

## THE EFFECT OF ATTENUATION ON SEISMIC BODY WAVES

BY VERNON F. CORMIER\*

### ABSTRACT

Both spectral and time domain studies indicate that the frequency dependence and regional variation of the attenuation of  $P$  waves parallels that of  $S$  waves with  $t_{\beta}^* = 4 t_{\alpha}^*$ . Scattering  $Q$  cannot be generally separated from intrinsic  $Q$  in the mantle. Forward scattering can generate time-dependent variations in the frequency content and complexity of body waves that affect the measurement of  $t^*$ . Assuming that lateral heterogeneity biases the apparent  $Q$  of surface waves, the frequency dependence of  $Q$  can be explained by a relaxation model of intrinsic  $Q$ . In this model,  $Q$  is constant with frequency up to a cutoff frequency  $1/(2\pi\tau_m)$  Hz, where  $\tau_m = 0.1$  to  $0.2$  sec.

Regional variations in mantle attenuation are consistent with radiometric or magnetic age and tectonic activity, regions of higher relative attenuation coincident with younger, tectonically active crust. Continental cratons are underlain by mantle having small attenuation at all depths. High attenuation usually correlates with slow travel times, lower  $P_n$  velocity, and inefficient  $P_n$  and  $S_n$  propagation. Measures of differential frequency content ( $\delta t^*$  and  $\delta Q$ ) generally correlate better with differential travel time than measures of differential amplitude and  $m_b$ . The regional pattern and intensity of both travel-time anomalies and  $t^*$  measurements suggest that both share a common origin due to the regional variation of the thermal structure of the upper 200 to 400 km of the mantle.

### INTRODUCTION

Reviews of body-wave attenuation have been conducted from the viewpoint of predicting the regional variation of strong ground motion or from the viewpoint of constructing path corrections to reduce the scatter in network estimates of  $m_b$ . Recent reviews include those by Lundquist *et al.* (1980), Chung and Bernreuter (1981a, b), and Der *et al.* (1982; unpublished observations) and contributions to a report on the effect of attenuation on yield estimation from  $m_b$  (Defense Advanced Research Projects Agency, 1982). This review will incorporate and synthesize the material of these reviews and attempt to give a balanced appraisal of experimental results. Further research will be suggested for resolving the frequency and regional dependence of attenuation and for reducing the scatter due to attenuation in network estimates of source size using  $m_b$  and other measures.

Attenuation will be assumed to be a combination of intrinsic anelasticity and scattering attenuation. In discussing quantities relevant to intrinsic anelasticity, a familiarity will be assumed with the terminology used in the theory of linear viscoelasticity (Gross, 1953; Liu *et al.*, 1976; Minster, 1978a, b). The possible physical mechanisms of intrinsic inelasticity will generally be considered to be a collection of thermally activated mechanisms. A review of these mechanisms is given by Jackson and Anderson (1970) and Minster (1980).

### METHODS OF ESTIMATING ATTENUATION FROM BODY WAVES

#### *Spectral decay methods*

Displacement spectra can be derived from the seismograms of one or more

\* Present address: Massachusetts Institute of Technology, Earth Resources Laboratory, 42 Carleton Street, Cambridge, Massachusetts 02142.

instruments that span the body-wave band. Source theories for earthquakes predict a spectral shape similar to that given by the theories for explosions: namely, a constant long-period level, followed by a decay above a corner frequency that scales inversely with source size. The constant long-period level is proportional to seismic moment. The decay power  $n$  above the corner frequency varies from  $-2$  to  $-3$  depending on the source model. The  $\omega^{-2}$  model gives the best fit to the  $m_b - M_s$  data from large earthquakes (Aki, 1967; Hanks, 1979). The source spectra predicted by most dynamic theories of rupture also exhibit a shift in the corner frequency of  $S$  waves to a lower value relative to that of  $P$  waves (Madariaga, 1976; Boatwright, 1981). The directivity of a finite source superimposes an additional azimuthal dependence on the high-frequency behavior, but an azimuthal average of the body-wave spectra of an earthquake source will generally exhibit a spectral decay of  $\omega^{-2}$  to  $\omega^{-3}$  and a corner frequency shift.

A measure of seismic attenuation can be defined from the rate of high-frequency decay above the corner frequency. The decay is assumed to be proportional to  $\omega^{-n}$  times an exponential factor  $e^{-\omega t^*/2}$ , where  $t^*$  represents the path-integrated effect of the quality factor  $Q$

$$t^* = \int_{\text{path}} Q^{-1}/v \, ds. \quad (1)$$

Experimental measures of  $t^*$  necessarily lump scattering losses together with intrinsic anelasticity. For a given frequency,  $t^*$  is observed to be nearly constant with distance, the increasing travel time of more deeply bottoming rays being compensated by increasing  $Q$  with depth. A differential  $\delta t^*$  can be determined from the difference in the  $t^*$ 's of two stations (e.g., Der *et al.*, 1977). In this case, the fundamental decay power  $n$  cancels for earthquake spectra at the same azimuth and explosion spectra at all azimuths. If the power  $n$  is assumed, then an absolute  $t^*$  can be determined at a single station (e.g., Sipkin and Jordan, 1979).

Spectra are normally determined by Fourier transforming a waveform and smoothing the result. Alternative methods of spectral analysis that have been used in attenuation measurements include amplitude and energy calculations in narrow bands. The method of narrow-band filters plots the peaks of the time domain output of a series of narrow-band filters. The filters can be chosen to be the phaseless Gaussian filters described in Dziewonski *et al.* (1969). By including only those peak amplitudes that are contained in a narrow "group arrival window," the analysis can exclude interfering depth phases which introduce holes in the spectrum (Archambeau *et al.*, 1982). The spectrum thus often requires no further smoothing. Proper choice of the parameters describing the Gaussian filters compromises between time and frequency resolution, and the resulting spectrum can be shown to reproduce the spectrum that would be obtained by conventional Fourier analysis and smoothing (Doornbos, 1974).

The ratios of the outputs of narrow-band filters can be used to determine an attenuation model. The simplest example of this procedure is to vary the source and attenuation model to reproduce the ratio of the amplitude of a body wave observed on a short-period instrument to that observed on a long-period instrument. A more formal application of the method takes an integral over the squared velocity in a window bracketing a seismic phase (Sipkin and Jordan, 1979). Burdick (1978) gives an example of both the simple amplitude and energy ratio methods.

*Biasing effects on spectral methods*

Estimates of attenuation based upon idealized models of the source spectrum can be biased by effects that interfere with the observation of the source spectrum as well as by the physical imperfections of the models. A summary of these effects follows.

*High-frequency asymptote.* Most of the spectral studies that will be reviewed assume a high-frequency asymptote in the range  $\omega^{-2}$  to  $\omega^{-3}$  and find that a variation within this range would not affect the calculation of absolute  $t^*$  by more than 0.1 sec. The existence of a lower decay rate, such as  $\omega^{-1}$ , in the frequency band of the asymptotic fit, may lead to an underestimate of absolute  $t^*$  by a slightly larger amount.

*Directivity.* The directivity factor of a finite earthquake source, first described by Ben-Menahem (1961), introduces oscillations in the raw spectral ratio of the body waves recorded by stations at different azimuths. These oscillations have zero mean over a broad-frequency band, and their amplitude decreases as the ratio of wavelength to fault length decreases. These effects can be minimized by measuring spectral slopes from broadband recordings that include frequencies significantly higher than the corner frequency.

*Surface reflections.* The *SH* phase of a shallow focus earthquake is composed of direct and reflected *SH* phases of the same polarity. The *P* phase is composed of a direct *P* phase, a surface reflected *pP* phase of opposite polarity, and an *sP* phase. Helmberger (1974) noted that the reversed polarity of the *pP* phase tends to shorten the long-period pulse of *P*, but the unreversed *sSH* phase tends to broaden the long-period pulse of *SH*. He suggested that these shallow focus effects may explain the corner frequency shift of *P* relative to *S* waves independent of any source properties. For sufficiently high-frequency and broadband recordings, the bias introduced by the *P* plus *pP* and *SH* plus *sSH* can be removed by spectral smoothing. When the *P* plus *sP* interference is important, its bias can be removed by examining the output of a sequence of narrow-band filters in the time domain.

*Corner frequencies.* Hanks (1981) noted that a comparison of *P*- with *S*-wave attenuation by the method of amplitude or energy ratios should account for the *P* to *S* corner frequency shift predicted by dynamic source theories. Corner frequencies have been experimentally determined from the intersection of lines fit to the asymptotic trends of the long- and short-period ends in a log-log plot of the *Q*-corrected displacement spectrum (Hanks and Wyss, 1972). The spectra of deep focus earthquakes are less sensitive to the *Q* correction and are uncontaminated by near-surface multiples. Application of this procedure to the spectra of deep focus earthquakes, which have been azimuthally averaged over the radiation pattern, confirms the predicted shift in corner frequencies (Wyss and Molnar, 1972; Linde and Sacks, 1972). The neglect of the corner frequency shift in a comparison of the spectral content of *P* and *S* waves will lead to an overestimate of  $t^*$ .

*Complexity and scattering.* The frequency content of the coda of a body wave can bias the spectral determination of  $t^*$  by making the spectrum depend on the length of the time window analyzed. This bias will always exist unless the frequency content of the coda closely matches that of the first several cycles.

The relative frequency content of the coda depends on the mechanism that generates the coda. In the mechanism proposed by Ward (1978), the arrivals between *P* and *PP* on a long-period record consist of reflected and converted phases at depths of rapid or discontinuous velocity change in the upper mantle. In the distance

range of  $10^\circ$  to  $30^\circ$ , complex body waves can be explained by multiple travel-time branches produced by upper mantle structure (e.g., Given and Helmberger, 1980). If the structure of the upper mantle is characterized by transition zones rather than discontinuities, body waves having wavelengths greater than the widths of the transition zones would be reflected or converted but body waves having shorter wavelengths would be transmitted without reflection or conversion. In this case, the coda generated by mantle structure would have lower frequency content.

In the mechanism described by Douglas *et al.* (1971), the  $P$  coda would have higher relative frequency content. They propose that complex  $P$  waves having large codas are generated by weak scattering along the entire ray path. In this mechanism, a complex  $P$  coda is made visible whenever the direct, unscattered,  $P$  wave is weak. The direct  $P$  wave may be weak because it arrives by a low  $Q$  path, is geometrically defocused by lateral structure, or corresponds to a nodal take off angle at the source. Douglas *et al.* have thus called this mechanism "the weak signal hypothesis." The weak scattering that generates the coda is assumed to be concentrated in the lithosphere and lowermost mantle. Localized strong scattering due to topographic and structural variations near the source and receiver may also contribute to the coda (Cleary *et al.*, 1975). Douglas *et al.* (1981), however, reject strong scattering at shallow depths as the primary mechanism that generates a  $P$  coda because the distribution of scatterers must have an unreasonably strong regional variation in order to explain observations of simple and complex seismograms over nearly identical paths.

In the low  $Q$  version of the weak signal hypothesis, the later arrivals of a complex body wave would have a relatively higher frequency content. The most complex signals shown by Douglas *et al.* (1973) are records in the distance range of  $10^\circ$  to  $30^\circ$ , where the effects of mantle triplications are strong. These records exhibit a higher relative frequency content of the coda, agreeing with Douglas *et al.*'s weak signal hypothesis but disagreeing with the frequency dependence expected for the reflectivity of transition zones.

In summary, the mechanism that generates a  $P$  coda is not completely understood. The biasing effect, however, that a frequency-dependent coda has on a spectral determination of  $t^*$  can be eliminated by examining the output in the time domain of a sequence of narrow-band filters.

*Instrument nonlinearities.* Sacks (1980) has suggested that low  $t^*$  values determined by spectral methods may, in fact, be due to spurious high frequencies generated by a nonlinear instrument response or by hand digitization of analog records. Peterson (1981) has determined the range of nonlinear response of the SRO instruments. Der *et al.* (1981) conducted a series of tests to check whether their  $t^*$  measurements were taken in the nonlinear range of the LRSM instruments. The tests included shake table tests at displacement levels in excess of those of teleseismic body waves, a check for cube root spectral scaling of underground explosions at close ranges, and observations of equal spectral levels of 1 Hz energy at stations having different levels of higher frequency energy.

#### *Results of spectral decay methods: frequency dependence of $Q$*

The earliest suggestion of the possible frequency dependence of  $Q$  was given by Gutenberg (1958). Gutenberg noted that the observations of phases such as  $P'P'P'$  requires that the product  $\omega t^*$  must increase at a rate less than  $\omega^1$ . Body waves having significant energy were observed on seismograph systems having responses peaked in the 1- to 10-Hz bands. This led many investigators to conclude that the

attenuation models based on surface-wave or free oscillation data were too attenuating to explain the observed frequency content (Asada and Takano, 1963; Archambeau *et al.*, 1969; Takano, 1971).

One of the best examples of such data is provided by the frequency content of short-period  $S$  waves from deep focus earthquakes. Models based on free oscillation data predict a  $t_\beta^*$  in excess of 3.0 for events having a focal depth of 600 km (Anderson and Hart, 1978; Sailor and Dziewonski, 1978). In the 0.2- to 0.4-Hz bands, Burdick (1978) determined a  $t_\beta^* = 3.0$  for  $S$  waves recorded in North America from deep focus events beneath South America. Figure 1 from Der *et al.* (1982), however, shows that in the 1- to 5-Hz bands, the spectral decay of  $S$  waves over these paths

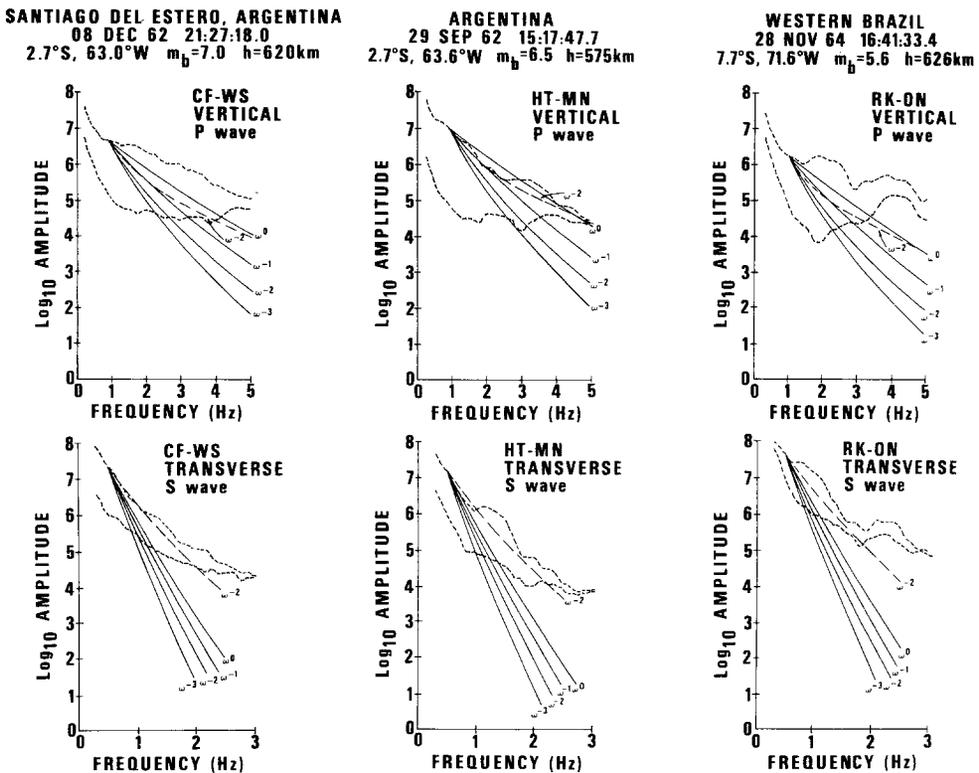


FIG. 1. Spectra of deep focus earthquakes in South America recorded by LRSM stations in North America from Der *et al.* (1982a). The upper dotted lines are signal, the lower noise. The spectral decay for  $t_\alpha^* = 1$  and  $t_\beta^* = 4$  cannot fit the observed spectra, even assuming a white spectrum ( $\omega^0$ ).  $t^*$  was determined from the decay by assuming a  $\omega^{-2}$  source.

is consistent with a much lower  $t_\beta^*$ . Even with the assumption of a white source spectrum, the spectral decay of the  $S$  waves cannot be fit by a  $Q$  model based on free oscillation data. By assuming a rate of spectral decay, Der *et al.* determined a  $t_\alpha^* = 0.2$  and a  $t_\beta^* = 0.8$  for these paths. They also conducted tests that indicated that these results were not biased by either the length of the time window analyzed or by nonlinearities of the instrument response. If anomalous path and receiver effects can also be eliminated as a cause of these observations, the data suggest that  $Q_\alpha$  and  $Q_\beta$  must be significantly higher in the 1- to 5-Hz bands than in lower frequency bands.

Intrinsic anelasticity allows  $Q$  to increase at a rate no faster than  $\omega^1$ . Thus, if the mantle  $Q$  were solely due to intrinsic anelasticity, combined low- and high-frequency

data could be used to place rather tight constraints upon both the frequency dependence of  $Q$  and the  $Q$ -source tradeoff in the short-period band. Sipkin and Jordan (1979) used this assumption to interpret the  $Q_\beta$  of the mantle determined from ScS phases. They determined  $Q_{ScS}$  using both an energy ratio method and the fit of a least-squares line to the spectral decay of ScS phases observed on short-period stations. They found a  $Q_{ScS}$  of 750 beneath the North Pacific in the frequency band 1 to 2 Hz. An earlier study in the frequency band 0.02 to 0.06 Hz yielded a  $Q_{ScS}$  of 250 (Jordan and Sipkin, 1977). By combining the results of the short- and long-period experiments, they concluded that  $Q_\beta$  in the mantle must increase with frequency as shown in Figure 2. The solid lines in Figure 2 show the predictions of an absorption band model of attenuation in the mantle of the type described by Minster (1978a). In this model,  $Q$  is constant in a band of frequencies up to a radian cutoff frequency  $1/(2\pi\tau_m)$ , where it then increases as the first power of frequency.

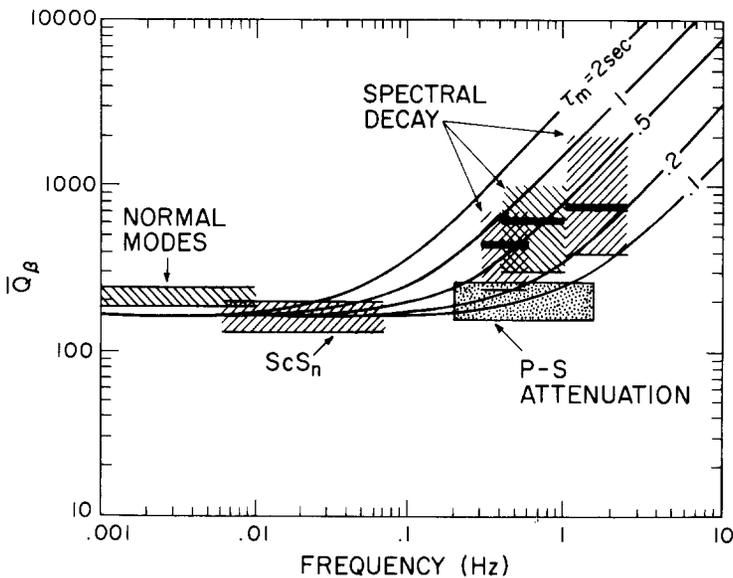


FIG. 2. The frequency dependence of  $Q_\beta$  determined by Sipkin and Jordan (1979) from source cancelling with  $ScS_n/ScS_{n-1}$  in the 0.001- to 0.06-Hz band, and from the spectral decay of ScS assuming a  $\omega^{-2}$  source in the band greater than 0.06 Hz. The dotted region represents the results of the ScS/ScP experiment of Burdick (1981).

A range of  $\tau_m$  equal to 0.1 to 2 sec is consistent with error bars based on the frequency band of the WWSSN data and spectral decay rates between  $\omega^{-2}$  and  $\omega^{-3}$ .

Der *et al.* (1981) determined  $t_\alpha^*$  and  $t_\beta^*$  in the 1- to 5-Hz bands from the spectral decay of body waves obtained from several hundred digital recordings in North America. The stations consisted of LRSM, SRO, and Special Data Collection System (deployed as part of an experiment to measure anelastic attenuation in the upper mantle beneath the NTS). From the results of this experiment and the results obtained by other investigators in the long- and short-period bands, Der *et al.* (1982) constructed a series of constraints for the regional and frequency dependence of  $t^*$ . These constraints are summarized in Figure 3 by giving the upper bounds of  $t_\alpha^*$  and  $t_\beta^*$  consistent with the data. The low-frequency behavior of  $t^*$  and its regional variation was constrained from studies of free oscillations (Anderson and Hart, 1978; Sailor and Dziewonski, 1978), surface waves (Solomon, 1972; Lee and Solomon,

1975, 1979; Mills, 1978; Nakanishi, 1979), and long-period body waves (Solomon and Toksöz, 1970). These studies combined with  $t^*$  estimates from the spectral decay of short-period body waves support a gradual rise in  $t_\alpha^*$  and  $t_\beta^*$  between 0.01 and 5 Hz. The frequency dependence of  $S$  waves closely parallels that of  $P$  waves, consistent with attenuation purely in shear ( $t_\beta^* = 4t_\alpha^*$ ).

Der *et al.* (1982; unpublished observations) note that a relaxation band of anelasticity of the type proposed by Minster (1978a, b) could fit the combined broadband data for shield-shield paths with  $\tau_m = 0.08$  sec. The shield-tectonic paths, however, by exhibiting a more gradual and continuous increase in  $t^*$  in the low-frequency band, could not be fit by so simple a parameterization of the frequency dependence of  $Q$ . A more complex parameterization (Figure 4) allows the high-frequency cutoff of an absorption band model to be a function of depth (Lundquist,

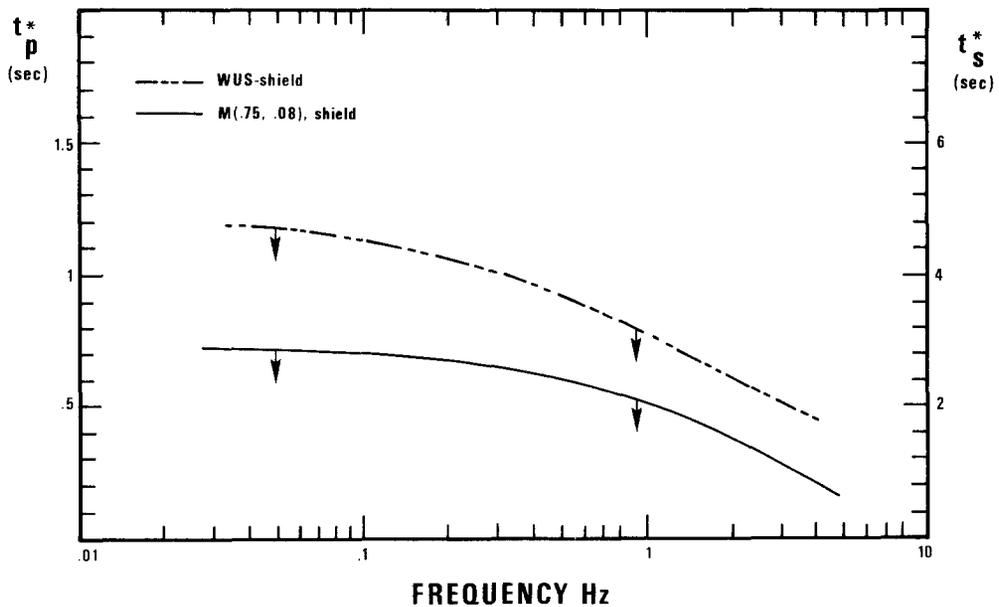


FIG. 3. The frequency dependence of  $t_\alpha^*$  and  $t_\beta^*$  proposed by Der *et al.* (1981, 1982; unpublished observations).

1979; Anderson and Given, 1982). These models propose that the depth variation in  $\tau_m$  is due to the effects of the pressure profile and geotherm on the activation parameters of the physical mechanisms of anelasticity. The  $t^*$  predicted by these models has a more gradual increase with frequency.

In an alternative resolution of the shield-tectonic constraints, Der *et al.* suggest that the low-frequency constraints obtained from free oscillations and surface waves may be biased toward slightly lower  $Q$  by the effects of lateral heterogeneity. Ultrasonic modeling experiments, for example, have shown that the apparent  $Q$  determined from the fundamental mode surface waves can be strongly influenced by topographic irregularities along the propagation path (Toksöz *et al.*, 1981). If the shield-tectonic curve can be corrected downward at the low-frequency end to remain parallel to the shield-shield curve, then all of the constraints can be satisfied by a relaxation model in which  $\tau_m$  is constant with depth and equal to 0.1 to 0.2 sec.

*Results of spectral decay methods: regional dependence of Q*

Studies by Der *et al.* (1975) and Der and McElfresh (1976) found that the regional variation of short-period attenuation is similar to the long-period variations found by Solomon and Toksöz (1970). The  $Q_\alpha$  in the mantle beneath the Eastern United States (EUS) averages 1600 to 2000 in the 1- to 5-Hz bands and 400 to 500 beneath the Western United States (WUS). The magnitude bias observed between the EUS and WUS can be explained by a low  $Q$  layer located at a depth of less than 100 km beneath the WUS. Further studies by Der and McElfresh (1977) and Der *et al.* (1979) quantitatively related the difference in the observed  $t^*$ 's between the EUS and WUS to a relative  $m_b$  bias. The station  $m_b$ 's at each LRSM station were corrected for crustal amplification. The  $P$ -wave magnitude anomalies were observed

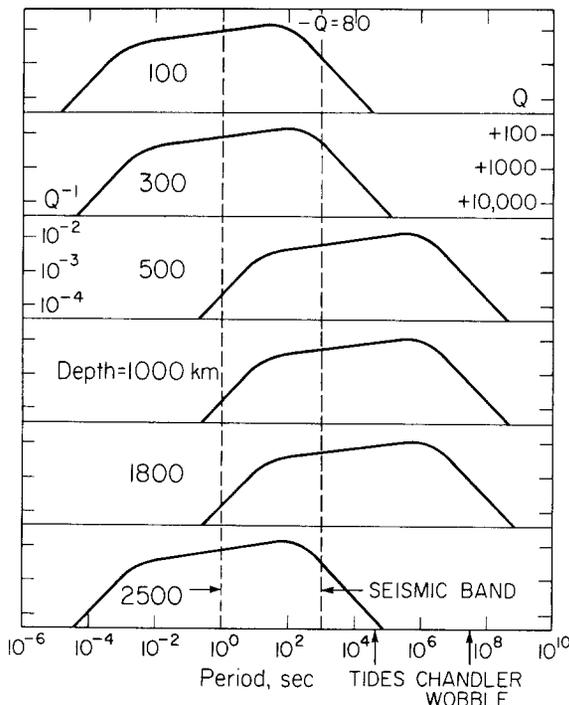


FIG. 4. The behavior of the seismic relaxation spectrum with frequency and depth proposed by Anderson and Given (1982).

to separate into a population of EUS stations and a population of WUS stations with a 0.3  $m_b$  difference between them. Noting that  $\delta t_\alpha^*(\text{WUS} - \text{EUS}) = 0.2$ , Der *et al.* proposed that the  $m_b$  bias due to attenuation can be estimated from  $t^*$  measurements by taking

$$\delta m_b = 1.35 \delta t^* \quad (2)$$

for explosions at NTS. A study of  $S$ -wave spectra from deep focus earthquakes (Der *et al.*, 1980a) found that  $\delta t_\beta^*(\text{WUS} - \text{EUS}) = 0.67$ , approximately consistent with attenuation occurring purely in shear. In both the  $P$  and  $S$  studies, some finer scale variations in  $\delta t^*$  and  $\delta m_b$  were observed superimposed on the WUS - EUS difference (Figure 5). Lowest amplitudes and lowest frequency content were observed in the southwestern United States. The highest amplitudes and highest



frequency content were observed in the north-central United States. The north-eastern United States and the rest of the WUS were intermediate in the amplitude and frequency content of body waves.

Fits to spectral decay in the 0.5- to 4-Hz bands (Der *et al.*, 1981, 1982; unpublished observations), assuming a frequency-independent  $Q$ , gave a  $t_\alpha^* = 0.1$  to 0.2 for purely shield paths and  $t_\alpha^* = 0.4$  to 0.5 for mixed shield-tectonic paths. The  $t^*$  values for deep events studied by Der *et al.* (1981, 1982; unpublished observations) are comparable to those that they determined for shallow events when the source and receiver were both on shields. They concluded that deep events under shields differ little from shallow events having shield-to-shield paths, indicating that the  $Q$  under shields is high throughout the entire mantle.

The results obtained in the Der *et al.* studies agree qualitatively and quantitatively with the spectral measurements of  $\delta t^*$  obtained by other investigators using different instruments and different combinations of source-receiver paths. Ward and Toksöz (1971) measured  $\delta t_\alpha^*$  (LASA – NORSAR) = 0.3 for deep focus earthquakes. Using this difference to construct models of intrinsic  $Q$  beneath the arrays, they predicted that  $m_b$ 's measured at NORSAR would be 0.4  $m_b$  units higher than those at LASA for 1-Hz waves arriving from 70°. This bias agrees with data from deep focus earthquakes arriving at the arrays and agrees with Der *et al.*'s empirical formula relating  $\delta t_\alpha^*$  to  $\delta m_b$ . Filson and Frasier (1972) assumed Haskell's (1967) spectral scaling of explosions to measure absolute  $t_\alpha^*$  from array recordings of  $P$  waves from underground nuclear explosions in east Kazakh. Filson and Frasier's absolute  $t_\alpha^*$ s and relative  $\delta t_\alpha^*$ s confirm the differences in attenuation between regions found in other studies. For example, their  $t_\alpha^*$  for east Kazakh to NORSAR paths (0.05) and their  $t_\alpha^*$  for east Kazakh to LASA paths (0.4) agree with the  $t_\alpha^*$ s found in the Der *et al.* studies for pure shield paths and mixed shield-tectonic paths, respectively. The  $t_\alpha^*$  for east Kazakh to NORSAR paths can be subtracted from the  $t_\alpha^*$  for east Kazakh to LASA paths to obtain a  $\delta t_\alpha^*$  (LASA – NORSAR) of 0.4. This value differs little from the  $\delta t_\alpha^*$  (LASA – NORSAR) of 0.3 obtained by Ward and Toksöz from deep focus events.

### *Source cancelling experiments*

Estimates of attenuation from observations of spectral decay require the assumption of a source spectrum. Experiments can be designed, however, to eliminate the uncertainty introduced by this assumption. These experiments are ones that cancel the source spectrum by taking the ratio of two seismic phases such that for each phase, the convolved source spectrum is evaluated at the same or nearly the same azimuth and takeoff angle. An attenuation model must satisfy the observed frequency dependence of the amplitude ratio of the two phases. A more complicated version of this experiment models the phase of the spectral ratio as well as its amplitude in determining the linear operator needed to convert one seismic phase into another.

### *Results of source cancelling experiments: regional dependence of $Q$*

Spectral ratios of multiple  $ScS$  phases have been used to determine the  $Q_{ScS}$  of the mantle locations (e.g., Kovach and Anderson, 1964; Yoshida and Tsujiura, 1975). Jordan and Sipkin (1977) noted that the spectral ratio studies that divide smoothed power spectra of two  $ScSn$  phases may be biased toward high  $Q_{ScS}$  values because incoherent as well as coherent parts of the two spectra contribute to the ratio. This contribution is greater for the phase having higher reflection numbers. To remove

this bias, Jordan and Sipkin developed a phase equalization and stacking procedure to determine the attenuation operator that converts an  $ScSn$  phase into an  $ScSn + 1$  phase. They applied this algorithm to multiple  $ScS$  phases sampling the north Pacific (Sipkin and Jordan, 1979) and later extended it to a broader data set (Sipkin and Jordan, 1980) in an investigation of the regional variation of  $Q_{ScS}$ . For high-gain long period and long-period ASRO data, Sipkin and Jordan found a representative value of  $Q_{ScS}$  of 150 for ocean basins and 225 for continents. Lower  $Q_{ScS}$  correlated with younger crustal age and tectonic activity. They estimated the average  $Q_{ScS}$  for the whole earth to be  $170 \pm 20$  per cent in the frequency band of 0.006 to 0.06 Hz.

An average  $Q_\beta$  of the mantle can also be determined from the spectral ratio of  $ScS$  to  $ScP$ . This ratio measures the difference in attenuation between the second  $S$  leg of  $ScS$  and the  $P$  leg of  $ScP$ . If attenuation is assumed to occur in pure shear or if the attenuation of the  $P$  leg is independently known, the average  $Q$  along the  $S$  leg can be estimated and compared against Sipkin and Jordan's  $Q_{ScS}$  values. Kanamori (1967) measured the  $ScS/ScP$  and the  $PcP/PcS$  spectral ratio at the TFSO array in Arizona from deep focus events beneath Central and South America. Using an average  $Q$  for the  $P$  legs determined in a separate  $PcP$  study, Kanamori determined a  $Q_\beta$  of 230 for the  $S$  legs in the frequency band 0.2 to 0.7 Hz. This  $Q_\beta$  value agrees with Sipkin and Jordan's (1980)  $Q_{ScS}$  value of 225 for continents in the frequency band 0.006 to 0.06 Hz. Kanamori estimated the ratio  $Q_\alpha/Q_\beta = 1.90$ . Assuming a  $P$  velocity/ $S$  velocity ratio equal to 3, this  $Q$  ratio gives a  $t_\beta^*/t_\alpha^*$  ratio equal to 3.3, close to the value of 4.0, expected if attenuation were purely in shear and agreeing exactly with the ratio of  $t_\beta^*/t_\alpha^*$  observed by Der *et al.* (1980a).

Another path average of mantle attenuation can be determined by taking the ratio of the spectrum of a  $P$  phase at a station to the spectrum of a  $PP$  phase observed at another station at the same azimuth and twice the distance. This ratio measures the attenuation of the second leg of the  $PP$  phase. Lundquist and Samowitz (1981) performed this experiment using long-period SRO recordings, obtaining a large scatter in  $t_\alpha^*$  around 1 for continental ray paths. Shore (1982a), using digital short-period data, determined a  $t_\alpha^* = 0.6$  for primarily shield paths. The  $\delta t^*$  between the different paths of these two studies is consistent with the  $\delta t^*$  between shield and mixed tectonic-shield paths found in other studies.

Using observations of the  $sS/sP$  ratio of the Borrego Mountain earthquake at WWSSN stations, Burdick determined a  $t_\alpha^* = 5.3$  for paths from Borrego Mountain in southern California to stations in the northeastern United States. This value is consistent with attenuation measurements in the period band of long-period body and surface waves for mixed shield-tectonic paths. Der *et al.* (1980b) notes that this result is not in conflict with measurements of significantly smaller  $t_\alpha^*$  in the 1- to 5-Hz bands for similar paths, given the frequency band of Burdick's data and the frequency dependence of attenuation shown in Figure 3. Although this result can thus be made consistent with the results of other studies, the exchange between Hanks (1981, 1982) and Burdick (1982) illustrates that the result critically depends on the proper identification of the  $sS$  and  $sP$  phases.

#### *Results of source cancelling experiments: frequency dependence of $Q$*

The average  $Q_{ScS}$  for the earth ( $170 \pm 20$  per cent) found by Sipkin and Jordan (1980) is lower than values of 200 to 240 derived from the inversion of lower frequency (0.002 to 0.01 Hz) normal mode data (Sailor and Dziewonski, 1978; Anderson and Hart, 1978). This led Sipkin and Jordan to suggest that  $Q_{ScS}$  may decrease with frequency in the vicinity of 0.01 Hz.

The data from source-cancelling experiments has been primarily limited to frequencies less than 0.7 Hz, but suggests that attenuation is nearly independent of frequency between 0.01 to 0.7 Hz.  $P/PP$  experiments in the short-period band of body waves give attenuation values comparable to those obtained from long-period body waves in the same region. Even on a short-period instrument, however, the dominant frequency of  $PP$  is always less than 0.7 Hz. Thus, nothing can be inferred about the frequency dependence at higher frequencies. The results from  $ScS/ScP$  and  $PcP/PcS$  experiments in the short-period band of body waves are also consistent with the results of attenuation experiments at longer periods, both giving comparable values in continental regions (Kanamori, 1967; Sipkin and Jordan, 1980) and in oceanic regions (Burdick, 1981; Sipkin and Jordan, 1979, 1980).

In an  $ScS/ScP$  experiment, Burdick (1981) sought to determine the high-frequency cutoff,  $1/2\pi\tau_m$ , in an absorption band model of  $Q$ . He assumed  $\tau_m$  to be constant with depth and attenuation to occur purely in shear. Burdick's results

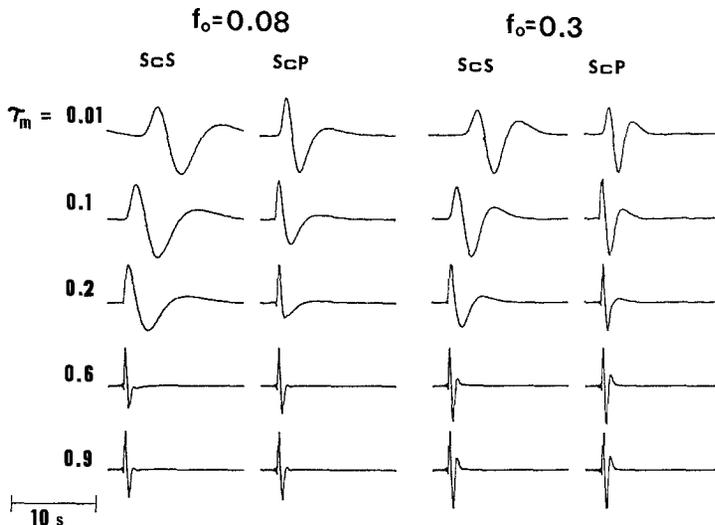


FIG. 6. A comparison of synthetic  $ScS$  and  $ScP$  waves for an earthquake at 600 km depth observed at  $50^\circ$  by a short-period WWSSN instrument. The lower time constant  $\tau_m$  of a mantle absorption band is varied between 0.01 and 0.9 sec [high-frequency cutoff is given by  $1/(2\pi\tau_m)$ ]. The corner frequency in a Brune (1970)  $\omega^{-2}$  source is taken as 0.08 and 0.3 Hz.

using oceanic  $ScS/ScP$  data can be compared with Sipkin and Jordan's (1979) results for  $\tau_m$  obtained from observations of the spectral decay of short-period  $ScS$  phases having paths beneath the western Pacific (Figure 2). The apparent period of  $ScS$  is nearly always observed to be at least twice that of  $ScP$  on short-period-WWSSN records. To satisfy this observation with synthetic seismograms (Figure 6), in which the  $S$  corner frequency is varied in the range commonly observed from deep focus earthquake,  $\tau_m$  must be less than 0.3 sec. Hanks (1982), however, noted that the overlap between the results from the two types of experiments in Figure 2 was poor because the lower error bounds of the spectral decay experiments were based on a delta function source. This may indicate that the error bounds in the  $ScS/ScP$  experiment may be larger than those shown in Figure 2 or that biases exist in one or both experiments. In any case, it should be noted that the  $ScS/ScP$  and all other source-cancelling experiments are consistent with an upper bound of  $\tau_m = 0.2 - 0.3$  sec and that this value is consistent with Der *et al.*'s (1982; unpublished observations) fit of  $\tau_m = 0.1 - 0.2$  sec to body- and surface-wave data. This simple

relaxation model assumes that the shield-tectonic curve for  $t^*$ , shown in Figure 3, represents an upper bound, the low-frequency end being biased toward higher  $t^*$  by the effect of lateral heterogeneity on the apparent  $Q$  of surface waves.

### *Time domain modeling*

The interpretation of the time domain studies that model phases convolved with an instrument response must recognize that the instrument response will emphasize the frequencies in its pass band. Thus, all phenomena that introduce spectral holes, such as depth phases, crustal reverberations, and multipathing, must be well understood or shown to be unimportant in the pass band of the instrument and time window analyzed before valid conclusions can be drawn regarding attenuation. Particularly in the frequency band near and above the corner frequency, care must be taken not to trade off the source model with the attenuation model.

### *Time domain modeling of earthquakes*

Most modeling studies of body waves have been confined to long-period data, which are relatively insensitive to attenuation. Good matches to observed waveforms can be obtained using a point source representation and  $Q$  models determined from free oscillations and surface waves, which predict  $t^*_\alpha = 1$  and  $t^*_\beta = 4$  for a surface focus event. Application of dynamic source theories to short-period and broadband data produces matches to observed waveforms with  $t^*$  less than 1 in the band 0.2 to 2 Hz (Bouchon, 1978; Choy and Boatwright, 1981). Such studies recognize that the source model can trade off with the attenuation model. Arguments similar to those used in spectral studies, which assume a decay rate of  $\omega^{-2}$  to  $\omega^{-3}$  above the corner frequency, however, can be used to place bounds on the range of permissible attenuation models. In modeling the body waves from two deep focus earthquakes, Choy and Boatwright found that, with a range of reasonable models of dynamic rupture, the  $t^*_\alpha$  had to be significantly less than 1 in the 1- to 5-Hz bands in order to achieve agreement between observed and synthetic  $P$  waves. They fit a  $t^* = 0.2$  to the direct  $P$  waves, 0.4 to 0.6 to the  $pP$  waves, and a  $t^* = 3.0$  to the  $sP$  waves.

Strong lateral variation of attenuation in the upper mantle is well documented in the vicinity of island arcs (e.g., Oliver and Isacks, 1967; Barazangi and Isacks, 1971). This variation can affect the  $t^*$  of the direct body waves as well as the  $t^*$ 's of the depth phases (Ward and Toksöz, 1971). The results of the Choy and Boatwright study, for example, can be interpreted using the results of these earlier studies. Barazangi *et al.* (1975) observed that the regions beneath the concave sides of the Kuril and Indonesian arcs, from which the Choy and Boatwright data were obtained, do not exhibit strong attenuation. This is consistent with the relative attenuation of  $P$  ( $t^* = 0.2$ ) and  $pP$  ( $t^* = 0.4$  to 0.6) observed in the study. The  $pP$  waves, having three legs through the upper mantle, should have a  $t^*$ , a factor of 3 higher than  $P$  waves, having one leg through the upper mantle. This assumes that the attenuation in the mantle beneath the bounce point equals the attenuation in the mantle beneath the receiver and that most of the attenuation occurs in the upper mantle. The strong attenuation of the  $sP$  phase ( $t^* = 3$ ), however, may indicate that the ray path of this phase samples a localized zone of intense attenuation behind the arc.

### *Time domain modeling of explosions*

In modeling of explosions, the uncertainties associated with the rate of spectral decay, relative corner frequencies of  $P$  and  $S$ , and directivity are essentially removed or minimized. The source function can be estimated from seismic measurements

close to the source. A simple parameterized model can be fit to the shape of the reduced displacement potential (RDP) observed in free-field experiments at 200 to 800 m (e.g., Haskell, 1967; Murphy, 1978). In the spectral domain, the parameters of such a model define a long-period level  $\psi_\infty$ , proportional to seismic moment, a radian corner frequency  $k$ , and a spectral overshoot parameter  $B$ .  $B$  influences the amount by which the peak spectral value exceeds the long-period spectral level. It is important to note that the shape parameters  $k$  and  $B$  are not true source parameters. The true source parameters are quantities such as yield  $Y$ , cavity radius  $r_c$ , and elastic radius  $r_e$ . Mueller and Murphy (1971) incorporated these source parameters in a theoretical source function for the pressure observed at the elastic radius of an explosion and used RDP observations to develop empirical scaling relations for these parameters in different media. von Seggern and Blandford (1972) developed relations between Mueller and Murphy's source model and simple parameterized models of the spectral shape. Using these relations and yield scaling relations determined from free-field measurements, the spectral shape can be estimated from events of known yield, source depth, and shot medium. The attenuation operator can be estimated from a comparison of observed with synthetic spectra or waveforms at teleseismic distances. When the precise source depth is not known, it is varied to match the observed waveform, subject to yield scaling relations for containment.

TABLE 1  
SPECTRAL SHAPE PARAMETERS SCALED TO GRANITE  
RDP

	Haskell $\omega^{-4}$	Helmberger-Hadley $\omega^{-3}$	von Seggern- Blandford $\omega^{-2}$
$k$	31.60	23.90	16.8
$B$	0.24	0.85	2.0

#### *Biasing effects on the determination of $t^*$ from explosion modeling*

The effects of the receiver crust, tectonic strain release (Bache, 1976), spall phases (Bache *et al.*, 1979), multipathing (Douglas *et al.*, 1973; Shumway and Blandford, 1978), and frequency-dependent suppression of the  $pP$  reflection coefficient (Shumway and Blandford, 1978; Bache *et al.*, 1979; Scott and Helmberger, unpublished observations) can be included in the modeling. Except when multipathing is mistaken for  $pP$  or when the spectral overshoot is improperly traded off against the  $pP$  reflection coefficient, these effects generally do not strongly affect the determination of  $t^*$ .

One problem in comparing the results obtained by different investigators is that they assume different spectral decay rates above the corner frequency, ranging from  $\omega^{-4}$  to  $\omega^{-2}$  (e.g.,  $\omega^{-4}$  in Haskell, 1967;  $\omega^{-3}$  in Burdick and Helmberger, 1979; and  $\omega^{-2}$  in von Seggern and Blandford, 1972). When a simple parametric form of the RDP is used, the yield scaling of the parameters  $\psi_\infty$ ,  $k$ , and  $B$  depends on the spectral rate. Table 1 compares the  $k$  and  $B$  parameters in  $\omega^{-4}$ ,  $\omega^{-3}$ , and  $\omega^{-2}$  spectral models for a 5-kt explosion in granite.  $\omega^{-4}$  and  $\omega^{-3}$  decay, in combination with a corner frequency that scales as  $Y^{-1/3}$  and predicts  $m_b$ -yield relations that conflict with observations. Above the corner frequency, the  $\omega^{-4}$  decay predicts  $m_b$  to decrease with increasing yield; the  $\omega^{-3}$  decay predicts  $m_b$  to remain constant with yield. The slope of the  $m_b$ -yield relation is observed to decrease above 100 kt (Evernden, 1970),

but does not become zero or negative. von Seggern and Blandford (1972) note that  $\omega^{-2}$  decay better approximates the boundary condition at the elastic radius (discontinuous velocity), improves the fine scale fit to the RDP observed at free-field distances, and agrees with the decay rate predicted from Mueller and Murphy's source model. Although physical and observational considerations favor  $\omega^{-2}$  spectral decay for explosions, the RDP can be adequately matched by assuming any  $\omega^{-n}$  decay. Care should be taken, however, that the dominant frequency content of teleseismic  $P$  waves is less than the corner frequency in the  $\omega^{-4}$  and  $\omega^{-3}$  models so that the decay rate does not bias the  $t^*$  determination.

Nonlinear behavior of the rheology in the high-strain region near the source may invalidate the source constraints obtained from seismic measurements close to the source. Laboratory measurements find  $Q$  to be strain dependent for strains exceeding  $10^{-6}$  (Johnson and Toksöz, 1980; Tittmann *et al.*, 1981). Additional evidence of nonlinear rheology near the source may come from free-field experiments in which the peak velocity of ground motion decays with distance faster than would be predicted by cube root scaling of the source parameters. This discrepancy occurs at the furthest ranges (1 km) of free-field measurements (Trulio, 1978, 1981). These results, were, however, not obtained in the frequency band important for teleseismic observation ( $\leq 5$  Hz). The laboratory measurements are usually made at 100 and greater Hertz, and strong attenuation of the RDP is observed primarily at frequencies at and above 10 Hz.

#### *Results for $t_\alpha^*$ from explosion modeling*

Using a  $\omega^{-4}$  source and yield scaling of the RDP, Frasier (1972) and Frasier and Filson (1972) obtained a  $t_\alpha^* = 0.3 - 0.5$  at 0.6 to 3 Hz for  $P$  waves observed from NTS at NORSAR. Using a  $\omega^{-2}$  source, Shumway and Blandford (1978) obtained a  $t_\alpha^* = 0.3$  for Aleutian explosions and  $t_\alpha^* = 0.6$  for NTS explosions recorded at LRSM stations at 0.5 to 5 Hz. Lundquist and Cormier (1980) used a relaxation model of  $Q$  in which  $\tau_m$  varied with depth to obtain a fit to  $P$  waves from nuclear tests. They fit short-period WWSSN and LRSM waveforms, while simultaneously satisfying spectral decay data. They judged the waveform fits to be as good as that obtained by assuming a frequency constant  $t_\alpha^* = 1$ .

Instead of using yield scaling of the RDP, the source may be constrained by matching velocity records obtained at the 5- to 10-km range. This procedure attempts to eliminate or minimize the biasing effect of nonlinear behavior near the source. The waveform at this range, however, is composed of crustal reverberations that act to lengthen the apparent pulse width and, thereby, introduce a tradeoff between crustal structure and  $t_\alpha^*$ . Using seven accelerograms at close range to constrain the source and a model of the crust in the source region, Helmberger and Hadley (1981) fit a  $t^* = 1.3$  to the ratio of long-period to short-period and absolute short-period amplitudes of  $P$  waves from NTS at WWSSN stations. Cole (1981) used this method to obtain  $t^* = 1$  for Aleutian tests recorded by short-period WWSSN stations.

Are these different results mutually consistent and are they consistent with spectral decay experiments that find  $t_\alpha^*$  less than 1 in the 1- to 5-Hz bands? In answering these questions, it is important to consider whether the source  $Q$  tradeoff has been completely eliminated by the modeling techniques. Figure 7 shows an example in which identical matches to short- and long-period WWSSN amplitudes can be obtained with slight changes in spectral shape parameters, and the  $pP$  reflection coefficient with  $t_\alpha^* = 0.7$  sec instead of  $t_\alpha^* = 1.0$  sec. The far-field source

spectra (Figure 8) for the two source models are nearly identical up to 1.2 Hz. This suggests that unless the seismic measurements near the source were enriched in frequencies higher than 1 Hz, the two source models would be indistinguishable at close range as well. The spectra in Figure 8 also demonstrate that the  $t^*$  inferred from modeling of short-period WWSSN records primarily applies to a narrow frequency band about 0.5 to 0.8 Hz. Given this frequency band and assuming an occasional uncertainty of as much as 0.3 sec in  $t_{\alpha}^*$ , most of the determinations of  $t_{\alpha}^*$  by time domain modeling are consistent with the frequency dependence of  $Q$  suggested by spectral decay experiments. Bache (1982) notes that in the few cases in which the results of spectral and time domain modeling are compared, they give the same  $t_{\alpha}^*$  in the spectral band where they overlap. Although narrow-bandedness introduces some uncertainty in the source  $Q$  tradeoff and the biasing effects on RDP estimates by nonlinear behavior near the source are unknown, the time domain results suggest that strong frequency dependence of  $Q$  does not begin until about 0.7 to 0.8 Hz. This observation can be matched by  $\tau_m = 0.2$  in a relaxation model of

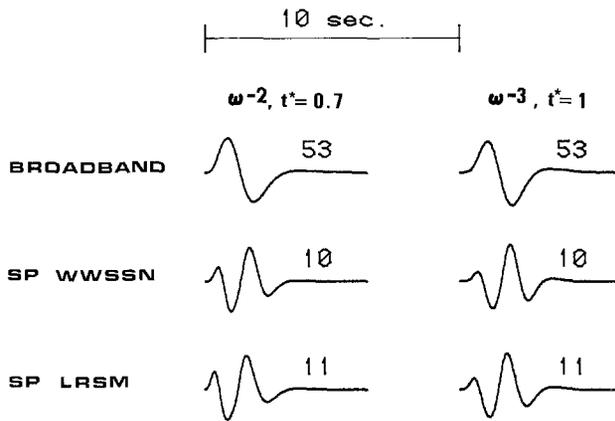


FIG. 7. A comparison of synthetic  $P$  waves constructed from a  $\omega^{-2}$  source model of Milrow ( $k = 2.87$ ,  $B = 2$ ) with those constructed from a  $\omega^{-3}$  source model ( $k = 5$ ,  $B = 2$ ). The  $pP$  reflection coefficient is taken as its elastic value in the  $\omega^{-3}$  source, but is suppressed by the factor  $F = 0.5 + 0.5 \exp(-f^2)$  in the  $\omega^{-2}$  source. The long-period level  $\psi_{\infty}$  in the  $\omega^{-2}$  source is taken as 1.55 times that of the  $\omega^{-3}$  source. (Note from Table I that equal  $B$  values in different  $\omega^{-n}$  sources do not imply equal overshoot in the RDP.) The numbers to the left of each pulse give peak-to-peak amplitude for comparison.

mantle  $Q$  and is consistent with the results obtained from both source cancelling and spectral decay experiments.

#### *Regional variation of attenuation from amplitudes in the time domain*

The most extensive data on the regional variation of body-wave amplitudes is concentrated in North America. Most of the studies find that amplitudes in the EUS are larger than those in the WUS. Evernden and Clark (1970), for example, find that WUS amplitudes average a factor of 3 lower than EUS amplitudes on short-period LRSM instruments. Lower amplitudes are generally found to correlate with mountainous areas and higher amplitudes with regions of crustal stability (Cleary, 1967). Later studies resolved a more detailed variation of amplitudes across North America, showing that the lowest amplitudes are associated with the Rocky Mountain Front and the largest amplitudes with the Central United States. The east and west coasts, including portions of the Basin and Range Province, have intermediate amplitudes. Figure 9 summarizes these variations for short-period WWSSN stations and two short-period LRSM stations. This regionalization is

consistent with studies of regional variations in the  $M_S - m_b$  relation (Evernden and Filson, 1971),  $m_b$  anomalies (Booth *et al.*, 1974; North, 1977), the spectral content measured by  $t^*$  (Solomon and Toksöz, 1972; Der *et al.*, 1982; unpublished observations), and short-period amplitudes (Butler, 1979; Butler and Ruff, 1980; Lay and Helmberger, 1981). The study by Lay and Helmberger (1981) found that the regional variation in amplitudes across North America could be fit by either a variation in the absolute level of  $t_{\alpha}^*$  of 0.5 sec or by a variation in the cutoff time  $\tau_m$  of 0.001 to 0.25 sec in a mantle absorption band. Variations in  $P$  amplitudes could be used to predict variations in  $S$  amplitudes by assuming  $t_{\beta}^* = 4 t_{\alpha}^*$ , and variations in both long- and short-period amplitudes agreed with the same attenuation models. In

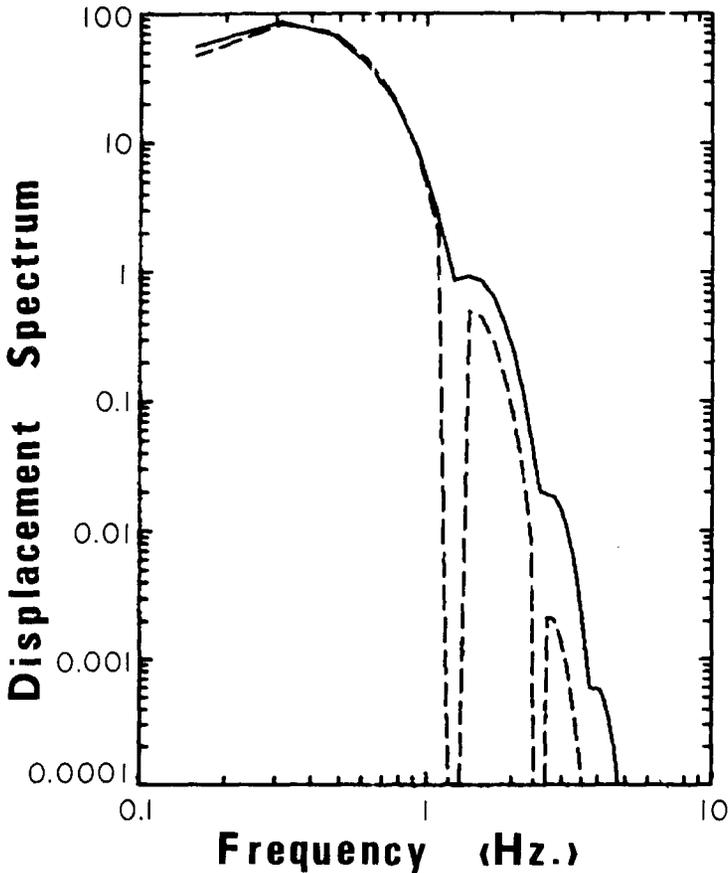


FIG. 8. A comparison of the  $P + pP$  spectrum for Milrow predicted by the  $\omega^{-2}$  source (solid line) and the  $\omega^{-3}$  source (dashed line) described in Figure 8.

summary, the results of amplitude studies are consistent with those obtained from spectral studies. When the data can be processed to remove the effects of the source time function, crustal structure, and three-dimensional structure, both types of studies show that a regional variation that remains is best explained by a variation in attenuation in the upper mantle.

#### *Measures of scattering and coda Q*

The regional variation of attenuation observed in spectral and time domain studies can be a combination of both intrinsic and scattering attenuation. Although the regional variation is generally consistent with the variation expected for ther-

SHORT PERIOD P WAVE AMPLITUDE ANOMALIES

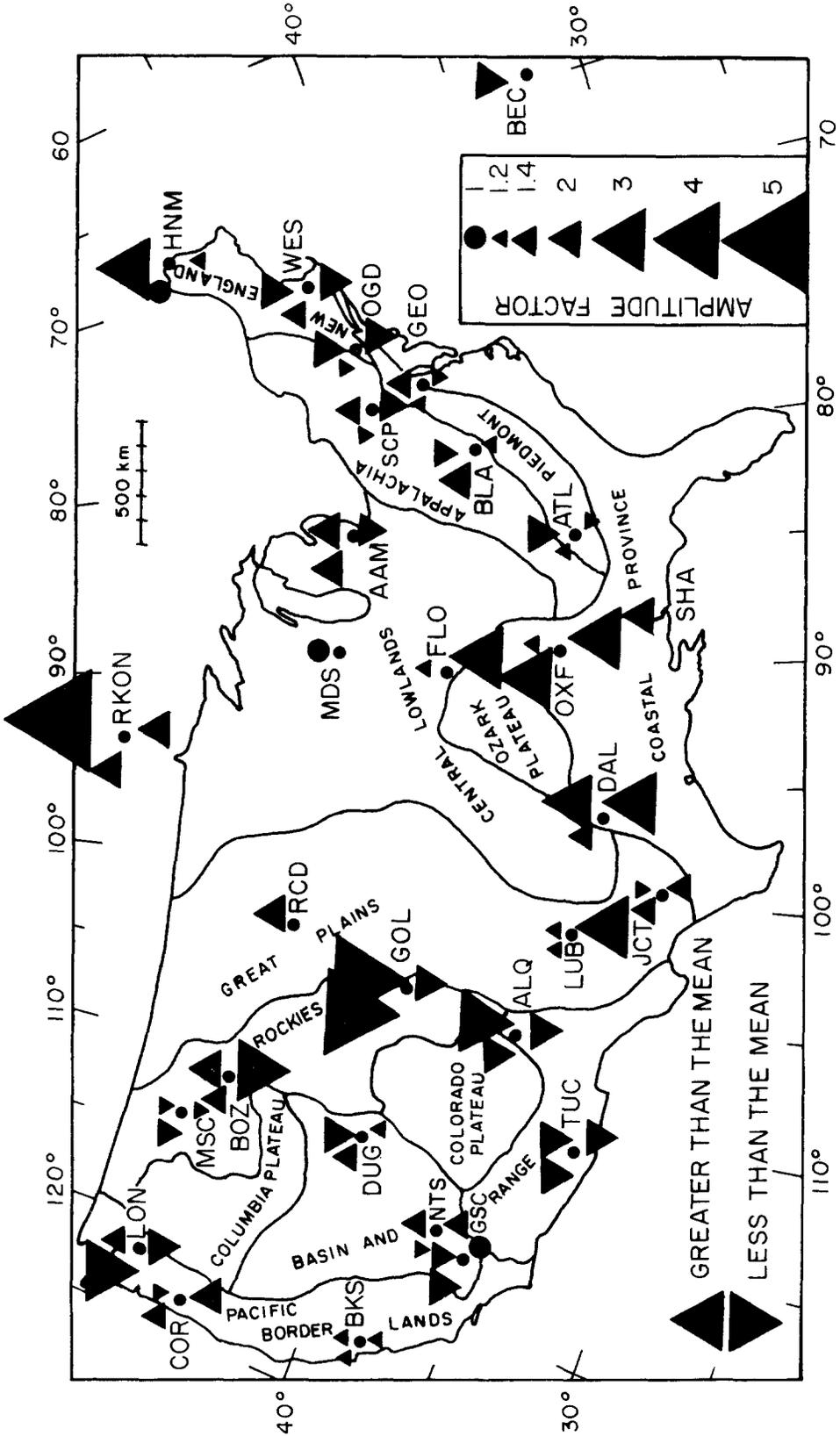


FIG. 9. Amplitude anomalies of short-period *P* waves from Soviet explosions and deep focus earthquakes. Upward pointing triangles are values greater than the mean, downward pointing triangles are values less than the mean. A circle indicates mean amplitude. The position of each triangle relative to each station indicates azimuth: north, northwest; and southeast (from Butler, 1979).

mally activated mechanisms of intrinsic anelasticity, the mechanisms responsible for scattering attenuation may also correlate with the thermal structure of the upper mantle and tectonic activity. The most extensive analysis of seismic data in terms of scattering has centered on the coda following the direct  $S$  phase of local earthquakes. Aki and Chouet (1975) interpreted this coda using the scattering theory of Chernov (1960). In this theory coda  $Q$ 's are determined by fitting the formula  $t^{-1} \exp(-\pi f t/Q)$  to the envelope of filtered coda waves. The coda  $Q$  represents the combined effects of intrinsic anelasticity and backscattering (Aki, 1980a). The agreement of coda  $Q$  with the  $Q_\beta$  determined from local earthquakes (Rautian and Khalturin, 1978) and the  $Q$  of  $Lg$  (Herrmann, 1980) confirms that coda  $Q$  can be equated with the  $Q_\beta$  of the crust and lithosphere. From a comparison of the  $Q_\beta$  determined from long-period surface waves with the  $Q_\beta$  determined from the coda of local earthquakes in the 1- to 20-Hz bands, Aki (1980a, b) concludes that

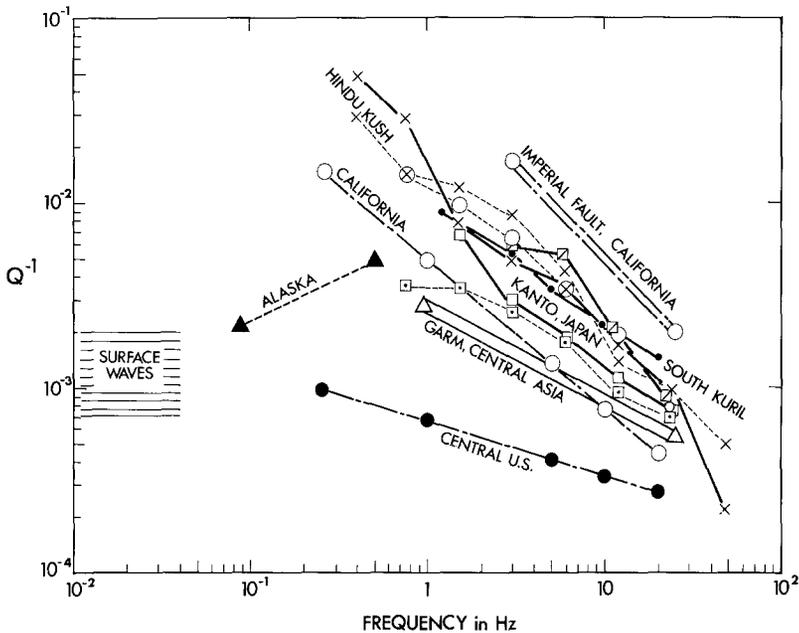


FIG. 10. The frequency dependence of the coda  $Q$  of local earthquakes as summarized by Aki (1980a, b). The lowest frequency data are those from fundamental surface waves.

there exists a peak in  $Q_\beta^{-1}$  in the lithosphere near 1 Hz (Figure 10). The  $Q$  of coda waves is observed to have a regional dependence such that  $Q^{-1}$  is largest at 1 Hz for tectonically active regions. The  $Q^{-1}$  curves converge to a low value at high frequency, independent of current tectonic activity (Aki and Chouet, 1975; Chouet, 1976; Herrmann, 1980).

If the coda  $Q$  is interpreted to be due entirely to the effects of single  $S$ -to- $S$  scattering, then the theory of Chernov predicts that the correlation distance between scattering inhomogeneities in the crust is 1 to 10 km (Aki, 1980a, b; Dainty, 1981). Some support for the scattering interpretation of coda  $Q$  is that the attenuation varies azimuthally such that attenuation is strongest perpendicular to structural trends. Structural boundaries and faults can be expected to more effectively scatter waves propagating perpendicular to their planes. The results obtained by Bache *et al.* (1980) for the synthesis of  $Lg$  phases indirectly favor the scattering interpretation of coda  $Q$ . They found that when the coda  $Q$  for the  $Lg$  phase is interpreted as an

intrinsic  $Q_\beta$ , the distance decay and frequency content of synthetic coda waves do not match those of the observed coda waves.

In plane-layered, laterally homogeneous structure, the waves guide of the coda wave portion of a local seismogram (0 to 500 km) is confined to the crust. This can be demonstrated by examining the depth behavior of the eigenfunctions of Rayleigh and Love modes in the group velocity window and frequency band of the coda waves (Bache *et al.*, 1980). Array studies of the amplitude and phase fluctuation of teleseismic  $P$  waves are consistent with the scatterers being confined to the upper 60 to 150 km of the earth (Aki, 1973; Capon, 1974). These studies determine a correlation length between scattering inhomogeneities of about 10 km, which agrees with the correlation lengths determined from coda waves.

Assuming that scattering is primarily concentrated in the crust and that coda  $Q$  measures,  $Q_\beta$ , upper bounds can be placed on the contribution of backscattering to the attenuation of teleseismic  $S$  waves. The contribution to the apparent  $t_\beta^*$  can be

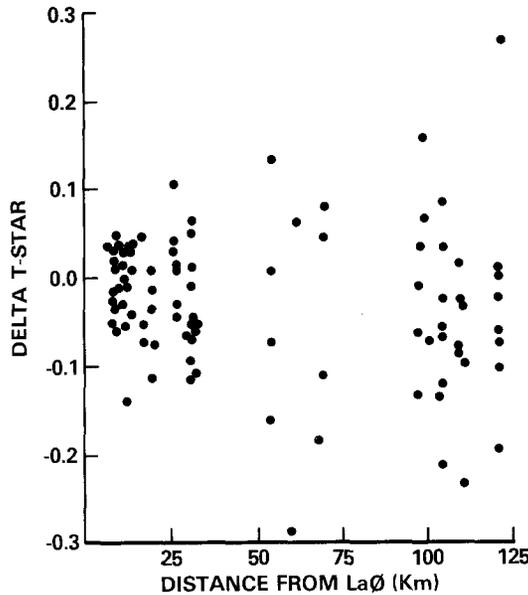


FIG. 11. The variation in differential  $t^*$  across the LASA array determined from the spectral decay of teleseismic  $P$  waves (from Shore, 1982a).

estimated by substituting the peak coda  $Q^{-1}$  shown in Figure 10 into equation (1) and integrating the ray path along the crustal legs of a teleseismic  $S$  wave. This gives a maximum contribution of 0.4 sec to the  $t_\beta^*$  of a teleseismic  $S$  wave having a surface focus. This calculation, however, also assumes that scattering in the crust is isotropic. In isotropic scattering, teleseismic body waves, which travel nearly vertically through the crust, are assumed to be affected by the same amount of scattering in a given path length as the coda waves of a local earthquake, which travel nearly horizontally through the crust. Julian (1982) notes that the assumption of isotropically distributed heterogeneities is "inappropriate in a crust, in which layered and foliated rocks are common."

Forward scattering introduces a random component to the  $\delta t^*$  calculated between stations. Der *et al.* (1982) observed a standard deviation in spectral slopes across NORSAR equivalent to 0.06 sec in  $\delta t_\alpha^*$ . Shore's (1982a) measurements of  $\delta t_\alpha^*$  between array elements at LASA (Figure 11) are consistent with a standard deviation in  $\delta t_\alpha^*$  of about 0.2 sec.

### *Stratigraphic attenuation*

Another scattering mechanism, distinct from scattering by randomly distributed heterogeneities, may contribute to the apparent attenuation of a body wave. Investigators in exploration seismology have noted that the transmission of body waves in finely layered media can be associated with an apparent attenuation that mimics the effects of intrinsic  $Q$  (O'Doherty and Anstey, 1971; Spencer *et al.*, 1977, 1982). Energy in the transmitted wave is lost to waves reflected backward. An apparent energy loss is also observed within the first several cycles of the transmitted wave due to the destructive interference of multiple reflections within the layers. The greatest losses occur in which the thin layer velocities cyclically alternate on the positive and negative sides of a mean velocity and for which the velocity contrast at the boundaries of the thin layers is large. Multiple reflections within each thin layer tend to compensate for the transmission losses through such a medium. To the extent that the cyclic variation of velocity in the thin layers is not random, the less efficient this compensation is and the greater the loss. The pulse broadening and phase delay introduced by this type of attenuation cannot be distinguished from those of intrinsic anelasticity.

Richards and Menke (1982) have suggested that stratigraphic attenuation in the mantle may contribute to the  $t^*$  inferred from teleseismic body waves. The size of this contribution, however, cannot be predicted without making some assumptions about the petrofabric of the mantle. Richards and Menke, however, do propose several diagnostics that may help distinguish the presence of stratigraphic attenuation and separate its effects from those of intrinsic anelasticity. One of these diagnostics is that when stratigraphic attenuation dominates intrinsic anelasticity, the apparent  $P$  and  $S$  wave  $Q$ 's are approximately equal. Since this is contrary to observations of apparent  $Q$ 's of body waves for frequencies up to 4 to 5 Hz, the contribution of stratigraphic attenuation to the attenuation of teleseismic body waves must be much smaller than those of intrinsic  $Q$  and scattering by randomly distributed heterogeneities.

### CORRELATION OF SEISMIC ATTENUATION WITH OTHER GEOPHYSICAL PARAMETERS

The attenuation structure of a region can often be correlated with the seismic velocity structure or values of other geophysical parameters of the region. The strength of these correlations determine their relative success in predicting the attenuation structure of a region for which waveform data are sparse.

#### *Heat flow*

Archambeau *et al.* (1969) noted a qualitative agreement between regions of high heat flow with regions underlain by low  $Q$ . Mikami and Hirahara (1981) measured differential attenuation in the United States and found a correlation of  $\delta t^*$  with the heat flow averaged over  $5^\circ$  grids. The correlation, however, was weak and the error bars in the heat flow measurements were large. These studies generally conclude that heat flow fails to be strongly correlated with attenuation because the variable distribution of radiogenic elements in the crust introduces variations in heat flow as large or larger than the mantle geotherm.

#### *$S_n$ and $P_n$ transmission*

Molnar and Oliver (1969) globally mapped regions of efficient and inefficient transmission of the  $S_n$  phase. Shields and ocean basins were found to transmit  $S_n$  efficiently.  $S_n$  transmission was found to be inefficient across tectonically younger

regions, such as the WUS. A danger in using this correlation to predict mantle  $Q$  is that  $Sn$  transmission is also strongly affected by the continuity of the  $Sn$  wave guide. Inefficient transmission is observed for paths crossing rift zones and subduction zones. Some disturbances in the wave guide, such as faults and mountain belts, may not necessarily be correlated with low mantle  $Q$ .

### *Crustal age*

The radiometric or thermal age of the crystalline basement of the continental crust or the age of magnetic stripes in the oceanic crust can be compared with deep seismic attenuation structure. Sipkin and Jordan (1980) found a correlation of  $Q_{ScS}$  with crustal age, younger oceans (less than 100 m.y.) having a  $Q_{ScS}$  of 135 to 142, and older oceans (greater than 100 m.y.) having a  $Q_{ScS}$  of 155 to 184. The highest  $Q_{ScS}$  occurred beneath stable pre-Cambrian shields. Studies of  $\delta t^*$  in the long-period band (Solomon and Toksöz, 1970) and the short-period band (Der *et al.*, unpublished observations) found that the highest mantle  $Q$  underlies Proterozoic crust and the lowest underlies crust tectonically active in the Cenozoic. The mantle  $Q$  beneath Caledonian age crust in the northeastern United States was found to be intermediate between those beneath Proterozoic and Cenozoic crusts. Studies of the coda  $Q$  of local earthquakes also indicate a lower  $Q$  (scattering and intrinsic) in regions having recent tectonic activity (Aki, 1980a, b). Although crustal age is observed to have a correlation with mantle  $Q$  and has fewer predictive exceptions than heat flow, the scatter in crustal age versus  $Q$  limits its usefulness in formulating station corrections to  $m_b$ .

### *Seismic travel time*

The intensity and regional pattern of travel-time anomalies correlates with the intensity and regional pattern of attenuation inferred from spectra and amplitudes. These variations are best seen in travel-time data processed to remove the effects of the source and receiver crusts. Relative to a reference baseline, Cleary and Hales (1966) determined that  $P$  waves are late by up to 1 sec in the Basin and Range Province and early by up to 1 sec in the Central United States. Intermediate travel times are found in the northeastern United States and California. The anomalies for  $P$  waves are observed to have a 3-sec variation across North America; those for  $S$  waves have a 8-sec variation (Hales and Doyle, 1967). Travel-time anomalies for  $S$  waves are generally about four times those found for  $P$  waves (Hales and Roberts, 1970). A comparison of regional with teleseismic travel times indicates that the anomalies can be mostly explained by variations in the thickness of a low-velocity zone in the upper mantle (Hales *et al.*, 1968). The combined results of these studies strongly suggest that both attenuation and travel-time anomalies share a common origin related to a zone of intrinsic attenuation in pure shear concentrated in the upper mantle.

The results of other studies confirm the correlation of travel time with attenuation, the best correlation seen when the attenuation is measured using spectral rather than amplitude data. Sipkin and Jordan (1980) found that  $Q_{ScS}$  correlates with the difference in travel time between  $ScSn$  and  $ScSn + 1$  (Figure 12). Der *et al.* (1981) compared relative travel-time residuals  $\delta T$ 's with  $\delta t^*$ 's and found a weak correlation. Shore (1982b), however, found a poor correlation between travel-time residuals from underground explosions in the western Soviet Union and  $m_b$  residuals determined by North (1977). This negative correlation may be explained using the results of array studies, which find amplitude variations of up to 0.5  $m_b$  units and travel-time

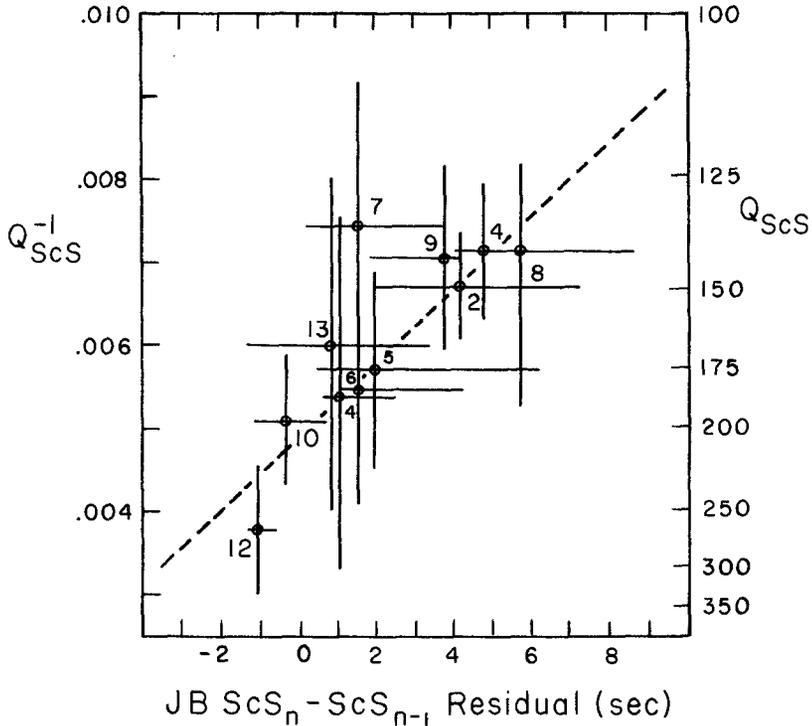


FIG. 12. The correlation of  $Q^{-1} ScS$  with  $ScS_n - ScS_{n-1}$  Jeffreys-Bullen residual determined by Sipkin and Jordan (1980).

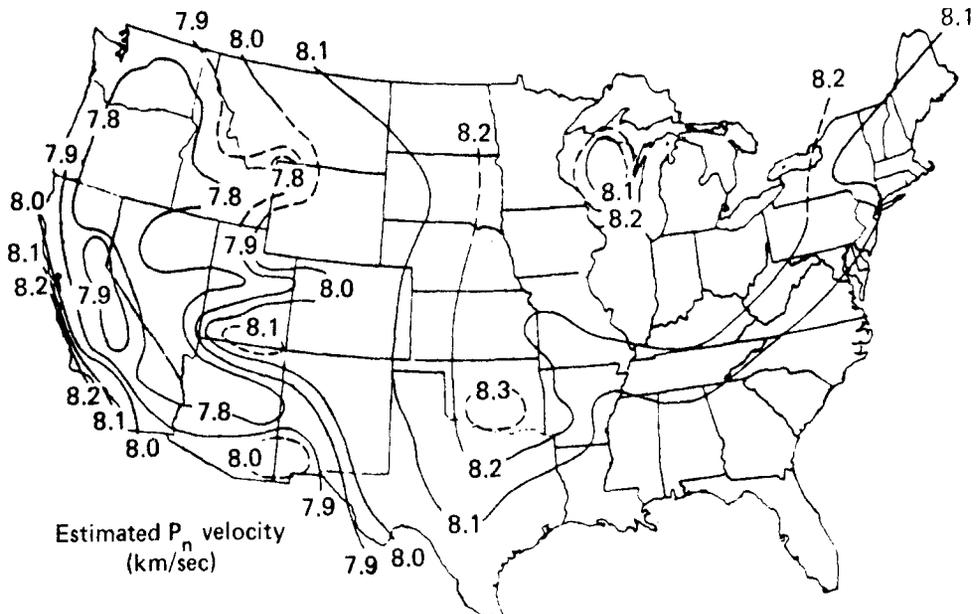


FIG. 13. The  $P_n$  velocity distribution in the conterminous United States (from Herrin and Taggart, 1962, 1968).

variations of up to 0.8 sec across arrays having dimensions of 50 to 100 km. At both NORSAR (Haddon and Husebye, 1978) and LASA (Chang and von Seggern, 1980), the amplitude variations correlate with the travel-time variations, but in the opposite sense observed in comparisons of mantle  $Q$  with travel time: slower travel time correlates with larger rather than smaller amplitudes. Haddon and Husebye showed

that these variations can be explained by the focusing-defocusing effects of velocity heterogeneities in the upper 150 to 200 km of the mantle beneath the array.

Marshall *et al.* (1979) proposed a relation between  $Pn$  velocity and  $Q_\alpha$ . The relation was derived using  $P$  velocity and  $Q$  values determined by Archambeau *et al.* (1969) and Helmberger (1974) beneath North America. The variation of  $Pn$  velocity in the range 7.75 to 8.3 used in this relation, however, is also of the same magnitude as azimuthal variations in  $Pn$  velocity seen over the Caltech seismic array in southern California (Vetter and Minster, 1981). A summary of  $Pn$  velocity data compiled by Herrin and Taggart, 1962, 1968 shows strong localized variations in  $Pn$  velocity near the California coast and the Rocky Mountain front (Figure 13). The study by Vetter and Minster concluded that the strong variation in  $Pn$  velocity in southern California was consistent with an anisotropy axis oriented parallel to the trend of the Transverse Ranges. These observations indicate that, while there may be a relation between  $Pn$  and  $Q$ , the  $Pn$  velocity data used in constructing the relation should be dense enough to identify strong localized variations and anisotropy near plate boundaries and boundaries of tectonic provinces.

Travel-time and  $Q$  data jointly support a model in which attenuation and its effects are greatest in the upper 200 to 400 km of the mantle. The effects of attenuation structure at greater depths are significantly weaker but still may be observable. Travel-time anomalies in the upper mantle are consistent with velocity variations of up to 10 per cent in  $P$  velocity and more in  $S$  velocity. In the lower mantle, travel-time studies have mapped regions having velocity contrasts of several per cent and scale lengths on the order of 1000 km (Julian and Sengupta, 1973; Sengupta and Toksöz, 1976; Dziewonski *et al.*, 1977; Lay, 1982). Stewart (1981), using a technique to remove  $m_b$  anomalies due to crust and upper mantle structure, found a good correlation between  $\delta m_b$  and travel-time anomalies due to the three-dimensional structure of the deep mantle. He suggests that the  $Q$  structure of the lower mantle correlates with the velocity structure and that lateral variations in the  $Q_\alpha$  of the deep mantle cause a 0.1  $m_b$  variation.

## CONCLUSIONS

### *Frequency dependence*

Spectral studies of attenuation all point to a  $t_\alpha^*$  less than 1 in the 1- to 5-Hz bands and hence a frequency dependence of  $Q$ . These studies measure  $t^*$  from the slope of the spectral decay above the corner frequency, assuming a  $\omega^{-2}$  decay for earthquakes and explosions.  $t^*$  is normally assumed to be frequency independent in the frequency band in which the slope is measured. Studies in the 1- to 5-Hz bands conclude that no serious bias is introduced by either the assumption of a frequency constant  $t^*$  over this band or the assumption of any decay rate between  $\omega^{-2}$  and  $\omega^{-3}$ . For shield-shield paths,  $Q$  data from surface-wave studies and spectral decay estimates of body waves can be satisfied by a relaxation band of frequency constant  $Q$ , in which  $Q$  increases as  $\omega^1$  above a cutoff frequency  $1/(2\pi\tau_m)$ , where  $\tau_m = 0.1$  to  $0.2$  and is constant with depth. The  $Q$  data for a shield-tectonic path, however, give a  $t^*$  that increases more gradually and slowly with frequency from 0.01 to 10 Hz. This behavior can be taken either as a constraint on the variation of depth or as an indication that the surface-wave data may be biased toward low  $Q$  by lateral heterogeneity.

Both forward and backward scattering can contribute to the apparent  $Q$  of a body-wave path. The coda  $Q^{-1}$  of local earthquakes, which is a measure of intrinsic plus backscattering attenuation in the lithosphere, peaks at 1 Hz (Aki, 1980a, b).

Backscattering may be important at deeper depths also, particularly in the lowermost mantle, based on evidence of low apparent  $Q$  (Mitchell and Helmberger, 1973) and strong lateral heterogeneity (Sacks *et al.*, 1979) in this region. The random focusing and defocusing effects of laterally varying structure in the crust and lithosphere may contribute up to 0.1 to 0.2 sec to the teleseismic  $t^*$  measured for any single path. The relative contribution of stratigraphic attenuation to the attenuation of teleseismic body waves seems to be small in the 0.01- to 5-Hz bands. The existence of either stratigraphic attenuation or forward scattering, however, will tend to make attenuation measurements based on the first cycle or two of a body wave, such as modeling in the time domain or equalizing the phase of a spectral ratio, give a lower apparent  $Q$  than measurements based on a wider time window following the first arrival, such as spectral decay estimates using smoothed spectra. Scattering and stratigraphic attenuation at all depths may account for a residual  $t_\alpha^*$  of 0.1 to 0.3 in the 5- to 10-Hz bands, where intrinsic anelasticity may be small. Combining scattering  $Q$  with intrinsic  $Q$  may improve the overall fit of relaxation models of mantle  $Q$  to surface- and body-wave data.

Experiments that cancel the source or model waveforms in the time domain have primarily been restricted to frequencies less than 0.7 Hz. Cancelling experiments using  $sS/sP$  and  $ScS/ScP$  phase pairs find that attenuation is nearly independent of frequency between 0.01 and 0.7 Hz. Due to narrow-bandedness and dependence on source constraints, most time domain studies cannot resolve a frequency dependence of  $Q$  even if it exists. Depending on the source constraints used, modeling of short-period WWSSN records gives a  $t_\alpha^* = 0.7$  to 1.3 in the 0.6- to 0.9-Hz bands for mixed shield and tectonic paths. The matching of broader band  $P$  waves requires a  $t_\alpha^*$  less than 0.7 in the 1- to 5-Hz bands. The results of source cancelling and time domain modeling experiments are compatible with a simple relaxation model of intrinsic attenuation in pure shear in which  $\tau_m = 0.3$  sec. This model closely agrees with the frequency dependence of attenuation determined from measurements of spectral decay.

### *Regional variation*

Regional variations in mantle attenuation are consistent with radiometric or magnetic age and tectonic activity, regions of higher relative attenuation coincident with younger, tectonically active crust. Continental cratons are underlain by mantle having small attenuation at all depths. High attenuation usually correlates with slow travel times, lower  $P_n$  velocity, and inefficient  $P_n$  and  $S_n$  propagation. Measures of differential frequency content ( $\delta t^*$  and  $\delta Q$ ) generally correlate better with differential travel time than measures of differential amplitude and  $m_b$ . The correlation of attenuation with travel time is most evident in studies in which differential attenuation is compared with travel-time residuals carefully constructed to remove near-surface effects. The regional pattern and intensity of both travel-time anomalies and  $\delta t^*$  measurements suggest that both share a common origin due to the regional variation of the thermal structure of the upper 200 to 400 km of the mantle.

Focusing and defocusing by lateral variations in the velocity structure of the lithosphere can cause variations in  $m_b$  across an array dimension of 50 to 100 km as large as those estimated for regional variations in intrinsic  $Q$  (Haddon and Husebye, 1978; Chang and von Seggern, 1980). A network average of  $m_b$  tends to cancel the variation due to the effects of focusing and the effects of differing crustal structure and mantle  $Q$  beneath the receivers, leaving a bias due to the mantle  $Q$  beneath the source region. Calculations using  $Q$  models (Chung and Bernreuter, 1981b) estimate

that this bias varies by up to  $0.3 m_b$  units across North America. Much larger biases may occur for  $P$  waves sampling zones of high attenuation beneath active spreading ridges (Solomon, 1973) and behind island arcs (Barazangi, 1971).  $Q$  variations in the lower mantle may contribute to an  $m_b$  bias of up to  $0.1 m_b$  units. The  $m_b$  bias will make the network  $m_b$  of two events having the same size differ if the events are located in tectonically different provinces. From comparisons of  $\delta t^*$  and  $\delta m_b$ , Der *et al.* (1981) suggest that the relation  $\delta m_b = 1.35 \delta t^*$  can be used to correct for the biasing effect of attenuation.

Studies in both the time and frequency domain indicate that  $t_\alpha^*$  and  $t_\beta^*$  vary in parallel with region and frequency such that attenuation occurs predominantly in shear, giving  $t_\beta^* = 4 t_\alpha^*$ . Although evidence exists for elastic anisotropy in the upper mantle (Dziewonski and Anderson, 1981), no detailed evidence yet exists either for substantial anisotropy in anelasticity or for substantial bulk anelasticity in the upper mantle. Thus, the  $S$  attenuation in a given frequency band can be used to predict the  $P$  attenuation in the same frequency band or vice versa.

Most of the evidence for frequency variation in attenuation indicates that the differential attenuation between two regions will be nearly independent of frequency. There is some suggestion, however, that the differential attenuation between regions may be slightly larger at lower frequency (surface waves) than at higher frequency (body waves). A similar behavior is seen in the 1- to 10-Hz bands for the coda wave  $Q$ 's at local and regional distances, in which regional differences in attenuation are greater at 1 Hz but converge to a uniformly low value at 10 Hz. If differential attenuation were shown to be independent of frequency, then corrections for differential attenuation at station sites could be derived by neglecting the frequency dependence of attenuation.

## RECOMMENDATIONS

### *Frequency dependence*

Further work is needed in the time and frequency domains to isolate the frequency dependence in the 0.5- to 1.5-Hz bands. Efforts in both time and frequency domain experiments should be made to report the frequency band and error bars that apply to each experiment, including the frequency weighting effect of the instrument response on time domain modeling, the effect of frequency-dependent  $t^*$  on spectral slope measurements, and the effects on both methods of the frequency dependence of incoherently scattered energy in the coda of body waves.

Explosions of known yield can be used for controlled source experiments to directly measure the attenuation suffered by teleseismic  $P$  waves. Assessments must be made of the biasing effects on the results of these experiments by nonlinear rheology and complicated crustal structure near the source. Emphasis should be placed on the frequency band 5 Hz and under, which is important for teleseismic monitoring. It seems little additional information can be learned about the frequency dependence of attenuation from continued modeling of narrow-band records. Broader band modeling, which more evenly weights frequencies in the 0.1- to 0.5- and 1.0- to 5-Hz bands, may give results more consistent with spectral estimates of  $t^*$  and highlight regional differences in attenuation.

The effect of lateral variation of velocity structure on estimates of mantle  $Q$  from surface waves is unknown. Comparison of surface wave  $Q$ 's with the deeper path average provided by long-period body waves, such as multiple  $ScS$  phases, may aid in measuring this effect. The results of such studies would clarify whether regional differences in attenuation are truly larger at lower frequencies.

*Regional variation*

The correlation of seismic velocities with attenuation appears to offer the best hope for deriving attenuation models for source regions for which the quality or quantity of seismic data precludes inversion for attenuation structure. Velocity anomalies, however, should be determined by methods that eliminate focusing effects and the effects of near-source or receiver structure, using differential travel times or relative residuals between stations for events from many different azimuths. Most work has concentrated on the variation of attenuation in the mantle beneath North America, defining distinct regions in the Central United States, the Basin and Range Province, and the east and west coasts. The regional variation of attenuation in the mantle beneath other continents has not yet been mapped on a comparably fine scale.

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LINCOLN LABORATORY  
APPLIED SEISMOLOGY GROUP  
MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
CAMBRIDGE, MASSACHUSETTS 02142

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