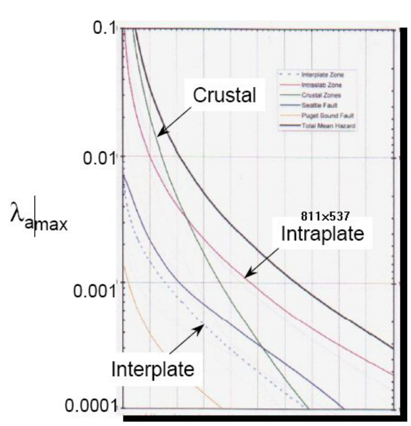
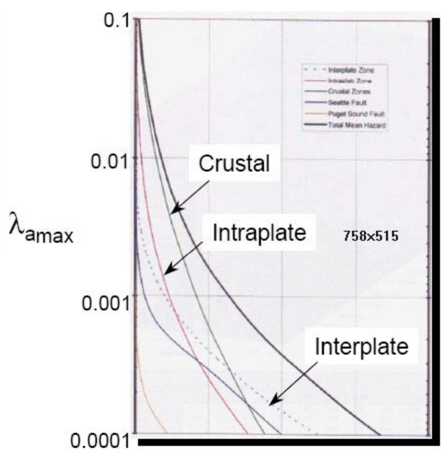
Using Seismic Hazard Curves



2% in 50 yrs

Peak acceleration

Using Seismic Hazard Curves

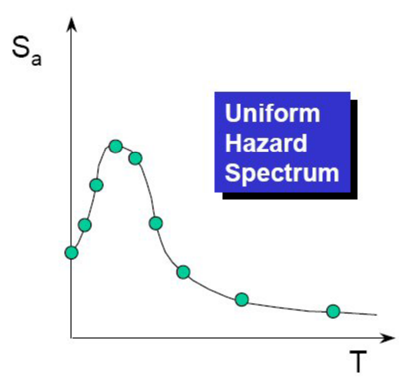


2% in 50 yrs

S_a(T = 3 sec)

Uniform Hazard Spectrum (UHS)

Find spectral acceleration values for different periods at constant λ
All S_a values have same λ-value → same probability of exceedance



Disaggregation (De-aggregation)

- Question?
  - What magnitude and distance does that a_{max} value correspond to?

	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5
25 km	0.01	0.01	0.02	0.03	0.03	0.02	0.01	0.01
50 km	0.02	0.03	0.04	0.04	0.05	0.04	0.03	0.02
75 km	0.03	0.03	0.05	0.06	0.09	0.06	0.05	0.02
100 km	0.03	0.03	0.05	0.05	0.08	0.05	0.05	0.02
125 km	0.02	0.02	0.03	0.04	0.05	0.03	0.02	0.01
150 km	0.01	0.01	0.02	0.03	0.05	0.02	0.01	0.00
175 km	0.00	0.00	0.01	0.01	0.03	0.01	0.01	0.00
200 km	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00

Total hazard includes contributions from all combinations of M & R.

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75 km	0.03	0.03	0.05	0.06	0.09	0.06	0.05	0.02
100 km	0.03	0.03	0.05	0.05	0.08	0.05	0.05	0.02
125 km	0.02	0.02	0.03	0.04	0.05	0.03	0.02	0.01
150 km	0.01	0.01	0.02	0.03	0.05	0.02	0.01	0.00
175 km	0.00	0.00	0.01	0.01	0.03	0.01	0.01	0.00
200 km	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00

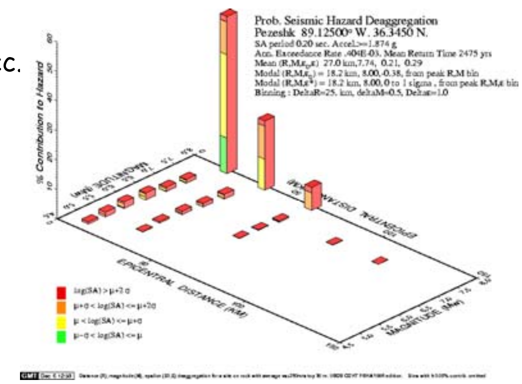
Total hazard includes contributions from all combinations of M & R.

Break hazard down into contributions to "see where hazard is coming from."

M=7.0 at R=75 km

Disaggregation (De-aggregation)

- USGS De-aggregation
  - West Tenn
  - 2%in50yrs
  - 0.2 spectral Acc.



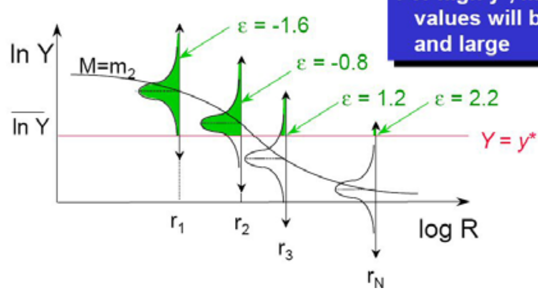
Disaggregation (De-aggregation)

Another disaggregation parameter

$$\varepsilon = \frac{\ln y^* - \overline{\ln y}}{\sigma_{\ln y}}$$

For low y*, most ε values will be negative

For high y*, most ε values will be positive and large

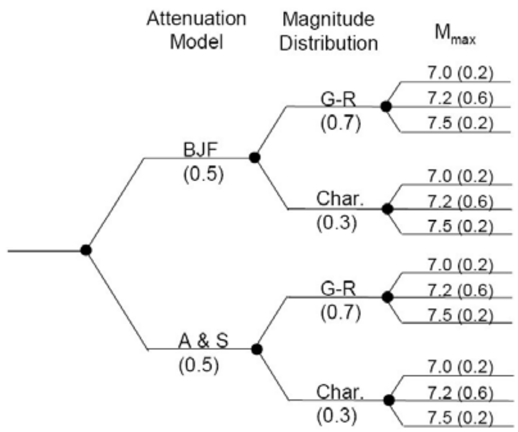


Disaggregation (De-aggregation)

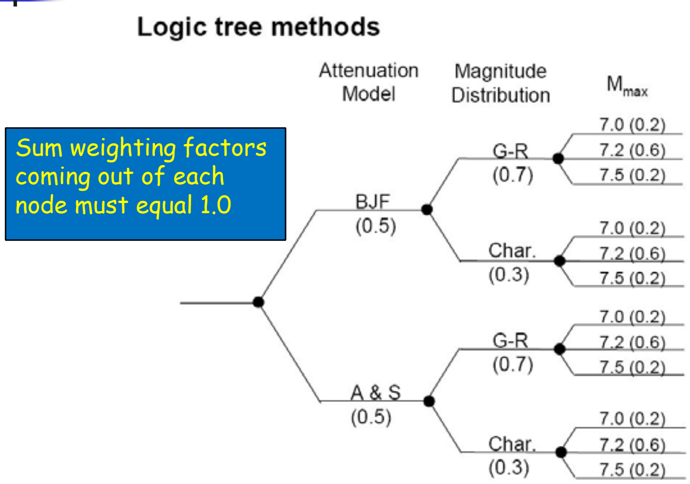
- Logic Tree Methods
  - Not all uncertainty can be described by probability distribution
    - Most appropriate model may not be clear
      - Attenuation relationship
      - Magnitude Distribution
      - Etc.
    - Expert may disagree on model parameters
      - Fault segmentation
      - Maximum magnitude
      - Etc.

Logic Tree Methods

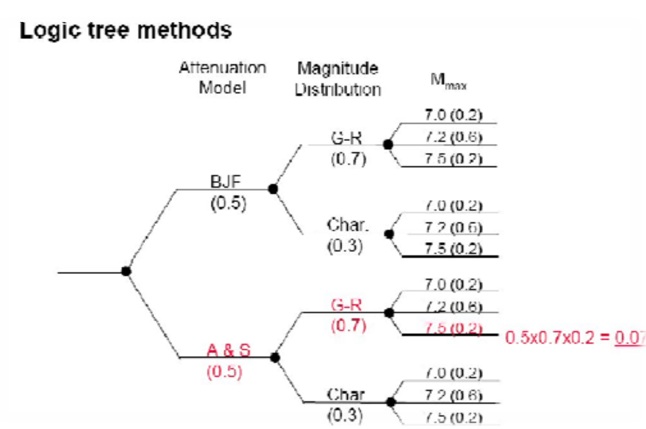
Logic tree methods



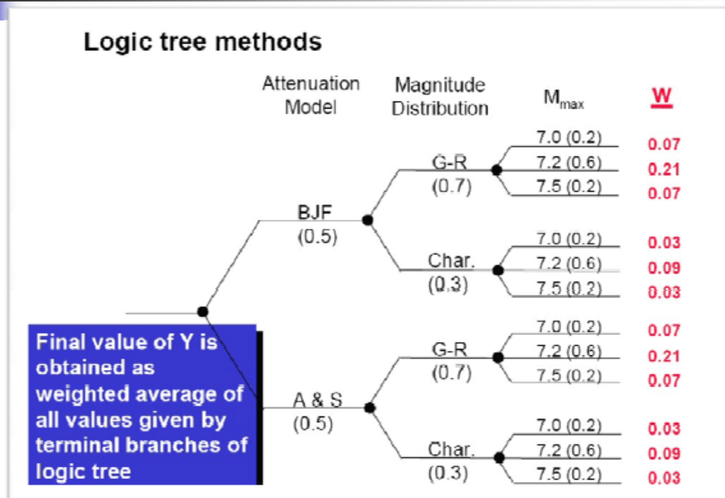
Logic Tree Methods



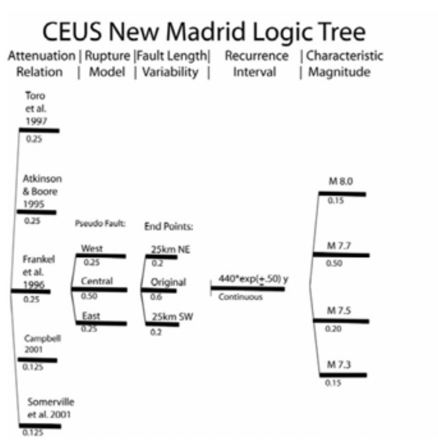
Logic Tree Methods



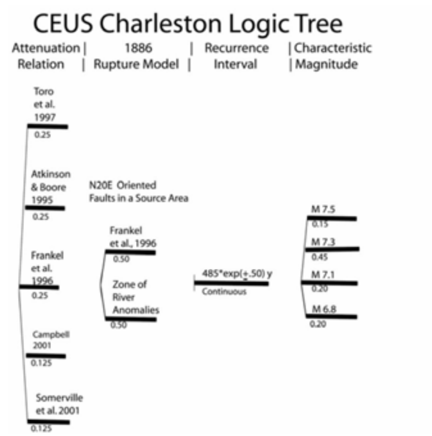
Logic Tree Methods



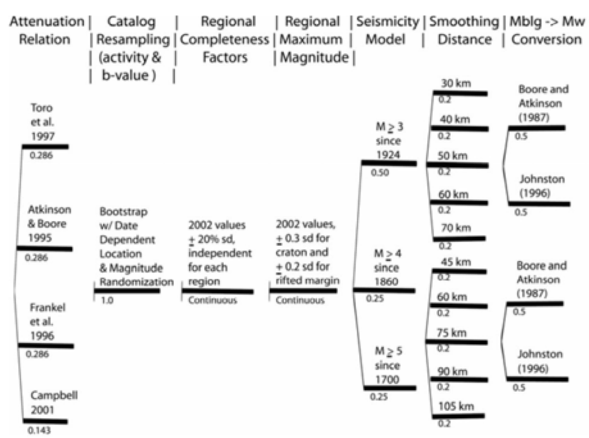
CEUS New Madrid Logic Tree



CEUS Charleston Logic Tree



CEUS Smoothed Seismicity Logic Tree



Seismic Hazard Curves

- For a a given earthquake occurrence, the probability that a ground motion parameter Y will exceed a particular value y* can be computed using the total probability theorem

$$P[Y > y^*] = \int\int P[Y > y^* | m, r] f_M(m) R_R(r) dmdr$$

$P[Y > y^*]$ Is obtained from the predictive

$f_M(m)$ and $f_R(r)$ Are the probability density functions for magnitude and distance

Seismic Hazard Curves

- If the site of interest is in a region of Ns potential earthquake sources, each of which has an average rate of threshold magnitude exceedance $\nu = e^{\alpha_i - \beta_i m_0}$ the total average exceedance rate for the region will be

$$P[Y > y^*] = \sum_{i=1}^{N_s} \nu_i \int\int P[Y > y^* \mid m, r] f_M(m) R_R(r) dm dr$$

Y = the ground motion level we are interested

ν = is the occurrence rate of earthquakes with magnitudes $m \geq m_0$

Hazard Curves

- Divide magnitude range into *M* sub range, and the seismic source zones into *Ns* sub zones and assume the distance from each sub zone to the site is the same, then

$$P[Y > y^*] = \sum_{i=1}^{N_s} \sum_{j=1}^{N_M} \sum_{k=1}^{N_R} \nu_i P[Y > y^* \mid m_j, r_k] f_{Mi}(m_j) f_{Ri}(r_k) \Delta m \Delta r$$

$$P[Y > y^*] = \sum_{i=1}^{N_s} \frac{\nu_i S_i}{S} \sum_{j=1}^{N_M} P[Y > y^* \mid m_i, r_i] f_M(m_j)$$

$$m_j = m_0 + (j - 0.5) \frac{(m_{\max} - m_0)}{N_M}$$

$$r_k = r_0 + (k - 0.5) \frac{(r_{\max} - r_{\min})}{N_R}$$

$$\Delta r = \frac{r_{\max} - r_{\min}}{N_R}$$